Practice Assessment (1) Unit 3 - Mathematics 3 (H)

Outcome 1 *Marks*

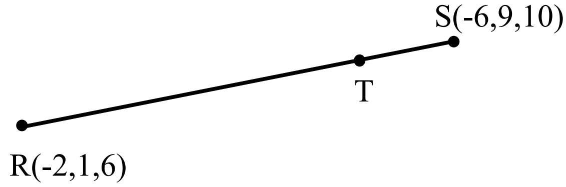
1. (*a*) P, Q and R have coordinates $(2, -3, 3)$, $(6, -2, 0)$ and $(14, 0, -6)$ respectively.

(i) Write down the components of $\overrightarrow{PQ}$. **(1)**

(ii) Hence show that the points P, Q and R are collinear. **(3)**

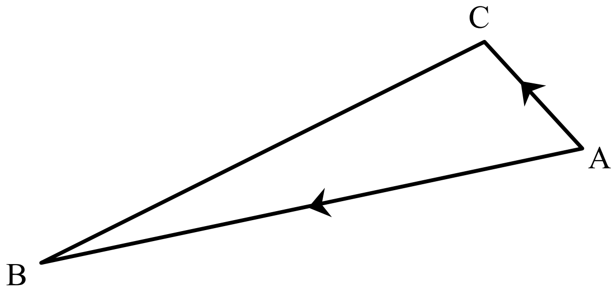
(*b*) The point T divides RS in the ratio 3:1 as shown in the diagram.

Find the coordinates of T. **(3)**



2. The diagram shows triangle ABC where

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix} \quad \text{and} \quad \overrightarrow{AC} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$



(*a*) Find the value of $\overrightarrow{AB} \cdot \overrightarrow{AC}$. **(1)**

(*b*) Use the result of (*a*) to find the size of angle BAC. **(4)**

Outcome 2

3. (*a*) Differentiate $-6\cos x$ with respect to x. **(1)**

(*b*) Given $y = \frac{1}{3}\sin x$, find $\frac{dy}{dx}$. **(1)**

4. Find $g'(x)$ when $g(x) = (x - 1)^{-4}$. **(2)**

5. (*a*) Find $\int 2\sin x \, dx$ **(2)**

(*b*) Integrate $\frac{1}{2}\cos x$ with respect to x. **(1)**

(*c*) Evaluate $\int_1^2 (x+3)^4 \, dx$ **(4)**

Outcome 3

6. (*a*) Simplify $\log_x 8 + \log_x 5$. **(1)**

(*b*) Simplify $5\log_9 3 - \log_9 27$ **(4)**

7. (*a*) If $x = \dfrac{\log_e 33}{\log_e 7}$, find an approximation for *x*. **(1)**

(*b*) Given that $\log_{10} y = 2\cdot 5$, write down an expression for the **exact** value of *y*. **(1)**

(*c*) If $y = 10^{1\cdot 66}$, find an approximation for *y*. **(1)**

Outcome 4

8. Express $4\cos x^\circ + \sin x^\circ$ in the form $k\cos(x - a)^\circ$ where $k > 0$ and $0 \le a < 360$. **(5)**

Practice Assessment (2) Unit 3 - Mathematics 3 (H)

Outcome 1 *Marks*

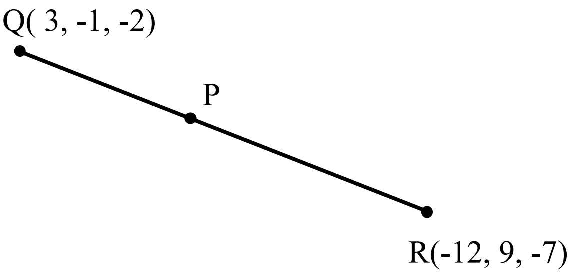
1. (*a*) E, F and G have coordinates (1, 4, -2) , (-1, 8, -1) and (-5, 16, 1) respectively.

