St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 7

(Contains Unit 3 Topics – Clearly marked)

Paper I - Non~calculator

Time allowed - 1 hour 30 minutes

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2007/2008 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

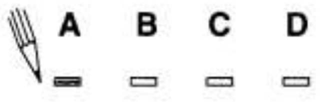
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k,7)$ lies on this line, the value of k is

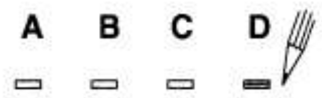
A 2
B 27
C 1·5
D − 2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



SECTION A

ALL questions should be attempted except those marked as Unit 3

1. A sequence is defined by the recurrence relation $u_{n+1} = -0.7u_n + 21$ with $u_0 = 10$.
   What is the limit of this sequence?

   A $\frac{210}{17}$
   B $\frac{210}{13}$
   C 30
   D 70

2. A line L is parallel to the line with equation $4x + 2y = 6$ and passes through the point $(-3, 1)$.
   What is the equation of L ?

   A $y - 1 = -2(x - 3)$
   B $y - 1 = 4(x - 3)$
   C $y - 1 = -2(x + 3)$
   D $y + 3 = -2(x - 1)$

3. f and g are functions defined by $f(x) = x^2 + 1$ and $g(x) = 2x$, where x is a real number.
   Find an expression for $f(g(x))$

   A $f(g(x)) = 2x^2 + 2$
   B $f(g(x)) = 4x^2 + 1$
   C $f(g(x)) = 2x^3 + 1$
   D $f(g(x)) = 2x^3 + 2x$

4. How many stationary points does the function $f(x) = 2x^3 - 9x^2 + 12x$ have?

   A 0
   B 1
   C 2
   D 3

5. Find the equation of the line passing through the points with coordinates $(1,-2)$ and $(-3,4)$.

A $3x+2y+1=0$
B $3x-2y-7=0$
C $2x+3y+4=0$
D $2x-3y-8=0$

6. A sequence is defined by the recurrence relation
$u_{n+1}=3u_n-7$ and $u_0=1$.
What is the value of u_2?

A -19
B -11
C -4
D -1

7. Express $\dfrac{11}{10-2\sqrt{3}}$ as a fraction with a rational denominator in its simplest form.

A $\dfrac{5+\sqrt{3}}{8}$

B $\dfrac{\sqrt{3}}{4}$

C $\dfrac{5-\sqrt{3}}{4}$

D $\dfrac{5+\sqrt{3}}{4}$

8. A curve has equation $y=x^3+2x^2+5$.
What is the gradient of the curve at the point where $x=1$?

A 7
B 8
C 10
D 12

9. If $\frac{dy}{dx} = 2x + 1$ and $y = 3$ when $x = 1$, express y in terms of x.

A $y = x^2$

B $y = x^2 + x + 1$

C $y = 2$

D $y = x^2 + 2$

10. What is the value of $\cos(45° - 30°)$

A $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

B $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

C $\frac{\sqrt{3} - 1}{2}$

D $\frac{\sqrt{2} - \sqrt{3}}{2}$

11. What are the coordinates of the centre of the circle with equation $3x^2 + 3y^2 - 12x - 9y + 1 = 0$

A $(12,9)$

B $(6,4\frac{1}{2})$

C $(4,3)$

D $(2,1\frac{1}{2})$

12. Find the solution of $x^2 + x - 12 < 0$.

A $x < -4$ *or* $x > 3$

B $x < -3$ *or* $x > 4$

C $-4 < x < 3$

D $-3 < x < 4$

13. Between what values does y lie where $y = \sin 240° + 3\cos x°$ and $0 \le x \le 360$?

A $\quad -\frac{5}{2} \le y \le \frac{7}{2}$

B $\quad -\frac{7}{2} \le y \le \frac{5}{2}$

C $\quad \dfrac{-\sqrt{3}-6}{2} \le y \le \dfrac{-\sqrt{3}+6}{2}$

D $\quad \dfrac{\sqrt{3}-6}{2} \le y \le \dfrac{\sqrt{3}+6}{2}$

14. Vectors u and v are defined by $u = \begin{pmatrix} g \\ 3 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ -4 \\ -g \end{pmatrix}$. If u and v are perpendicular, what is the value of g?

A $\quad 4$

B $\quad \dfrac{13}{3}$

C $\quad -4$

D $\quad \dfrac{12}{7}$

Unit 3 – Vectors!

15. If $x+5$ is a factor of the polynomial $x^3 + 4x^2 + kx - 10$ what is the value of k?

A $\quad -3$

B $\quad -7$

C $\quad -35$

D $\quad -43$

16. Find $\dfrac{dy}{dx}$ when $y = \sqrt[4]{(5x-3)}$.

A $\quad 5(5x-3)^{-3/4}$

B $\quad \dfrac{1}{20}(5x-3)^{-3/4}$

C $\quad \dfrac{5}{4}(5x-3)^{3/4}$

D $\quad \dfrac{5}{4}(5x-3)^{-3/4}$

Unit 3 – Further Calculus!

17. $2\sin x - 3\cos x$ in the form $k\cos(x-\alpha)$ where k and α are constants and $0 \le \alpha \le 2\pi$.
What is the value of $\tan\alpha$?

