TEEJAY PUBLISHERS

Level F
Textbook

Produced by members of the TeeJay Writing Group

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PUPIL BOOK
F

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First Edition published by TeeJay Publishers - April 2004

Level F Textbook

The book can be used in both Primary and Secondary with pupils who have gained a Level E.

- In secondary schools, it can be used to condense the S1/2 course into a **ONE** year Level F course for those pupils who had already gained a National Test level E in Primary or early Secondary. It can also be used as a follow on from our Level E Textbook if this has been used to take pupils through Level E successfully in S1.

    - It should prepare pupils to sit maths Level F national test, **or equivalent**, approximately 1 year after gaining a Level E pass.
    - There are no A and B exercises. It basically covers the entire Level F course without the teacher having to pick and choose which questions to leave out and which exercises are important. They all are !
    - It covers the important work of Level F in ONE textbook.
    - It should prove to be an invaluable aid to the "fast tracking" of pupils in S1/2 and allow them to begin their Credit or Intermediate 2 course at the beginning of S2 or at the latest by Christmas time of S2.
    - It contains an 8 page "Chapter (–1)" which primarily revises every topic at level E and can be used as a diagnostic tool. This could be followed by a diagnostic assessment of the work of Level E.
    - It is then followed by 17 chapters, **ending with Chapter 59 !**
    - Non-calculator skills will be emphasised and encouraged throughout the book.
    - Each topic will have a "Topic in a Nutshell" exercise as a summary.
    - Homework will be available as a photocopiable pack along with an Assessment pack which can be used topic by topic or combined to form a series of Level F cumulative Tests.

Pupils should then be able to complete their Credit or Intermediate 2 course leisurely by the end of S3 or early in S4. This could allow Unit 1 of Higher Maths to be tackled and assessed before beginning the revision for their Credit or Intermediate 2 May examination.

This might also help eradicate the two term dash needed to complete the Higher course in S5.

Tom Strang and Jim Geddes

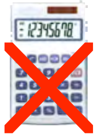
(*April 2004*)

Contents

Chapter –1

Level E Consolidation

The following questions (pages 1 - 8) cover every topic from Level E.
(No calculator unless stated)



1. Do the following mentally :-

(a) 52×100 (b) 203×30 (c) $6200 \div 10$ (d) 43×300

(e) $168000 \div 800$ (f) $10 \times 3{\cdot}27$ (g) $15{\cdot}3 \div 10$ (h) $100 \times 0{\cdot}216$

(i) $4{\cdot}3 \div 100$ (j) $8{\cdot}7 + 3{\cdot}4$ (k) $9 - 3{\cdot}6$ (l) $1 - 0{\cdot}07$

2. Set down and find the following :-

(a) $\begin{array}{r} 19{\cdot}8 \\ +\ 37{\cdot}75 \\ \hline \end{array}$ (b) $\begin{array}{r} 40{\cdot}2 \\ -\ 13{\cdot}87 \\ \hline \end{array}$ (c) $\begin{array}{r} 14{\cdot}63 \\ \times\ 8 \\ \hline \end{array}$ (d) $7 \overline{)\,12{\cdot}88}$

3. Write as a decimal :- (a) $\frac{3}{10}$ = ... (b) $\frac{3}{4}$ = ... (c) $\frac{2}{5}$ = ...

4. What does the **4** represent in :- (a) 5·451 (b) 0·042 (c) 10·034 ?

5. Share £87·52 equally amongst 8 people.

6. Round to 1 decimal place :-

(a) 19·68 (b) 0·273 (c) 0·7499 (d) 10·96.

7. £1 = 1·65 American dollars. How many dollars would I receive for £6 ?

8. If £5 = 7·50 euros, how many euros will I receive for £3 ?

9. Two blocks of cheese, each 1·7 kilograms in weight, are cut from a 5 kilogram round of cheese. What weight of cheese is left ?

10. Simplify these fractions as far as possible :-

(a) $\frac{12}{18}$ (b) $\frac{20}{45}$ (c) $\frac{16}{48}$ (d) $\frac{36}{84}$.

11. Find the following :-

(a) $\frac{2}{3}$ of 60 (b) $\frac{3}{4}$ of £120 (c) $\frac{4}{5}$ of 35 kg (d) $\frac{3}{10}$ of 1200.

12. Of the 180 adults living in Stewart Street, $\frac{8}{9}$ voted at the last election.
How many of the adults did <u>**not**</u> vote ?

13. (a) 50% of £120 (b) 25% of £800 (c) 10% of £60 (d) $33\frac{1}{3}$% of 75p
(e) 20% of £60 (f) 30% of £90 (g) 1% of 2600 (h) 75% of 24 kg

14. Simplify these ratios :-

(a) 5 : 10 (b) 18 : 24 (c) 27 : 45 (d) 108 : 63.

15. The ratio of C.D.'s : Tapes in a boy's collection is 24 : 18.

Simplify this ratio as far as possible.

16. 

The ratio of broken legs : broken arms in the Accident & Emergency unit last week was 2 : 5.

If there were 12 broken legs, how many broken arms were there ?

17. Fill in the next 3 terms in each of these patterns of numbers :-

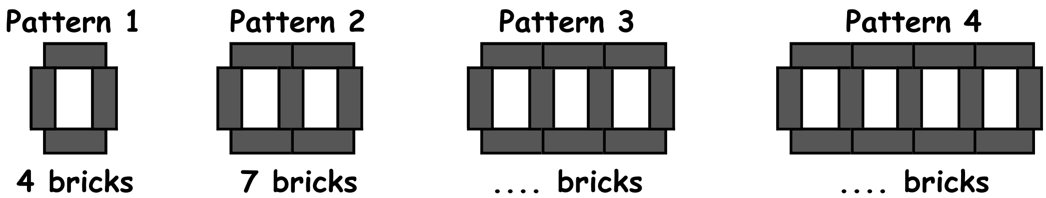
(a) 3, 7, 11, 15, ... (b) 70, 64, 58, 52, ...

(c) 9, 16, 25, 36, 49 (d) 1, 3, 6, 10, 15, ...

18. Find the 20th number in the pattern :- 2, 5, 8, 11, 14, ...

19. List all the Prime Numbers less than 40.

20. A boy makes patterns with wooden bricks.

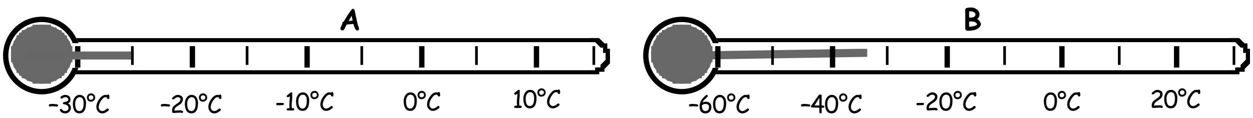


(a) How many bricks are needed for each of the pattern numbers 3, 4 and 5 ?

(b) Describe in words (or symbols) a formula which will allow you to calculate the number of bricks needed, once you are given the pattern number.

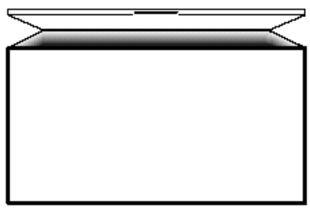
(c) How many bricks are needed for pattern number 100 ?

21. What temperature is represented on these thermometers ?



22. The temperature one day rose from –15°C to 12°C.

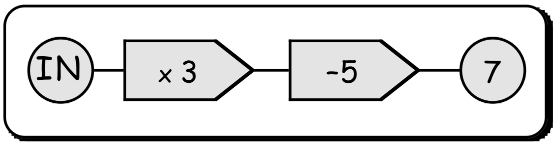
By how much had it risen ?



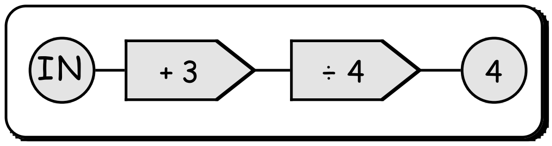
23. When a freezer was switched on, its temperature dropped by 25°C.
If its temperature began at 7°C, what was the final temperature ?

24. What numbers must have gone “IN” the following number machines :-

(a)



(b)



25. Solve the following equations for x :-

(a) $x - 5 = 12$ (b) $x + 7 = 11$ (c) $4x = 18$ (d) $2x = 17$

(e) $3x + 1 = 19$ (f) $4x - 5 = 15$ (g) $5x + 5 = 5$ (h) $6x - 2 = 13$.

26. Choose all the numbers from this list which make these inequalities true :- $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(a) $x < 5$ (b) $x \geq 3$ (c) $x - 2 > 7$ (d) $x + 6 < 8$.

27. The area of a boy’s bedroom is known to be one of the following :-

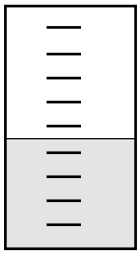
$2\ m^2$, $12\ m^2$, $60\ m^2$, $100\ m^2$

Which is it most likely to be ?

28. Measure the length of this line in millimetres.

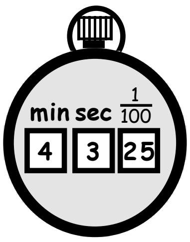
29. How many kilograms are there in 3 tonnes ?

30. Estimate the volume of liquid in this container. (*in millilitres*)

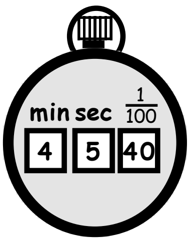


Capacity = 1 litre

31.



winner



last horse

Shown are the times of the winning horse and the last horse in a race.

By how much did the first horse beat the last horse ?

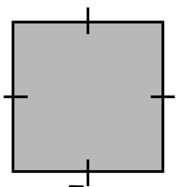
32. Calculate the area of the following shapes :-

(a)

5 cm

12cm

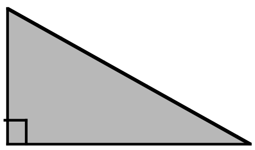
(b)



7 cm

(c)

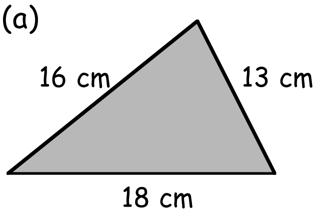
6 cm



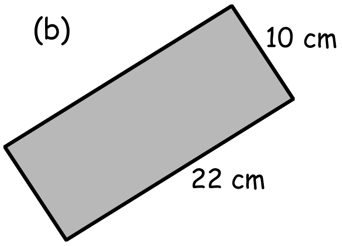
10 cm

33. Calculate the perimeters of the following shapes :-

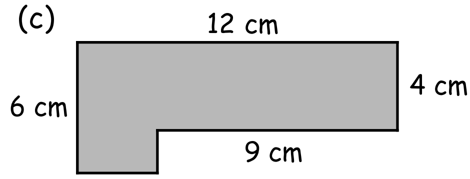
(a)



(b)

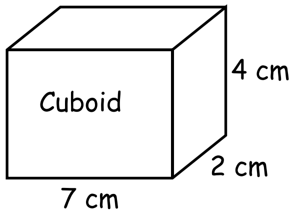


(c)

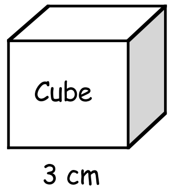


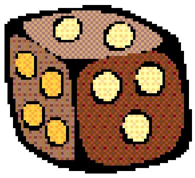
34. Calculate the volumes of these shapes (in cm^3) :-

(a)

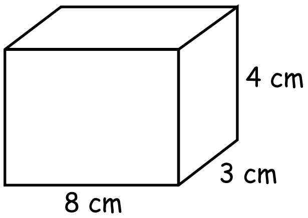


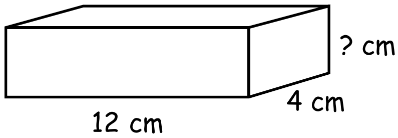
(b)





35. The following 2 cuboids have the **SAME** volume.

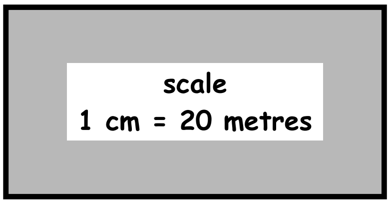




Calculate the height of the 2nd cuboid.

36. The scale of this drawing of a rectangular field is 1 cm to 20 metres.

you will require a ruler here



(a) Measure the length of the field in this drawing.

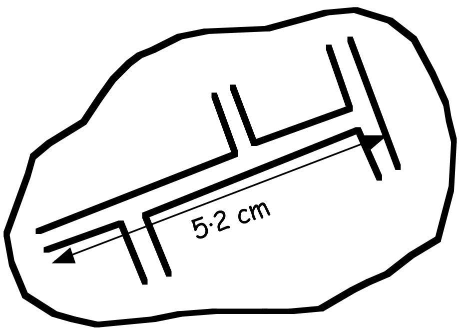
(b) Use the scale to determine the **real length** of the field in metres.

(c) Calculate the real **perimeter** of the field.

37. The scale on a map is 1 : 10 000.

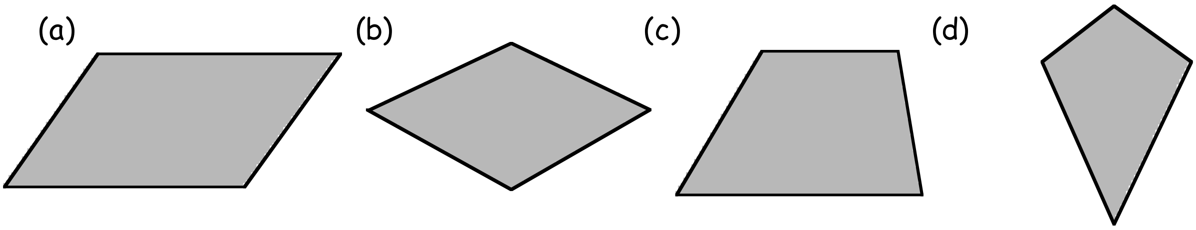
A street on the map is 5·2 cm long.

Calculate the real length of the street in metres.



38. Name these quadrilaterals :-

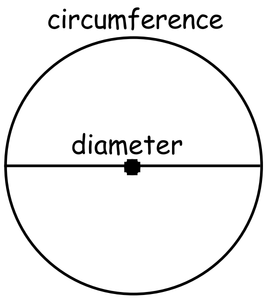
(a) (b) (c) (d)



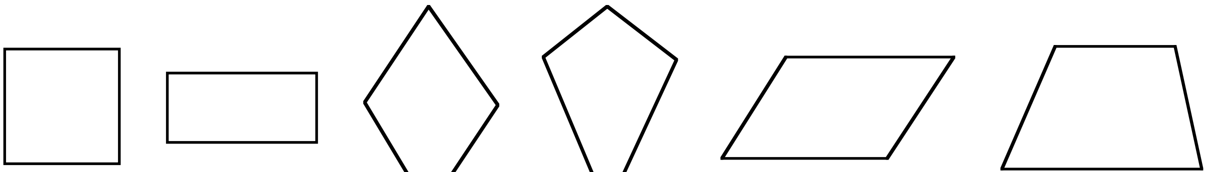
39. It is possible to **approximately** calculate the circumference of a circle if you know its diameter.

You simply multiply the diameter by :- {2, 3, 4, 5 or 10}

Which one ?



40. From the list of quadrilaterals shown below, answer the following questions :-



Square Rectangle Rhombus Kite Parallelogram Trapezium

(a) Which of the above have all 4 sides the same length ?

(b) Which of them have **exactly** 2 lines of symmetry ?

(c) Which of them do **NOT** have $\frac{1}{2}$ turn symmetry ?

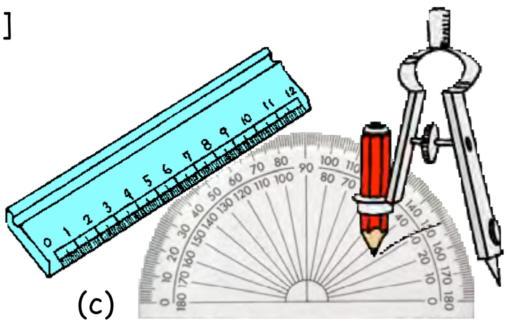
(d) Which of them have their diagonals the same length ?

[Note :- you may like to sketch them if it helps]

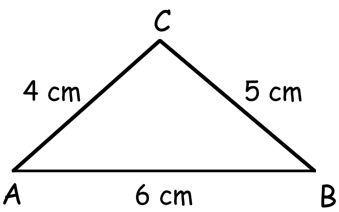
41. (You will need **compasses**, a **protractor** and a **ruler**)

Shown below are sketches of 3 triangles.

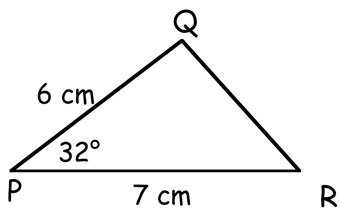
Show how to construct (draw) them accurately.



