X056/302

NATIONAL QUALIFICATIONS 2000

THURSDAY, 25 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 There are three Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

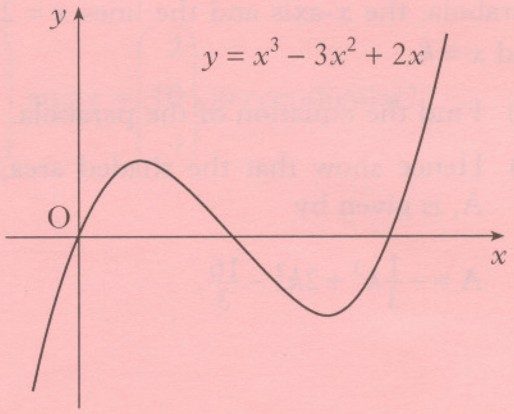
3 Full credit will be given only where the solution contains appropriate working.

4 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

ALL candidates should attempt this Section.

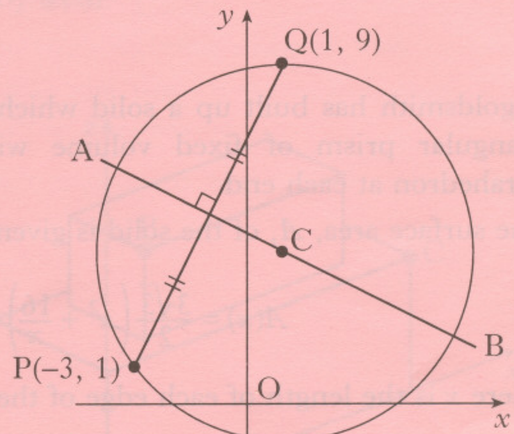
A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. 5

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. 5

A2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). 4



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. 3

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. 2

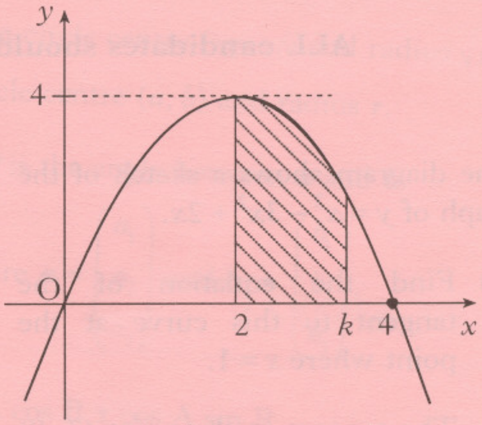
A3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. 2

(*b*) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. 3

A4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



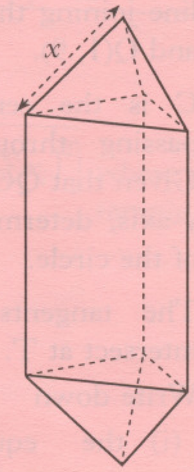
(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

A5. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \leq x \leq 360$. **5**

A6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.



The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**

[*END OF SECTION A*]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Pages five and six*

OR Section C (Statistics) on *Pages seven and eight*

SECTION B (Mathematics 3)

Marks

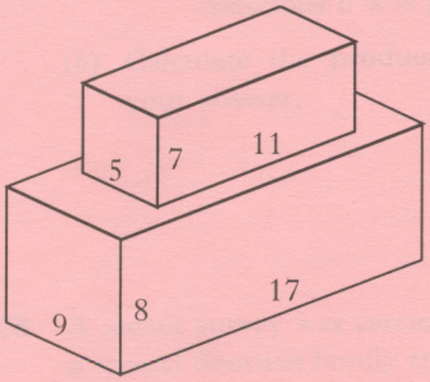
ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

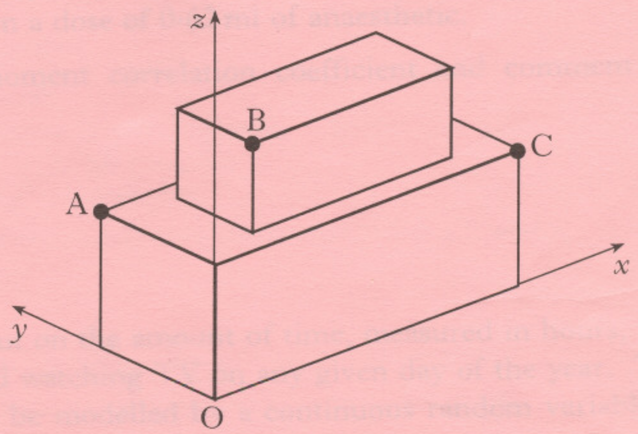
B7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

B8. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

B9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

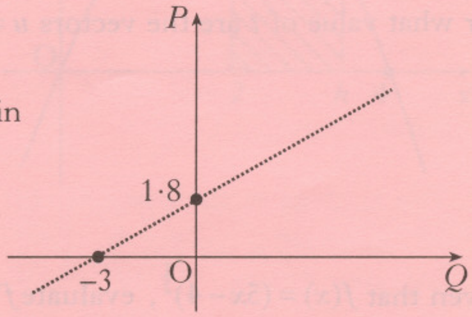
[Turn over

Marks

B10. Find $\int \frac{1}{(7-3x)^2}\,dx$. 2

B11. The results of an experiment give rise to the graph shown.

(*a*) Write down the equation of the line in terms of P and Q. 2



It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. 4

[*END OF SECTION B*]