Vectors Past Papers Unit 3 outcome 1

Written Questions

[SQA] 1. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find

(*a*) the components of vector $\overrightarrow{AB}$; 1

(*b*) the length of AB. 2

[SQA] 2. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by

$$\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}, \quad \boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}.$$

(*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. 2

(*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ 1

(*c*) Find $|\boldsymbol{r}|$. 1

[SQA] 3. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows:

$$\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}, \; \boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}, \; \boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}.$$

(*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. **1**

(*b*) Find the value of $|2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}|$. **2**

[SQA] 4. The vector $a\boldsymbol{i} + b\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to both the vectors $\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $-2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$.
Find the values of a and b. 3

[SQA] 5. The position vectors of the points P and Q are $\boldsymbol{p} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 7\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. 2

(*b*) Find the length of PQ. 1

[SQA] 6. Calculate the length of the vector $2\mathbf{i} - 3\mathbf{j} + \sqrt{3}\mathbf{k}$. 2

[SQA] 7. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined as follows:

$$\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{k}, \quad \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}, \quad \boldsymbol{c} = -\boldsymbol{j} + \boldsymbol{k}.$$

(*a*) Evaluate $\boldsymbol{a}.\boldsymbol{b} + \boldsymbol{a}.\boldsymbol{c}$. 3

(*b*) From your answer to part (*a*), make a deduction about the vector $\boldsymbol{b} + \boldsymbol{c}$. 2

[SQA] 8. Show that the vectors $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ are perpendicular. **3**

[SQA] 9. If $\boldsymbol{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, write down the components of $\boldsymbol{u} - \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$. Hence show that $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular. **3**

[SQA] 10. (a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. **4**
(b) Find the ration in which M divides LN. **1**

[SQA] 11. Show that P(2, 2, 3), Q(4, 4, 1) and R(5, 5, 0) are collinear and find the ratio in which Q divides PR. **4**

[SQA] 12. A is the point (2, –5, 6), B is (6, –3, 4) and C is (12, 0, 1). Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

[SQA] 13. The point Q divides the line joining $P(-1, -1, 0)$ to $R(5, 2, -3)$ in the ratio $2 : 1$. Find the coordinates of Q. **3**

[SQA] 14. For what value of t are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

[SQA] 15. $A(4, 4, 10)$, $B(-2, -4, 12)$ and $C(-8, 0, 10)$ are the vertices of a right-angled triangle.
Determine which angle of the triangle is the right angle. **3**

[SQA] 16. Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$ are perpendicular. **3**

[SQA] 17. PQRS is a parallelogram with vertices $P(1,3,3)$, $Q(4,-2,-2)$ and $R(3,1,1)$.

Find the coordinates of S. **3**

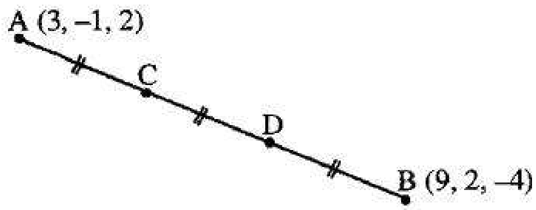
[SQA] 18. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

(*a*) Find the coordinates of M, the midpoint of AB. **1**

(*b*) Find the coordinates of the point T, which divides CM in the ratio $2:1$. **3**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

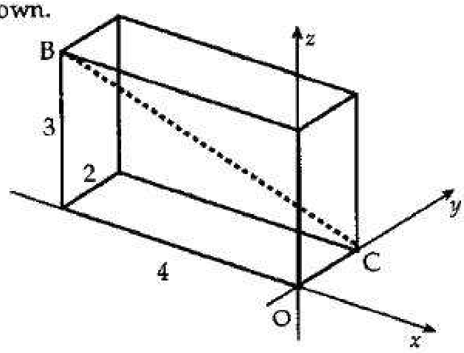
[SQA] 19. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).