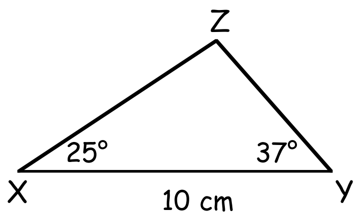
(a)



(b)



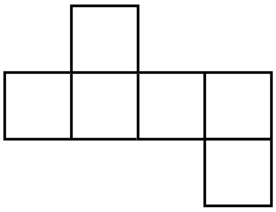
(c)



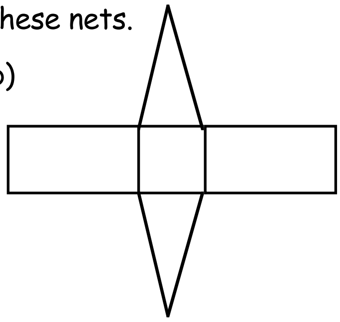
42. Shown are nets of 3 solid shapes.

Name the solid shapes formed from these nets.

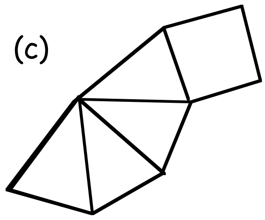
(a)



(b)

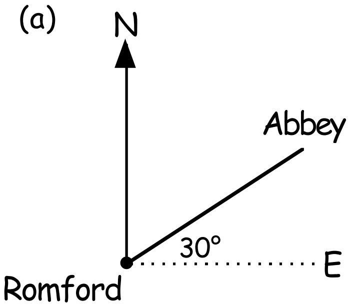


(c)

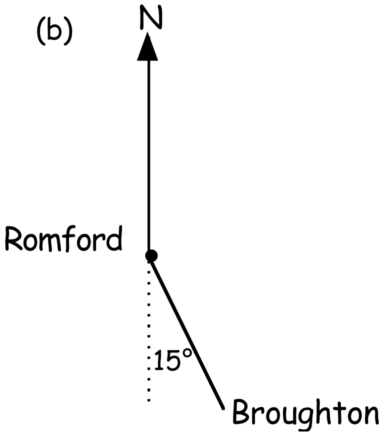


43. Write down the **3 figure bearings** of each town from Romford.

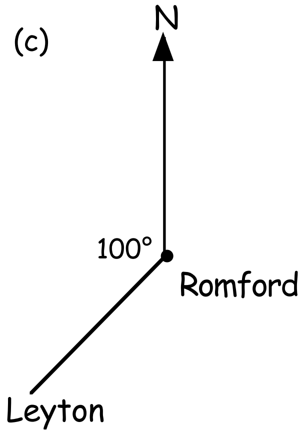
(a)



(b)



(c)

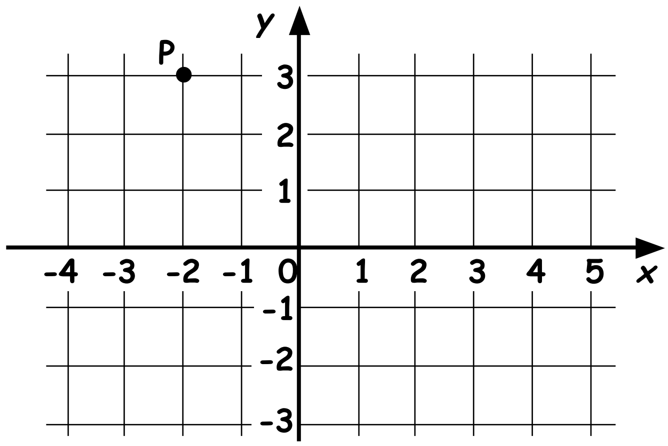


44. (a) Write down the coordinates of point P.

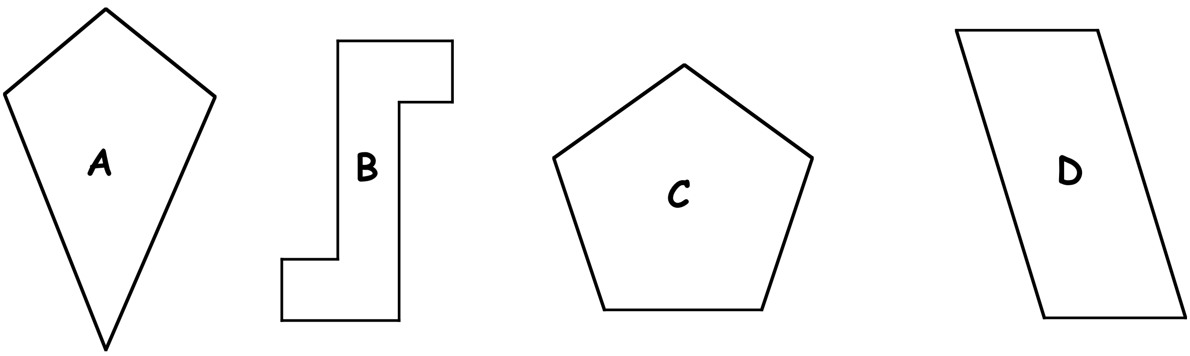
(b) Copy this diagram and plot the points :–

Q(5, -2) and R(-2, -1).

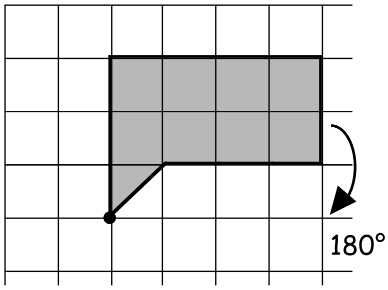
(c) Write down the distance, (in boxes), from P to R.



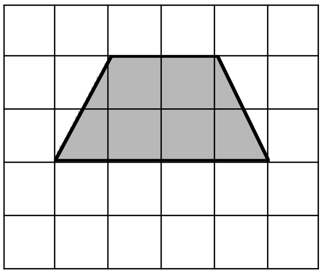
45. Which of the following shapes have **rotational** symmetry ?



46. Rotate this shape by 180° around the "dot".



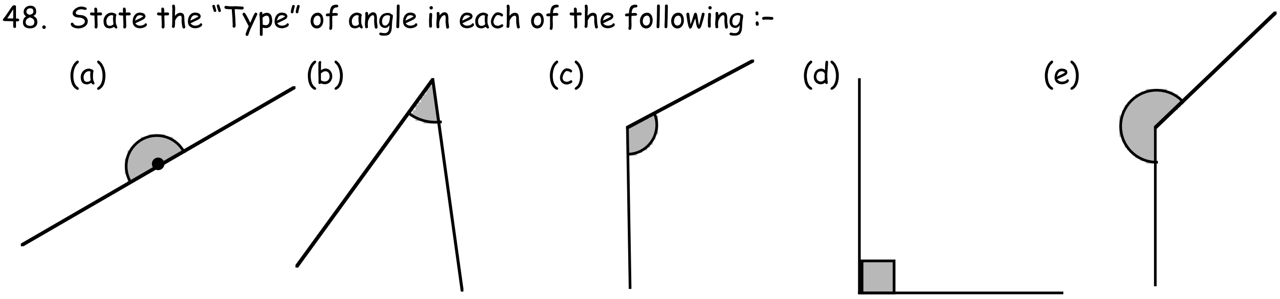
47.



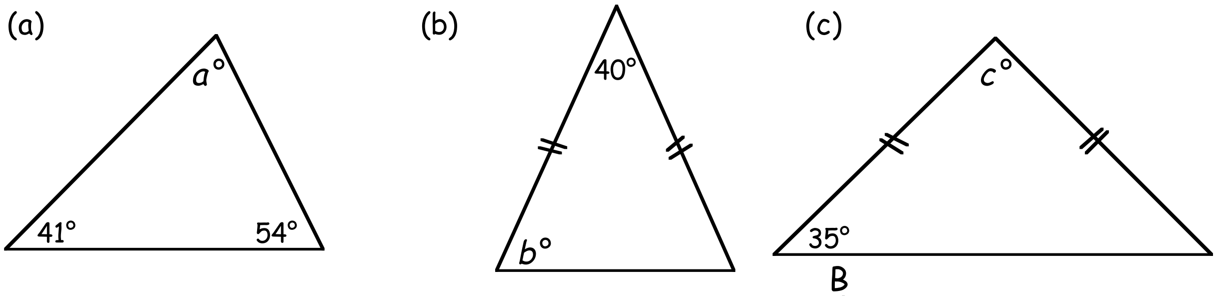
Make a neat copy of this shape on squared paper.

Show how to cover the surface with tiles congruent to this one.

48. State the "Type" of angle in each of the following :–

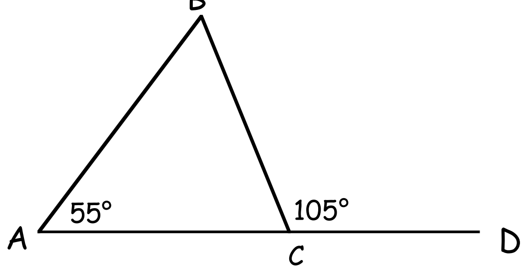


49. **Calculate** the size of the angles marked *a*, *b* and *c* :–

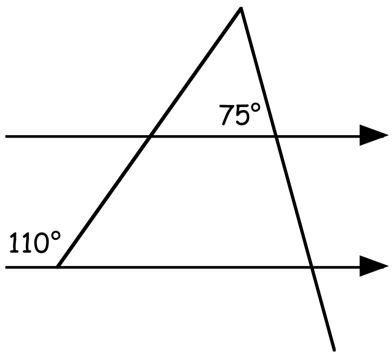


50. Make a sketch of this shape and fill in the sizes of **ALL** the angles.

(*Do not measure them*)



51.



Make a sketch of this shape and fill in the sizes of **ALL** the angles.

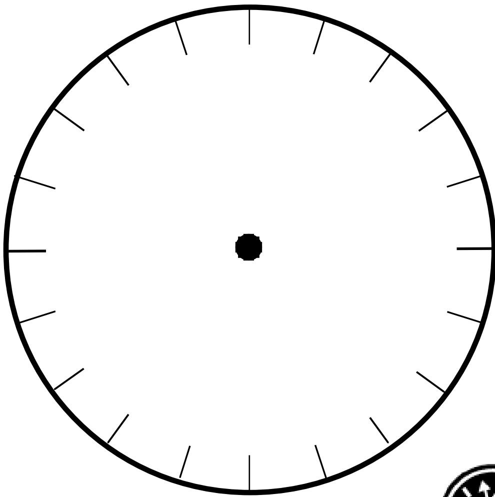
(*Do not measure them*)

52. Draw a neat LABELLED bar graph to show this information about the make of digital cameras owned by a group of tourists.

Make	Olympus	Cannon	Fuji	Kodak	Sony
Number	10	14	6	9	3

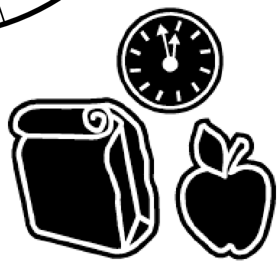
53. The table below shows what a group of 2nd year pupils do for lunch.

School Lunch	45%
Packed Lunch	10%
Go Home	20%
Van/Shops	.....%



(a) What percentage of the pupils go to the van or shops for their lunch ?

(b) Copy or trace this blank pie chart, complete and label it to show the information from the table.



54. 5 women discuss how much money each of them spent when buying their last pair of shoes.

Ruth – £70, Joan – £35, Alice – £45, Mary – £50, Kim – £100

Calculate the **mean** amount each woman paid.

55. The temperature was recorded every 2 hours in a living room.

(a) Describe the general trend of the graph.

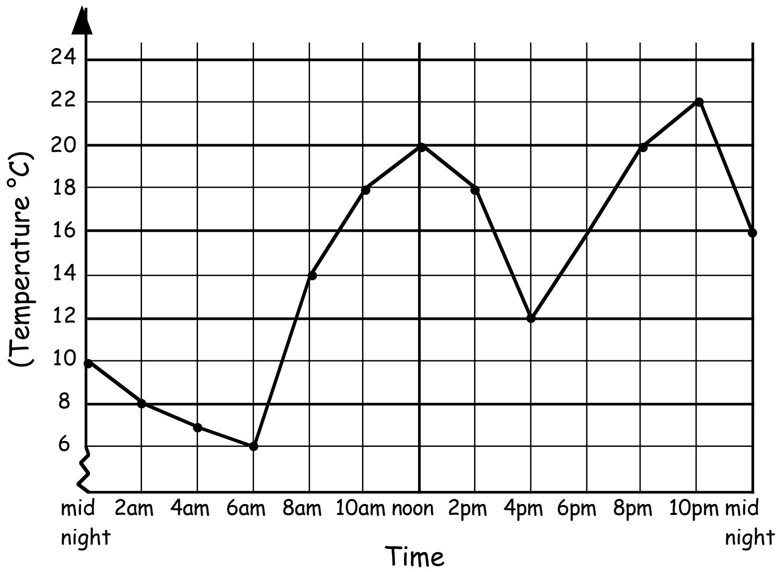
(b) What was the :–

(i) maximum temp ?

(ii) minimum temp ?

(c) The central heating is switched on twice daily.

At what times (approx) ?



Chapter 2

Integers

Integers

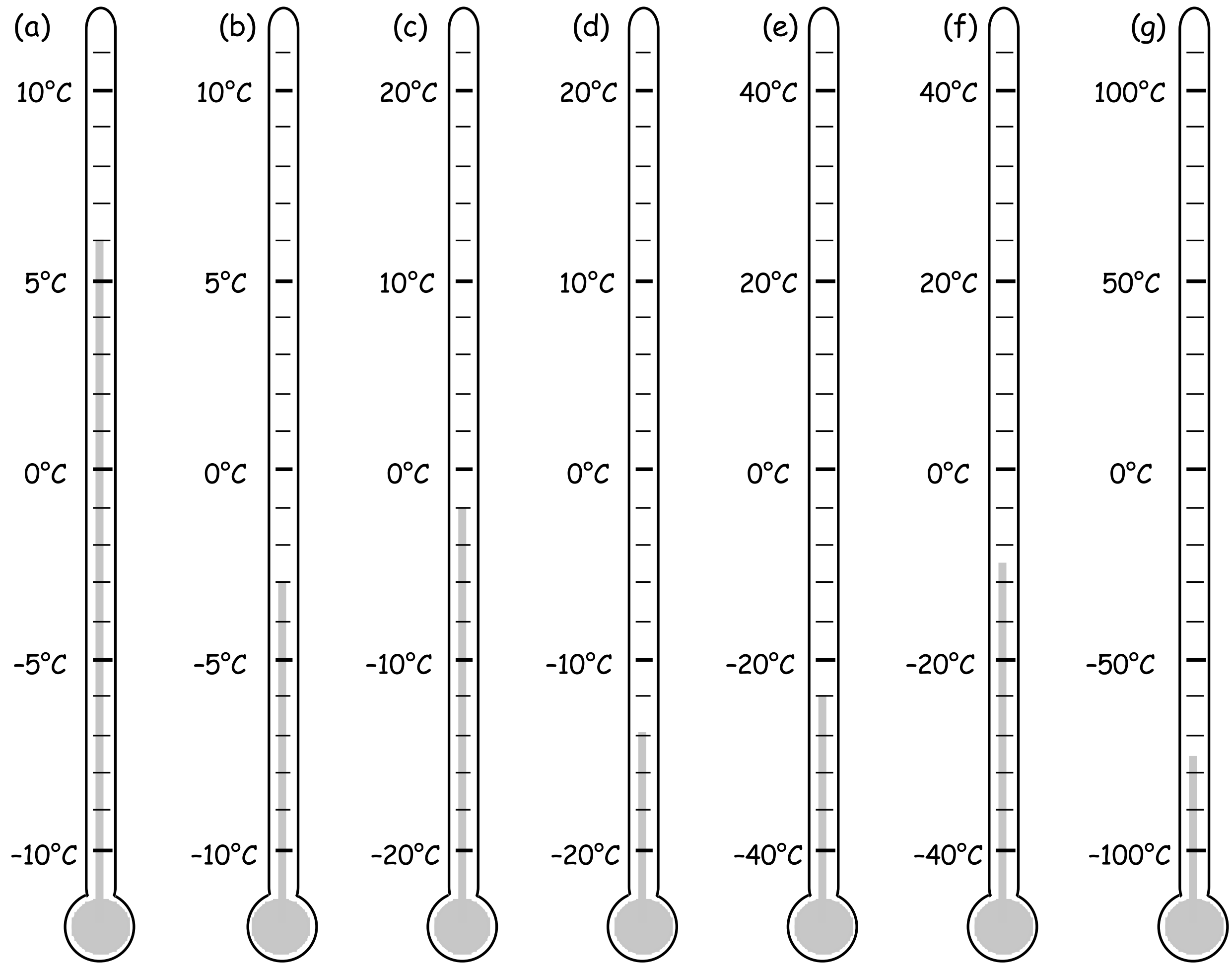
The **Positive** and **Negative** whole numbers, along with **Zero** are called the set of **INTEGERS.**

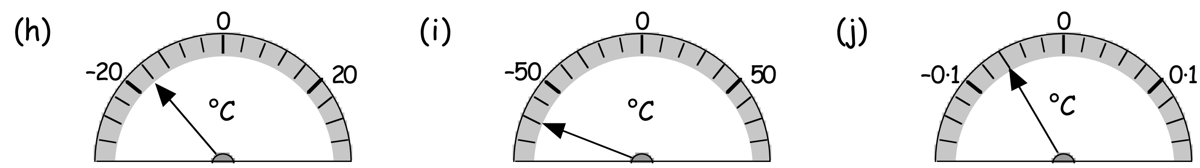
Examples of Integers :- –5, –37, 11, 45, 0, –13, 2000, –5014.

