

St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 3

Paper I - Non-calculator

Time allowed - 1 hour 10 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

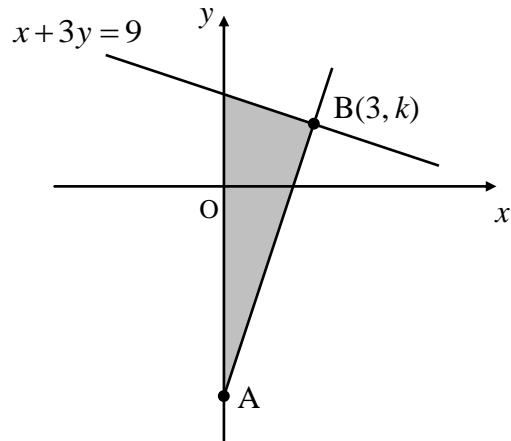
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

All questions should be attempted

1. Part of the line with equation $x + 3y = 9$ is shown in the diagram. B lies on this line and has coordinates $(3, k)$.



- (a) Find the value of k . 1
- (b) Given that the line AB is perpendicular to the line $x + 3y = 9$, find the equation of the line AB. 3
- (c) Hence write down the coordinates of A. 1
- (d) Calculate the area of the shaded triangle. 4

2. (a) A function f has as its derivative $f'(x) = x^3 - ax^2 - 4ax$.

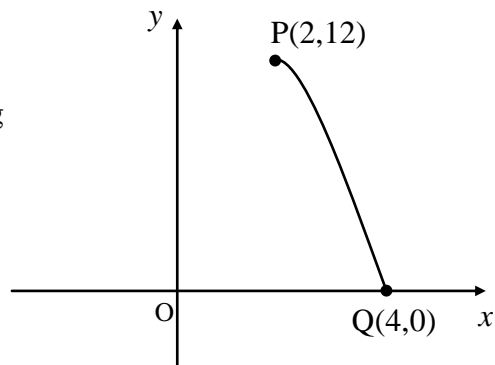
Find a if the function has a stationary point at $x = 4$. 4

- (b) Hence find the rate of change of this function at $x = -2$ and comment on your result. 2

3. A quadratic function, defined on a suitable domain, is given as $f(x) = 12x - 3x^2$.

The diagram shows part of the graph of this quadratic function, $y = f(x)$.

The graph passes through the points P(2,12) and Q(4,0) as shown.



- (a) Sketch the graph of $y = -f(x) + 6$ marking clearly the image points of P and Q and stating their coordinates. 3

- (b) Given that $g(x) = -f(x) + 6$, write down a formula for $g(x)$. 2

4. Find a given that $\int_a^2 (4 + 2x) dx = 0$, where $a < 2$. 5

5. Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\sin 2x = -\cos x$. 6

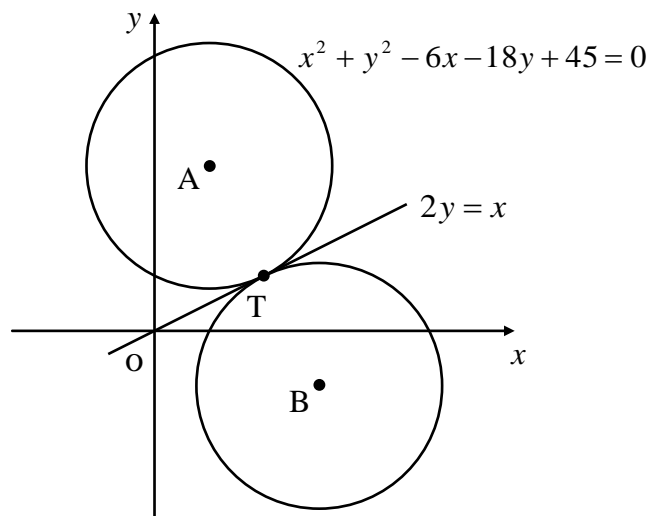
6. Two functions are defined on suitable domains as $f(x) = x + 1$ and $g(x) = x^2 + 6x + 13$.
Given that the function h is such that $h(x) = g(f(x))$, express h in the form

$$h(x) = (x + a)^2 + b, \text{ where } a \text{ and } b \text{ are integers,}$$

and hence write down the minimum value of h and the corresponding replacement for x .