(i) Write down the components of $\overrightarrow{EF}$. **(1)**

(ii) Hence show that the points E, F and G are collinear. **(3)**

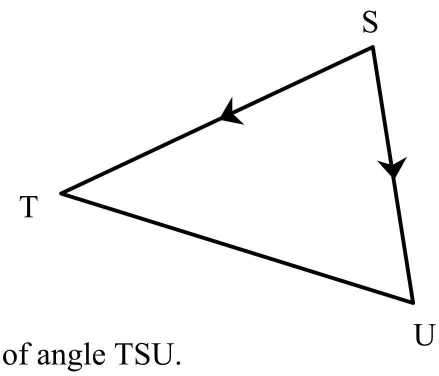
(*b*) The point P divides QR in the ratio 2:3 as shown in the diagram.

Find the coordinates of P. **(3)**



2. The diagram shows triangle STU where

$$\overrightarrow{ST} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{SU} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$$



(*a*) Find the value of $\overrightarrow{ST} \cdot \overrightarrow{SU}$. **(1)**

(*b*) Use the result of (*a*) to find the size of angle TSU. **(4)**

Outcome 2

3. (*a*) Differentiate $2\cos x$ with respect to x. **(1)**

(*b*) Given $y = 5\sin^3 x$, find $\frac{dy}{dx}$. **(1)**

4. Find $h'(x)$ when $h(x) = (2+3x)^3$. **(2)**

5. (*a*) Find $\int \frac{1}{2}\cos x \; dx$ **(2)**

(*b*) Integrate $\sin 4x$ with respect to x. **(1)**

(*c*) Evaluate $\int_2^4 (x-2)^3 \; dx$ **(4)**

Outcome 3

6. (*a*) Simplify $\log_p 6 + \log_p 3$. **(1)**

(*b*) Simplify $2\log_2 6 - \log_2 9$ **(4)**

7. (*a*) If $x \log_e 9 = \log_e 11$, find an approximation for x. **(1)**

(*b*) Given that $\log_3 y = 1{\cdot}6$, write down an expression for the **exact** value of y. **(1)**

(*c*) If $y = 10^{0{\cdot}8}$, find an approximation for y. **(1)**

Outcome 4

8. Express $5\sin x^\circ + 2\cos x^\circ$ in the form $k\cos(x - a)^\circ$ where $k > 0$ and $0 \leq a < 360$. **(5)**

Practice Assessment (3)   Unit 3 - Mathematics 3 (H)

Outcome 1 *Marks*

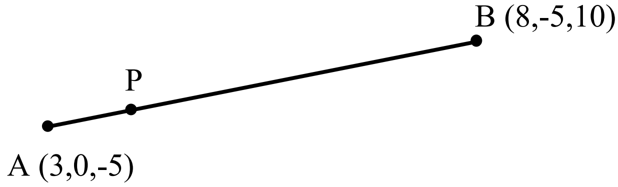
1. (*a*) S, T and U have coordinates $(1, 2, -5)$, $(-3, 4, 1)$ and $(-5, 5, 4)$ respectively.

(i) Write down the components of $\overrightarrow{ST}$. **(1)**

(ii) Hence show that the points S, T and U are collinear. **(3)**

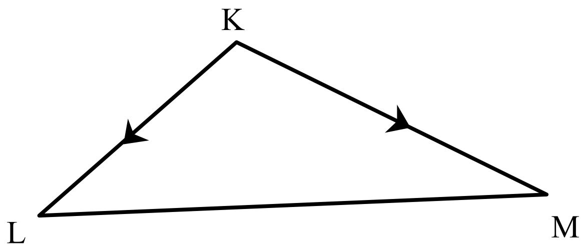
(*b*) The point P divides AB in the ratio 1 : 4 as shown in the diagram.