4·5, $\frac{1}{4}$, –3·7, $-2\frac{1}{3}$, -29·45, etc are **NOT** integers.

Exercise 1 **(Revision Work) – to be done orally !!**

1. What temperatures are shown on the following thermometers ?





2. Banks deal with positive and negative values of money.

If you have £70 in your bank account, the computer records this as

+ £70·00

If you are "overdrawn" by £70, the computer records this as

– £70·00.

(a) Describe, in words, what each of these bank balances mean :-

(i) **Dave Smith**
15/09/02
balance **+ £39·50**

(ii) **Karly Davis**
22/11/03
balance **– £22·00**

(iii) **Angie Douglas**
07/02/04
balance **+ £119·75**

(iv) **Rob Lowe**
13/03/03
balance **£0·00**

(b) Billy had £25·00 in his bank account and withdrew £30·00.

What will his balance now show on the computer ?

(c) Lena's bank balance is shown opposite.
She pays £5 into her account.

balance – £18·00

What will her new balance be ?

(d) The Wilson's bank balance was £0·00.
Mr Wilson withdrew £45.

What will their new balance show as ?

(e) Last week my bank statement showed a balance of – £25·00. (I was in the "red").
I withdrew a further £15.

What was my new balance ?

(f) If my bank statement said my balance was (– £85), how much must I deposit to "clear my overdraft" ?

(g) Karen's bank balance showed + £35·00.
She signed a cheque for £25·00 and another cheque for £16·00.

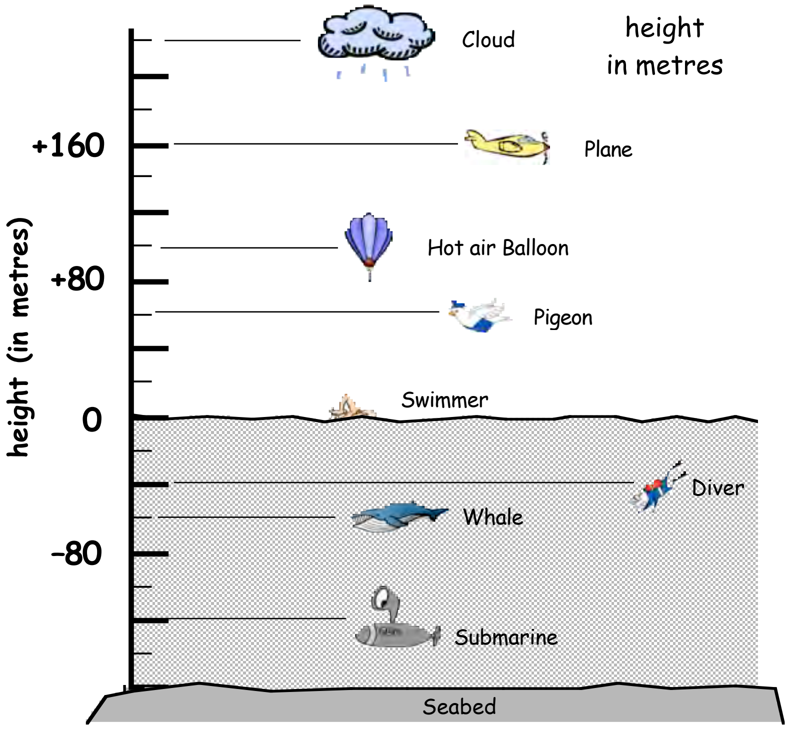
What will her new balance now show ?

3. We can use positive and negative numbers to describe heights above or below sea-level.

Heights **ABOVE** sea level are ***positive*** (+)

Heights **BELOW** sea level are ***negative*** (–)

(a) Use "+" or "–" to describe the heights (depths) of the following :-



(i) the pigeon
(ii) the plane
(iii) the swimmer
(iv) the hot air balloon
(v) the seabed
(vi) the whale
(vii) the diver
(viii) the submarine
(ix) the cloud

(b) How high is the pigeon above the whale ?

4. We can also use negative numbers in the context of **TIME**.

Christians say we live in the year 2004 A.D. (anno domini)

This means <u>2004 years since the birth of Christ</u>. (or **+2004**)

If a man was born in the year 80 B.C. (before Christ),
we say he was born in the year (– **80**).

(a) Use "+" or "–" signs to describe the following dates :-

(i) 1527 A.D. (ii) 655 A.D. (iii) 35 B.C. (iv) 850 B.C.

(b) Julius was born in 69 A.D. and died in 117 A.D.
How old was Julius when he died ?

(c) Marcus Antonius was born in 96 B.C. and died in 46 B.C.
How old was Marcus when he died ?

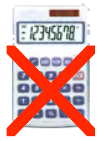
(d) Lucretia was born in 22 B.C. and died in 45 A.D.
How old was she on her death ?

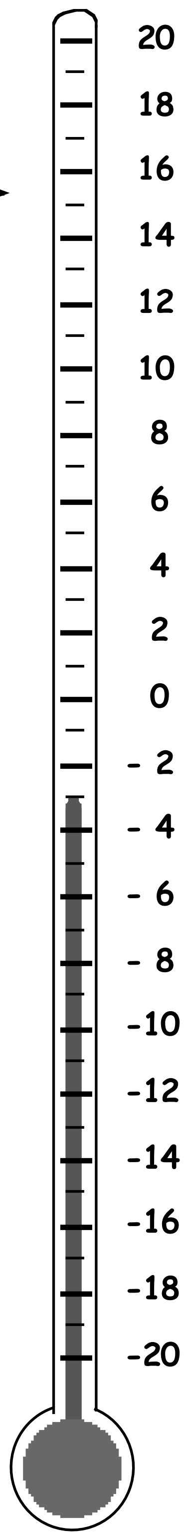
(e) Bilikus was born in 35 B.C. and lived to the ripe old age of 70.
In which year did Bilikus die ?

(f) Tomaticus lived to the age of 65. He died in the year 8 B.C.
In which year was Tomaticus born ?

Integer Calculations

A thermometer is an extremely useful way of looking at integers and it can also be useful in helping with integer calculations.





Exercise 2 (no calculator)

1. Copy this thermometer neatly into your jotter using a ruler.

2. Use the thermometer to help you do the following :-

   Find the temperature that is :-

   (a) 7°C <u>up</u> from 12°C (b) 22°C up from 0°C
   (c) 15°C up from 13°C (d) 11°C <u>down</u> from 23°C
   (e) 16°C down from 16°C (f) 7°C up from -5°C
   (g) 10°C down from -2°C (h) 15°C up from -11°C
   (i) 8°C down from 3°C (j) 37°C down from 0°C
   (k) 12°C down from -8°C (l) 30°C down from -20°C
   (m) 9°C up from -16°C (n) 35°C up from -40°C.

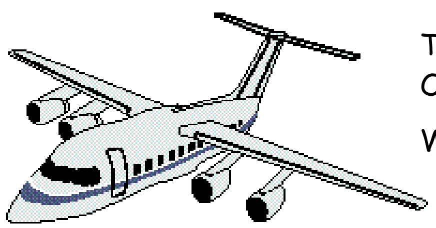
3. Look carefully at the thermometer.
   Can you see that 6°C is "**<u>8°C up from</u>**" -2°C ?

   Do the same for each of these :-

   (a) 25°C is .... from 20°C (b) 4°C is .... from 16°C
   (c) 0°C is .... from 80°C (d) 10°C is .... from -5°C
   (e) -13°C is .... from 0°C (f) 6°C is .... from -8°C
   (g) -25°C is .... from -5°C (h) -12°C is .... from 12°C
   (i) 50°C is .... from -50°C (j) -65°C is .... from -45°C

4. The temperature in the fridge is -5°C.
   The freezer was set to be 13°C colder.

   What was the temperature in the freezer ?

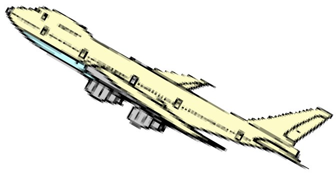
5. 

   The temperature inside the jet plane was 22°C.
   Outside, the temperature was -54°C.

   What was the difference in the two temperatures ?

6. When Borsi travelled from Moscow to Stalingrad to play in the chess tournament, the temperature rose from –31°C to –18°C.

By how much had the temperature risen ?

7. 

As the plane rose from the airport into the night sky, the outside temperature fell by a **steady amount** every 1000 metres.

At ground level, the temperature was 20°C and it fell by 8°C for every 1000 metres ascent.

What would the temperature be at :-

(a) 1000 metres (b) 3000 metres

(c) 5000 metres (d) 10 000 metres ?

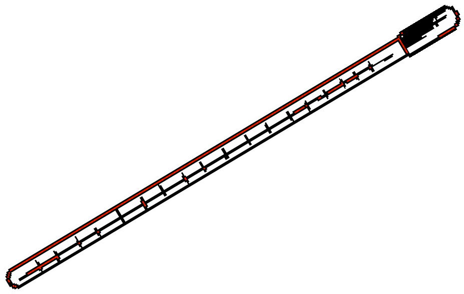
8. A chemical is stored at a temperature of –200°C.

When it is exposed to the air, it rises in temperature at the rate of 30°C per minute.

What will the chemical's temperature be after :-

(a) 2 mins (b) 5 mins

(c) 7 mins (d) $6\frac{2}{3}$ mins ?



9. The temperature was taken on the 1st of the month, every month, in a storeroom in a factory.

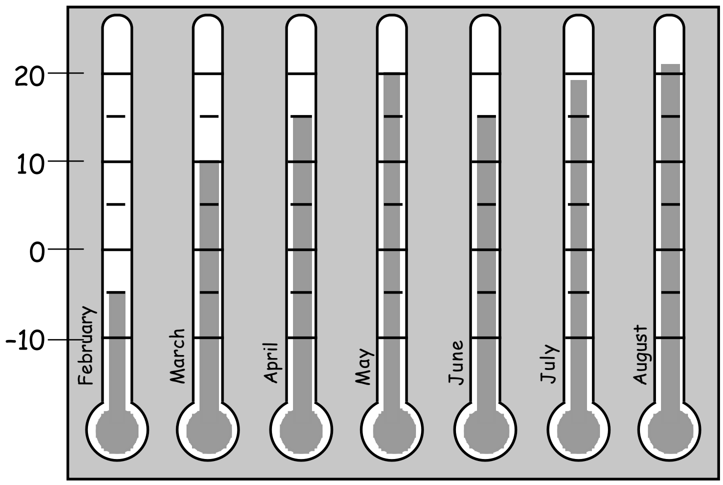
This is shown opposite.

(a) What was the temp. on 1st Feb ?

(b) What was the temp. on 1st May ?

(c) Between which 2 months was there the biggest rise in temperature ?

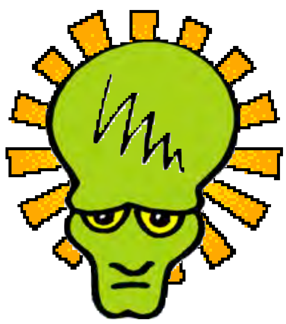
(d) From August to September, the temperature had dropped by 22°C. What was the temperature on 1st September ?



10. When "Captain Icicle" enters a room, the temperature drops by 125°C.

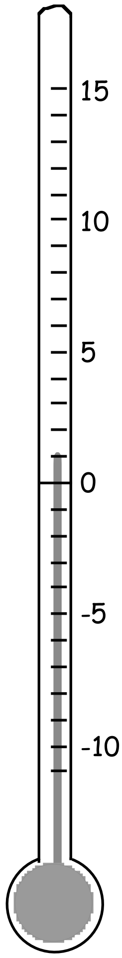
The temperature in the Daily Planet newspaper office was 25°C when Captain Icicle breezed in.

To what temperature did the room drop ?



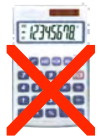
Adding and Subtracting Integers

When adding and subtracting integers, consider a thermometer and use the following two step method :-



Example 1	To find (-4) + 7 =>	• picture the **first** number (-4) • then move (**up**) by 7 => 3
Example 2	To find 2 - 9 =>	• picture the **first** number (2) • then move (**down**) by 9 => -7
Example 3	To find 3 + (-8) =>	• picture the **first** number (3) • then move (**down**) by 8 => -5
Example 4	To find (-2) + (-5) =>	• picture the **first** number (-2) • then move (**down**) by 5 => -7

Exercise 3 (no calculator)



1. Draw a thermometer to help you here.

   Write down each question first, then the answer :-

(a) 6 + 9	(b) 2 + 11	(c) 0 + 23	(d) 10 + (-7)
(e) 8 + (-2)	(f) 7 + (-7)	(g) 2 + (-6)	(h) 3 + (-13)
(i) 0 + (-20)	(j) (-5) + 11	(k) (-6) + 6	(l) (-3) + 15
(m) (-9) + 5	(n) (-11) + 4	(o) 1 + (-17)	(p) (-8) + (-5)
(q) (-9) + (-9)	(r) (-13) + (-17)	(s) (-15) + 7	(t) (-21) + (-19)
(u) (-80) + 60	(v) (-35) + (-55)	(w) 10 + (-45)	(x) (-3·6) + (-2·4)

2. Again use your thermometer to help here :-

   (remember :- 2 - 7 means "go to 2, then move <u>down</u> by 7" => -5).

(a) 9 - 3	(b) 10 - 10	(c) 4 - 1	(d) 3 - 5
(e) 5 - 10	(f) 2 - 12	(g) 0 - 15	(h) (-1) - 4
(i) (-7) - 3	(j) (-11) - 5	(k) (-1) - 21	(l) 0 - 35
(m) 19 - 39	(n) (-15) - 25	(o) 100 - 300	(p) (-71) - 29
(q) 0 - 22	(r) (-10) - 10	(s) 6 - 22	(t) (-25) - 35
(u) (-1) - 1	(v) (-63) - 27	(w) (-13) - 13	(x) (-2·5) - 3·5

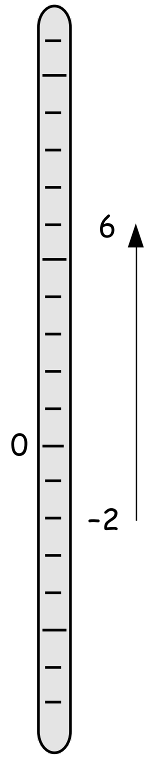
3. Here is a mixture. Remember the "two step" rule :-

step 1 :-	**"Picture the first number on your thermometer".**
step 2 :-	• If you add (a positive) number move **UP**.
	• If you add (a negative) number, or take away a number move **DOWN**.

(a) 2 + 8	(b) 3 + (-10)	(c) 1 - 11	(d) (-5) + 15
(e) -7 + (-8)	(f) 6 - 14	(g) (-5) - 7	(h) (-40) + (-60)
(i) (-20) + 35	(j) 0 - 27	(k) 0 + (-27)	(l) (-18) + (-12)
(m) 22 + (-15)	(n) (-10) + 3	(o) (-41) + 41	(p) 45 - 75
(q) (-27) + 14	(r) 0 + (-35)	(s) (-101) + 99	(t) 19 + (-21)

The Double Negative !

When asked "what is 6 - (-2)", the obvious answer is 4. But 4 is the wrong answer !



Think of a simpler question :-

7 - 2 means "how far is it from 2 up to 7 ?"

=> by counting, we can see the answer is 5.

This means that the question :-

6 - (-2) means "how far is it from (-2) up to 6 ?"

=> by counting, we see the answer is **8** (not 4).

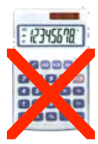
Notice that 6 + 2 also gives **8**

A Rule for **DOUBLE NEGATIVES** :- **"Two negatives make a positive"**

Examples

7 - (-5)	20 - (-10)	(-2) - (-5)
= 7 + 5	= 20 + 10	= -2 + 5
= 12	= 30	= 3

Exercise 4



1. Copy and complete the following :-

(a) 8 - (-3) = 8 + 3 = ...	(b) 11 - (-9) = 11 + 9 = ...
(c) 6 - (-5) = 6 + ... = ...	(d) 30 - (-20) = 30 + ... = ...
(e) 13 - (-6) = ... + ... = ...	(f) 4 - (-4) = ... + ... = ...