(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(*b*) Find the coordinates of C and D. 2

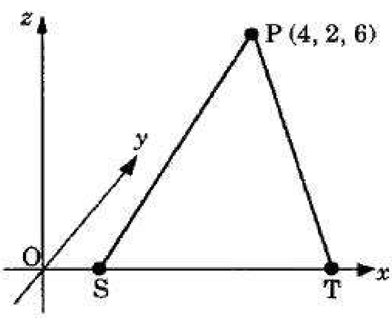
[SQA] 20. A cuboid crystal is placed relative to the coordinate axes as shown.



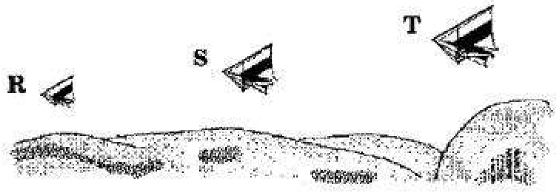
(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $|\overrightarrow{BC}|$. 2

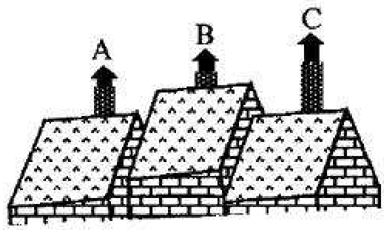
[SQA] 21. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 3



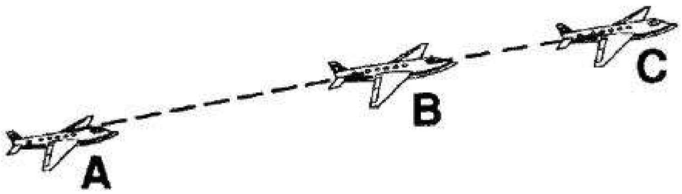
[SQA] 22. Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6). Prove that R, S and T are collinear. 3



[SQA] 23. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1, 4) and C(4, –9, 8). Show that A, B and C are collinear. 3



[SQA] 24. An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point (–1, 3, 4) and B is the point (3, 1, –2). Find the co-ordinates of the point C. 3