Find the coordinates of P. **(3)**



2. The diagram shows triangle KLM where

$$\overrightarrow{KL} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{KM} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$$



(*a*) Find the value of $\overrightarrow{KL} \cdot \overrightarrow{KM}$. **(1)**

(*b*) Use the result of (*a*) to find the size of angle LKM. **(4)**

Outcome 2

3. (*a*) Differentiate $-\frac{1}{2}\sin x$ with respect to x. **(1)**

(*b*) Given $y = \cos 2x$, find $\dfrac{dy}{dx}$. **(1)**

4. Find $f'(x)$ when $f(x) = (3x+1)^{-6}$. **(2)**

5. (*a*) Find $\int -3\sin x \; dx$ **(2)**

(*b*) Integrate $\cos 4x$ with respect to x. **(1)**

(*c*) Evaluate $\int_1^3 (2x+1)^3 \; dx$ **(4)**

Outcome 3

6. (*a*) Simplify $\log_y 16 - \log_y 8$. **(1)**

(*b*) Simplify $3\log_4 2 + \log_4 8$ **(4)**

7. (*a*) If $p\log_e 12 = 2\log_e 17$, find an approximation for p. **(1)**

(*b*) Given that $\log_2 x = 3\cdot 4$, write down an expression for the **exact** value of x. **(1)**

(*c*) If $y = 4^{2\cdot 7}$, find an approximation for y. **(1)**

Outcome 4

8. Express $7\sin x^\circ + 4\cos x^\circ$ in the form $k\cos(x - a)^\circ$ where $k > 0$ and $0 \le a < 360$. **(5)**

Unit 3 - Practice Assessments **Answers**

Practice Assessment 1

Outcome 1 : **1.** (a) i) $\vec{PQ} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$ ii) proof (b) T(-5 ,7 ,9)

2. (a) $\vec{AB}.\vec{AC} = 22$ (b) $55 \cdot 7^\circ$

Outcome 2 : **3.** (a) $6\sin x$ (b) $\frac{dy}{dx} = \frac{1}{3}\cos x$ **4.** $g'(x) = -4(x-1)^{-5}$

5. (a) $-2\cos x + C$ (b) $\frac{1}{2}\sin x + C$ (c) $420 \cdot 2$

Outcome 3 : **6.** (a) $\log_x 40$ (b) 1 **7.** (a) $1 \cdot 80$ (b) $y = 10^{2 \cdot 5}$ (c) $45 \cdot 71$

Outcome 4 : **8.** $\sqrt{17}\cos(x - 14 \cdot 0)^\circ$

Practice Assessment 2

Outcome 1 : **1.** (a) i) $\vec{EF} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ ii) proof (b) P(-3 ,3 ,-4)

2. (a) $\vec{ST}.\vec{SU} = 2$ (b) $79 \cdot 1^\circ$

Outcome 2 : **3.** (a) $-2\sin x$ (b) $\frac{dy}{dx} = 15\sin^2 x.\cos x$ **4.** $h'(x) = 9(2+3x)^2$

5. (a) $\frac{1}{2}\sin x + C$ (b) $-\frac{1}{4}\cos 4x + C$ (c) 4

Outcome 3 : **6.** (a) $\log_p 18$ (b) 2 **7.** (a) $1 \cdot 09$ (b) $y = 3^{1 \cdot 6}$ (c) $6 \cdot 3$

Outcome 4 : **8.** $\sqrt{29}\cos(x - 68 \cdot 2)^\circ$

Practice Assessment 3

Outcome 1 : **1.** (a) i) $\vec{ST} = \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$ ii) proof (b) P(4 ,-1 ,-2)

2. (a) $\vec{KL}.\vec{KM} = -5$ (b) $104 \cdot 4^\circ$

Outcome 2 : **3.** (a) $-\frac{1}{2}\cos x$ (b) $\frac{dy}{dx} = -2\sin 2x$ **4.** $f'(x) = -18(3x+1)^{-7}$

5. (a) $3\cos x + C$ (b) $\frac{1}{4}\sin 4x + C$ (c) 290

Outcome 3 : **6.** (a) $\log_y 2$ (b) 3 **7.** (a) $2 \cdot 28$ (b) $x = 2^{3 \cdot 4}$ (c) $42 \cdot 2$

