

Vectors are quantities that have magnitude and direction.

They are usually expressed mathematical in component form and graphically as a straight line with an arrow on it. The arrow represents the direction and the length of the line, the magnitude.

In component form:-

$$\overrightarrow{AB} = \overrightarrow{V} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$
 2 units parallel to the x-axis
3 units parallel to the y-axis
4 units parallel to the z-axis

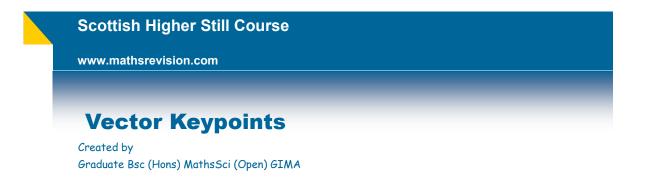
1. To calculate the magnitude (size) of a vector we use the formula below.

$$|\underline{V}| = \sqrt{(a^2 + b^2 + c^2)}$$
 where $\underline{V} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

2. Vectors are equal if and only if they have the SAME magnitude AND direction.

3. The only vector that does not have a direction by definition is the zero vector.

$$V = 0$$



4. We can use the normal rules of addition and subtraction for vectors as long as we apply them to each component in turn.

$$\underline{a} = \begin{bmatrix} 4\\3\\2 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1\\0\\2 \end{bmatrix} \qquad \underline{a} + \underline{b} = \begin{bmatrix} 4\\3\\2 \end{bmatrix} + \begin{bmatrix} 1\\0\\2 \end{bmatrix} = \begin{bmatrix} 5\\3\\4 \end{bmatrix} \qquad \underline{a} - \underline{b} = \begin{bmatrix} 4\\3\\2 \end{bmatrix} - \begin{bmatrix} 1\\0\\2 \end{bmatrix} = \begin{bmatrix} 3\\3\\0 \end{bmatrix}$$

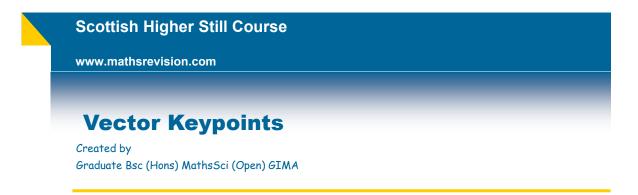
- 5. The negative of a vector \underline{V} is $-\underline{V}$. It has the same magnitude as \underline{V} but points in the opposite direction.
- **6**. The scalar multiple of a vector \underline{V} in general is given by

$$\mathbf{k} \, \underline{V} = k \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} k \cdot a \\ k \cdot b \\ k \cdot c \end{bmatrix}$$

k is simply a number and has the following effect:-

k>1 increases the vector \underline{V} by a factor of k.

k<-1 increases the vector \underline{V} by a factor of k in the opposite direction of \underline{V} .



7. Position vector: OA written as \underline{a} is the position vector of A.

A (a, b, c) is an address using coordinates

$$\underline{a} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$
 instruction from the origin to the point A

In general to find the position between A and B we have

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underline{a} + \underline{b} = \underline{b} - \underline{a}$$

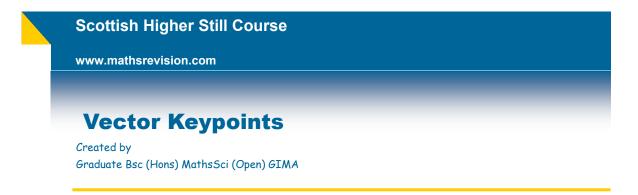
8. Points x, y, z are collinear if they lie on the same straight line.

Using the scalar multiply above we can say that if x, y, z are collinear then the following is true.

$$\overrightarrow{XY} = k \cdot \overrightarrow{YZ}$$

9. The mid-point of A and B in terms of position vectors is given by

$$\underline{m} = \frac{1}{2} \cdot (\underline{a} + \underline{b})$$



10. If the point P divides the length AB in the ratio m: n then

$$\overrightarrow{AP} = \frac{m}{(m+n)} \cdot \overrightarrow{AB}$$

11. To find the angle between 2 vectors a and b say, we use the scalar/dot product formula.

$$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\theta$$

or in component form

$$\underline{a} \cdot \underline{b} = (a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2)$$

REMEMBER: WHEN USING THE FORMULA THE VECTORS HAVE TO BE TAIL TO TAIL

12. Two vectors a and b say, are perpendicular to each other if

scalar/dot product formula equals zero.

$$\underline{a} \cdot \underline{b} = \underline{0}$$

13. Vectors obey the rules of algebraic addition and subtraction but you cannot multiple or divide by vectors as it does not have any meaning.!



14. A unit vector has a magnitude of 1. The 3 unit vectors parallel to the x, y, z axis are given by

$$\underline{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \underline{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

These 3 vectors form a set of basis vectors since any vector v can be written in terms of them.

	2	1 1	1		0		$\begin{bmatrix} 0 \end{bmatrix}$	
$\underline{\mathbf{V}} =$	3	= 2 ·	0	$+3 \cdot$	1	$+4 \cdot$	0	
	4		0		0		1	

$$\underline{\mathbf{V}} = 2 \cdot \underline{\mathbf{i}} + 3 \cdot \mathbf{j} + 4 \cdot \underline{\mathbf{k}}$$

Note that $\underline{\textit{i}},\ \underline{\textit{j}},\ \underline{\textit{k}}$ are perpendicular to each other since

$$\underline{\mathbf{i}} \cdot \underline{\mathbf{j}} = \underline{\mathbf{j}} \cdot \underline{\mathbf{k}} = \underline{\mathbf{i}} \cdot \underline{\mathbf{k}} = \underline{\mathbf{0}}$$

 $\underline{i} \cdot \underline{i} = j \cdot j = \underline{k} \cdot \underline{k} = \underline{1}$

Also