[SQA] 25. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

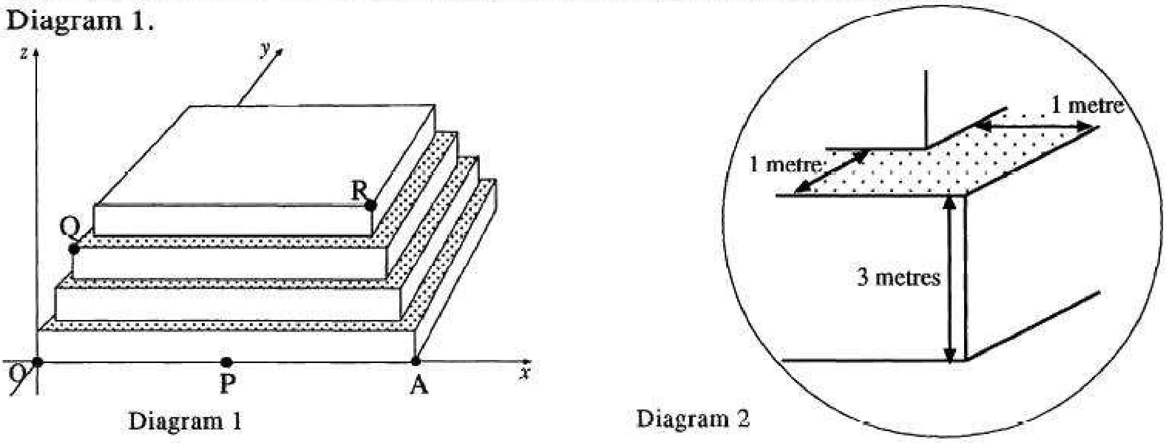


Diagram 1 Diagram 2

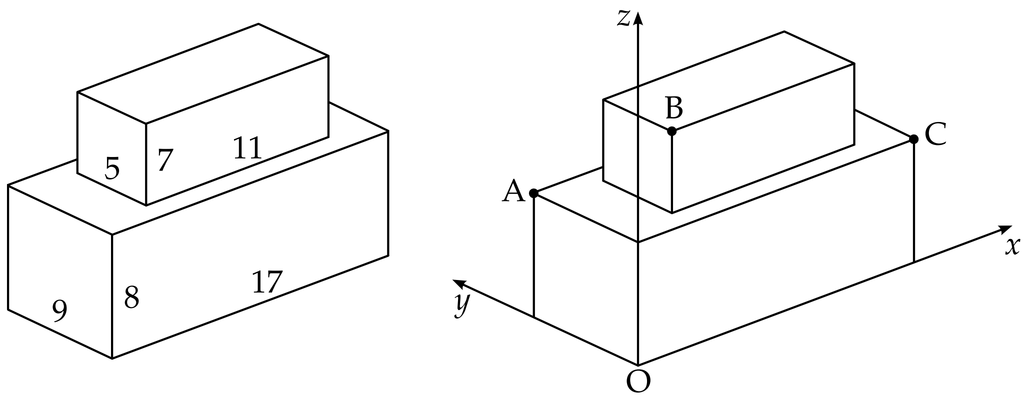
Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**

(*b*) Find the size of angle QPR. **(7)**

[SQA] 26. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

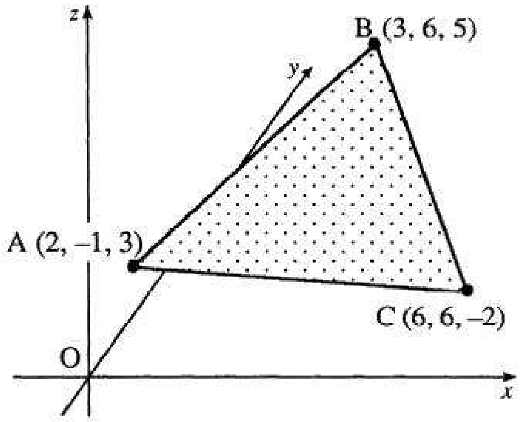
Coordinates axes are taken as shown.



(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

[SQA] 27. A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

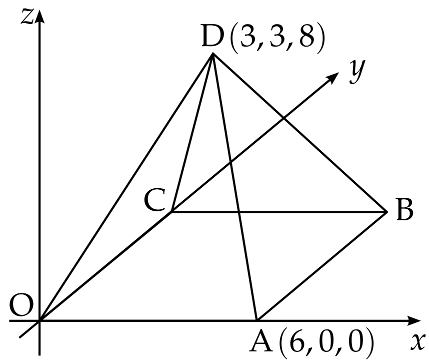
(*c*) Hence find the area of the triangle. **(2)**

[SQA] 28. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are $(6, 0, 0)$ and $(3, 3, 8)$.

C lies on the y-axis.

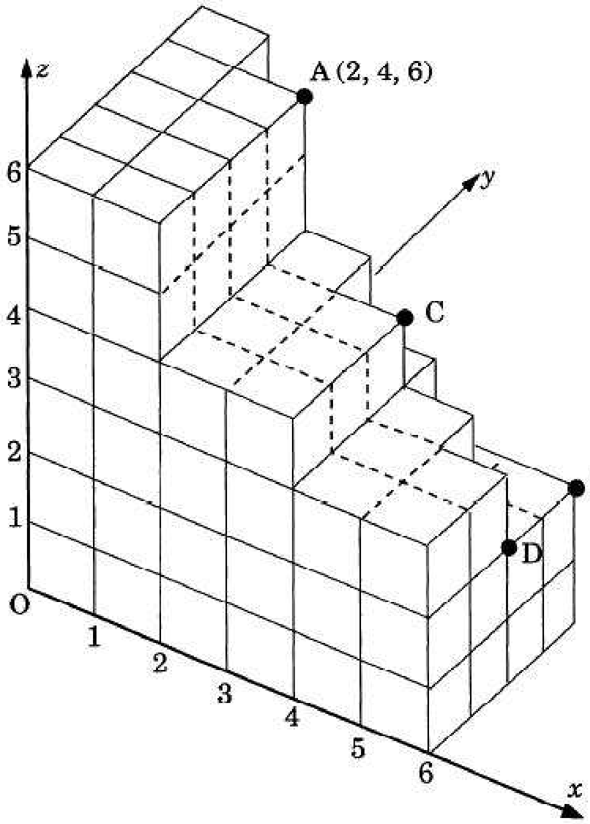


(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

[SQA] 29. With coordinate axes as shown, the point A is (2,4,6).