Outcome 4 : **8.** $\sqrt{65}\cos(x - 60 \cdot 3)^\circ$

Higher

Mathematics

UNIT 3

Specimen NAB Assessment

HSN23510

UNIT 3

Specimen NAB Assessment

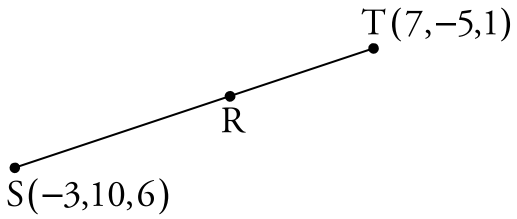
Outcome 1

1. (a) Points A, B and C have coordinates $(-4,-3,1)$, $(0,-1,0)$ and $(4,1,-1)$ respectively.

   (i) Write down the components of $\overrightarrow{AC}$.

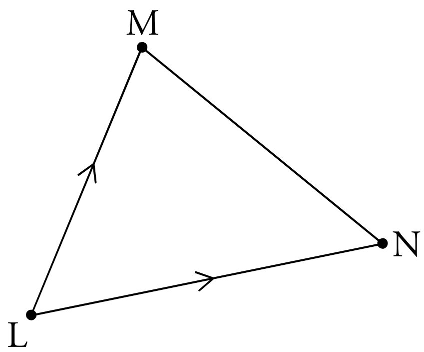
   (ii) Hence show that the points A, B and C are collinear. **4**

   (b) The point R divides $\overrightarrow{ST}$ in the ratio $3:2$, as shown below.



   Find the coordinates of R. **3**

2. The diagram shows triangle LMN where $\overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ and $\overrightarrow{LN} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$.



   (a) Find the value of $\overrightarrow{LM}.\overrightarrow{LN}$. **1**

   (b) Use your answer from part (a) to find the size of angle $M\hat{L}N$. **4**

Outcome 2

3. (a) Differentiate $-2\sin x$ with respect to x. **1**

   (b) Given $y = 5\cos x$, find $\dfrac{dy}{dx}$. **1**

4. Find $f'(x)$ when $f(x) = (2x+7)^{\frac{1}{3}}$. **2**

5. (a) Find $\int \left(\frac{\sqrt{3}}{2}\cos x\right) dx$. **2**

   (b) Integrate $3\sin x$ with respect to x. **1**

   (c) Evaluate $\int_4^6 (x-3)^3 \, dx$. **4**

Outcome 3

6. (a) Simplify $\log_a 7 + \log_a 3$. **1**

   (b) Simplify $\log_3 5 - 3\log_3 2$. **3**

   (c) Evaluate $\log_2 2$. **1**

7. (a) Given $x = \dfrac{\log_e 7}{\log_e 4}$, find an approximation for x. **1**

   (b) Given $\log_{10} y = 3{\cdot}1$, write an expression for the exact value of y. **1**

   (c) Given $y = 10^{2{\cdot}9}$, find an approximation for y. **1**

Outcome 4

8. Express $12\cos x + 5\sin x$ in the form $k\cos(x - a)$ where $k > 0$ and $0 \le a \le 360$. **5**

Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{9}{12}$	$\frac{8}{11}$	$\frac{5}{8}$	$\frac{3}{5}$

Outcome 1 – Vectors		
1. (a) (i) $\overrightarrow{AC} = \boldsymbol{c} - \boldsymbol{a} = \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} \checkmark = 2\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$	• Components of $\overrightarrow{AC}$	**1**
(ii) $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} \checkmark = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \checkmark$ Since $2\overrightarrow{AB} = \overrightarrow{AC}$ and A is a common point, A, B and C are collinear. ✓	• Know to find $\overrightarrow{AB}$ or $\overrightarrow{BC}$ • Components of $\overrightarrow{AB}$ or $\overrightarrow{BC}$ • Conclusion	**3**
(b) $\frac{SR}{RT} = \frac{3}{2} \checkmark$ $2\overrightarrow{SR} = 3\overrightarrow{RT}$ $2(\boldsymbol{r} - \boldsymbol{s}) = 3(\boldsymbol{t} - \boldsymbol{r})$ $5\boldsymbol{r} = 3\boldsymbol{t} + 2\boldsymbol{s} \checkmark$ $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$ R is the point $(3,1,3)$.✓	• Strategy for finding R • Process • State the coordinates of R	**3**
2. (a) $\overrightarrow{LM}.\overrightarrow{LN} = (3 \times -2) + (4 \times 4) + (2 \times 5) = 20$ ✓	• Calculate scalar product	**1**

