Higher Mathematics

UNIT 2

Specimen NAB Assessment

HSN22510

UNIT 2

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Outcome 1

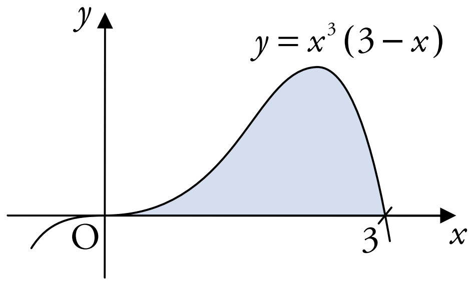
1. Show that $(x+2)$ is a factor of $f(x)=x^3-2x^2-4x+8$ and hence factorise fully $f(x)$. **4**

2. Use the discriminant to determine the nature of the roots of the equation $3x^2+4x-2=0$. **2**

Outcome 2

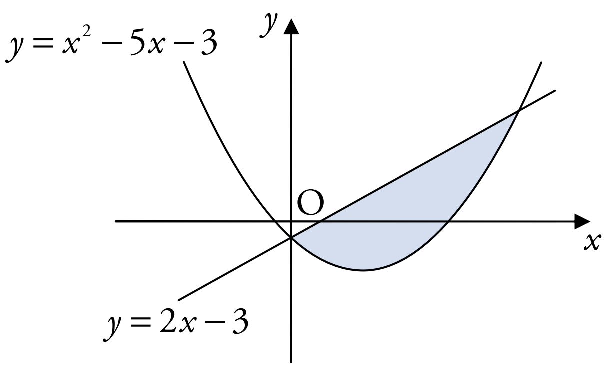
3. Find $\int \frac{6}{x^3}\,dx$, where $x \neq 0$. **3**

4. The curve $y=x^3(3-x)$ is shown in the diagram below.



Calculate the shaded area enclosed between the curve and the x-axis between $x=0$ and $x=3$. **5**

5. The diagram shows the line with equation $y=2x-3$ and the curve with equation $y=x^2-5x-3$.

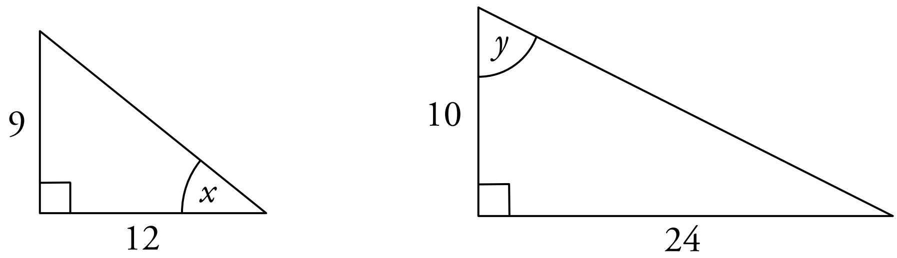


Write down the integral which represents the shaded area.
Do not carry out the integration. **3**

Outcome 3

6. Solve the equation $\sqrt{2}\sin 2x = 1$ for $0 \leq x < \pi$. **3**

7. The diagram below shows two right-angled triangles.



(a) Write down the values of $\sin x$ and $\cos y$. **2**

(b) By expanding $\cos(x+y)$ show that the exact value of $\cos(x+y)$ is $-\frac{16}{65}$. **2**

8. (a) Express $\sin 15\ \cos x\ + \cos 15\ \sin x$ in the form $\sin(a\ + b\ )$. **1**

(b) Use your answer from part (a) to solve the equation

$\sin 15\ \cos x\ + \cos 15\ \sin x\ = \frac{\sqrt{3}}{2}$ for $0 < x < 360$. **4**

Outcome 4

9. (a) A circle has radius 7 units and centre $(2,-3)$.