6

7. The diagram below shows two congruent circles which touch at a single point T.
The circle, centre A, has as its equation $x^2 + y^2 - 6x - 18y + 45 = 0$.
The line with equation $2y = x$ is the common tangent to the two circles through T.



- (a) **Show algebraically** that T has coordinates (6,3).

5

- (b) Hence establish the the coordinates of B, the centre of the lower circle, and find the equation of this circle.

6

8. Find $f'(x)$ when $f(x) = \frac{x^2 - 2\sqrt{x}}{x}$, expressing your answer with positive indices, and hence calculate the value of the gradient of the tangent to the curve $y = f(x)$ at $x = \frac{1}{4}$.

6

9. What can you say about p if the equation, in x , $\frac{x}{p} + \frac{9}{px} = 1$ has **no real** roots?

6

St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 3

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

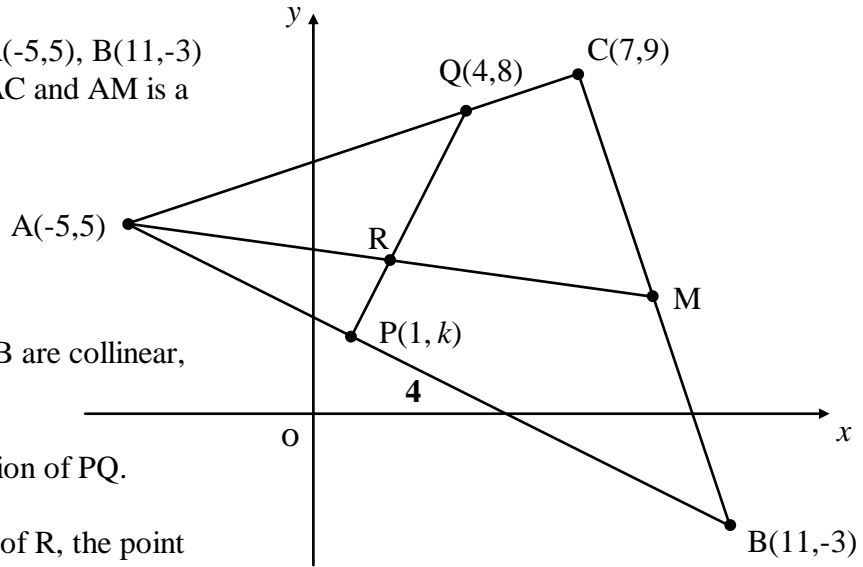
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

All questions should be attempted

1. Triangle ABC has vertices A(-5,5), B(11,-3) and C(7,9). Q(4,8) lies on AC and AM is a median of the triangle.

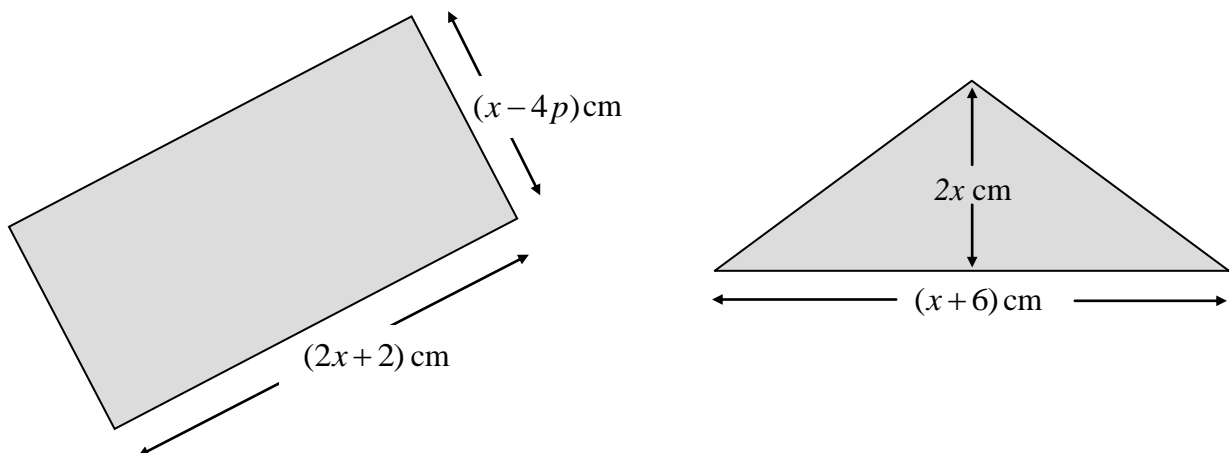


- (a) Given that A, P and B are collinear, find the value of k . 2
- (b) Hence find the equation of PQ. 2
- (c) Find the coordinates of R, the point of intersection between the line PQ and the median AM. 5