2. Show your steps in finding the following :-

(a) $6 - (-9)$ (b) $12 - (-13)$ (c) $0 - (-11)$ (d) $4 - (-16)$

(e) $15 - (-7)$ (f) $35 - (-15)$ (g) $7 - (-7)$ (h) $600 - (-400)$

(i) $23 - (-37)$ (j) $6{\cdot}5 - (-3{\cdot}5)$ (k) $2{\cdot}1 - (-3{\cdot}2)$ (l) $\frac{1}{2} - (-\frac{1}{2})$

3. Copy and complete :- (*Remember to use a thermometer scale if it helps*)

(a) $-4 - (-6) = -4 + 6 = ...$ (b) $(-2) - (-7) = -2 + 7 = ...$

(c) $(-10) - (-15) = -10 + ... = ...$ (d) $(-8) - (-12) = -8 + ... = ...$

(e) $(-40) - (-30) = ... + ... = ...$ (f) $(-5) - (-5) = ... + ... = ...$

4. Show your steps in finding the following :-

(a) $(-2) - (-6)$ (b) $(-3) - (-9)$ (c) $(-8) - (-11)$ (d) $(-9) - (-6)$

(e) $(-1) - (-2)$ (f) $(-13) - (-7)$ (g) $(-14) - (-14)$ (h) $(-50) - (-120)$

(i) $(-24) - (-4)$ (j) $(-2{\cdot}5) - (-4{\cdot}5)$ (k) $(-0{\cdot}9) - (-0{\cdot}4)$ (l) $(-\frac{1}{2}) - (-\frac{1}{2})$

5. The same idea works with algebraic expressions. Find :-

(a) $4x - (-3x)$ (b) $8x - (-10x)$ (c) $0 - (-5x)$ (d) $4a - (-9a)$

(e) $5p - (-8p)$ (f) $7w - (-13w)$ (g) $8g - (-12g)$ (h) $60f - (-20f)$

(i) $(-3m) - (-7m)$ (j) $(-9k) - (-4k)$ (k) $(-5n) - (-5n)$ (l) $(-b) - (-2b)$

(m) $(-6q) - (q)$ (n) $(-11z) - (-15z)$ (o) $(-6c) - (-12c)$ (p) $(-23g) - (-23g)$

6. A great big **MIXTURE**.

Find :-

(a) $(-3) + 8$ (b) $(-4) - 6$ (c) $2 - (-9)$ (d) $(-11) + 15$

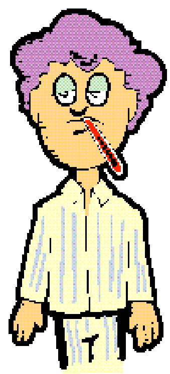
(e) $(-17) + 17$ (f) $8 - 22$ (g) $0 - 13$ (h) $(-7) + 17$

(i) $8 - (-12)$ (j) $(-3) - (-4)$ (k) $7 - (-7)$ (l) $(-22) + 42$

(m) $3x - (-4x)$ (n) $(-5p) + 11p$ (o) $10a - (-2a)$ (p) $(-3g) - 12g$

(q) $a - (-a)$ (r) $0 - (-5p)$ (s) $101 - (-99)$ (t) $65f - 95f$

(u) $2a^2 - 5a^2$ (v) $(-7t^2) + 15t^2$ (w) $(-1000) + 3000$ (x) $(-2\frac{1}{2}) - 3\frac{1}{2}$



Simple Multiplication and Division of Integers

Since $4 \times 6 = 24$, then obviously $4 \times (-6)$ cannot also be 24.

$4 \times (-6)$ means "4 lots of -6" = -24.

Examples $3 \times (-8) = -24$ $7 \times (-2) = -14$.

$(-9) \times 3 = -27$ $(-6) \times 6 = -36$.

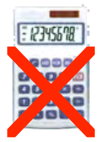
Similarly :- since $12 \div 3 = 4$, then obviously $(-12) \div 3$ cannot also be 4.

$(-12) \div 3$ = "-12 shared by 3" = -4.

Examples $(-20) \div 5 = -4$ $(-21) \div 7 = -3$.

$(-32) \div 8 = -4$ $(-45) \div 9 = -5$.

Exercise 5 (no calculator)



1. Write down each of the following and find the answers :-

(a) $4 \times (-5)$	(b) $6 \times (-7)$	(c) $2 \times (-9)$	(d) $5 \times (-5)$
(e) $(-8) \times 3$	(f) $(-9) \times 4$	(g) $(-11) \times 2$	(h) $(-10) \times 7$
(i) $6 \times (-8)$	(j) $8 \times (-3)$	(k) $4 \times (-12)$	(l) $7 \times (-7)$
(m) $9 \times (-1)$	(n) $(-9) \times 3$	(o) $(-2) \times 10$	(p) $(-9) \times 5$

2. Write down each of the following and find the answers :-

(a) $(-30) \div 6$	(b) $(-20) \div 5$	(c) $(-56) \div 7$	(d) $(-63) \div 9$
(e) $(-40) \div 2$	(f) $(-90) \div 10$	(g) $(-33) \div 3$	(h) $(-32) \div 4$
(i) $(-8) \div 8$	(j) $(-5) \div 1$	(k) $(-54) \div 6$	(l) $(-100) \div 5$

3. Find the answers to the following :-

(a) $(4 \times 9) \div 6$	(b) $(2 \times (-10)) \div 5$	(c) $3 \times (-2) \times 4$	(d) $5 \times (-1) \times 6$
(e) $3 \times (-8) \div 6$	(f) $(-6) \times 6 \div 4$	(g) $6 \times (-4) \div 2$	(h) $10 \times (-10) \div 5$

4. Find the following :- (*hint : find the bit in brackets first*)

(a) $(8 + (-5)) \times 7$	(b) $6 \times (4 - 7)$	(c) $((-10) + 2) \times 2$
(d) $((-4) - 8) \div 2$	(e) $10 \times (12 - 14)$	(f) $(8 - 3) \times (-5)$
(g) $((-3) - 4) \times 5$	(h) $(6 + (-12)) \div 3$	(i) $((-9) - 11) \div 5$

5. (a) What do you think the answer to $10 \div (-2)$ will be ? 5 or -5 ?

(b) If you think 5, check if $5 \times (-2)$ really takes you back to the original 10.

(c) If it doesn't, then the answer must be **-5** !

note :- $12 \div (-3) = -4$ (not 4) $28 \div (-4) = -7$ (not 7)

in other words, if you divide two integers, where one of them is positive and one of them is negative => the answer is always **negative**.

6. Write down each of the following and find the answers :-

(a) $20 \div (-5)$	(b) $24 \div (-6)$	(c) $18 \div (-9)$	(d) $25 \div (-5)$
(e) $36 \div (-4)$	(f) $40 \div (-8)$	(g) $7 \div (-1)$	(h) $42 \div (-3)$
(i) $96 \div (-8)$	(j) $100 \div (-5)$	(k) $120 \div (-6)$	(l) $49 \div (-7)$
(m) $1 \div (-1)$	(n) $7 \div (-2)$	(o) $30 \div (-4)$	(p) $3 \div (-6)$

the DOUBLE NEGATIVE again :-

=> remember :- since $4 \times (-3) = -12$ => $(-4) \times (-3)$ cannot also be -12 !

=> the only other possiblilty is that $(-4) \times (-3) = (+)12$

RULE 1:- "when **two negatives** are multiplied => the answer is **positive**"

Examples :- $(-5) \times (-6) = 30$ $(-3) \times (-8) = 24$ $(-8) \times (-10) = 80$

RULE 2:- "when **two negatives** are divided => the answer is **positive**"

Examples :- $(-21) \div (-3) = 7$ $(-32) \div (-8) = 4$ $(-48) \div (-6) = 8$

7. Write down each of the following and find the answers :-

(a) $(-4) \times (-3)$	(b) $(-5) \times (-2)$	(c) $(-7) \times (-9)$	(d) $(-8) \times (-4)$
(e) $(-7) \times (-8)$	(f) $(-8) \times (-8)$	(g) $(-1) \times (-14)$	(h) $(-10) \times (-9)$
(i) $(-5) \times (-5)$	(j) $(-20) \times (-3)$	(k) $(-4) \times (-50)$	(l) $(-400) \times (-10)$

8. Find the answers to the following :-

(a) $(-20) \div (-5)$	(b) $(-18) \div (-3)$	(c) $(-32) \div (-4)$	(d) $(-22) \div (-2)$
(e) $(-36) \div (-9)$	(f) $(-40) \div (-8)$	(g) $(-54) \div (-6)$	(h) $(-80) \div (-4)$
(i) $(-84) \div (-7)$	(j) $(-120) \div (-6)$	(k) $(-200) \div (-10)$	(l) $(-168) \div (-3)$

9.

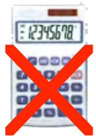
(a) $(4 \times (-9)) \div 6$	(b) $((-2) \times (-10)) \div 5$	(c) $3 \times (-2) \times (-4)$
(d) $3 \times (-8) \div (-6)$	(e) $(-8) \times (-3) \div (-4)$	(f) $(-5) \times 6 \div (-2)$
(g) $(5 + (-8)) \times (-6)$	(h) $(-7) \times (3 - 9)$	(i) $((-10) + (-2)) \div (-3)$

10.

(a) $(-2) \times (-3) \times (-4)$	(b) $(-3) \times (-4) \times (-5)$	(c) $(-4) \times (-5) \times (-6)$
(d) $(-3)^2$	(e) $(-5)^2$	(f) $(-10)^2$
(g) $(-1)^2$	(h) $(-1)^3$	(i) $(-1)^4$

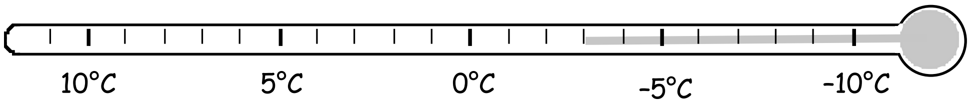
Integers

Topic in a Nutshell

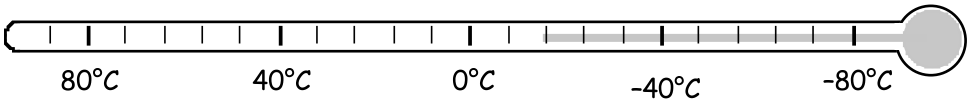


1. What temperatures are showing on the following thermometers ?

(a)



(b)



2. (a) Jason had £50 in his bank account. He then withdrew £80.

   What will the balance be on his account now ?

   (b) On Monday, Mary had a bank balance of –£145.
   On Tuesday, she withdrew £35.
   On Wednesday she paid £50 into her account.

   How much must she deposit on Thursday to clear her overdraft ?

   (c) Maximus Tritus was born in 42 B.C. and died 15 A.D. How old was he when he died ?

   (d) The temperature last night was –5°C. This morning it has risen by 9°C.

   What is the temperature this morning ?

3. Calculate :-

(a) 9 – 3	(b) 4 – 6	(c) 7 – 12	(d) 13 – 25
(e) (-5) + 10	(f) (-2) + 9	(g) (-1) – 2	(h) (-5) – 7
(i) 6 + (-1)	(j) 3 + (-1)	(k) 18 + (-12)	(l) 19 – (-1)
(m) 15 – (-25)	(n) 10 – (-43)	(o) (-71) – 19	(p) (-3) + (-2)
(q) (-7) + (-12)	(r) (-10) – (-10)	(s) (-43) – (-12)	(t) (-0·6) – (-0·4)

4. Find :-

(a) 2x + (-x)	(b) 5y – (-2y)	(c) 5t + (-4t)	(d) (-3u) + 7u
(e) (-7h) + (-10h)	(f) (-2p) + (-9p)	(g) 5pq + (-2pq)	(h) $(-5w^2) - (-7w^2)$.

5. Write down each of the following and find the answers :-

(a) 3 × (-4)	(b) 6 × (-7)	(c) (-5) × 8	(d) (-36) ÷ 9
(e) (-56) ÷ 7	(f) (-24) ÷ 12	(g) (3 × 8) ÷ 2	(h) (8 × 8) ÷ 4
(i) 10 × (10 – 15)	(j) 9 × (10 – 19)	(k) ((-4) – 11) ÷ 3	(l) (-6) × (-3)
(m) (-33) × (-1)	(n) (-56) ÷ (-4)	(o) 2 × (-3) × (-4)	(p) ((-10) + (-2)) ÷ (-3)
(q) $(-2)^2$	(r) $(-3)^3$	(s) (-1) × (-2) × (-3) × (-2) × (-1)	

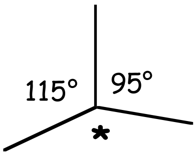
Chapter 3

Angle Properties

Revision of Level E Angle Work

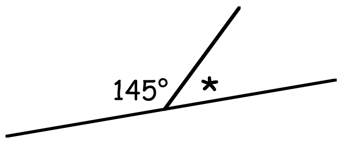
Some basic reminders

The angles round a point always add to give 360°.



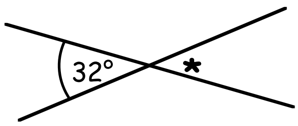
* must be 150° here.

Two angles making a straight line always add to give 180°.



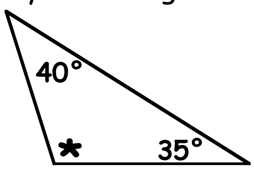
* must be 35° here.

Angles opposite each other at a cross are equal.



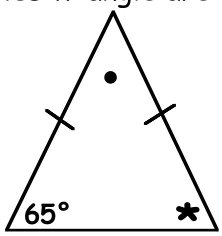
* must be 32° here.

The 3 angles of every triangle always add to give 180°.



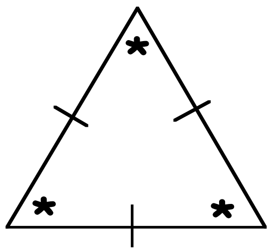
* must be 105° here.

Two of the angles in an **isosceles** triangle are equal.



* must be 65° here.

All three of the angles in an equilateral triangle are equal.



* must be 60° here.

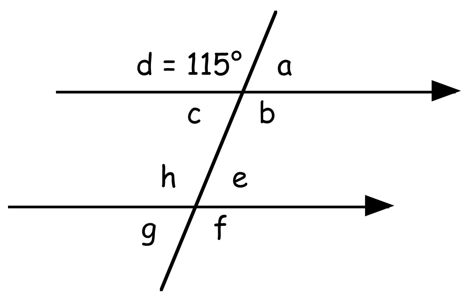
The following should also be known :-

h is **corresponding** to d and must be 115°.

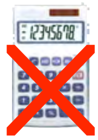
b is **(vertically) opposite** d and must be 115°.

c must be 65°, (**it adds to 115 to give 180**).

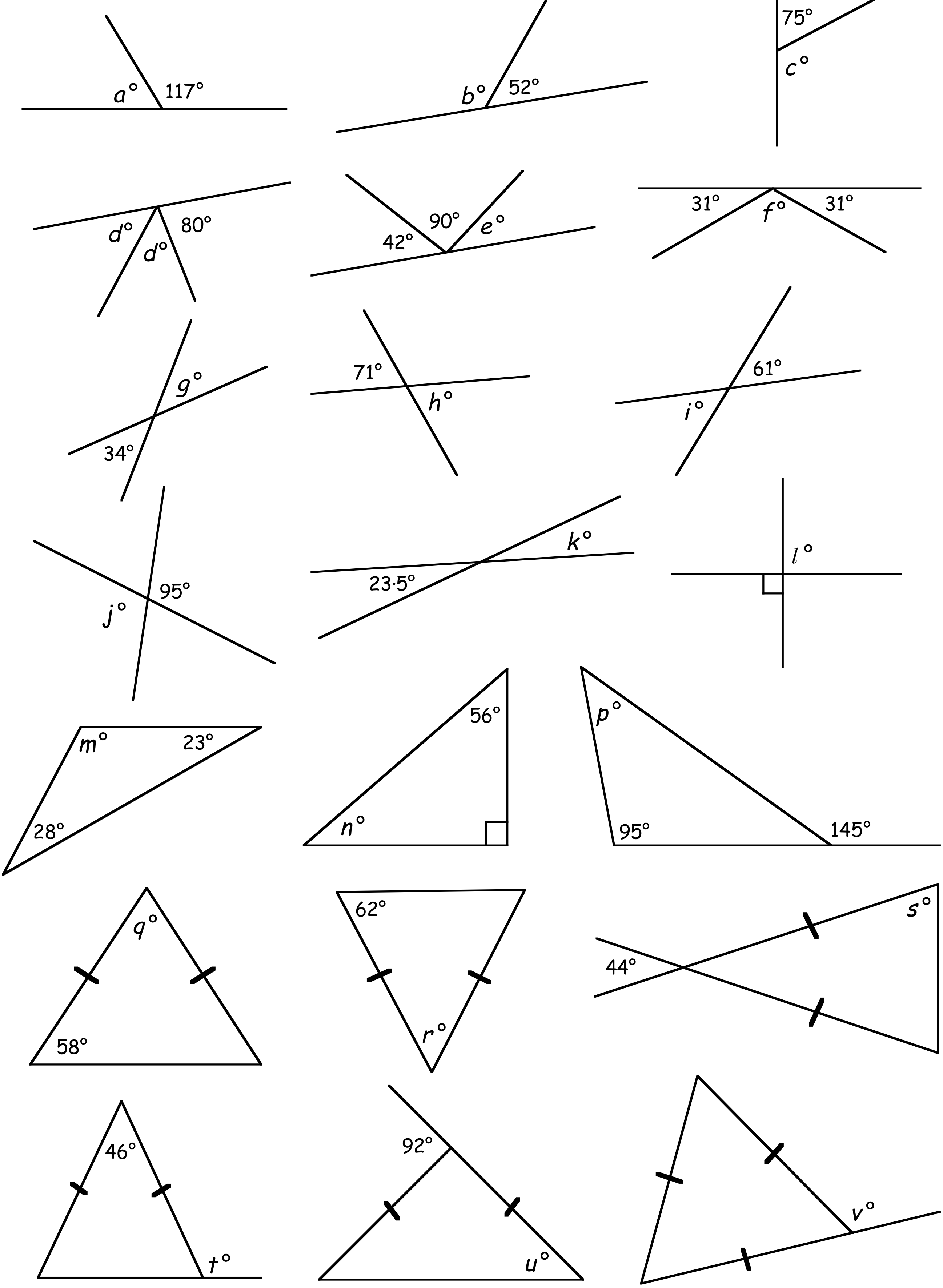
e is **alternate** to c and must be 65° also.