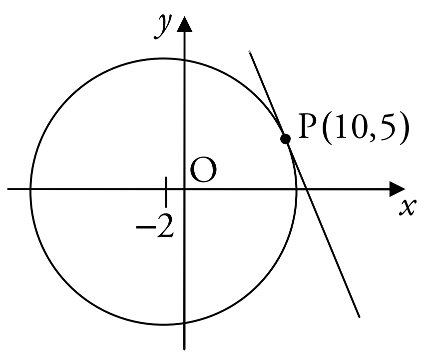
Write down the equation of the circle. **2**

(b) A circle has equation $x^2 + y^2 - 10x + 6y - 3 = 0$.

Write down its radius and the coordinates of its centre. **3**

10. Show that the straight line $y = -2x - 3$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. **5**

11. The point $P(10,5)$ lies on the circle with centre $(-2,0)$, as shown in the diagram below.



Find the equation of the tangent to the circle at P. **4**

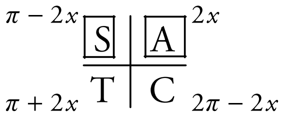
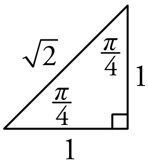
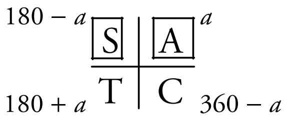
Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{4}{6}$	$\frac{8}{11}$	$\frac{7}{12}$	$\frac{10}{14}$

Outcome 1 – Polynomials and Quadratics		
1. -2 ✓ $\begin{array}{c\|cccc} & 1 & -2 & -4 & 8 \\ & & -2 & 8 & -8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$ Since $f(-2)=0$, $(x+2)$ is a factor. ✓ $f(x)=(x+2)(x-4x+4)$ ✓ $=(x+2)(x-2)(x-2)$ ✓	• Know to evaluate $f(-2)$ • Complete evaluation and conclusion • Quadratic factor • Factorise quadratic	**4**
2. b^2-4ac ✓ $=4^2-4\times3\times(-2)$ $=40$ Since $b^2-4ac>0$, the roots are real and distinct. ✓	• Use the discriminant • Calculate discriminant and state nature of roots	**2**

Outcome 2 – Integration		
3. $\int \frac{6}{x^3}\,dx=\int\left(6x^{-3}\right)dx$ ✓ $=\frac{6x^{-2}}{-2}+c$ $=-3x^{-2}$ ✓ $+c$ ✓	• Express in standard form • Integrate term with negative power • Constant of integration	**2**
4. $\int_0^3$ ✓ $x^3(3-x)\,dx=\int_0^{3}$ ✓ $\left(3x^3-x^4\right)dx$ $=\left[\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^3$ ✓ $=\left(\frac{3}{4}(3)^4-\frac{1}{5}(3)^5\right)-0$ ✓ $=\frac{243}{20}$ ✓ $\left(\text{or } 12\frac{3}{20}\right)$	• Know to integrate with limits • Use correct limits • Integrate • Process limits • Complete process	**5**

5. $x^2 - 5x - 3 = 2x - 3$ ✓ $x^2 - 7x = 0$ $x(x-7) = 0$ $x = 0$ or $x = 7$ ✓ Shaded area is $\int_0^7 \left((2x-3)-(x^2-5x-3)\right)dx$ ✓ square units.	• Strategy to find intersection • Solve quadratic • Use $\int(\text{upper} - \text{lower})\,dx$ with limits from quadratic	**3**
Outcome 3 – Trigonometry		
6. $\sin 2x = \frac{1}{\sqrt{2}}$ ✓   $2x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ $2x = \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}$ $x = \frac{\pi}{8}$✓ or $\frac{3\pi}{8}$✓	• Rearrange to standard form • One solution • Second solution	**3**
7. (a) $AC = \sqrt{9^2 + 12^2} = 15$ $DF = \sqrt{10^2 + 24^2} = 26$ ✓ $\sin x = \frac{9}{15} = \frac{3}{5}$ and $\cos x = \frac{10}{26} = \frac{5}{13}$ ✓	• Calculate remaining sides • $\sin x$ and $\cos y$	**2**
(b) $\cos(x+y) = \cos x \cos y - \sin x \sin y$ ✓ $= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13}$ ✓ $= \frac{20}{65} - \frac{36}{65}$ $= -\frac{16}{65}$	• Use compound angle formula • Substitute values	**2**
8. (a) $\sin 15 \ \cos x \ + \cos 15 \ \sin x \ = \sin(15 \ + x \ )$ ✓	• Use compound angle formula	**1**
(b) $\sin(15 \ + x \ ) = \frac{\sqrt{3}}{2}$✓  $a = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60$ ✓ $15 + x = 60$ or $180 - 60$ $x = 45$✓ or 105✓	• Substitute $\sin(15 \ + x \ )$ • Process $\sin^{-1}$ • One solution • Second solution	**4**