2. A scientist studying a large colony of bats in a cave has noticed that the fall in the population over a number of years has followed the recurrence relation $U_{n+1} = 0.75U_n + 200$, where n is the time in years and 200 is the average number of bats born each year during a concentrated breeding week.

- (a) He estimates the colony size at present to be 2100 bats with the breeding week just over. Calculate the estimated bat population in 4 years time immediately **before** that years breeding week. 3
- (b) The scientist knows that if in the **long term** the colony drops, at any time, below 700 individuals it is in serious trouble and will probably be unable to sustain itself. Is this colony in danger of extinction? **Explain your answer with words and appropriate working.** 4

3. The diagram below shows a rectangle and an isosceles triangle. The letter p is a constant. All lengths are in centimetres.



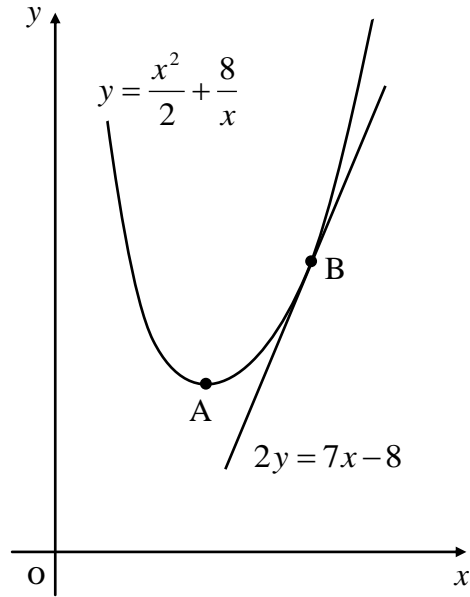
- (a) Taking A_1 as the area of the rectangle, and A_2 as the area of the triangle, show clearly that the difference between the two areas can be written in the form

$$A_1 - A_2 = x^2 - (8p + 4)x - 8p \quad 4$$

- (b) Given that $A_1 - A_2 = 1 \text{ cm}^2$, establish the value of p , where p is > -1 , for this equation to have **only one solution** for x . 6

- (c) Hence find x when p takes this value. 2

4. The diagram shows part of the graph of the curve with equation $y = \frac{x^2}{2} + \frac{8}{x}$, $x \neq 0$.



- (a) Find the coordinates of the stationary point A. 5

- (b) Also shown is the line with equation $2y = 7x - 8$ which is a tangent to the curve at B. Establish the coordinates of B. 6

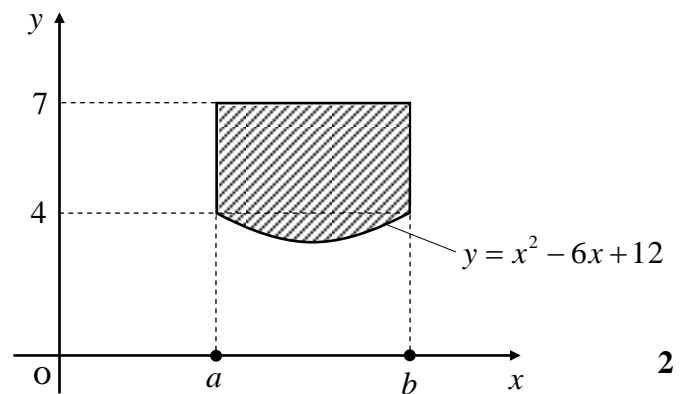
5. A function is defined on a suitable domain as $f(x) = x^2 - a$, where a is a constant.