A $-\frac{3}{2}$

B $-\frac{2}{3}$

C $\frac{2}{3}$

D $\frac{3}{2}$

Unit 3 – Wave Function!

18. v is the vector with components $\begin{pmatrix} x \\ 2 \\ \sqrt{4x} \end{pmatrix}$. u is a unit vector parallel to v.

Find the scalar product $u.v$

A $\sqrt{x^2-4x-4}$
B $\sqrt{x^2+4x-4}$
C $(x-2)$
D $(x+2)$

Unit 3 – Vectors!

19. Evaluate $\int_0^{\frac{\pi}{2}} \cos(2x+\pi)\,dx$

A 0

B $-\frac{1}{2}$

C 1

D $-\frac{1}{4}$

Unit 3 – Further Calculus!

20. $f(x) = 2\log_{10} x + 5$

What is the value of x when $f(x) = 11$?

A 100
B 1000
C 10000
D 100000

Unit 3 – Logs!

SECTION B

ALL questions should be attempted except those marked as Unit 3

21. A circle $x^2 + y^2 - 4x - 8y + 7 = 0$ is cut by the line $x - 5y + 5 = 0$
    a. Find the points of intersection
    b. These two points form the diameter of another circle. Find the equation of the circle.
    c. Determine whether the point (1,0) lies inside, outside or on the circle.

22. A canal loch holds 6000000 gallons of water.
When a boat passes the loch is emptied by opening a sluice gate. The number of gallons of water G, in the loch t minutes after opening is modelled by

$$G(t) = 6000000 - k\ln(t-1)$$

where k is a number which can be altered by opening the sluice gate more.

x	$\ln x$
15	2.71
16	2.77
17	2.83
18	2.89
19	2.94
20	3.00
21	3.04

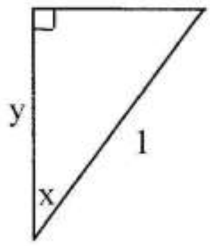
Unit 3 – Logs!

a. The loch keeper thinks it is safe to empty the loch in 21 minutes. What would the value of k have to be to achieve this?
b. Using this value of k how much water would be left after 16 minutes?
c. How long would it take for there to be 4000000 gallons left in the lock, using the same value for k?

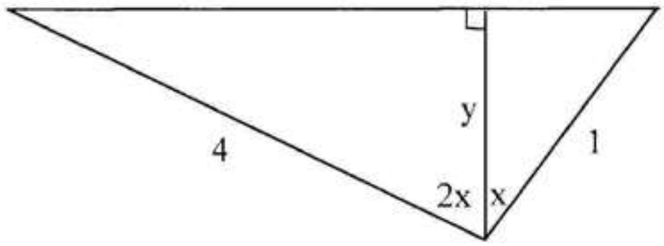
23. Consider $4\cos 2x = 2\cos x - 1$.

a. Find all the possible values of $\cos x$ for $-1 \le \cos x \le 1$

b. Hence, calculate the exact value of y, justifying your answer.



c. Prove that the base of this triangle is equal to $7\sin x$ and hence find an exact value for the base expressed as a surd in its simplest form.



[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 7

(Contains Unit 3 Topics – Clearly marked)

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

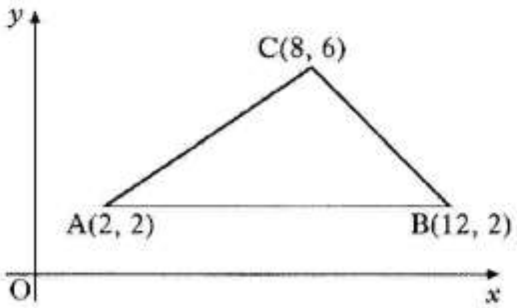
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

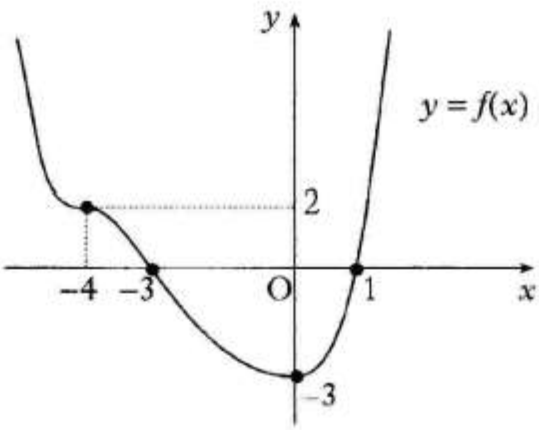
ALL questions should be attempted except those marked as Unit 3

1. Triangle ABC has vertices as shown
   a. Find the perpendicular bisectors of AB and AC.
   b. Hence, find the equation of the circle passing through A, B and C.