Outcome 4 – Circles		
9. (a) $(x-2)^2+(y+3)^2$ ✓ $=49$ ✓	• Centre • Square of radius	**2**
(b) The centre is $(5,-3)$ ✓ The radius is $\sqrt{(-5)^2+3^2-(-3)}$ ✓ $=\sqrt{37}$ ✓	• State centre • Know how to calculate radius • Process radius	**3**
10. $x^2+y^2+6x+4y+8=0$ $x^2+(-2x-3)^2+6x+4(-2x-3)+8=0$ ✓ $5x^2+10x+5=0$ ✓ $x^2+2x+1=0$ b^2-4ac ✓ $=2^2-4\times1\times1$ $=16-16$ $=0$ ✓ Since the discriminant is zero, the line is a tangent to the circle. ✓	• Strategy for finding intersection • Express in standard form • Know to calculate discriminant • Calculate discriminant • Conclusion	**5**
11. $m_{\text{PC}}=\frac{5-0}{10+2}$ ✓ $=\frac{5}{12}$ ✓ So $m_{\text{tgt}}=-\frac{12}{5}$ ✓ since the radius and tangent are perpendicular. $y-5=-\frac{12}{5}(x-10)$ ✓ $12x+5y-145=0$	• Know how to find gradient of radius • Process gradient of radius • Gradient of tangent • Equation of tangent	**4**

Practice Assessment (1) Unit 2 - Mathematics 2 (H)

Outcome 1 *Marks*

1. Show that $(x+2)$ is a factor of $g(x) = x^3 + 4x^2 + x - 6$, and express $g(x)$ in fully factorised form. **(4)**

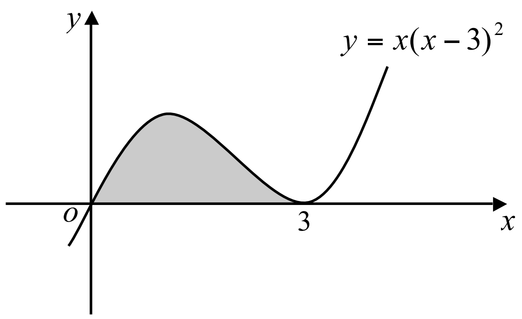
2. Use the discriminant to determine the nature of the roots of the equation

$$3x^2 + 5x + 1 = 0.$$ **(2)**

Outcome 2

3. Find $\int \frac{1}{x^3}\, dx$. **(3)**

4. Calculate the shaded area shown in the diagram.

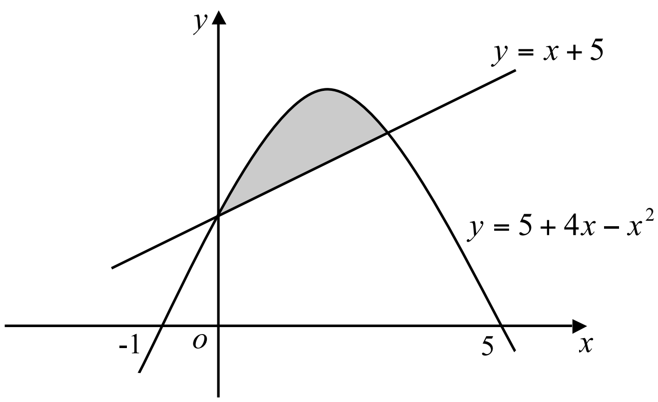


(5)

5. The diagram shows the line with equation $y = x + 5$ and the curve with equation $y = 5 + 4x - x^2$.

Write down the integral which represents the shaded area.