- (a) Find a formula for h given that $h(x) = f(f(x))$. 2

- (b) Given now that $h'(2) = 8$, find a . 3

6. The diagram, which is not drawn to scale, shows the cross-section of an iron bar. The units are in centimetres.

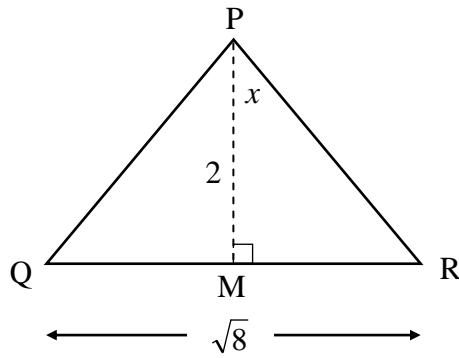
When placed in the coordinate diagram the curved section of the rod has as its equation $y = x^2 - 6x + 12$.



- (a) **Show algebraically** that the values of a and b are 2 and 4 respectively. 2

- (b) Calculate the shaded area in square centimetres. 6

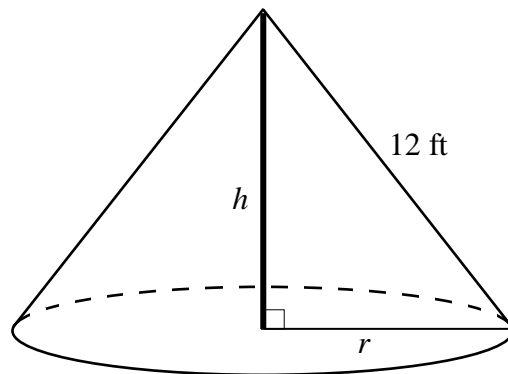
7. Triangle PQR is isosceles with $PQ = PR$ as shown. M is the midpoint of QR. $QR = \sqrt{8}$ units, $PM = 2$ units and $\angle RPM = x$.



- (a) Show clearly that the exact value of $\sin x$ is $\frac{1}{3}\sqrt{3}$. 4
- (b) Hence, or otherwise, show that $\sin \angle RPQ = \frac{2}{3}\sqrt{2}$. 4

8. An old fashioned bell tent is in the shape of a cone.

The tent has radius r , vertical height h and a slant height of 12 feet as shown.

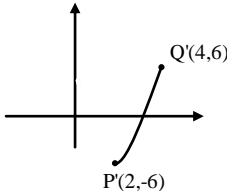


- (a) Write down an expression for r^2 in terms of h . 1
- (b) Hence show clearly that a function, in terms of h , for the volume of this cone can be expressed as

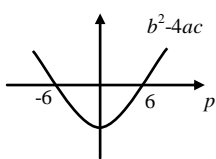
$$V(h) = 48\pi h - \frac{1}{3}\pi h^3$$

[note: the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$] 2

- (c) Hence find the exact value of h , the height of the tent, which would maximise the volume of the tent. 5
Justify your answer.

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	<p>(a) ans: $k = 2$ 1 mark</p> <ul style="list-style-type: none"> •1 sub. to answer <p>(b) ans: $y = 3x - 7$ (or equiv.) 3 marks</p> <ul style="list-style-type: none"> •1 for gradient of line •2 for gradient of AB •3 for equation <p>(c) ans: $A(0, -7)$ 1 mark</p> <ul style="list-style-type: none"> •1 answer (y intercept) <p>(d) ans: Area = 15 square units 4 marks</p> <ul style="list-style-type: none"> •1 y intercept of top line •2 length of base •3 perpendicular length •4 calculation to answer 	<p>(a) •1 $3 + 3k = 9 \therefore k = 2$</p> <p>(b) •1 $m = -\frac{1}{3}$ •2 $m_{AB} = 3$ •3 $y - 2 = 3(x - 3)$</p> <p>(c) •1 $A(0, -7)$</p> <p>(d) •1 $(0, 3)$ •2 distance between y intercepts = 10 •3 y-axis to B ... 3 units $A = \frac{1}{2}bh$ •4 $= \frac{1}{2} \times 10 \times 3 = 15 \text{ units}^2$</p>
2.	<p>(a) ans: $a = 2$ 4 marks</p> <ul style="list-style-type: none"> •1 knowing to solve deriv. to zero •2 setting up synthetic division •3 completing synth. div. •4 solving equ. to zero and answer <p>(b) ans: zero, another stat. point 2 marks</p> <ul style="list-style-type: none"> •1 substituting for a and -2 •2 calculation finds zero + conclusion <p>** pupils may complete part (a) by substitution.</p>	<p>(a) •1 at s.p. $f'(x) = 0$ (stated or implied) •2 $4 \begin{array}{r rrrr} & 1 & -a & -4a & 0 \\ & & & & \\ \hline & 1 & -a & -4a & 0 \\ & & 4 & 16-4a & 64-32a \\ \hline & & 1 & 4-a & 16-8a & 0 \end{array}$ •3 $4 \begin{array}{r rrrr} & 1 & -a & -4a & 0 \\ & & & & \\ \hline & 1 & -a & -4a & 0 \\ & & 4 & 16-4a & 64-32a \\ \hline & & 1 & 4-a & 16-8a & 0 \end{array}$ •4 $64 - 32a = 0, a = 2$</p> <p>(b) •1 $f'(-2) = (-2^3) - 2(2)(-2^2) - 4(2)(-2)$ •2 $f'(-2) = 0 \therefore$ another stat. point</p>
3.	<p>(a) ans: see sketch 3 marks</p> <ul style="list-style-type: none"> •1 for reflecting •2 for translation up 6 •3 for annotating, coordinates <p>(b) ans: $g(x) = 3x^2 - 12x + 6$ 2 marks</p> <ul style="list-style-type: none"> •1 for strategy •2 for answer (any equivalent form) 	<p>(a) •1  •2 •3</p> <p>(b) •1 strategy •2 $g(x) = -(12x - 3x^2) + 6$</p>