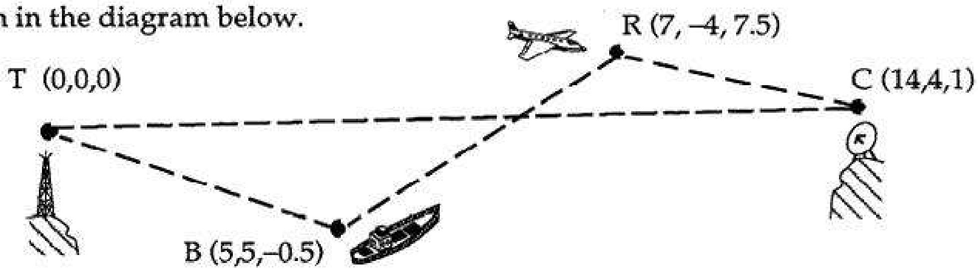
(a) Write down the coordinates of B,C and D. **(3)**

(b) Show that C is the midpoint of AD. **(1)**

(c) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(d) Hence calculate the size of angle OAB. **(2)**

[SQA] 30. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point $(5, 5, -0{\cdot}5)$, the centre C of the dish on the top of a mountain is the point $(14, 4, 1)$ and the reflector R on the aircraft is the point $(7, -4, 7{\cdot}5)$.

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point $M(-2, 4, 8{\cdot}5)$. Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

[SQA] 31. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

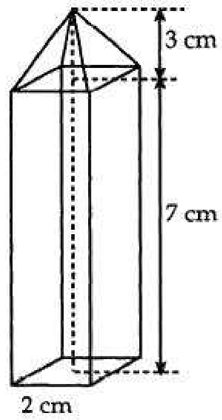


Diagram 1

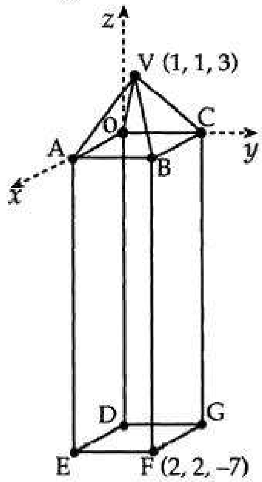


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

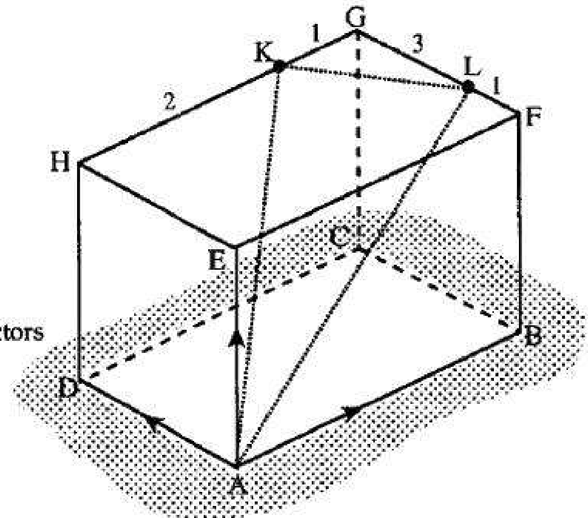
Calculate the area of the glass triangle VEF. **(3)**

[SQA] 32. ABCDEFGH is a cuboid.

K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ can be represented by the vectors

$\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ respectively.