Exercise 1

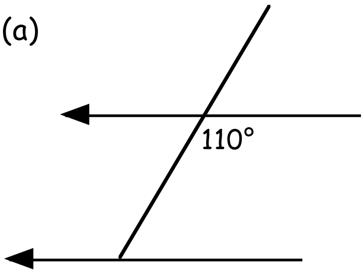


1. Calculate the sizes of the angles marked *a*, *b*, *c*, *d*, ...

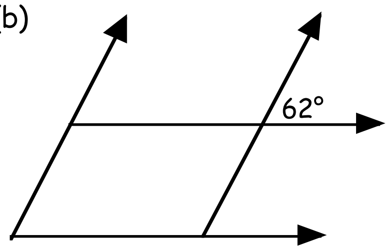


2. Copy each of the following and fill in the sizes of **all** the missing angles

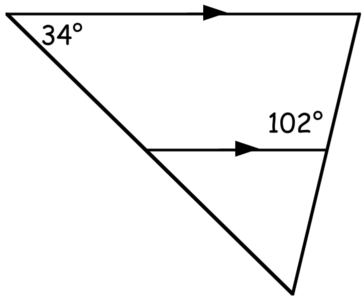
(a)



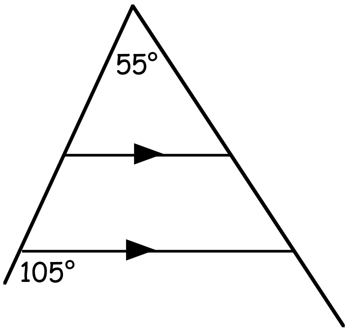
(b)



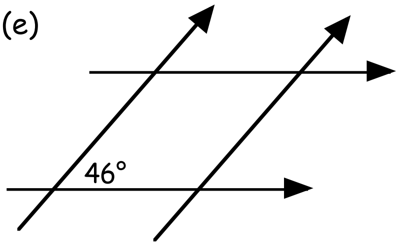
(c)



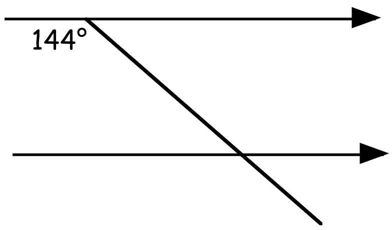
(d)



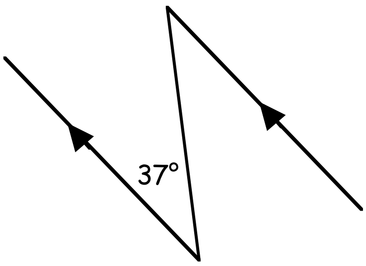
(e)



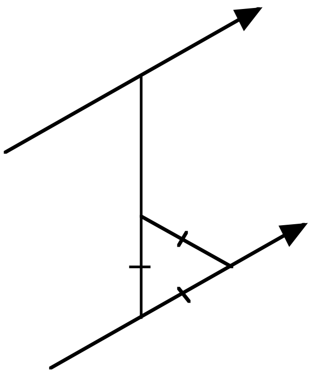
(f)



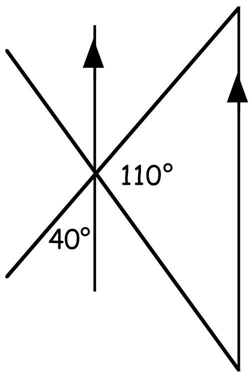
(g)



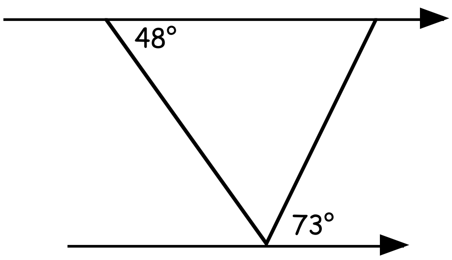
(h)



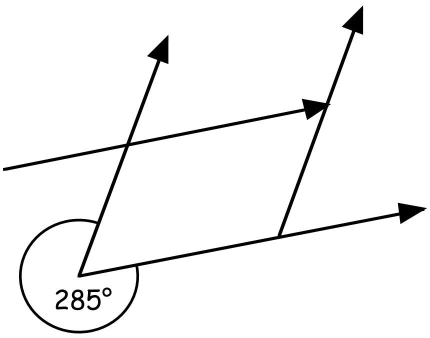
(i)



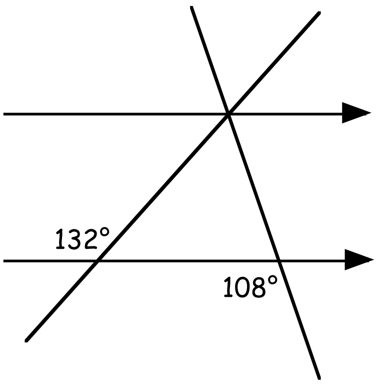
(j)



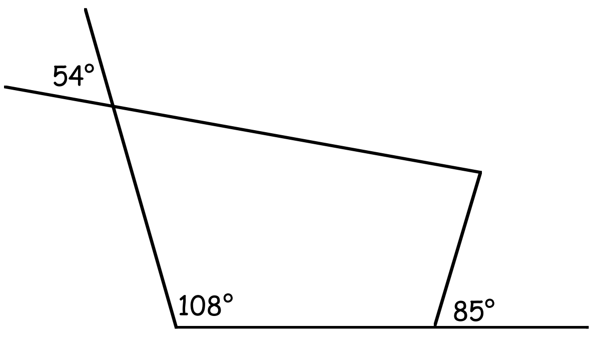
(k)



(l)



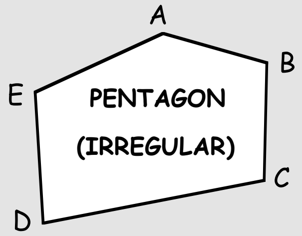
(m)



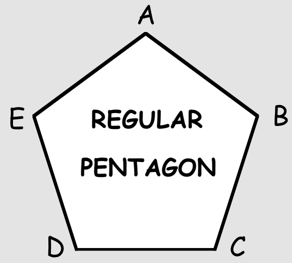
Polygons

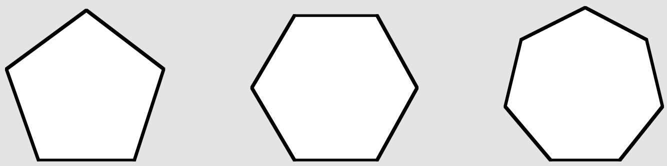
A polygon is a "many-sided closed straight-lined figure".

This 5-sided (polygon) is called a **PENTAGON**.



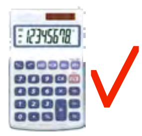
If all the angles are the same size and all the sides are the same length, it is referred to as a **REGULAR PENTAGON**.





pentagon **hex**agon **hept**agon

Exercise 2



1. (a) Trace, or copy the above 3 polygons and write their names below them

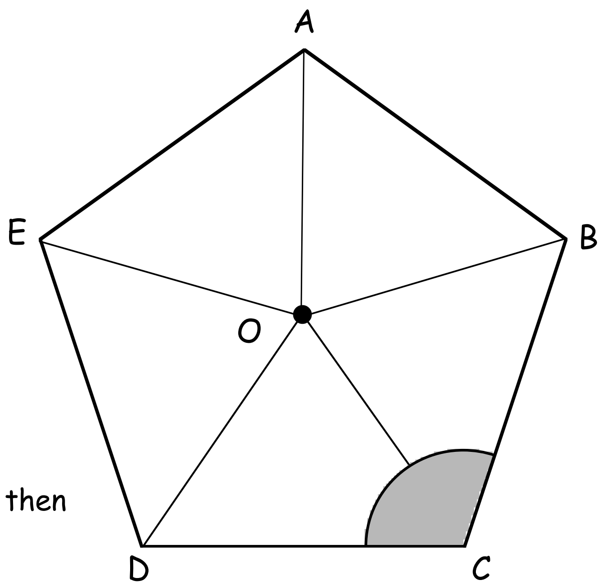
   (b) Make a sketch of an regular 8 sided polygon. What is its special name ?

   (c) Repeat for a regular 9 sided polygon.

   (d) Repeat for the regular 10 sided polygon. (*learn the names of these polygons*)

2. Look at this regular **pentagon** with centre O.

   Trace it into your jotter.



   (a) Copy and complete :-

   "Since a whole turn is 360°, the size of ∠DOC must be (360° ÷ 5) = ...°".

   (b) Copy and complete :-

   "Since DOC is an isosceles triangle, both ∠OCD and ∠ODC must be .....°".

   (c) Copy and complete :-

   "Since ∠OCD and ∠OCB are the same size, then the **Interior** (shaded) ∠BCD must = ...°".

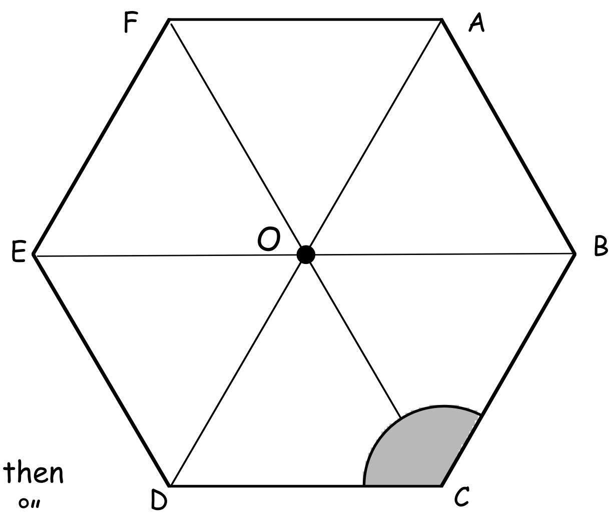
   (d) Copy and complete :-

   "Since the five angles of the polygon are the same size, then :-

   THE FIVE ANGLES OF A PENTAGON ADD TO GIVE ....°".

3. Look at this regular **hexagon** with centre O.

Trace it into your jotter.



(a) Copy and complete :-

"Since a whole turn is 360°, the size of ∠DOC must be (360° ÷ 6) = ...°".

(b) Copy and complete :-

"Since DOC is an isosceles triangle, both ∠OCD and ∠ODC must each be .....°".

(c) Copy and complete :-

"Since ∠OCD and ∠OCB are the same size, then the **Interior** (shaded) angle ∠BCD must = ...°".

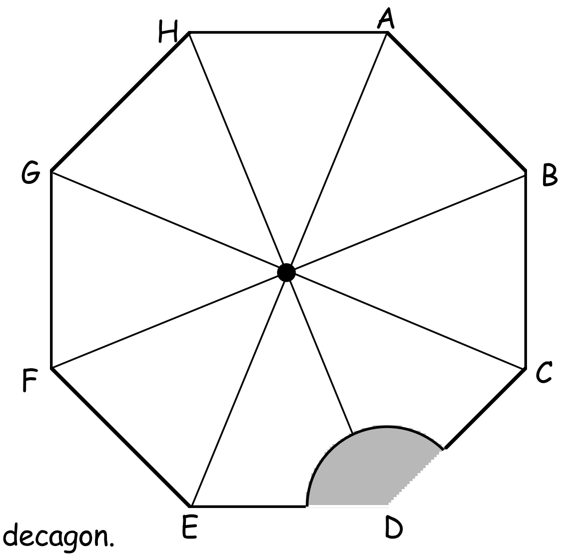
(d) Copy and complete :-

"Since the six angles of the polygon are the same size, then

THE SIX ANGLES OF A HEXAGON ADD TO GIVE ....°".

4. The polygon this time is an **octagon**.

Go through the same 4 steps as shown in Questions 2 and 3 to find :-

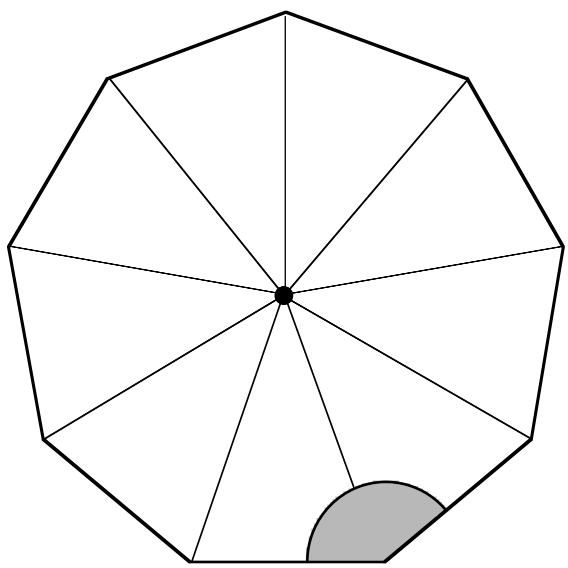


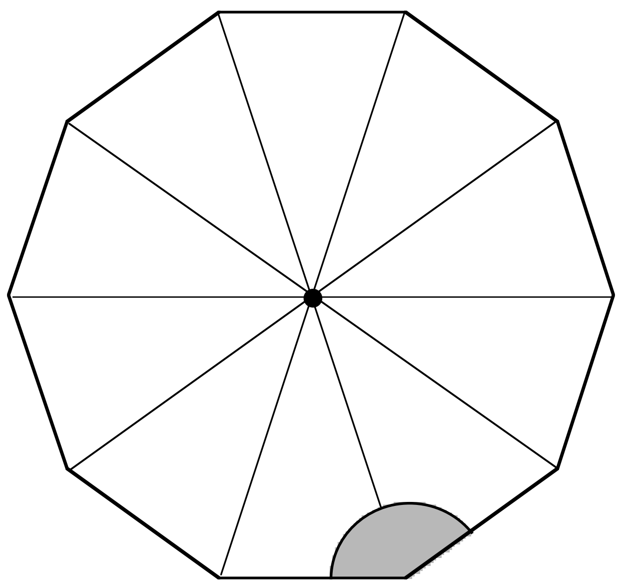
(a) the size of each of the eight **interior** angles of the octagon.

(b) the **sum** of all eight angles of the octagon.

5. Repeat for the :-

(a) nonagon (b) decagon.





6. Copy and complete this table, filling in the values for a 4, 7, 9, 10, 11 and 12 sided polygon.

Polygon sides	4	5	6	7	8	9	10	11	12
Sum of all angles	....	540°	720°	....	1080°	....	....	....	....
Size of each **interior** angle *	....	108°	....	....	....	....	....	....	....

(* this can be found by dividing the sum of all angles by the actual number of angles, e.g. 540° ÷ 5 = 108°)

A rule :- If the number of sides in the polygon is ***n***, the size of the **interior** angle is found by :-

interior angle = 180 – (360 ÷ ***n***).

Example :- For a hexagon (n = 6) => interior angle = 180 – (360 ÷ 6) = 180 – 60 = 120°.

7. Check the above rule works for a pentagon, heptagon, octagon and nonagon.

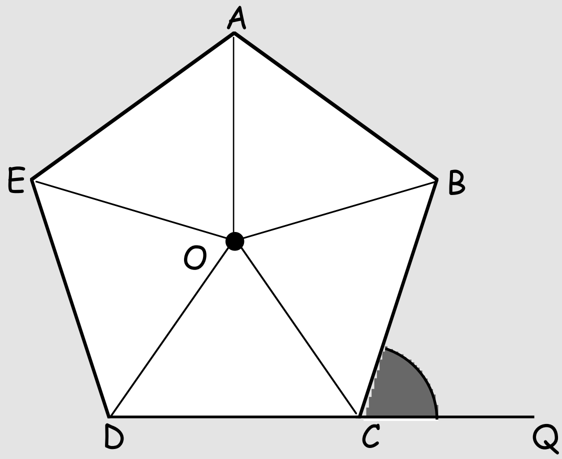
The **exterior** angles of a Polygon :-

From questions 2 and 7, you should have discovered that the **INTERIOR** angles of a regular PENTAGON were each 108°.

The diagram shows the **exterior** angle is found by :-

exterior angle = 180° – **interior** angle

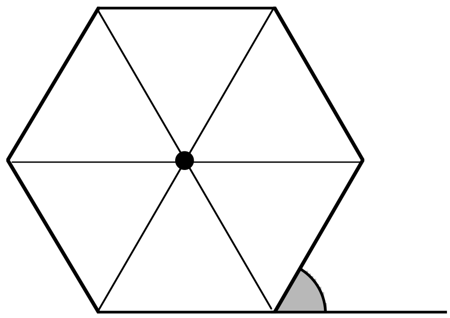
=> **exterior** angle of <u>pentagon</u> = 180° – 108° = 72°.



8. (a) Calculate the size of the exterior angles of a regular hexagon.

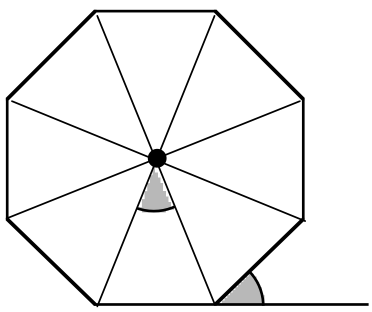
(b) Calculate the size of the exterior angles of a regular octagon.