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	<p>ans: $a = -6$ 5 marks</p> <ul style="list-style-type: none"> •1 integrating first term •2 integrating 2nd term •3 sub. in limits •4 simplifying to quad. equation •5 solving and choosing answer 	<ul style="list-style-type: none"> •1 $4x \dots\dots$ •2 $\dots\dots + \frac{2x^2}{2}$ •3 $(8+4) - (4a + a^2) = 0$ •4 $a^2 + 4a - 12 = 0$ •5 $(a-2)(a+6) = 0$ $a = 2$ or $a = -6$
5.	<p>ans: $\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$ 6 marks</p> <ul style="list-style-type: none"> •1 knowing to solve to zero •2 replacement •3 factorising and 2 solutions •4 1st angle from one solution •5 1st angle from other solution •6 for remaining two angles 	<ul style="list-style-type: none"> •1 $\sin 2x + \cos x = 0$ •2 $2\sin x \cos x + \cos x = 0$ •3 $\cos x(2\sin x + 1) = 0$ $\therefore \cos x = 0$ or $\sin x = -\frac{1}{2}$ •4 $x = \frac{\pi}{2}$ •5 $x = \frac{7\pi}{6}$ •6 $x = \frac{3\pi}{2}, \frac{11\pi}{6}$
6.	<p>ans: $h(x) = (x+4)^2 + 4$, $h_{\min} = 4$ @ $x = -4$ 6 marks</p> <ul style="list-style-type: none"> •1 for f into g •2 for expansion and simplifying •3 bracket term •4 number term •5 for minimum value •6 for x 	<ul style="list-style-type: none"> •1 $g(f(x)) = (x+1)^2 + 6(x+1) + 13$ •2 $h(x) = x^2 + 8x + 20$ •3 $[(x+4)^2 \dots\dots$ •4 $\dots\dots -16] + 20$ •5 $\min = 4$ •6 @ $x = -4$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	<p>(a) ans: proof 5 marks</p> <ul style="list-style-type: none"> •1 know to solve a system •2 combining equations •3 simplifying to quad. •4 for 1st coordinate •5 for 2nd coordinate <p>(b) ans: B(9,-3) , $(x-9)^2 + (y+3)^2 = 45$ 6 marks</p> <ul style="list-style-type: none"> •1 knowing T mid-pt between centres •2 drawing out centre of top circle •3 finding B •4 knowing r the same •5 finding r^2 •6 writing down equation of lower circle 	<p>(a)</p> <ul style="list-style-type: none"> •1 set up a system •2 $(2y)^2 + y^2 - 6(2y) - 18y + 45 = 0$ •3 $5y^2 - 30y + 45 = 0$ •4 $5(y-3)(y-3) = 0 \therefore y = 3$ •5 $x = 2(3) = 6$ (or equivalent) <p>(b)</p> <ul style="list-style-type: none"> •1 strategy •2 A(3,9) •3 A(3,9) \rightarrow T(6,3) \rightarrow B(9,-3) •4 stated or implied $r_1 = r_2$ •5 $r^2 = \sqrt{9+81-45} = 45$ •6 $(x-9)^2 + (y+3)^2 = 45$
8.	<p>ans: $f'(x) = 1 + \frac{1}{x^{\frac{3}{2}}}$, 9 6 marks</p> <ul style="list-style-type: none"> •1 preparing to differentiate •2 diff. 1st term •3 diff. 2nd term •4 writing with positive indices •5 substituting •6 answer 	<ul style="list-style-type: none"> •1 $f(x) = x^{-1}(x^2 - 2x^{\frac{1}{2}})$ $= x - 2x^{-\frac{1}{2}}$ •2 1..... •3 $+1x^{-\frac{3}{2}}$ •4 $f'(x) = 1 + \frac{1}{x^{\frac{3}{2}}}$ •5 $f'(\frac{1}{4}) = 1 + \frac{1}{(\frac{1}{4})^{\frac{3}{2}}}$ •6 $f'(\frac{1}{4}) = 1 + \frac{1}{\frac{1}{8}} = 9$
9.	<p>ans: $-6 < p < 6$ (or equivalent) 6 marks</p> <ul style="list-style-type: none"> •1 dealing with the fractions •2 manipulation to quad. form •3 discriminant statement •4 for a, b and c •5 finding discriminant •6 solution from quad. inequat. 	<ul style="list-style-type: none"> •1 strategy $\times px$ (or equiv.) •2 $x^2 + 9 = px$ $x^2 - px + 9 = 0$ •3 for no real roots $b^2 - 4ac < 0$ •4 $a = 1, b = -p, c = 9$ •5 $p^2 - 36 < 0$ •6 $-6 < p < 6$ 