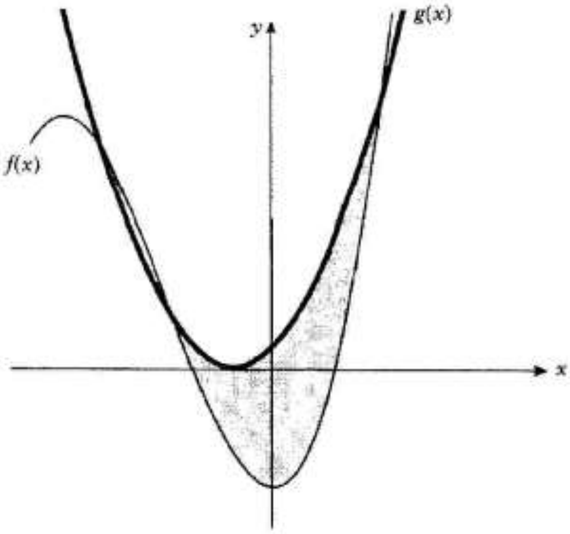


2.
   a. Write $x^2 - 10x + 27$ in the form $(x+b)^2 + c$.
   b. Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.
   c. Use differentiation to establish the value of x that gives the minimum gradient.

3.
   a. State the coordinates of the four points on the graph for
      I. $y = 3f(x)$.
      II. $y = f(3x)$.
   b. Sketch the graph of $y = 3 - f(x)$.



4. The graph shows the functions
   $f(x) = x^3 + 5x^2 - 10$ and
   $g(x) = 2x^2 + 4x + 2$.
   a. Show that $f(-3) = g(-3)$.
   b. Hence find all three points where the graphs intersect.
   c. Calculate the grey area trapped between the curves.



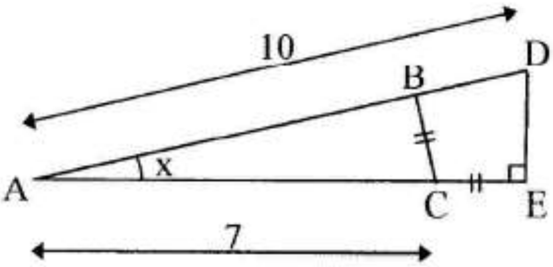
5.

a. Write $3x^2 - 8x + 11$ in the form $(x+b)^2 + c$.

b. Hence, write down the coordinates of the turning point of the parabola with equation $y = 3x^2 - 8x + 11$.

6. A right angled triangle ABC has been constructed with the right angle at B and AC = 7 cm. AC is extended to E so that CE = BC.
It is found that AD = 10 cm and triangle AED is right angled at E.

Show that $10\cos x - 7\sin x = 7$



7.

a. Express $10\cos x - 7\sin x$ in the form $k\sin(x-a)$, where $k \succ 0$ and $0 \le a \le 2\pi$.

Unit 3 – Wave Function!

b. Hence solve $10\cos x - 7\sin x = 7$ to find all solutions in the region $0 \le x \le 2\pi$.

8. In a controlled experiment the number of caterpillars and birds present in a garden were closely monitored. It was decided that the number of birds present was related to the number of caterpillars present on the previous day. Further, it was noticed that the number of caterpillars was related to the number of birds present the previous day.
Let C_n be the number of caterpillars present on day n.
Let B_n be the number of birds present on day n.

The situation has been modelled by the following equations

$$C_{n+1} = 0.1B_n + 100$$
$$B_{n+1} = 2C_n - 100$$

a) Initially there were 200 caterpillars and no birds. Calculate how the situation changed over 5 days.

b) Write down an equation relating C_n to B_{n-1}

c) Write down an equation relating B_{n+1} to B_{n-1}

d) In the long term what does this model predict will happen to the ratio of caterpillars to birds on any particular day?

9. P,Q and R have coordinates (1,3,-1), (2,0,1) and (-3,1,2) respectively.

a. Calculate the distance QP and QR
b. Hence or otherwise find the size of the angle PQR

Unit 3 – Vectors!

10. The fraction of a radioactive substance remaining after t years can be modelled by the equation

$$f(t) = 1 - e^{-0.0072t}$$

Unit 3 – Logs!

a. If there were 750 micrograms present initially, calculate the amount remaining after 60 years (to the nearest microgram)
b. After how many years will the model predict only 80% of the substance will remain ($f(t) = 0.8$)?

[**END OF QUESTION PAPER**]

Mathematics

Higher Prelim Revision 7

Paper 1 - Section A - Answer Sheet

NAME :	CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ **40**	
Section B	/ **30**	
Total (P1)	/ **70**	
Total (P2)	/ **60**	
Overall Total	/ **130**	**%**

Please make sure you have filled in all your details above before handing in this answer sheet.

Higher Prelim Revision 7 Paper 1 ANSWERS - Section A

1	A
2	C
3	B
4	C
5	A
6	A
7	D
8	A
9	B
10	B
11	D
12	C
13	C
14	A
15	B
16	D
17	B
18	D
19	A
20	B

	A	B	C	D
1	■	□	□	□
2	□	□	■	□
3	□	■	□	□
4	□	□	■	□
5	■	□	□	□
6	■	□	□	□
7	□	□	□	■
8	■	□	□	□
9	□	■	□	□
10	□	■	□	□
11	□	□	□	■
12	□	□	■	□
13	□	□	■	□
14	■	□	□	□
15	□	■	□	□
16	□	□	□	■
17	□	■	□	□
18	□	□	□	■
19	■	□	□	□
20	□	■	□	□

RED denote Unit 3 Topics

21.