(c) Calculate the size of the exterior angles of a regular decagon.

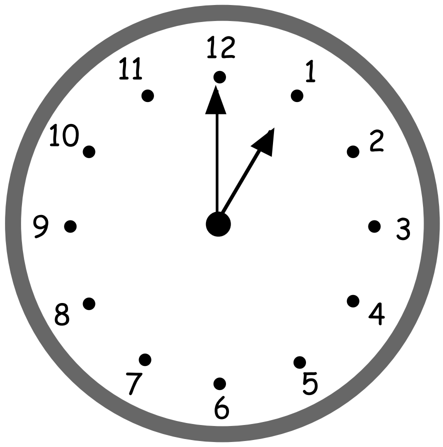


9. Can you see a connection between the size of the **external** angle of a polygon, (say an octagon), and the **angle at the centre** between any 2 adjacent "spokes" ?

(Try to explain why this should be so)

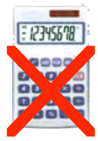


The Mathematics of the Analogue Clock Face



Lots of angle work can be done by studying the face of an **analogue** clock, (as opposed to digital one).

Exercise 3



1. (a) How many degrees are there in 1 full turn ?

   (b) How many hours are there on the face of this clock ?

   (c) Copy and complete this sentence :-

   "The size of the angle between 12 and 1 (as shown) or between any 2 "adjacent" hours must be $360° \div ... = ...°$".

2. What must the size of the (smaller) angle be between the hands of a clock at :-

   (a) 2 o'clock ? (b) 3 o'clock ? (c) 4 o'clock ?

   (d) 5 o'clock ? (e) 6 o'clock ? (f) 9 o'clock ?

3. (a) Trace the above clock into your jotter and neatly show the time "half past 2".

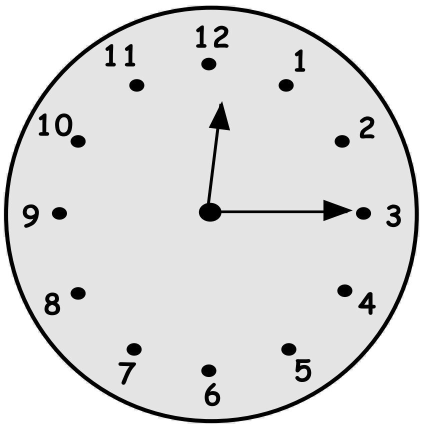
   (b) Calculate the size of the angle between the hour and minute hand.

   (*note :- it is not 120° or 90°*)

4. Calculate the size of the (smaller) angle between the hands of a clock at :-

   (a) half past one (b) half past 5 (c) 3.30

   (d) 6.30 (e) half past 11 (f) 9.30.



5. The clock shows "quarter past 12" this time.

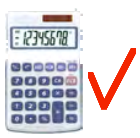
   Think carefully !!

   (a) Calculate the size of the angle between the hour and the minute hand of the clock at this time.

   (b) Calculate the angle between the hands at 2.15.

   (c) Harder :- Find the angle between the hands at 1.20.

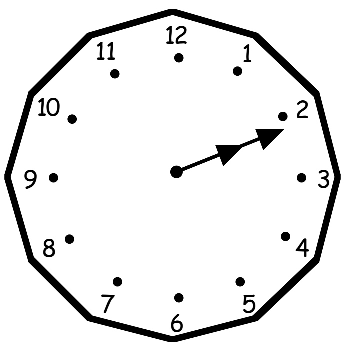
6. Really hard :-



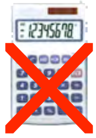
   There is a time between 2.00 o'clock and quarter past two, when the hour hand and the minute hand lie exactly on top of one another.

   Find the time to the nearest second.

   (*This will require a fair bit of calculation*)



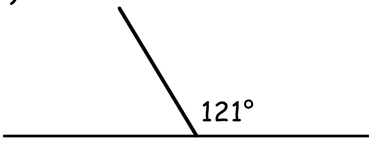
Angles



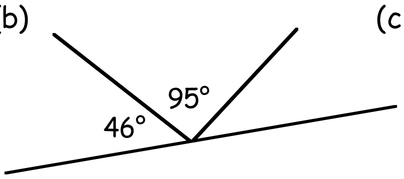
Topic in a Nutshell

1. Copy each of the following diagrams and fill in the sizes of **all** the missing angles.

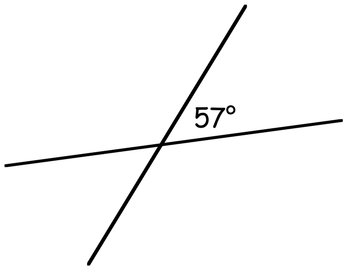
(a)



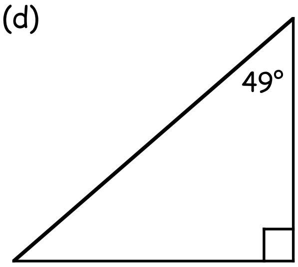
(b)



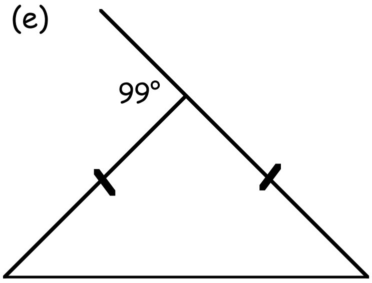
(c)



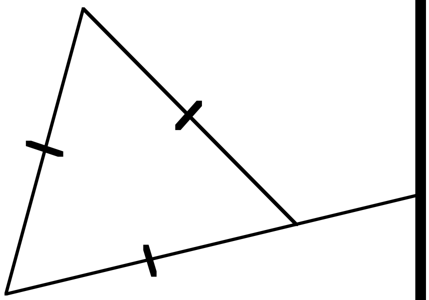
(d)



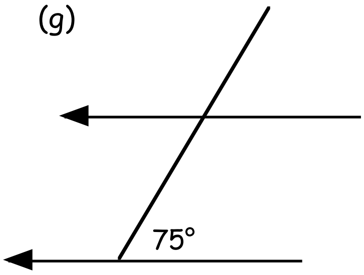
(e)



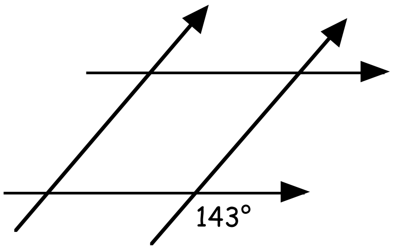
(f)



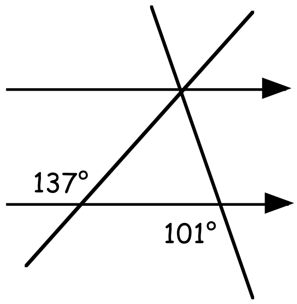
(g)



(h)

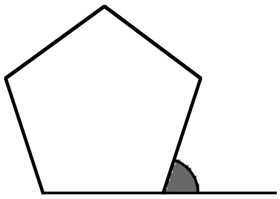


(i)



2. (a) An interior angle of a regular polygon is found to be 120°.
What is the regular polygon called ?

(b) Calculate the size of the exterior (shaded) angle of this regular pentagon.



(c) Calculate the external angle of a regular icosagon. (twenty sided shape).

3. Calculate the size of the acute angle between the hands of a clock at :-

(a) 1 o'clock
(b) half past 3
(c) half past 7
(d) quarter past 3

4. Calculate the size of the **reflex** angle between the hands of a clock at :-

(a) 2020 hours
(b) one minute past midnight.

Chapter 5

Types of Numbers

Multiples

In Primary school, you **should have learned** your tables or "**stations**" .

For example :-   8 x 1 = <u>8</u>, 8 x 2 = <u>16</u>, 8 x 3 = <u>24</u>, 8 x 4 = <u>32</u>,.......

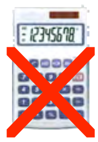
Some schools referred to the 8, 16, 24, 32, 40, ... as the "stations of 8".

We will now refer to them by their proper name - the **MULTIPLES** of 8.

Examples :- The first six multiples of 8 are (0), 8, 16, 24, 32, 40,...
The first eight multiples of 3 are (0), 3, 6, 9, 12, 15, 18, 21, ...

Since "0" is always a multiple, (the trivial multiple), for the rest of this chapter we will ignore it.

Exercise 1



1. (a) List the first **ten** multiples of **5** (excluding 0).
   (b) List the first **eight** multiples of **6** (excluding 0).
   (c) List the first **six** multiples of **10** (excluding 0).

2. (a) List all the multiples of **3** between **10** and **40**.
   (b) List all the multiples of **4** between **30** and **50**.
   (c) List all the multiples of **7** between **20** and **50**.

3. (a) List the first **ten** multiples of 2.
   (b) There is a special name for the "**multiples of 2**". What is it ?
   (c) Subtract 1 from each of the numbers you have in part (a) and write them down. Is this a set of multiples ?
   (d) What is the special name for this set of numbers ?

4. {24, 30, 36, 42, 48} could be described as "**the multiples of 6 from 24 to 48**".
   Describe the following sets of numbers in a similar way :-
   (a) {40, 45, 50, 55, 60, 65, 70} (b) {22, 24, 26, 28, 30, 32, 34, 36}
   (c) {84, 90, 96, 102, 108} (d) {150, 160, 170, 180, 190}
   (e) {28, 42, 56, 70, 84} (f) {200, 250, 300, 350, 400}

5. (a) List the first ten multiples of **3**.

   (b) List the first ten multiples of **4**.

   (c) From (a) and (b), write down the multiples which are "**common**" to both lists.
   (*the numbers that are multiples of 3 and 4*)

   (d) What is the **lowest** number that is a multiple of both 3 and 4 ?

   This is called the "**lowest common multiple**" of 3 and 4 (the **l.c.m.**)

6. (a) List the first twelve multiples of **6**.

   (b) List the first twenty multiples of **4**.

   (c) List the common multiples of 6 and 4.

   (d) What is the l.c.m. of 6 and 4 ?

7. (a) List the first ten multiples of **5**.

   (b) List the first twenty multiples of **2**.

   (c) List the common multiples of 5 and 2.

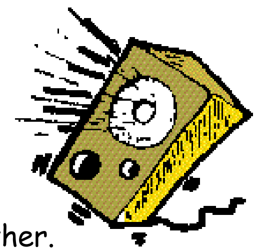
   (d) What is the l.c.m. of 5 and 2 ?

8. Find the l.c.m. of each of the following pairs of numbers.
   (hint :- *go through the multiples of the larger of the two numbers until you reach a number into which the smaller number divides exactly*)

(a) 3 and 5	(b) 6 and 2	(c) 4 and 7	(d) 3 and 6
(e) 8 and 6	(f) 6 and 10	(g) 5 and 6	(h) 7 and 9
(i) 10 and 7	(j) 8 and 9	(k) 4 and 12	(l) 7 and 11.

9. Find the l.c.m. of :-

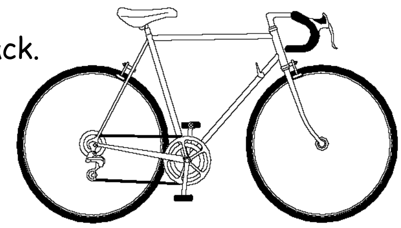
	(a) 2, 3 and 4	(b) 3, 4 and 6	(c) 2, 5 and 6
(d) 3, 5 and 10	(e) 2, 3 and 5	(f) 4, 6 and 8	(g) 3, 6 and 9



10. Jamie has a set of flashing disco lights.

   - The red light flashes every 9 seconds.
   - The yellow light flashes every 12 seconds.
   - The blue light flashes every 15 seconds.

   When Jamie switches all of them on, the yellow, red and blue flash together.
   After how many seconds will all the lights flash at the same time again ?



11. 3 speed cyclists set off together round a circular speedway track.

   - Jacques can complete a lap in 20 seconds.
   - Louise can complete a lap in 25 seconds.
   - Henri can complete a lap in 30 seconds.

   How many seconds will it take for all 3 cyclists to pass the starting line at the same time ?

12. When we come to adding fractions like $\frac{1}{6} + \frac{1}{8}$, it is important that we know how to find the l.c.m. of the 2 denominators 6 and 8.

(* we will meet fraction work later)

Simple example :-

(l.c.m. = 24)

$$\frac{1}{6} + \frac{1}{8}$$
$$= \frac{?}{24} + \frac{?}{24}$$
$$= \frac{4}{24} + \frac{3}{24}$$
$$= \boxed{\frac{7}{24}}$$

(Change both denominators to 24)

(Can you see ?)

Try the following :-

(a) $\frac{1}{3} + \frac{1}{5}$
$= \frac{?}{15} + \frac{?}{15}$
$=$

(b) $\frac{1}{8} + \frac{1}{4}$
$= \frac{?}{8} + \frac{?}{8}$
$=$

(c) $\frac{1}{4} + \frac{1}{3}$
$= \frac{?}{12} + \frac{?}{?}$
$=$

(d) $\frac{1}{2} + \frac{1}{5}$
$= \frac{?}{?} + \frac{?}{?}$
$=$

(e) $\frac{1}{4} - \frac{1}{10}$

(f) $\frac{1}{2} + \frac{1}{6}$

(g) $\frac{1}{3} - \frac{1}{7}$

(h) $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$

Factors

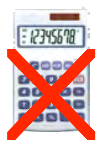
We can make a list of all the numbers that divide exactly into 10. They are {1, 2, 5, 10}.

These numbers that divide exactly into 10 are called **FACTORS** of 10.

- The factors of 6 are {1, 2, 3, 6}
- The factors of 20 are {1, 2, 4, 5, 10, 20}

Note that : the factors of any number "*x*" will always include **1** and ***x*** itself.

Exercise 2



1. The number 15 has FOUR factors. What are they ?
2. List all six factors of 28.
3. List all eight factors of 24.

Factors usually occur in "pairs". In the example below, 1 and 24 are a pair, as are 2 and 12. 3 and 8 and 4 and 6 are also pairs.

1 2 3 4 6 8 12 24

Using this "**pairing**" helps you not to miss out any of the factors.

4. Copy and complete the following, showing all the factors of 18.

1, 2, 3, ..., ..., ...

5. Use this method to find all the factors of :-

(a) 8	(b) 12	(c) 22	(d) 27
(e) 29	(f) 30	(g) 32	(h) 40
(i) 45	(j) 50	(k) 60	(l) 61.

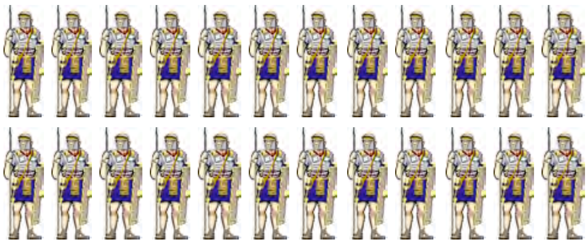
6. Look at all twelve answers to Q5. Check that in each case, there is an **EVEN** number of factors.

7. For each of the following
   - list all the factors
   - state how many factors each number has

(a) 4	(b) 25	(c) 36	(d) 9
(e) 49	(f) 16	(g) 100	(h) 64.

8. (a) Check that each number in Q7 had an **ODD** number of factors.
   (b) What is the special name for these numbers ? {4, 9, 16, 25, 36, ...}
   (c) Can you explain why there will always be an odd number of factors for this type of number ?

9. 24 Roman soldiers line up in rows. One way is to have 2 rows of 12 soldiers.
   State a few other ways of grouping the 24 soldiers. (3 rows of ...., etc)



10. (a) List all the factors of 12. {1, 2. .... }
    (b) List all the factors of 16. {1, 2. .... }
    (c) Make a list of the common factors of 12 and 16. (those that appear in both lists)
    (d) What is the largest of these numbers ?
    This number is referred to as the **HIGHEST COMMON FACTOR** (or **h.c.f.**) of 12 and 16.

11. (a) List all the factors of 20.
    (b) List all the factors of 30.
    (c) Make a list of the common factors.
    (d) What is the **h.c.f.** of 20 and 30 ?

12. Find the highest common factor for each of the following :-

(a) 6 and 8	(b) 12 and 20	(c) 15 and 20	(d) 24 and 30
(e) 24 and 36	(f) 40 and 60	(g) 13 and 26	(h) 12 and 42.

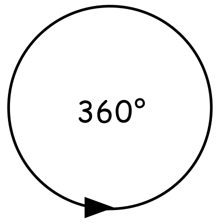
13. Find the h.c.f. of :-

(a) 13 and 17 (b) 23 and 29 (c) 11 and 19 (d) 7 and 3.

14. Find the h.c.f. of :-

(a) 8, 12, 16 (b) 10, 20, 25 (c) 14, 35, 42 (d) 24, 32, 48.

15. A full revolution is divided into 360 parts. Each part is called "1 degree".
The choice of 360 is no accident.
The reason is that 360 has many factors (**24** in fact) and this means a circle can be divided equally in lots of ways.



Find all 24 factors of 360.

16. There is only one number which is both a **multiple** and a **factor** of 12.

What is the number ?

Prime Numbers

Look at the factors of the following 2 group of numbers.