Do **not** carry out the integration.

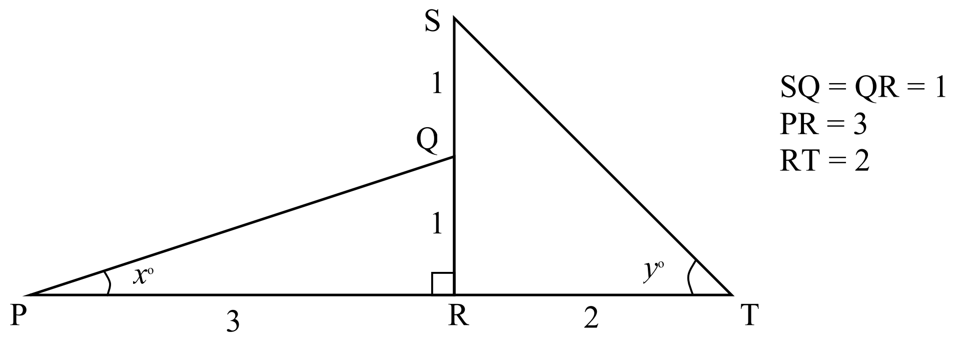


(3)

Outcome 3

6. Solve algebraically the equation $2\sin 2x = \sqrt{3}$ for $0 \le x < \pi$. **(3)**

7. The diagram below shows two right-angled triangles PQR and SRT.



(a) Write down the values of $\cos x°$ and $\sin y°$. **(2)**

(b) By expanding $\sin(x+y)°$ show that the **exact** value of $\sin(x+y)°$ is $\frac{8}{\sqrt{80}}$. **(2)**

8. *(a)* Express $\cos x° \cos 30° - \sin x° \sin 30°$ in the form $\cos(A+B)°$. **(1)**

(b) Hence solve the equation $\cos x° \cos 30° - \sin x° \sin 30° = \frac{1}{4}$ for $0 < x < 360$. **(4)**

Outcome 4

9. *(a)* A circle of radius 6 units has as its centre the point C(4,-1).
Write down the equation of this circle. **(2)**

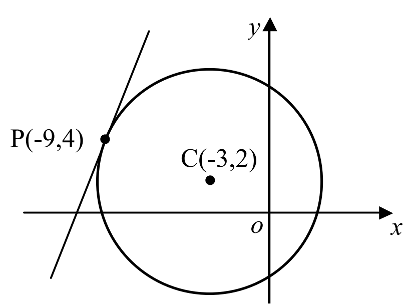
(b) A circle has equation $x^2 + y^2 - 4x + 2y - 4 = 0$. Write down the coordinates of its centre and calculate its radius. **(3)**

10. Show that the line with equation $y = 5 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 2y - 10 = 0$. **(5)**

11. A circle has as its centre the point C(-3,2) , as shown in the diagram.

The point P(-9,4) lies on the circumference of the circle.

Find the equation of the tangent at P.



(4)

Outcome 1 *Marks*

1. Show that $(x-1)$ is a factor of $f(x) = x^3 + 3x^2 - 4$, and express $f(x)$ in fully factorised form. **(4)**

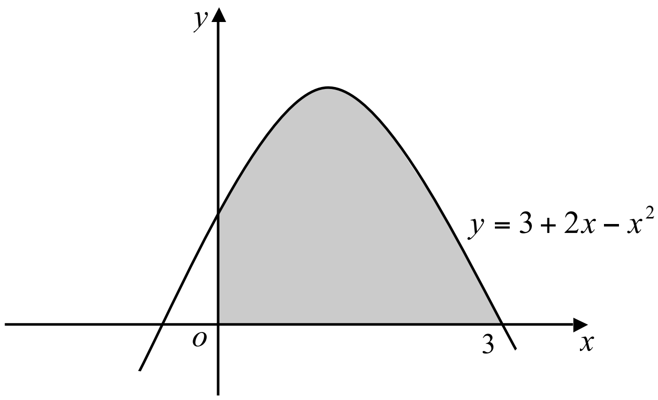
2. Use the discriminant to determine the nature of the roots of the equation