(a)

$$x^2+y^2-4x-8y+7=0$$
$$x=5y-5$$

- For circle problems substitution is used for where two graphs meet.
- Straight line rearranged to read $x=$

$$(5y-5)^2+y^2-4(5y-5)-8y+7=0$$
$$25y^2-50y+25+y^2-20y+20-8y+7=0$$
$$26y^2-78y+52=0$$
$$y^2-3y+2=0$$
$$(y-1)(y-2)=0$$
$$\therefore\ y=1 \text{ or } 2$$

For $y=1 \Rightarrow x=5y-5=5\times1-5=0\ (0,1)$
For $y=2 \Rightarrow x=5y-5=5\times2-5=5\ (5,2)$

$\therefore$ Points of intersection are $(0,1)$ and $(5,2)$

(b) For equation of a circle you need centre and radius and use $(x-a)^2+(y-b)^2=r^2$

Centre is midpoint of diameter

$$\text{Centre}=\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)\quad (0,1)(5,2)$$
$$=\left(\frac{5}{2},\frac{3}{2}\right)$$

Radius is distance from centre to either $(0,1)$ or $(5,2)$

$$\text{Radius}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\quad \text{using } \left(\frac{5}{2},\frac{3}{2}\right)(0,1)$$

$$= \sqrt{\left(\frac{5}{2}-0\right)^2+\left(\frac{3}{2}-1\right)^2}$$

$$= \sqrt{\frac{25}{4}+\frac{1}{4}}$$

$$= \sqrt{\frac{26}{4}}$$

$\therefore$ using radius $\sqrt{\frac{26}{4}}$ and centre $\left(\frac{5}{2}, \frac{3}{2}\right)$

$$(x-a)^2+(y-b)^2 = r^2$$

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = \cancel{\frac{26}{4}}\frac{13}{2}$$

(c) Sub in $(1,0)$ to find inside $\left(< \frac{13}{2}\right)$ outside $\left(> \frac{13}{2}\right)$ or on $\left(= \frac{13}{2}\right)$ the circle.

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = \frac{13}{2}$$

$$\left(1-\frac{5}{2}\right)^2+\left(0-\frac{3}{2}\right)^2 = \frac{13}{2}$$

$$\left(-\frac{3}{2}\right)^2+\left(-\frac{3}{2}\right)^2$$

$$= \frac{9}{4}+\frac{9}{4} = \frac{18}{4} = \frac{9}{2}$$

$\therefore$ since $\frac{9}{2} < \frac{13}{2}$

$\therefore (1,0)$ lies inside the circle

22.

Unit 3 – Logs!

(a) $G(t) = 6000000 - k\ln(t-1)$

Sub in $t = 21$

$= 6000000 - k\ln 20$

$\Rightarrow 6000000 - 3k = 0$ ($G(t) = 0$ is loch emptied)

$3k = 6000000$

$k = 2000000$

(b) $G(t) = 6000000 - 2000000\ln 15$ (sub in $t = 16$)

$= 6000000 - 5420000$

$= 580000$ gallons

$$\begin{array}{r} 2.71 \\ \times 2 \\ \hline 5.42 \end{array} \times 1000000$$

$= 5420000$

$$\begin{array}{r} 6000000 \\ -\ 5420000 \\ \hline 580000 \end{array}$$

(c) $G(t) \Rightarrow 6000000 - 2000000\ln(t-1) = 4000000$

$2000000 = 2000000\ln(t-1)$

$\ln(t-1) = 1$

$\log_e(t-1) = \log_e e$

$t - 1 = e$

$t = 2.72 + 1$

$= 3.72$ minutes.

① Note
Solve log/ln equations one log/ln on each side

② Note
$e = 2.72$
You need to know this value

23.

(a) $4\cos 2x = 2\cos x - 1$

$$4\cos 2x - 2\cos x + 1 = 0$$

HARD PROBLEM → FORMULAE SHEET

$$4(2\cos^2 x - 1) - 2\cos x + 1 = 0$$

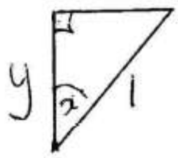
$$8\cos^2 x - 2\cos x - 3 = 0$$

$$(4\cos x - 3)(2\cos x + 1) = 0$$

$$\cos x = \frac{3}{4} \text{ or } -\frac{1}{2}$$

[Note
Question asked for $\cos x$ values]

(b)



From SOHCAHTOA
↓
$$\cos x = \frac{y}{1}$$

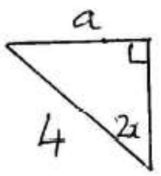
$$\therefore y = \cos x$$

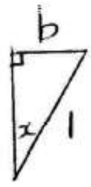
$$\therefore \cos x = \frac{3}{4} = y$$

$$\therefore y = \frac{3}{4}$$

[Note
x must be less than 180° as its in a triangle
$\therefore -\frac{1}{2}$ cannot be used]

(c)