Total 60 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	<p>(a) ans: $k = 2$ 4 marks</p> <ul style="list-style-type: none"> •1 gradient of AB •2 gradient of AP •3 equating gradients •4 finding k <p>(b) ans: $y = 2x$ 2 marks</p> <ul style="list-style-type: none"> •1 gradient of PQ •2 equation <p>(c) ans: R(2,4) 5 marks</p> <ul style="list-style-type: none"> •1 coordinates of M •2 equation of median •3 setting up a system •4 finding first coordinate •5 finding 2nd coordinate 	<p>(a)</p> <ul style="list-style-type: none"> •1 $m_{AB} = \frac{-3-5}{11+5} = -\frac{1}{2}$ •2 $m_{AP} = \frac{k-5}{6}$ •3 $\frac{k-5}{6} = -\frac{1}{2}$ •4 $k = 2$ <p>(b)</p> <ul style="list-style-type: none"> •1 $m_{PQ} = \frac{8-2}{4-1} = 2$ •2 $y - 2 = 2(x - 1)$ <p>(c)</p> <ul style="list-style-type: none"> •1 M(9,3) <i>no mark given for gradient of AM ... $m_{AM} = -\frac{1}{7}$</i> •2 $y - 3 = -\frac{1}{7}(x - 9)$ •3 $7y = -x + 30$ $y = 2x$ •4 $x = 2$ •5 $y = 4$
2.	<p>(a) ans: 1011.33 bats (ignore rounding) 3 marks</p> <ul style="list-style-type: none"> •1 first two lines of calculation •2 lines 3 and 4 of calculations •3 answer <p>(b) ans: Colony is in danger. 600 prior to breeding week is less than 700 bats 4 marks</p> <ul style="list-style-type: none"> •1 knows to calculate limit + knows formula •2 calculates limit correctly •3 knows to subtract 200 •4 explanation 	<p>(a)</p> <ul style="list-style-type: none"> •1 Low High $U_1 = 0.75(2100) = 1575 + 200 = 1775$ $U_2 = 0.75(1775) = 1331.25 + 200 = 1531.25$ •2 $U_3 = 0.75(1531.25) = 1148.44 + 200 = 1348.44$ $U_4 = 0.75(1348.44) = 1011.33$ •3 1011.33 <p>(b)</p> <ul style="list-style-type: none"> •1 $L = \frac{b}{1-a}$ •2 $L = \frac{200}{1-0.75} = 800$ •3 low population $800 - 200 = 600$ •4 600 prior to breeding week is less than 700 bats so colony in danger