(b) $\cos M\hat{L}N = \dfrac{\overrightarrow{LM}.\overrightarrow{LN}}{\lvert\overrightarrow{LM}\rvert\lvert\overrightarrow{LN}\rvert}$ ✓ $= \dfrac{20}{\sqrt{3^2+4^2+2^2}\sqrt{(-2)^2+4^2+5^2}}$ ✓ $= 0.554$ ✓ $M\hat{L}N = 56.4$ ✓ (to 1 d.p.)	• Use $\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ • Start to process • Complete process • State angle	**4**
Outcome 2 – Further Calculus		
3. (a) $\frac{d}{dx}(-2\sin x) = -2\cos x$ ✓	• Differentiate	**1**
(b) $\dfrac{dy}{dx} = -5\sin x$ ✓	• Differentiate	**1**
4. $f'(x) = \frac{1}{3}(2x+7)^{-\frac{2}{3}}$ ✓ $\times 2$ ✓ $= \dfrac{2}{3\sqrt[3]{(2x+7)^2}}$	• Differentiate term with fractional power • Use chain rule	**2**
5. (a) $\int\left(\frac{\sqrt{3}}{2}\cos x\right)dx = \frac{\sqrt{3}}{2}\sin x$ ✓ $+c$ ✓	• Integrate • Constant of integration	**2**
(b) $\int(3\sin x)\,dx = -3\cos x$ ✓ $+c$	• Integrate	**1**
(c) $\int_4^6 (x-3)^3\,dx = \left[\dfrac{(x-3)^4 ✓}{4 ✓}\right]_4^6$ $= \frac{1}{4}(6-3)^4 - \frac{1}{4}(4-3)^4$ ✓ $= \dfrac{80}{4}$ $= 20$ ✓	• Raise power • Correct multiplier • Substitute limits • Process	**4**
Outcome 3 – Exponentials and Logarithms		
6. (a) $\log_a 7 + \log_a 3 = \log_a 21$ ✓	• $\log_a x + \log_a y = \log_a xy$	**1**
(b) $\log_3 5 - 3\log_3 2 = \log_3 5 - \log_3 2^3$ ✓ $= \log_3 \dfrac{5}{2^3}$ ✓ $= \log_3 \frac{5}{8}$ ✓	• $k\log_a x + \log_a x^k$ • $\log_a x - \log_a y = \log_a \frac{x}{y}$ • Complete	**3**
(c) $\log_2 2 = 1$ ✓	• Know that $\log_a a = 1$	**1**
7. (a) 1·404 ✓ (to 3 d.p.)	• Process	**1**

(b) $y = 10^{3 \cdot 1}$ ✓	• Use $\log_a y = x \Leftrightarrow y = a^x$	**1**
(c) $794 \cdot 3$ ✓ (to 1 d.p.)	• Process	**1**
Outcome 4 – Wave Functions		
8. $k\cos(x - a) = k\cos a \cos x + k\sin a \sin x$ ✓ $\left.\begin{aligned} k\cos a &= 12 \\ k\sin a &= 5 \end{aligned}\right\}$ ✓ $\Rightarrow k = \sqrt{12^2 + 5^2} = 13$ ✓ $\tan a = \frac{5}{12}$ ✓ $a = 22 \cdot 6$ ✓ (to 1 d.p.) So $12\cos x + 5\sin x = 13\cos(x - 22 \cdot 6)$.	• Use compound angle formula • Extract $k\cos a$ and $k\sin a$ • Calculate k • State $\tan a$ • Calculate a	**5**