$a + b$ = base of triangle

Using SOHCAHTOA

$$\sin 2x = \frac{a}{4} \quad \text{and} \quad \sin x = \frac{b}{1}$$

$$b = \sin x$$

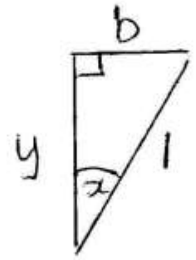
$$\therefore a = 4\sin 2x$$
$$= 4(2\sin x\cos x)$$
$$= 8\sin x\cos x \quad \Rightarrow \cos x = \frac{3}{4}$$

$$\therefore a = 8\sin x \times \frac{3}{4}$$

$$a = 6\sin x$$

$$\therefore a + b = 6\sin x + \sin x = 7\sin x$$

Exact Value



[Note
Need to find exact value for $\sin x$]

[Previous answer] $y = \frac{3}{4}$

Pythagoras → $\therefore\ b = \sqrt{1^2 - \left(\frac{3}{4}\right)^2}$

$$= \sqrt{1 - \frac{9}{16}}$$

$$= \sqrt{\frac{7}{16}}$$

$$= \frac{\sqrt{7}}{\sqrt{16}}$$

$$= \frac{\sqrt{7}}{4}$$

[From triangle at top of page] $\therefore\ \sin x = \frac{\frac{\sqrt{7}}{4}}{1} \left(\frac{opp}{hyp}\right)$

$$= \frac{\sqrt{7}}{4}$$

$\therefore$ Exact value of base $= 7\sin x$

$$= 7 \times \frac{\sqrt{7}}{4}$$

$$= \frac{7\sqrt{7}}{4}$$

1.

(a) Perpindicular Bisector of AB

Note Lines parallel to x or y axis do not use $y - b = m(x - a)$ Answer by inspection

Mid pt of AB $= \left(\frac{x_2 + x_1}{2}, \frac{y_2 - y_1}{2}\right) = (7, 2)$

$\therefore$ Equation of $AB_{PB} \Rightarrow x = 7$

Perpendicular Bisector of AC

Mid point of AC $= \left(\frac{2+8}{2}, \frac{6+2}{2}\right)$

$= (5, 4)$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{8-2} = \frac{2}{3}$ with $(2,2)\,(8,6)$

$\therefore m_{PB} = -\frac{3}{2}$ (since $m_1 m_2 = -1$)

$\therefore$ Using $m = -\frac{3}{2}$ and $(5,4)$

$y - b = m(x - a)$

$y - 4 = -\frac{3}{2}(x - 5)$

$2y - 8 = -3(x - 5)$

$2y + 3x = 23$

(b) Note:

- AC and AB are chords of circle if circle passes through A, B and C.
- Also perpendicular bisector of chords pass through the centre.
- Two things required for the equation of a circle are <u>CENTRE</u> and <u>RADIUS</u>.

Point of intersection (centre)

$$x = 7 \quad \text{and} \quad 2y + 3x = 23$$
$$2y + 3\times 7 = 23$$
$$2y = 2$$
$$y = 1$$
$$\therefore \text{ Centre is } (7, 1).$$

<u>Radius</u> (distance between 7, 1 and either A, B or C)

Distance between Centre and A

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (2,2)(7,1)$$
$$= \sqrt{(7-2)^2 + (1-2)^2}$$
$$= \sqrt{25 + 1}$$
$$= \sqrt{26}$$

$\therefore$ Using $(x-a)^2 + (y-b)^2 = r^2$

and centre $(7, 1)$ and $r = \sqrt{26}$

$$(x-7)^2 + (y-1)^2 = 26$$

Note: Check answer by sub in of either A, B or C and you would get 26

2.

(a) $x^2 - 10x + 27$

$= \boxed{x^2 - 10x + (-5)^2} - (-5)^2 + 27$

$= (x-5)^2 - 25 + 27$

$= \underline{\underline{(x-5)^2 + 2}}$

(b) For function increasing $g'(x) > 0$

$g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$

$g'(x) = x^2 - 10x + 27$

$\therefore$ For $g(x)$ increasing

$x^2 - 10x + 27 > 0$

using previous answer for (a) (Hence in the question)

$(x-5)^2 + 2 > 0$

Communication: For any value of x the squaring of the bracket will ensure an answer alway 0 or greater. The $+2$ will ensure answer alway greater than zero.

(c) Gradient equation is $x^2 - 10x + 27$

For any question asking for max/min, greatest/least, biggest/smallest find $f'(x) = 0$

$x^2 - 10x + 27$

$f'(x) = 0 \Rightarrow 2x - 10 = 0 \quad \therefore x = 5$

$\therefore$ minumum gradient is 5

(Always confirm answer with NATURE TABLE)

	5^-	5	5^+
$2x-10$	$-$	0	$+$
$f'(x)$	\	—	/

min

3. (a) $y = 3f(x)$ means y coordinate changing and y multiplied by 3.

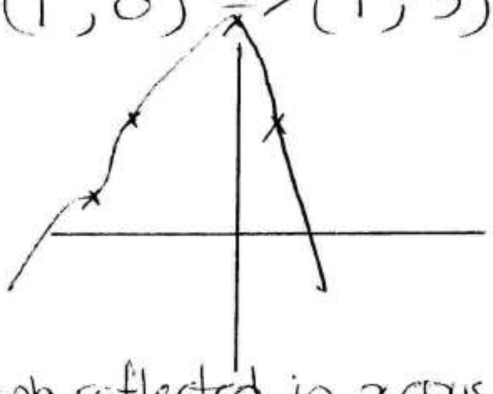
(i) $(-4,2) \Rightarrow (-4,6)$
$(-3,0) \Rightarrow (-3,0)$
$(0,-3) \Rightarrow (0,-9)$
$(1,0) \Rightarrow (1,0)$

(ii) $y = f(3x)$ means x coordinate changing x coordinate divided by 3.