	Give 1 mark for each •	Illustration(s) for awarding each mark
3.	<p>(a) ans: proof 4 marks</p> <ul style="list-style-type: none"> •1 area of rectangle •2 area of triangle •3 subtracting areas •4 tidy up and common factor <p>(b) ans: $p = -\frac{1}{4}$ 6 marks</p> <ul style="list-style-type: none"> •1 equating to zero •2 discriminant statement •3 a, b and c •4 substitution •5 to quadratic form •6 answer <p>(c) ans: $x = 1$ 2 marks</p> <ul style="list-style-type: none"> •1 substitution •2 solving to answer 	<p>(a)</p> <ul style="list-style-type: none"> •1 $A_{rec} = (2x + 2)(x - 4p)$ $= 2x^2 + 2x - 8px - 8p$ •2 $A_{tri} = \frac{1}{2}(x + 6) \times 2x = x^2 + 6x$ •3 $A_1 - A_2 =$ $= 2x^2 + 2x - 8px - 8p - (x^2 + 6x)$ •4 $A_1 - A_2 = x^2 - (8p + 4)x - 8p$ <p>(b)</p> <ul style="list-style-type: none"> •1 $x^2 - (8p + 4)x - 8p - 1 = 0$ •2 $b^2 - 4ac = 0$ for equal roots •3 $a = 1, b = -(8p + 4), c = -8p - 1$ •4 $(8p + 4)^2 - 4(-8p - 1) = 0$ •5 $64p^2 + 96p + 20 = 0$ •6 $4(4p + 5)(4p + 1) = 0$ $p = \frac{5}{4}$ or $p = -\frac{1}{4}$ <p>(c)</p> <ul style="list-style-type: none"> •1 $x^2 - (8(-\frac{1}{4}) + 4)x - 8(-\frac{1}{4}) - 1 = 0$ •2 $(x - 1)(x - 1) = 0, x = 1$
4.	<p>(a) ans: A(2,6) 5 marks</p> <ul style="list-style-type: none"> •1 knowing and preparing to differentiate •2 differentiating •3 solving to zero •4 x coordinate •5 y coordinate <p>(b) ans: B(4,10) 6 marks</p> <ul style="list-style-type: none"> •1 know to form a system •2 combining equations •3 manipulation to polynomial form •4 sets up synthetic division •5 finds x coordinate •6 for y coordinate 	<p>(a)</p> <ul style="list-style-type: none"> •1 $y = \frac{1}{2}x^2 + 8x^{-1}$ •2 $\frac{dy}{dx} = x - 8x^{-2} = x - \frac{8}{x^2}$ •3 $x - \frac{8}{x^2} = 0$ •4 $x^3 - 8 = 0 \therefore x = 2$ •5 $y = 6$ <p>(b)</p> <ul style="list-style-type: none"> •1 $y = \frac{x^2}{2} + \frac{8}{x}$ $2y = 7x - 8$ •2 $7x - 8 = x^2 + \frac{16}{x}$ •3 $x^3 - 7x^2 + 8x + 16 = 0$ •4 $\begin{array}{r rrrr} & 1 & -7 & 8 & 16 \\ & & & & 0 \\ \hline & 1 & -7 & 8 & 16 \\ & & 4 & -12 & -16 \\ \hline & 1 & -3 & -4 & 0 \end{array}$ •5 $\begin{array}{r rrrr} & 1 & -7 & 8 & 16 \\ & & 4 & -12 & -16 \\ \hline & 1 & -3 & -4 & 0 \end{array} \quad x = 4$ •6 $2y = 7(4) - 8 \therefore y = 10$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5.	<p>(a) ans: $h(x) = x^4 - 2ax^2 + a^2 - a$ 2 marks</p> <ul style="list-style-type: none"> •1 for dealing with composition •2 for formula (any equivalent form) <p>(b) ans: $a = 3$ 3 marks</p> <ul style="list-style-type: none"> •1 for differentiating •2 substituting and solving to 8 •3 answer 	<p>(a)</p> <ul style="list-style-type: none"> •1 $f(x^2 - a) = \dots$ •2 $h(x) = (x^2 - a)^2 - a$ $= x^4 - 2ax^2 + a^2 - a$ <p>(b)</p> <ul style="list-style-type: none"> •1 $h'(x) = 4x^3 - 4ax$ •2 $4(2^3) - 4(2)a = 8$ •3 $a = 3$
6.	<p>(a) ans: proof 2 marks</p> <ul style="list-style-type: none"> •1 for solving to 4 •2 for answer <p>(b) ans: $7\frac{1}{3} \text{ cm}^2$ 6 marks</p> <ul style="list-style-type: none"> •1 setting up integral •2 integrating •3 substituting limits •4 area under curve •5 area of rectangle •6 subtraction to answer 	<p>(a)</p> <ul style="list-style-type: none"> •1 $x^2 - 6x + 12 = 4$ •2 $(x-4)(x-2) = 0 \therefore x = 2 \text{ or } 4$ <p>(b)</p> <ul style="list-style-type: none"> •1 $A = \int_2^4 x^2 - 6x + 12 \, dx$ •2 $= \left[\frac{x^3}{3} - 3x^2 + 12x \right]_2^4$ •3 $= \left(\frac{64}{3} - 3(16) + 12(4) \right) - \left(\frac{8}{3} - 12 + 24 \right)$ •4 $= 6\frac{2}{3} \text{ cm}^2$ •5 $A_{\text{rec}} = 2 \times 7 = 14$ •6 $A = 14 - 6\frac{2}{3} = 7\frac{1}{3} \text{ cm}^2$