(*a*) Calculate the components of $\overrightarrow{AK}$. **2**

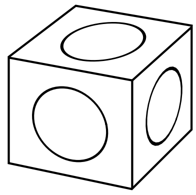
(*b*) Calculate the components of $\overrightarrow{AL}$. **2**

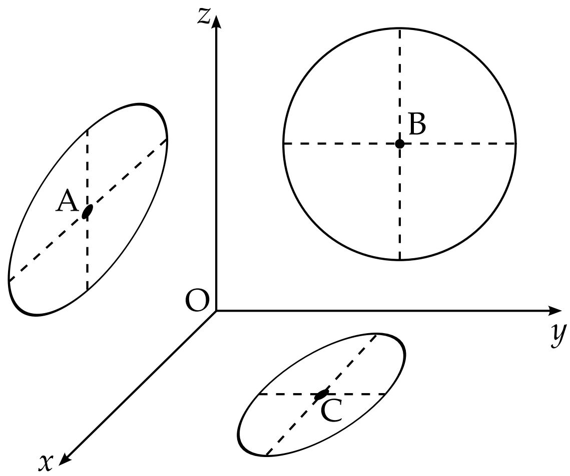
(c) Calculate the size of angle KAL. **5**

[SQA] 33. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A$(6,0,7)$, B$(0,5,6)$ and C$(4,5,0)$.

Find the size of angle ABC. **7**

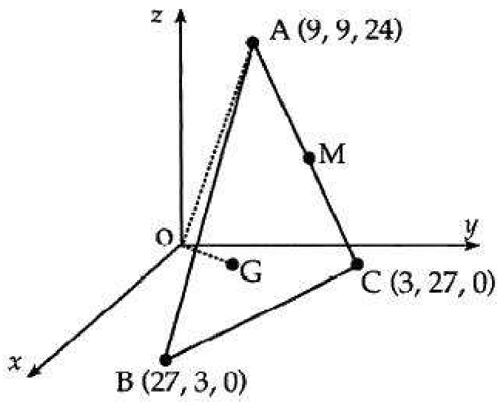




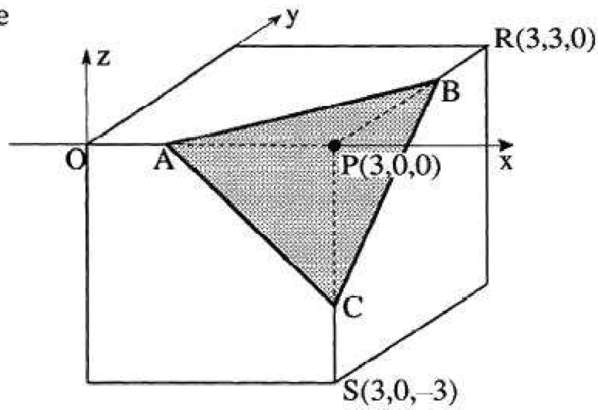
[SQA] 34. (*a*) Relative to mutually perpendicular axes O*x*, O*y* and O*z*, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.

Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**



[SQA] 35. A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC.
Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.

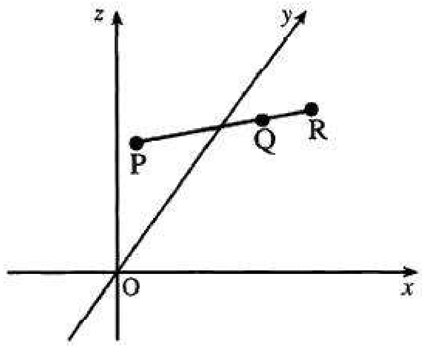


(*a*) Find the coordinates of A, B and C. **(3)**

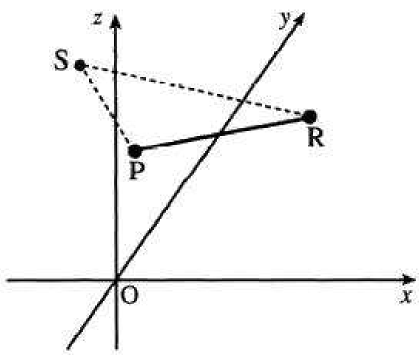
(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

[SQA] 36. Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.