$(-4,2) \Rightarrow (-\frac{4}{3},2)$
$(-3,0) \Rightarrow (-1,0)$
$(0,-3) \Rightarrow (0,-3)$
$(1,0) \Rightarrow (\frac{1}{3},0)$

(b) $y = 3 - f(x)$ means coordinate changing coordinate multiplied by -1 and add 3 to answer.

$(-4,2) \Rightarrow (-4,1)$
$(-3,0) \Rightarrow (-3,3)$
$(0,-3) \Rightarrow (0,6)$
$(1,0) \Rightarrow (1,3)$



[Graph reflected in x axis and vertical movement +3]

4.

(a) $f(-3) = x^3 + 5x^2 - 10$

$= (-3)^3 + 5(-3)^2 - 10$

Graphical calculator could be used using $-3 \rightarrow x$

$= -27 + 45 - 10$

$= \underline{\underline{8}}$

$g(-3) = 2x^2 + 4x + 2$

$= 2(-3)^2 + 4(-3) + 2$

$= 18 - 12 + 2$

$= \underline{\underline{8}}$ $\quad \therefore \underline{f(-3) = g(-3)}$

(b)

To find where two graphs meet make the equations equal to each other

$f(x) = g(x)$

$x^3 + 5x^2 - 10 = 2x^2 + 4x + 2$

$x^3 + 3x^2 - 4x - 12 = 0$

Higher than quadratic use synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -4 & -12 \\ & & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$\therefore x^3 + 3x^2 - 4x - 12 = (x+2)(x^2 + x - 6)$

$= (x+2)(x+3)(x-2)$

$\therefore x = -2, -3, 2$

Sub in except for $x = -3$ (worked out earlier).

For $x = -2$, $y = 2x^2 + 4x + 2 = 2$

For $x = -3$, $y = 8$

For $x = 2$, $y = 2x^2 + 4x + 2 = 18$

Three points are $(-2,2)$, $(-3,8)$ and $(2,18)$

(c) $\int_{\substack{x \text{ coordinate} \\ \text{where shading} \\ \text{starts}}}^{\substack{x \text{ coordinate} \\ \text{where shading finishes}}} (\text{graph above}) - (\text{graph below})\,dx$

$$= \int_{-2}^{2} \left(2x^2 + 4x + 2 - (x^3 + 5x^2 - 10)\right) dx$$

$$= \int_{-2}^{2} \left(2x^2 + 4x + 2 - x^3 - 5x^2 + 10\right) dx$$

$$= \int_{-2}^{2} \left(-x^3 - 3x^2 + 4x + 12\right) dx$$

$$= \left[-\frac{x^4}{4} - \frac{3x^3}{3} + \frac{4x^2}{2} + 12x \right]_{-2}^{2}$$

$$= \left(-\frac{16}{4} - \frac{24}{3} + \frac{16}{2} + 24\right) - \left(-\frac{16}{4} + \frac{24}{3} + \frac{16}{2} - 24\right)$$

$$= \left(\cancel{-4} - 8 \cancel{+8} + 24 \cancel{+4} - 8 - 8 + 24\right)$$

$$= 48 - 16$$

$$= 32 \text{ units}^2$$

5.

(a) $3x^2 - 8x + 11$

Note Coefficient of x^2 must be 1 before completing the square

$= 3\left[\left(x^2 - \frac{8}{3}x + \frac{11}{3}\right)\right]$

$= 3\left[\boxed{x^2 - \frac{8}{3}x + \left(\frac{4}{3}\right)^2} - \left(-\frac{4}{3}\right)^2 + \frac{11}{3}\right]$

$\frac{1}{2} \times \frac{8}{3}$
$= \frac{8}{6} = \frac{4}{3}$

$\frac{-4}{3} \times \frac{-4}{3} = \frac{16}{9}$

$= 3\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{11}{3}\right]$

$= 3\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{33}{9}\right]$

$= 3\left[\left(x - \frac{4}{3}\right)^2 + \frac{17}{9}\right]$

$= 3\left(x - \frac{4}{3}\right)^2 + \frac{17}{3}$

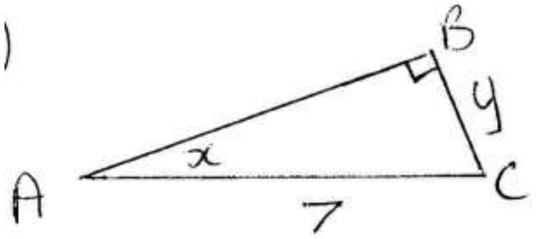
$\cancel{3} \times \frac{17}{\cancel{9}_3}$
$= \frac{17}{3}$

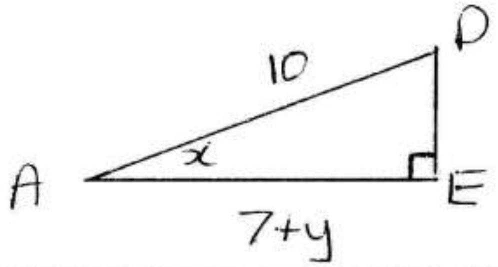
(b)

Coordinates of turning point is $\left(\frac{4}{3}, \frac{17}{3}\right)$ minimum

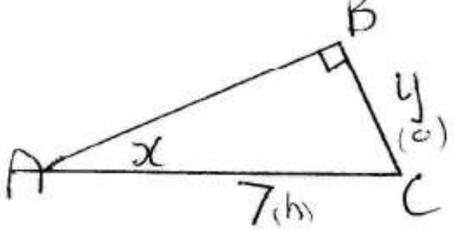
6.