$$4x^2 - 6x + 3 = 0.$$ **(2)**

Outcome 2

3. Find $\int \left(\frac{3}{x^4} + 1 \right) dx$. **(3)**

4. Calculate the shaded area shown in the diagram.

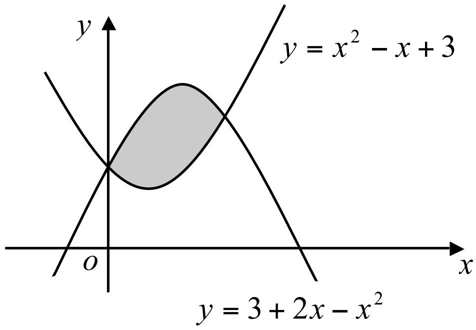


(5)

5. The diagram shows the curves with equations $y = x^2 - x + 3$ and $y = 3 + 2x - x^2$.

Write down the integral which represents the shaded area.

Do **not** carry out the integration.



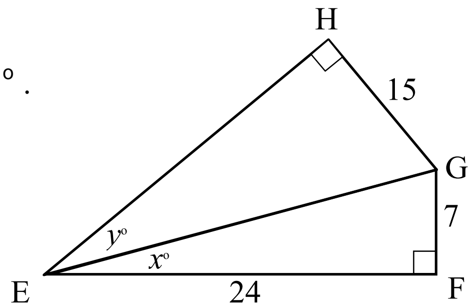
(3)

Outcome 3

6. Solve algebraically the equation $\sqrt{3}\tan 2x = 1$ for $0 \le x < \pi$. **(3)**

7. The diagram shows two right-angled triangles EFG and EHG.

$\angle FEG = x^\circ$ and $\angle HEG = y^\circ$.



(a) Write down the values of $\sin x^\circ$ and $\cos y^\circ$. **(2)**

(b) By expanding $\cos(x+y)^\circ$ show that the **exact** value of $\cos(x+y)^\circ$ is $\frac{3}{5}$. **(2)**

8. (a) Express $\sin x^\circ \cos 20^\circ - \cos x^\circ \sin 20^\circ$ in the form $\sin(A-B)^\circ$. **(1)**

(b) Hence solve the equation $\sin x^\circ \cos 20^\circ - \cos x^\circ \sin 20^\circ = \frac{4}{9}$ for $0 < x < 180$. **(4)**

Outcome 4

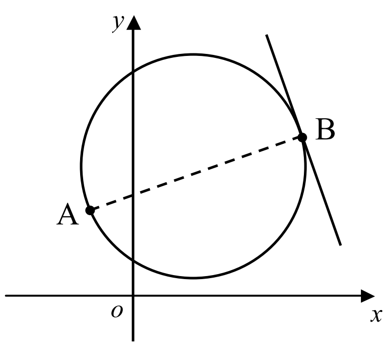
9. (a) A circle has radius 10 units and centre (5,-2).
Write down the equation of the circle. **(2)**

(b) A circle has equation $x^2 + y^2 - 2x + 10y + 1 = 0$.
Write down its radius and the coordinates of its centre. **(3)**

10. Show that the line with equation $y = x - 10$ is a tangent to the circle with equation $x^2 + y^2 - 6x + 6y + 10 = 0$. **(5)**

11. A circle has AB as a diameter, as shown in the diagram.
A and B have coordinates (-2,5) and (10,8) respectively.

Find the equation of the tangent at B.