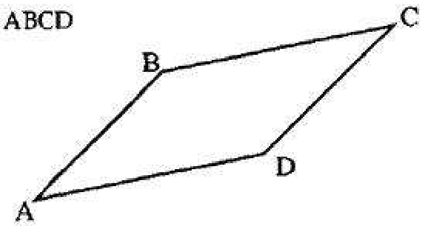


(*a*) Find the coordinates of R. **(3)**



(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**

[SQA] 37. A is the point $(2, -1, 4)$, B is $(7, 1, 3)$ and C is $(-6, 4, 2)$. If ABCD is a parallelogram, find the coordinates of D. 3



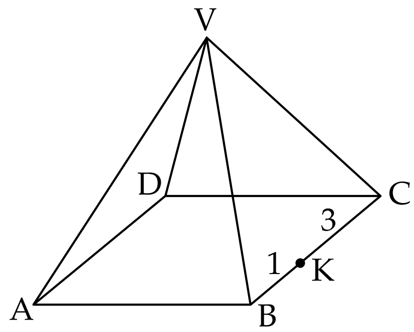
[SQA] 38. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.



K divides BC in the ratio $1 : 3$.

Find $\overrightarrow{VK}$ in component form. **3**

[SQA] 39. VABCD is a pyramid with rectangular base ABCD.

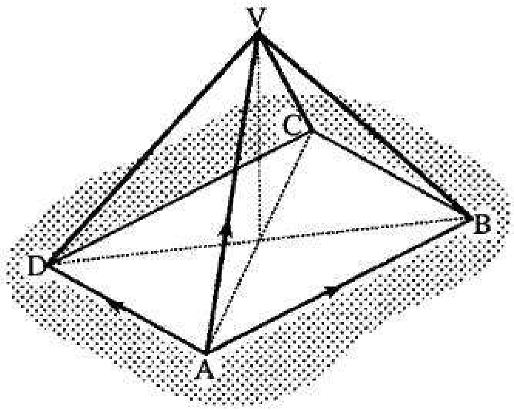
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8i + 2j + 2k$

$\overrightarrow{AD} = -2i + 10j - 2k$ and

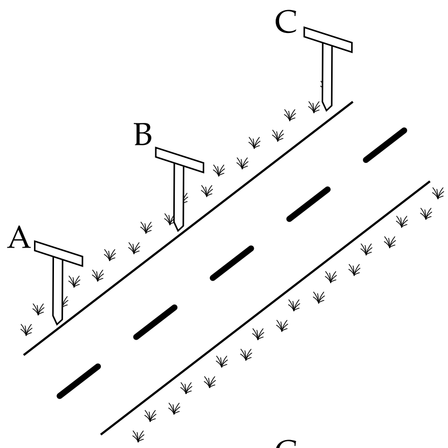
$\overrightarrow{AV} = i + 7j + 7k$.

Express $\overrightarrow{CV}$ in component form. 3



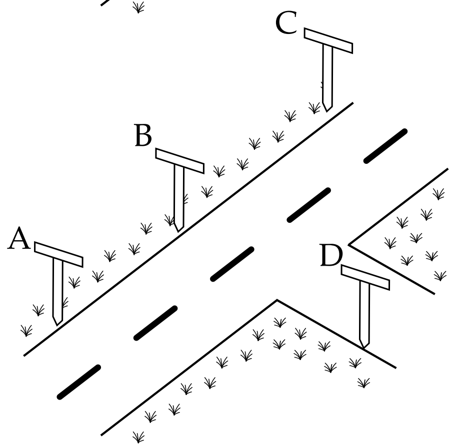
[SQA] 40. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8,-10,-2)$, $B(-2,-1,1)$ and $C(6,11,5)$.