)





Since BC = CE have called them both y
$\therefore AE = 7 + y$



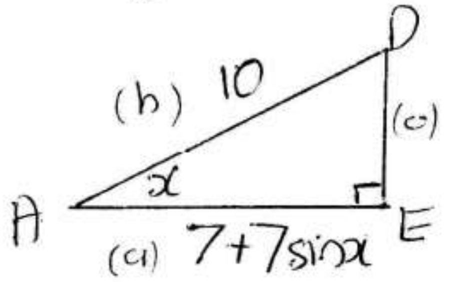
$\Rightarrow$ using SOHCAHTOA to obtain expression for y

$$\sin x = \frac{opp}{hyp}$$

$$\sin x = \frac{y}{7}$$

$$y = 7\sin x \qquad \therefore AE = 7 + y$$

$$= 7 + 7\sin x.$$



using SOHCAHTOA to obtain expression for final answer.

$$\cos x = \frac{adj}{hyp}$$

$$\cos x = \frac{7 + 7\sin x}{10}$$

$$10\cos x = 7 + 7\sin x$$

Rearranging to match question answer

$$10\cos x - 7\sin x = 7$$

7.

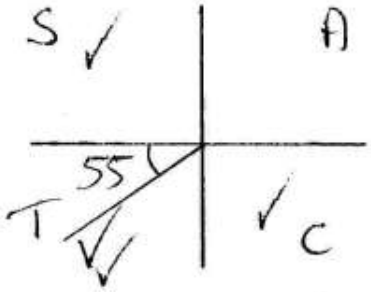
Unit 3 – Wave Function!

(a) $10\cos x - 7\sin x = k\sin(x - a)$

$$\boxed{\begin{aligned} k\sin(x-a) &= k\sin x\cos a - k\sin a\cos x \\ k\cos a &= -7 \\ k\sin a &= -10 \end{aligned}}$$

$$k = \sqrt{a^2 + b^2}$$
$$= \sqrt{10^2 + (-7)^2}$$
$$= \sqrt{149}$$

$$\tan\alpha = \frac{a}{b}$$
$$= \frac{-10}{-7} = \frac{10}{7} \quad \text{acute angle} = 55^\circ$$



$$a = 235^\circ$$

$$\therefore \quad 10\cos x - 7\sin x = k\sin(x - a)$$
$$= \sqrt{149}\,(x - 235)$$

$$10\cos x - 7\sin x = \sqrt{149}\left(x - \frac{235 \times \pi}{180}\right)$$
$$= \sqrt{149}\left(x - \frac{47\pi}{36}\right)$$

Note a fraction that is "awkward" should be written as a decimal ie replace $\pi = 3.14$

$$10\cos x - 7\sin x = \sqrt{149}\,(x - 4.1)$$

(b) Solve $10\cos x - 7\sin x = 7$

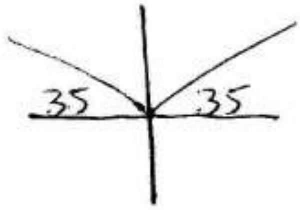
$$\sqrt{149}\,\sin(x - 235) = 7$$

$\left(\begin{array}{c}\text{SIMPLE}\\ \downarrow\\ \text{CAST}\end{array}\right)$

$$\sin(x - 235) = \frac{7}{\sqrt{149}}$$

$$\sin(x - 235) = \frac{7}{\sqrt{149}}$$

acute angle $= 35°$



$x - 235 = 35$

$x = 270°$

$x - 235 = 145$

$x = 380$

(Remember you can add or subtract the period for more answers)

$x = 380 - 360$

$= 20°$.

$\therefore\ x = 20°$ or $270°$

$$= \frac{20 \times \pi}{180} \text{ or } \frac{270 \times \pi}{180}$$

$$= \frac{\pi}{9} \text{ or } \frac{3\pi}{2}$$

8.