6 has 4 factors	{1, 2, 3, 6}
8 has 4 factors	{1, 2, 4, 8}
10 has 4 factors	{1, 2, 5, 10}
18 has 6 factors	{1, 2, 3, 6, 9, 18}
24 has 8 factors	{1, 2, 3, 4, 6, 8, 12, 24}

3 has 2 factors	{1, 3}
5 has 2 factors	{1, 5}
7 has 2 factors	{1, 7}
17 has 2 factors	{1, 17}
23 has 2 factors	{1, 23}

There is a special name for numbers which have exactly 2 factors.

They are called **PRIME** numbers,

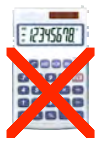
and are the most important group of numbers in the study of arithmetic.

Every number can be divided by itself and 1.

Every **prime** number can **ONLY** be divided by itself and 1.

Definition :- **A prime number is a number with EXACTLY TWO FACTORS.**

Exercise 3



1. Write all the factors of 12. Why is 12 **not** a prime number ?
2. Write all the factors of 11. Why **is** 11 a prime number ?
3. How many factors has the number 8 ? Is it a prime number or not ?
4. Explain why the number **1** is **not** a prime number.

5. For each of the following numbers :-

- list all of its factors.
- say whether or not it is a prime number.

(a) 7	(b) 10	(c) 9	(d) 13
(e) 15	(f) 21	(g) 23	(h) 29
(i) 50	(j) 49	(k) 47	(l) 45.

6. A number which is **not** a prime, is called a **COMPOSITE** number.

State which of the following numbers are composite :-

30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40.

7. A gardener uses a sieve to separate the soil from the stones.

Eratosthenes came up with a "mathematical sieve" which allows you to sift out the non-primes (the composites) and find all the primes (from 1 – 100).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) Make a neat large copy of this number square showing all the numbers from 1 to 100.

(b) **On your copy**, score out 1 – it is not a prime.

(c) **Circle 2** – score out every other multiple of 2 – (4, 6, 8, ... 100).

(d) **Circle 3** – score out every other multiple of 3 – (6, 9, 12, ... 99).

(e) **Circle 5** – score out every other multiple of 5 – (10, 15, 20, ... 95).

(f) **Circle 7** – score out every other multiple of 7 – (14, 21, 28, ... 98).

(g) Now circle every remaining number in the square. These are all the prime numbers.

Eratosthenes

(h) Make a neat list of all the primes from 1 to 100. (there are exactly 25 of them !)

8. (a) Make up a grid similar to the one in Q7 for all the numbers from 101 to 200.

(b) Score out every multiple of 2, 3, 5 and 7 as in Q7.

(c) Now score out every multiple of 11.

(d) Lastly score out every multiple of 13.

(e) Circle all the remaining numbers – these are the primes from 101 to 200.

(f) Make a neat list of all the primes. (There are 21 of them).

It is not difficult to check whether a large (but not too large) number is a prime or not.

You simply have to check if the number can be divided by all the primes 2, 3, 5, 7, 11, 13, etc, smaller than the number and if none of the primes (below it) divide into it, then the number must be prime.

The study of prime numbers has fascinated mathematicians for hundreds of years.

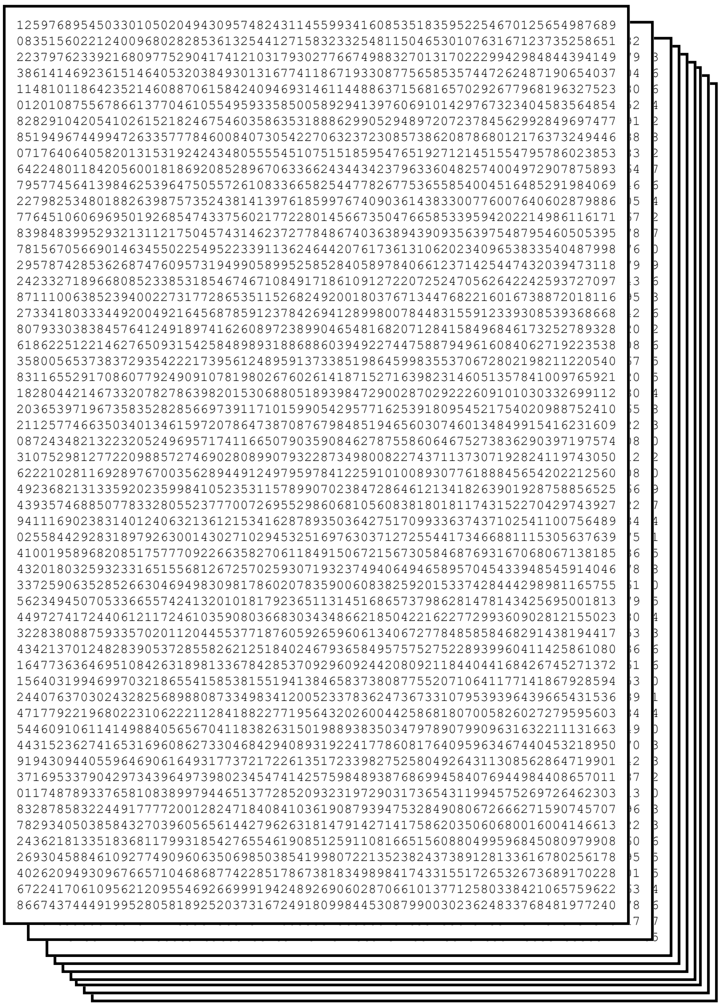
The largest prime number, as of January 2004, was the number

$2^{26996011} - 1$,   a massive number which contains 6320430 digits !

It begins 1259768954503 .... and ends 386177431990

6 320 430 digits

Want to check it ???



Takes 1590 pages to print this prime out !!!!

9. There are obvious reasons why the following are **<u>not</u>** primes.
In each case, say why :–

(a) 37495   (b) 1264572   (c) 89479480   (d) 3396303.

Prime Decomposition

A number is either Prime or Composite. (except for 0 which is neither).

If it is composite, like 12, it can be expressed as a "product of primes".

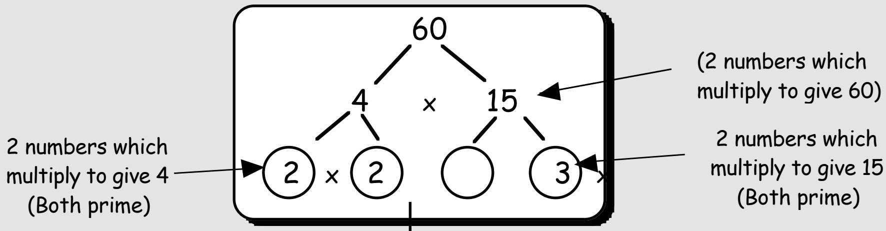
This means we can break down composite numbers and show they are made up by multiplying certain primes together.

$12 = 2 \times 6 = 2 \times 2 \times 3$ (3 prime numbers)

$30 = 2 \times 3 \times 5$ (3 prime numbers)

$32 = 2 \times 2 \times 2 \times 2 \times 2$ (5 prime numbers)

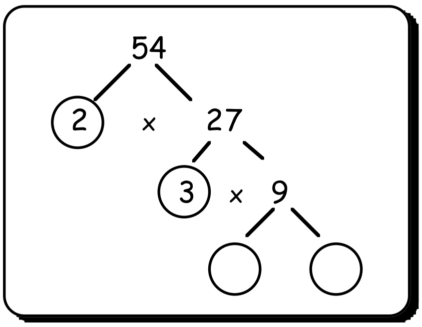
Here is an easy way of doing it for the number 60 :-

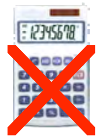


$\boxed{60 = 2 \times 2 \times 3 \times 5}$

This is called a "**Prime Factor Tree**".

Exercise 4

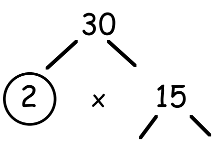




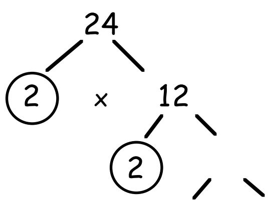
1. Copy this diagram and complete it to show the "**prime decomposition**" of the number 54.

   $\longrightarrow$ $54 = 2 \times 3 \times ...$

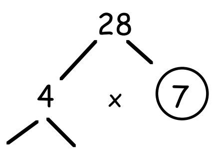
2. Copy and complete these prime factor trees :-

   (a) 

   $30 = 2 \times ... \times ...$

   (b) 

   $24 = 2 \times 2 \times ... \times ...$

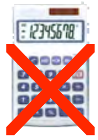
   (c) 

   $28 = 7 \times ... \times ...$

3. Use a similar method to find the prime decomposition of the following numbers :-

(a) 8	(b) 10	(c) 12	(d) 20
(e) 25	(f) 36	(g) 45	(h) 50
(i) 64	(j) 80	(k) 98	(l) 100.

Types of Numbers



Topic in a Nutshell

1. Excluding zero, list the first 8 multiples of :-

   (a) 6 (b) 11 (c) 15 (d) 25.

2. From the numbers shown below,

   2, 4, 5, 7, 12, 14, 15, 16, 18, 20, 24, 27, 33, 36, 40, 45, 51, 55, 58, 60

   List all the multiples of :-

   (a) 3 (b) 4 (c) 5 (d) 6.

3. Find the lowest common multiple (l.c.m.) of :-

   (a) 2 and 3 (b) 6 and 8 (c) 2, 3 and 4 (d) 5, 6 and 8.

4. Three geysers, Tom, Dick and Harry erupt every 6 minutes, 9 minutes and 12 minutes respectively.
   At noon they all erupt together.

   At what time will they next erupt together ?

5. List all the factors of :-

   (a) 6 (b) 23 (c) 48 (d) 99.

6. Find the highest common factor (h.c.f.) of :-

   (a) 6 and 10 (b) 15 and 33 (c) 21 and 29 (d) 32 and 52
   (e) 20, 24 and 60 (f) 12, 60 and 90 (g) 13 and 23 (h) $6x$ and $8x$.

7. How many factors does a prime number have ?

8. Look at the list of numbers in question 2. Write down all the **prime** numbers from the list.

9. List all the prime numbers between 50 and 100.

10. Explain why each of the following large numbers **cannot** possibly be prime :-

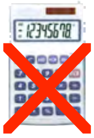
    (a) 12 543 672 (b) 22 334 455 (c) 123 111 970 (d) 909 636 963.

11. List each of the following numbers as a product of primes :-
    (Hint : *construct a "Prime Factor Tree"*).

    (a) 40 (b) 75 (c) 1000 (d) 660.

Chapter 7

The "Long" Way



Scientific Notation

A number like 3800 can be written in a different way.

$3800 = 380 \times 10$ (can you see this ?)

$= 38 \times 10 \times 10$ (follow this ?)

$= 3{\cdot}8 \times 10 \times 10 \times 10$ (still following ?) $= \boxed{3{\cdot}8 \times (10^3)}$

$3{\cdot}8 \times 10^3$ is called the "**standard form**" of 3800.

It is also said to be in **<u>scientific notation</u>** when the number at the start,

(the 3·8) lies between 1 and 10.

Examples

(a) 52000 $= (5200 \times 10) = (520 \times 10 \times 10) = (52 \times 10 \times 10 \times 10)$

$= (5{\cdot}2 \times 10 \times 10 \times 10 \times 10) = \boxed{5{\cdot}2 \times 10^4}$ (5·2 lies between 1 and 10).

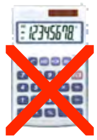
(b) 264000 $= (26400 \times 10) = (2640 \times 10 \times 10) = (264 \times 10 \times 10 \times 10)$

$= (26{\cdot}4 \times 10 \times 10 \times 10 \times 10)$

$= (2{\cdot}64 \times 10 \times 10 \times 10 \times 10 \times 10) = 2{\cdot}64 \times 10^5$ (2·64 lies between 1 and 10).

This is a handy way (a standard way) of writing large numbers.

Exercise 1



1. Copy and complete the following :-

   46 000 = 4600 × 10 = 460 × .... × .... = 46 × .... × .... × ....

   = 4·6 × .... (Stop here, since 4·6 lies between 1 and 10).

   = $4{\cdot}6 \times 10^{\cdots}$

2. Write the following numbers in scientific notation :-

   (a) 5900 = 590 × 10 = ... —> = $5{\cdot}9 \times 10^{\cdots}$

   (b) 340 = 34 × 10 = ... —> = $3{\cdot}4 \times 10^{\cdots}$

   (c) 81 000 = 8100 × 10 = ... —> = 8·1 × ....

3. Write the following in scientific notation :-

(a)	5200	(b)	4530	(c)	27 000
(d)	35 900	(e)	82 750	(f)	750 000
(g)	378 000	(h)	204 700	(i)	8 600 000

A Much Quicker Method

Here are further example showing how to use the long method :-

$3200 = 320 \times 10 = 32 \times 10 \times 10 = 3{\cdot}2 \times 10 \times 10 \times 10 = (3{\cdot}2 \times 10^3)$

$48\,000 = 4800 \times 10 = 480 \times 10 \times 10 = 48 \times 10 \times 10 \times 10 = 4{\cdot}8 \times 10 \times 10 \times 10 \times 10 = (4{\cdot}8 \times 10^4)$

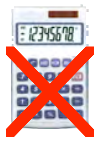
Can you see that changing 3200 to 3·2 meant you had to move the decimal point <u>3</u> places, and changing 48 000 to 4·8 meant you moved the decimal point <u>4</u> places ?

$(3{\cdot}2 \times 10^3)$ and $(4{\cdot}8 \times 10^4)$

Here is a method that allows you to change to scientific notation much faster :-

278 000 ->	Step 1	move the decimal point till it comes between the 1st and the 2nd digits.	2·78 ↑
	Step 2	now count how many places that you have moved the point.	$\overset{5}{\longleftarrow}$ 2·78000
	Step 3	finally write this number (the 5) as the power of 10.	$2{\cdot}78 \times 10^{\underline{5}}$

Exercise 2



1. Change 5700 to scientific notation using the above "quick" method.

   5700 => $\overset{3}{\longleftarrow}$ (5·700) => $5{\cdot}7 \times 10^{\cdots}$

2. Use the above method to write the following numbers in scientific notation :-

(a)	69 000	(b)	9300	(c)	234 000
(d)	520	(e)	4287	(f)	260 000
(g)	8000	(h)	47 000	(i)	20 000
(j)	9 320 000	(k)	4 800 000	(l)	25 700 000

3. This table gives the land areas of various countries (in square kilometres).

Country	*Area (km^2)*
Austria	84 200
Australia	7 687 000
Canada	9 976 000
France	547 200
Iceland	103 300
USSR	22 402 000
Hong Kong	987

4. You should you know that 12 million = 12 000 000

and 3·54 million = 3 540 000

and $8\frac{1}{2}$ million = 8 500 000

Write each of the following out in full, then write each in scientific notation :-

(a) 6 million = 6 000 000 = $6{\cdot}0 \times 10^{\cdots}$

(b) 8·5 million = 8 500 000 = ...

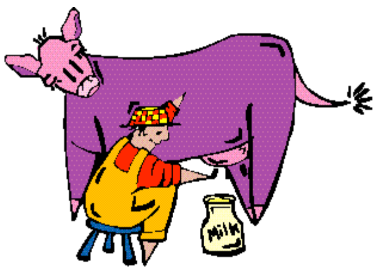
(c) 1·98 million = 1 980 000 =

(d) $3\frac{1}{2}$ million = ... = ...

(e) 11 million	(f) 27 million	(g) 7·4 million
(h) 6·23 million	(i) $10\frac{1}{2}$ million	(j) $15\frac{1}{2}$ million
(k) 2·745 million	(l) $3\frac{1}{4}$ million	(m) $5\frac{3}{4}$ million

5. (a) The rollover in last week's lottery was £$3\frac{1}{2}$ million.

Write this amount in scientific notation.



(b) A dairy sells 45 000 litres of milk each year.

Write this in scientific notation.

(c) Liverpool paid £$22\frac{1}{4}$ million for a striker.

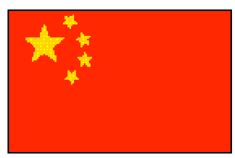
Write this in scientific notation.

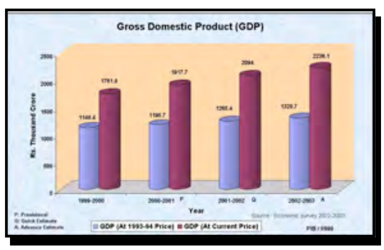
(d) The government spent "half a billion pounds" on Education last year.

Write this in scientific notation.

(e) The population of China was 852 million in 1995.

Write this in scientific notation.





(f) The **Gross Domestic Product** of Britain in 1995 was $228 billion.

Write this amount in scientific notation.

