Scottish Higher Still Course

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Vector Keypoints

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Vectors are quantities that have magnitude and direction.

They are usually expressed mathematical in component form and graphically as a straight line with an arrow on it. The arrow represents the direction and the length of the line, the magnitude.

In component form:-

$$\overrightarrow{AB} = \underline{V} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{matrix} \text{2 units parallel to the x-axis} \\ \text{3 units parallel to the y-axis} \\ \text{4 units parallel to the z-axis} \end{matrix}$$

1. To calculate the magnitude (size) of a vector we use the formula below.

$$|\underline{V}| = \sqrt{(a^2 + b^2 + c^2)} \quad \textit{where} \ \underline{V} = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix}$$

2. Vectors are equal if and only if they have the SAME magnitude AND direction.

3. The only vector that does not have a direction by definition is the zero vector.

$$\underline{V} = 0$$

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4. We can use the normal rules of addition and subtraction for vectors as long as we apply them to each component in turn.

$$\underline{a} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad \underline{a} + \underline{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} \qquad \underline{a} - \underline{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

5. The negative of a vector $\underline{V}$ is $-\underline{V}$. It has the same magnitude as $\underline{V}$ but points in the opposite direction.

6. The scalar multiple of a vector $\underline{V}$ in general is given by

$$\mathrm{k}\,\underline{V} = k \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} k \cdot a \\ k \cdot b \\ k \cdot c \end{bmatrix}$$

k is simply a number and has the following effect:-

$k>1$ increases the vector $\underline{V}$ by a factor of k.

$k<-1$ increases the vector $\underline{V}$ by a factor of k in the opposite direction of $\underline{V}$.

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7. Position vector: OA written as $\underline{a}$ is the position vector of A.

A (a, b, c) is an address using coordinates

$$\underline{a} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$ instruction from the origin to the point A

In general to find the position between A and B we have

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underline{a} + \underline{b} = \underline{b} - \underline{a}$$

8. Points x, y, z are **collinear** if they lie on the same straight line.

Using the scalar multiply above we can say that if x, y, z are collinear then the following is true.

$$\overrightarrow{XY} = k \cdot \overrightarrow{YZ}$$

9. The mid-point of A and B in terms of position vectors is given by

$$\underline{m} = \frac{1}{2} \cdot (\underline{a} + \underline{b})$$

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10. If the point P divides the length AB in the ratio m: n then

$$\overrightarrow{AP} = \frac{m}{(m+n)} \cdot \overrightarrow{AB}$$

11. To find the angle between 2 vectors a and b say, we use the scalar/dot product formula.

$$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\theta$$

or in component form

$$\underline{a} \cdot \underline{b} = (a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2)$$

REMEMBER: WHEN USING THE FORMULA THE VECTORS HAVE TO BE TAIL TO TAIL

12. Two vectors a and b say, are perpendicular to each other if

scalar/dot product formula equals zero.

$$\underline{a} \cdot \underline{b} = \underline{0}$$

13. Vectors obey the rules of algebraic addition and subtraction but you cannot multiple or divide by vectors as it does not have any meaning.!

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14. A unit vector has a magnitude of 1. The 3 unit vectors parallel to the x, y, z axis are given by

$$\underline{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \underline{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

These 3 vectors form a set of basis vectors since any vector v can be written in terms of them.

$$\underline{V} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{V} = 2 \cdot \underline{i} + 3 \cdot \underline{j} + 4 \cdot \underline{k}$$

Note that $\underline{i}$, $\underline{j}$, $\underline{k}$ are perpendicular to each other since

$$\underline{i} \cdot \underline{j} = \underline{j} \cdot \underline{k} = \underline{i} \cdot \underline{k} = \underline{0}$$

Also $\quad \underline{i} \cdot \underline{i} = \underline{j} \cdot \underline{j} = \underline{k} \cdot \underline{k} = \underline{1}$