(a) $C_0 = 200$ and $B_0 = 0$

After 1 day

$C_1 = 0.1B_0 + 100 = 0.1 \times 0 + 100 = 100$

$B_1 = 2C_0 - 100 = 2 \times 200 - 100 = 300$

After 2 days

$C_2 = 0.1B_1 + 100 = 0.1 \times 300 + 100 = 130$

$B_2 = 2C_1 - 100 = 2 \times 100 - 100 = 100$

After 3 days

$C_3 = 0.1B_2 + 100 = 0.1 \times 100 + 100 = 110$

$B_3 = 2C_2 - 100 = 2 \times 130 - 100 = 160$

After 4 days

$C_4 = 0.1B_3 + 100 = 0.1 \times 160 + 100 = 116$

$B_4 = 2C_3 - 100 = 2 \times 110 - 100 = 120$

After 5 days

$C_5 = 0.1B_4 + 100 = 0.1 \times 120 + 100 = 112$

$B_5 = 2C_4 - 100 = 2 \times 116 - 100 = 132$

(b) C_n to B_{n-1}

Note	M_{n+1} to M_n V_n to V_n C_{n+1} to C_n B_n to B_{n-1}	} All mean next term on LHS. Doesn't always have to be written as M_{n+1} to M_n

$\therefore$ If $C_{n+1} = 0.1B_n + 100$

then $\underline{\underline{C_n = 0.1B_{n-1} + 100}}$ is the same.

(c) B_{n+1} to B_{n-1} means

B_3 to B_1
or
B_4 to B_2
etc.

$$C_2 = 0.1B_1 + 100$$
$$B_3 = 2C_2 - 100$$
$$\therefore \; B_3 = 2(0.1B_1 + 100) - 100$$

(Sub in $C_2 = 0.1B_1 + 100$)

$$= 0.2B_1 + 200 - 100$$
$$B_3 = 0.2B_1 + 100$$
$$\therefore \; B_{n+1} = 0.2B_{n-1} + 100$$

(d) LIMIT since $-1 < 0.2 < 1$

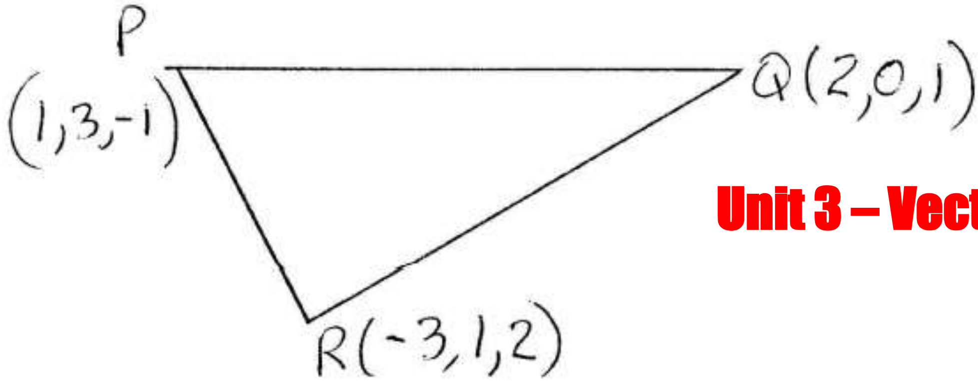
$$L_B = 0.2L + 100$$
$$0.8L_B = 100$$
$$L_B = \frac{100}{0.8} = \frac{1000}{8} = \underline{\underline{125}}$$

$$\therefore \; L_C = 0.1L_B + 100 = 0.1 \times 125 + 100$$
$$= \underline{\underline{112.5}}$$

$\therefore$ 125 Birds to 112 caterpillars on any particular day.

9.

(a)



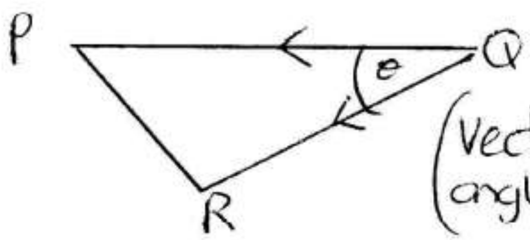
$$\overrightarrow{QP} = p - q = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$

$$|\overrightarrow{QP}| = \sqrt{(-1)^2 + (3)^2 + (2)^2} = \underline{\sqrt{14}}$$

$$\overrightarrow{QR} = r - q = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$$

$$|\overrightarrow{QR}| = \sqrt{(-5)^2 + (1)^2 + (1)^2} = \underline{\sqrt{27}}$$

(b)



(Vectors always away from angle being calculated)

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$= \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{|\overrightarrow{QP}||\overrightarrow{QR}|}$$

$$\boxed{\overrightarrow{QP} \cdot \overrightarrow{QR} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix} = 5 + 3 - 2 = \underline{6}}$$

$$\therefore \cos\theta = \frac{6}{(\sqrt{14}\sqrt{27})} = \frac{6}{\sqrt{378}}$$

$$\therefore \theta = 72^\circ$$

10.

Unit 3 – Logs!

10 (a) $f(t) = 1 - e^{-0.0072t}$

For $t = 60$

$f(60) = 1 - e^{-0.0072 \times 60}$

$= 1 - e^{-0.432}$

$= 1 - 0.649209$

$= 0.3508$

$\therefore$ Since $f(t)$ is the fraction of the remaining then

Amount remaining $= 0.3508 \times 750$

$= 263$ micrograms. (to the nearest microgram)

(b) $0.8 = 1 - e^{-0.0072t}$

$e^{-0.0072t} = 0.2$

$\log_e e^{-0.0072t} = \log_e 0.2$

$\log_e e = 1$

$-0.0072t \log_e e = \log_e 0.2$

$-0.0072t = \log_e 0.2$

$t = \dfrac{\log_e 0.2}{-0.0072}$

$= 223.5$ years