(4)

Practice Assessment (3) Unit 2 - Mathematics 2 (H)

Outcome 1 *Marks*

1. Show that $(x+3)$ is a factor of $f(x) = x^3 + x^2 - 5x + 3$, and express $f(x)$ in fully factorised form. **(4)**

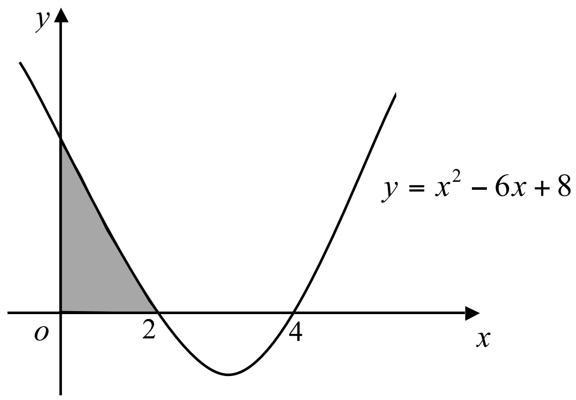
2. Use the discriminant to determine the nature of the roots of the equation

$$2x^2 - 5x + 3 = 0.$$ **(2)**

Outcome 2

3. Find $\int \frac{12}{x^5}\, dx$. **(3)**

4. Calculate the shaded area shown in the diagram.

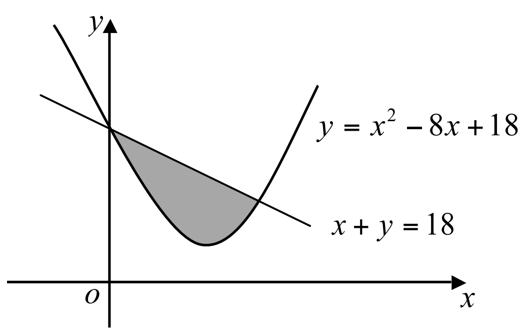


(5)

5. The diagram shows the curve with equation $y = x^2 - 8x + 18$ and the line $x + y = 18$.

Write down the integral which represents the shaded area.

Do **not** carry out the integration.



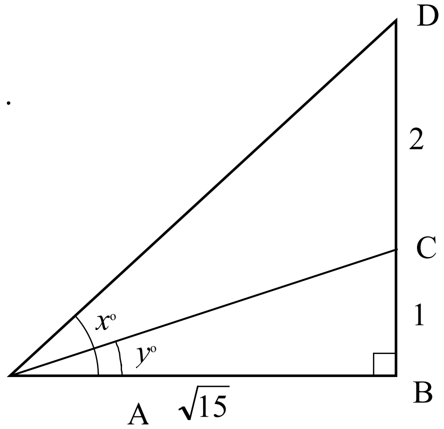
(3)

Outcome 3

6. Solve algebraically the equation $\sqrt{2}\cos 2x = 1$ for $0 \le x < \pi$. **(3)**

7. The diagram shows two right-angled triangles ABC and ABD.

$\angle DAB = x^\circ$ and $\angle CAB = y^\circ$.



(a) Write down the values of $\cos x^\circ$ and $\sin y^\circ$. **(2)**

(b) By expanding $\cos(x-y)^\circ$ show that the **exact** value of $\cos(x-y)^\circ$ is $\frac{18}{4\sqrt{24}}$. **(2)**

8. (a) Express $\sin x^\circ \cos 35^\circ + \cos x^\circ \sin 35^\circ$ in the form $\sin(\mathrm{A}+\mathrm{B})^\circ$. **(1)**

(b) Hence solve the equation $\sin x^\circ \cos 35^\circ + \cos x^\circ \sin 35^\circ = \frac{7}{11}$ for $0 < x < 180$. **(4)**

Outcome 4

9. (a) A circle has a radius of 1 unit and centre (-2,6).
Write down the equation of this circle. **(2)**

(b) A circle has equation $x^2 + y^2 - 6x + 5 = 0$.
Write down its radius and the coordinates of its centre. **(3)**

10. Show that the line with equation $y = 17 - 4x$ is a tangent to the circle with equation $x^2 + y^2 + 8x + 2y - 51 = 0$. **(5)**