Determine whether or not the section of road ABC has been built in a straight line. **3**

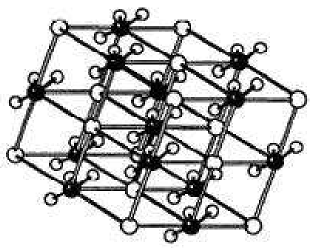


(*b*) A further T-rod is placed such that D has coordinates $(1,-4,4)$.

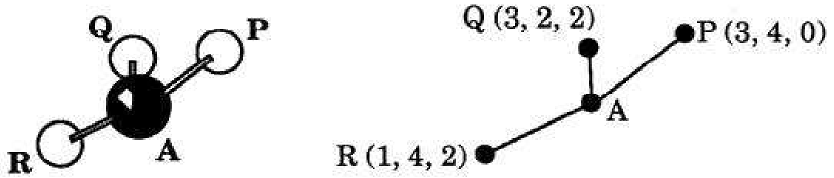
Show that DB is perpendicular to AB. **3**



[SQA] 41. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.



(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

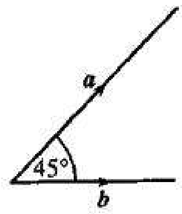
(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

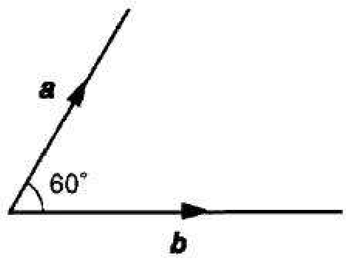
[SQA] 42. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$. These vectors are inclined at an angle of 45° to each other.



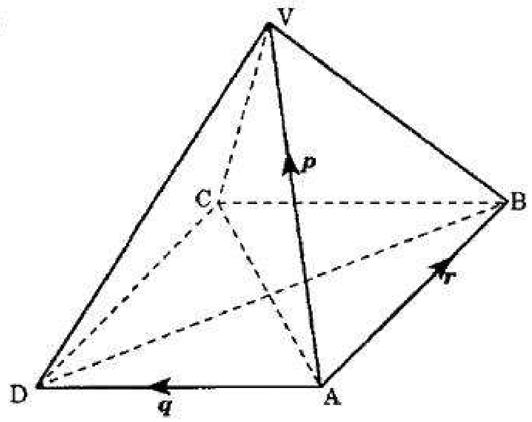
(a) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$
(ii) $\boldsymbol{b}.\boldsymbol{b}$
(iii) $\boldsymbol{a}.\boldsymbol{b}$ 2

(b) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.
Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $|\boldsymbol{p}|$. 4

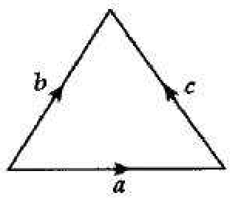
[SQA] 43. The diagram shows representatives of two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, inclined at an angle of 60°.
If $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = 3$, evaluate $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b})$ 3



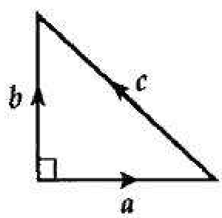
[SQA] 44. In the square-based pyramid, all the eight edges are of length 3 units.
$\vec{AV} = \boldsymbol{p}$, $\vec{AD} = \boldsymbol{q}$, $\vec{AB} = \boldsymbol{r}$.
Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. 4



[SQA] 45. The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown.
Evaluate $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$. 5



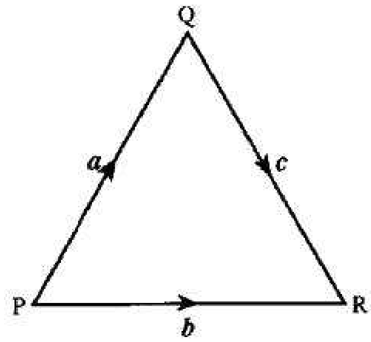
[SQA] 46. The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.
The two equal sides have length 2 units.
Find the value of $\boldsymbol{b}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$ 5



[SQA] 47. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. 4



[END OF WRITTEN QUESTIONS]