	Give 1 mark for each •	Illustration(s) for awarding each mark																				
7.	<p>(a) ans: proof 4 marks</p> <ul style="list-style-type: none"> •1 length of MR •2 length of PR •3 value of $\sin x$ •4 required form <p>(b) ans: proof 4 marks</p> <ul style="list-style-type: none"> •1 knowing angle is equiv. to $\sin 2x$ •2 use replacement •3 for value of $\cos x$ and substitution •4 required answer 	<p>(a)</p> <ul style="list-style-type: none"> •1 $MR = \sqrt{2}$ or $\frac{\sqrt{8}}{2}$ •2 $PR^2 = 2^2 + \sqrt{2}^2 = 6 \therefore PR = \sqrt{6}$ •3 $\sin x = \frac{\sqrt{2}}{\sqrt{6}}$ •4 $\sin x = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$ <p>(b)</p> <ul style="list-style-type: none"> •1 $\sin RPQ = \sin 2x$ •2 $= 2\sin x \cos x$ •3 $= 2 \times \frac{\sqrt{3}}{3} \times \frac{2}{\sqrt{6}}$ •4 $\frac{4\sqrt{3}}{3\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2}{3}\sqrt{2}$ 																				
8.	<p>(a) ans: $r^2 = 144 - h^2$ 1 mark</p> <ul style="list-style-type: none"> •1 answer <p>(b) ans: proof 2 marks</p> <ul style="list-style-type: none"> •1 substituting for r^2 •2 required form <p>(c) ans: $h = \sqrt{48} = 4\sqrt{3}$ ft 5 marks</p> <ul style="list-style-type: none"> •1 know to differentiate •2 solve derivative to zero •3 differentiate •4 solve for h •5 justification 	<p>(a)</p> <ul style="list-style-type: none"> •1 $r^2 = 144 - h^2$ <p>(b)</p> <ul style="list-style-type: none"> •1 $V = \frac{1}{3}\pi(144 - h^2)h$ •2 $V(h) = 48\pi h - \frac{1}{3}\pi h^3$ <p>(c)</p> <ul style="list-style-type: none"> •1 strategy to diff. (stated or implied) •2 solve $V'(h) = 0$ (stated or implied) •3 $V'(h) = 48\pi - \pi h^2$ •4 $48\pi - \pi h^2 = 0$ $h^2 = 48$ $h = \sqrt{48} = 4\sqrt{3}$ •5 <div style="text-align: center; margin-top: 10px;"> <table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border: none; padding: 0 5px;">→</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">$4\sqrt{3}$</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">→</td> </tr> <tr> <td style="border: none; padding: 0 5px;">+</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">0</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">-</td> </tr> <tr> <td style="border: none; padding: 0 5px;">/</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">—</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">\</td> </tr> <tr> <td style="border: none; padding: 0 5px;"></td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;">max</td> <td style="border: none; padding: 0 5px;"> </td> <td style="border: none; padding: 0 5px;"></td> </tr> </table> </div> 	→		$4\sqrt{3}$		→	+		0		-	/		—		\			max		
→		$4\sqrt{3}$		→																		
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