11. A circle has as its centre the point C(5,1). The point P(9,3) lies on its circumference.
Find the equation of the tangent at P. **(4)**

Unit 2 - Practice Assessments **Answers**

Practice Assessment 1

Outcome 1 : **1.** proof , $g(x) = (x+2)(x+3)(x-1)$ **2.** $b^2 - 4ac = 13 \quad \therefore$ real, distinct and irrational

Outcome 2 : **3.** $\dfrac{x^{-2}}{-2} + C \quad \left(-\dfrac{1}{2x^2} + C\right)$ **4.** $6\frac{3}{4}$ units^2

5. $\int_0^3 5 + 4x - x^2 - (x+5)\ dx \Rightarrow \int_0^3 3x - x^2\ dx$

Outcome 3 : **6.** $\left\{\frac{\pi}{6}, \frac{\pi}{3}\right\}$ **7.** (a) $\cos x = \dfrac{3}{\sqrt{10}}$, $\sin y = \dfrac{2}{\sqrt{8}}$ (b) proof

8. (a) $\cos(x+30)^\circ$ (b) $\{45\cdot5^\circ, 254\cdot5^\circ\}$

Outcome 4 : **9.** (a) $(x-4)^2 + (y+1)^2 = 36$ (b) C(2,-1) , $r = 3$

10. 1 root, $x = 1$, $\therefore$ a tangent **11.** $y - 4 = 3(x+9) \Rightarrow y = 3x + 31$

Practice Assessment 2

Outcome 1 : **1.** proof , $f(x) = (x-1)(x+2)(x+2)$ **2.** $b^2 - 4ac = -12 \quad \therefore$ not real

Outcome 2 : **3.** $\dfrac{3x^{-3}}{-3} + x + C \quad \left(-\dfrac{1}{x^3} + x + C\right)$ **4.** 9 units^2

5. $\int_0^{3/2} 3 + 2x - x^2 - (x^2 - x + 3)\ dx \Rightarrow \int_0^{3/2} 3x - 2x^2\ dx$

Outcome 3 : **6.** $\left\{\frac{\pi}{12}, \frac{7\pi}{12}\right\}$ **7.** (a) $\sin x = \dfrac{7}{25}$, $\cos y = \dfrac{20}{25}$ (b) proof

8. (a) $\sin(x-20)^\circ$ (b) $\{46\cdot4^\circ, 173\cdot6^\circ\}$

Outcome 4 : **9.** (a) $(x-5)^2 + (y+2)^2 = 100$ (b) C(1,-5) , $r = 5$

10. 1 root, $x = 5$, $\therefore$ a tangent **11.** $y - 8 = -4(x-10) \Rightarrow y = -4x + 48$

Practice Assessment 3

Outcome 1 : **1.** proof , $g(x) = (x+3)(x-1)(x-1)$ **2.** $b^2 - 4ac = 1 \quad \therefore$ real, distinct and rational

Outcome 2 : **3.** $\dfrac{12x^{-4}}{-4} + C \quad \left(-\dfrac{3}{x^4} + C\right)$ **4.** $6\frac{2}{3}$ units^2

5. $\int_0^7 18 - x - (x^2 - 8x + 18)\ dx \Rightarrow \int_0^7 7x - x^2\ dx$

Outcome 3 : **6.** $\left\{\frac{\pi}{8}, \frac{7\pi}{8}\right\}$ **7.** (a) $\cos x = \dfrac{\sqrt{15}}{\sqrt{24}}$, $\sin y = \dfrac{1}{4}$ (b) proof

8. (a) $\sin(x+35)^\circ$ (b) $\{4\cdot5^\circ, 105\cdot5^\circ\}$

Outcome 4 : **9.** (a) $(x+2)^2 + (y-6)^2 = 1$ (b) C(3,0) , $r = 2$

10. 1 root, $x = 4$, $\therefore$ a tangent **11.** $y - 3 = -2(x-9) \Rightarrow y = -2x + 21$