Changing from Scientific Notation back to Number Form

Look at the following :-

$1{\cdot}87 \times 10^4$ stands for

$1{\cdot}87 \times 10 \times 10 \times 10 \times 10$

$= 18{\cdot}7 \times 10 \times 10 \times 10$

$= 187 \times 10 \times 10$

$= 1870 \times 10$

$= \boxed{18\,700}$

$7{\cdot}1 \times 10^5$ means

$7{\cdot}1 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 71 \times 10 \times 10 \times 10 \times 10$

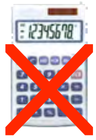
$= 710 \times 10 \times 10 \times 10$

$= 7100 \times 10 \times 10$

$= 71000 \times 10$

$= \boxed{710\,000}$

Exercise 3



1. Copy the following and show how to change $4{\cdot}56 \times 10^3$ to number form

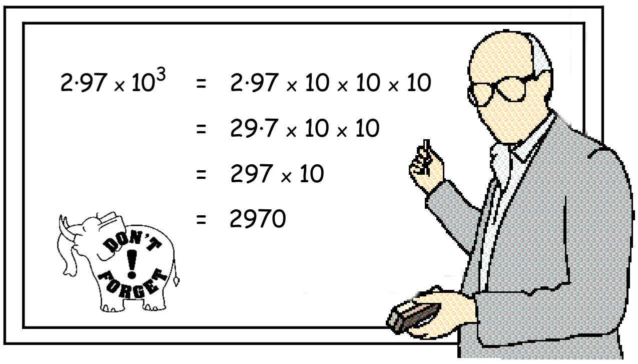
$$
\begin{aligned}
4{\cdot}56 \times 10^3 &= 4{\cdot}56 \times 10 \times 10 \times 10 \\
&= 45{\cdot}6 \times 10 \times 10 \\
&= 456 \times 10 \\
&= \text{............}
\end{aligned}
$$

2. Express each of the following in number form using the above method :-

(a) $7{\cdot}03 \times 10^4$

$$
\begin{aligned}
&= 7{\cdot}03 \times 10 \times 10 \times 10 \times 10 \\
&= \text{............} \\
&= \text{............} \\
&= \text{............}
\end{aligned}
$$

(b)	$2{\cdot}73 \times 10^6$	(c)	$1{\cdot}99 \times 10^2$	(d)	$3{\cdot}475 \times 10^7$
(e)	$8{\cdot}4 \times 10^5$	(f)	$3{\cdot}81 \times 10^3$	(g)	$6{\cdot}23 \times 10^4$
(h)	$9{\cdot}99 \times 10^4$	(i)	$1{\cdot}45 \times 10^8$	(j)	$7{\cdot}835 \times 10^5$
(k)	7×10^4	(l)	$5{\cdot}67 \times 10^9$	(m)	1×10^{12}



A quicker way

Instead of writing each of these long steps, it is easier to do the following :-

Example $2{\cdot}91 \times 10^5$ —> Step 1 Write down the 291 <u>without the point</u>

Step 2 Move the point (5) places to the right

5 places

$2{\cdot}91 \times 10^5$ = 2 9 1 0 0 0 = 291 000

(can you see why we need the extra zero's ?)

3. Change $3{\cdot}07 \times 10^3$ using this method

$3{\cdot}07 \times 10^3$ = 3 0 7 ..... = ?

4. Change each of the following to number form using the "quicker" method :-

(a) $2{\cdot}7 \times 10^4$	(b) $3{\cdot}4 \times 10^2$	(c) $5{\cdot}27 \times 10^5$	(d) $2{\cdot}85 \times 10^3$
(e) $4{\cdot}523 \times 10^4$	(f) 4×10^1	(g) 6×10^6	(h) $3{\cdot}5 \times 10^6$
(i) $9{\cdot}017 \times 10^4$	(j) 8×10^7	(k) $4{\cdot}37 \times 10^7$	(l) $1{\cdot}111 \times 10^8$

5.

Country	*Population*
Portugal	$9{\cdot}449 \times 10^6$
Malta	$3{\cdot}042 \times 10^5$
Israel	$3{\cdot}5971 \times 10^7$
Greece	$3{\cdot}548 \times 10^6$
Turkey	$4{\cdot}016 \times 10^7$
China	$8{\cdot}521 \times 10^8$
Greenland	$5{\cdot}013 \times 10^4$

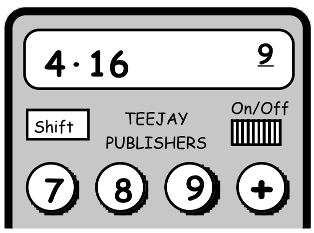
This table shows populations for several countries.

Write each of the populations out in full.

6. When large numbers turn up on a scientific calculator, they sometimes do so in scientific notation.

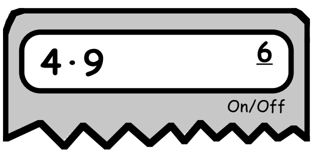
This calculator shows the number $4{\cdot}16 \times 10^9$

= **4 160 000 000**

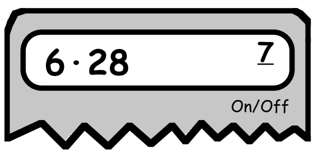


What numbers are shown on the following calculators :-

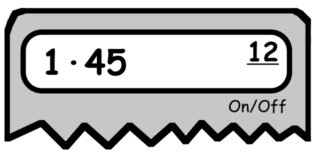
(a)



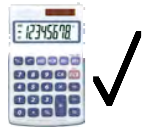
(b)



(c)



Very large numbers !!!!!



Do you remember the following :-

10^3	$= 10 \times 10 \times 10 = 1000$	(1 thousand)
10^6	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$	(1 million)
10^9	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000\,000$	(1 billion)
10^{12}	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000\,000\,000$	(1 trillion)

A **thousand** :- If you write all the numbers from 1 to 1000, how many **digits** will be needed ?

Answer :-

From 1 to 9	-	9×1	=	9 digits	(1, 2, 3, 4, 5, 6, 7, 8, 9)
From 10 to 99	-	90×2	=	180 digits	
From 100 to 999	-	900×3	=	2700 digits	
The 1000	-	1×4	=	4 digits	=> Total = **2893** digits

7. (a) Calculate how many digits are needed to write all the numbers from 1 to 1000 000 ?

   (b) Assume you began to write all the numbers from 1 to 1000 at a rate of 1 digit per second, how long would it take you in minutes and seconds ?

   (c) Assume you began to write all the numbers from 1 to 1000 000 at a rate of 1 digit per second, how long would it take you in days, hours, minutes and seconds ?

   (d) An A4 sheet of half centimetre squared paper has approximately 2500 squares. If you wrote all the numbers from 1 to 1000 000, with 1 digit in each square and 1 space between each actual number, how many sheets of paper would you need ?

A ***GOOGLE*** was the name invented by a scientist's son to stand for the number :-

1000000.......................................00000 (1, followed by 100 zeros)

that's 1×10^{100}.

An even bigger number is a **GOOGLEPLEX**.

A googleplex is $1 \times 10^{\text{google}}$.

This is a 1, followed by a google no. of 0's.

an awfully BIG number !!!!!!!!

"as close to infinity as you'll get"

one	1
ten	10
hundred	100
thousand	1,000
million	1,000,000
billion	1,000,000,000
trillion	1,000,000,000,000
quadrillion	1,000,000,000,000,000
quintillion	1,000,000,000,000,000,000
sextillion	1,000,000,000,000,000,000,000
septillion	1,000,000,000,000,000,000,000,000
octillion	1,000,000,000,000,000,000,000,000,000
nonillion	1,000,000,000,000,000,000,000,000,000,000
decillion	1,000,000,000,000,000,000,000,000,000,000,000
undecillion	1,000,000,000,000,000,000,000,000,000,000,000,000
duodecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000
tredecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000
quattuordecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
quindecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
sexdecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
septendecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
octodecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
novemdecillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
vigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
unvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
duovigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
trevigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
quattuorvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
quinvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
sexvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
septenvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
octovigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
novemvigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
trigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
untrigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
duotrigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
googol (google)	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
tretrigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
quattuortrigintillion	1,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

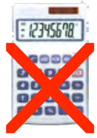
Scientific Notation for Very Small Numbers

It is also possible to write very small (decimal) numbers in the same way - in this standard form.

=> It is, yet again, a process of moving the decimal point to a position just after the first <u>non-zero</u> whole number.

Example 1	0·000 36 =>	$\overset{\text{4 places}}{00003{\cdot}6} \times 10^{-4}$	=	$3{\cdot}6 \times 10^{-4}$
Example 2	0·0912 =>	$\overset{\text{2 places}}{009{\cdot}12} \times 10^{-2}$	=	$9{\cdot}12 \times 10^{-2}$
Example 3	0·000 0025		=	$2{\cdot}5 \times 10^{-6}$

Exercise 4



1. Write the following small numbers in scientific notation :-

(a) 0·008 (b) 0·000 067 (c) 0·0931 (d) 0·000 007

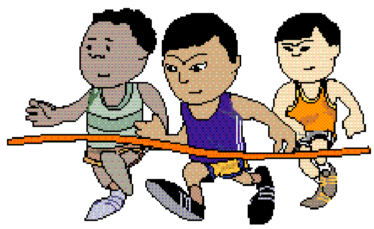
(e) 0·000 558 (f) 0·182 (g) 0·003 15 (h) 0·000 09

2. Rewrite each sentence expressing the number in scientific notation :-

(a) The diameter of the lead in a pencil is 0·0025 m.

(b) The weight of a single eye-lash is 0·000 000 25 kg.

(c) The winner beat the runner-up by 0·017 seconds.



In reverse, we simply move the point **left** to express the number in full.

Example 1	$1{\cdot}9 \times 10^{-3}$	=>	$\overset{\text{3 places}}{{\cdot}0\,0\,1\,9}$	=	0·0019
Example 2	$9{\cdot}12 \times 10^{-6}$	=>	$\overset{\text{6 places}}{{\cdot}0\,0\,0\,0\,0\,9\,1\,2}$	=	0·000 009 12.

3. Write the following numbers in full :-

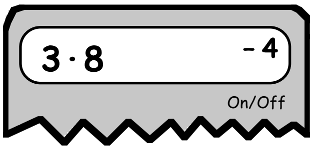
(a) $3{\cdot}4 \times 10^{-2}$ (b) $5{\cdot}7 \times 10^{-4}$ (c) $6{\cdot}38 \times 10^{-3}$ (d) $4{\cdot}05 \times 10^{-5}$

(e) $1{\cdot}8236 \times 10^{-1}$ (f) 9×10^{-3} (g) 4×10^{-5} (h) $2{\cdot}002 \times 10^{-6}$

4. A packet of crisps weighs $2{\cdot}4 \times 10^{-2}$ kilograms.

Is this more or less than 25 grams ?

A calculator shows

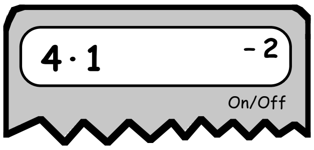


This means

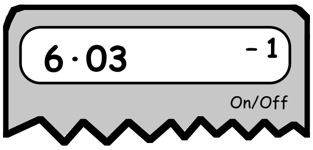
$3{\cdot}8 \times 10^{-4} = \underline{\underline{0{\cdot}00038}}$

5. What do the following mean :-

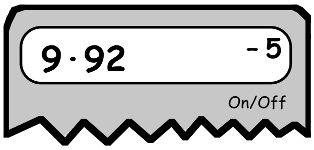
(a)



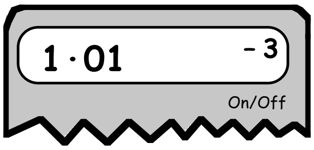
(b)



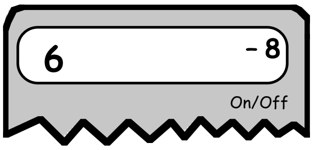
(c)



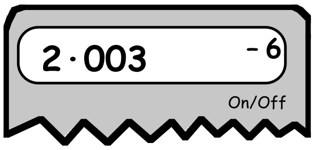
(d)



(e)



(f)



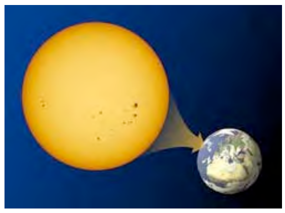
6. (a) Write out in full :-

(i)	$1{\cdot}7 \times 10^{-3}$	(ii)	$2{\cdot}9 \times 10^{3}$	(iii)	8×10^{-5}
(iv)	3×10^{4}	(v)	$1{\cdot}67 \times 10^{-6}$	(vi)	$2{\cdot}085 \times 10^{5}$

(b) Write in scientific notation :-

(i)	0·009	(ii)	264	(iii)	0·000 81
(iv)	5200	(v)	0·0139	(vi)	390 000
(vii)	0·000 007	(viii)	125 000 000	(ix)	0·9

7. The distance to the Sun is approximately 93000000 miles.
This number can be written as $9{\cdot}3 \times 10^{p}$.
What is the value of p ?



8. Write these numbers out in full :-

(a) The outer diameter of the rings of Saturn is $2{\cdot}74 \times 10^{5}$ kilometres.

(b) The time taken for a humming-bird to flap its wings once is $3{\cdot}35 \times 10^{-6}$ seconds.

(c) A blade of grass is $1{\cdot}03 \times 10^{-3}$ metres thick.

9. Write these numbers in scientific notation :-

(a) The distance from Pluto to the Sun is 5 950 000 000 kilometres.

(b) The country of Senegal received \$165 000 000 world aid in 1992.

(c) The record attendance for Hampden Park before it was modernised was 149400.

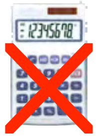
(d) A beam of light travels 1 kilometre in 0·000 003 3 seconds.

(e) The population of the World on 16th Feb. 2004 at 20:35:51 was 6,348,781,574.

Scientific Notation

Topic in a Nutshell

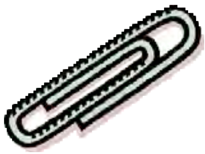
1. Write the following numbers in scientific notation :-



(a)	800	(b)	70 000		
(c)	93 000	(d)	1 400 000	(e)	547 000
(f)	34 700 000	(g)	103	(h)	ten million

2. (a) A biscuit factory produces two and a half million biscuits each week.

Write this number in scientific notation.



(b) A quarter of a billion paper clips were sold last year.

Write this number in scientific notation.

3. Write each of the following numbers in full :-

(a)	$4{\cdot}3 \times 10^{2}$	(b)	$8{\cdot}1 \times 10^{3}$	(c)	$1{\cdot}08 \times 10^{4}$	(d)	$1{\cdot}2 \times 10^{5}$
(e)	$5{\cdot}15 \times 10^{6}$	(f)	4×10^{9}	(g)	6×10^{1}	(h)	5×10^{4}

4. Write the following numbers in scientific notation :-

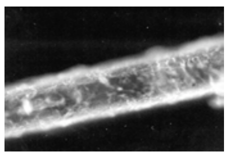
(a)	0·001	(b)	0·025	(c)	0·4	(d)	0·000 5
(e)	0·013 54	(f)	0·000 123	(g)	one tenth	(h)	one millionth.

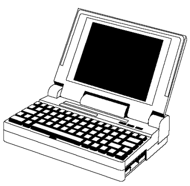
5. Write each of the following numbers in full :-

(a)	$1{\cdot}7 \times 10^{-2}$	(b)	$8{\cdot}2 \times 10^{-3}$	(c)	$5{\cdot}04 \times 10^{-4}$	(d)	$4{\cdot}55 \times 10^{-5}$
(e)	9×10^{-1}	(f)	$6{\cdot}1 \times 10^{-3}$	(g)	4×10^{-6}	(h)	$1{\cdot}0 \times 10^{-6}$

6. (a) The thickness of a hair was measured and found to be 0·0086 millimetres.

Write this number in scientific notation.





(b) A computer took $8{\cdot}702 \times 10^{-6}$ seconds to complete a calculation.

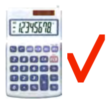
Write this number in full.

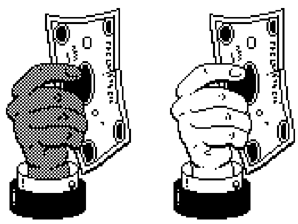
7. Write these numbers in scientific notation :-

(a) 0·000 000 000 000 000 000 000 000 1

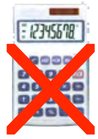
(b) A million billion dollars.

(c) The number of seconds in a 365 day year.





Chapter 11



In this Chapter

Algebra

Simplifying Algebraic Expressions

Examples

Collecting Like Terms

$x + x + x + x = \underline{\underline{4x}}$

$9p - 7p = \underline{\underline{2p}}$

$3a + b - a + 6b = \underline{\underline{2a + 7b}}$ (NOT $= 9ab$)

$10 + 6w - 1 = \underline{\underline{9 + 6w}}$ (NOT $= 15w$)

$x^2 + x^2 + x^2 = \underline{\underline{3x^2}}$ (NOT x^6)

Multiplying Terms

$7 \times a = \underline{\underline{7a}}$

$w \times 5 = \underline{\underline{5w}}$

$b \times b = \underline{\underline{b^2}}$ (NOT $2b$)

$2m \times 4m = \underline{\underline{8m^2}}$ (NOT $8m$)

Exercise 1

1. Simplify these expressions by collecting like terms :-

(a) $x + x$ (b) $w + w + w$ (c) $m + m + m + m + m$

(d) $c - c + c$ (e) $f + f - f + f$ (f) $x + x + x - x - x$

(g) $x + 4x + 3x - x$ (h) $5e + 4e - 8e + e$ (i) $2x + 2x + y + y$

(j) $a + b - a + b$ (k) $3p + 4q - 3p + q$ (l) $5x + 3w - 2x$

(m) $4g + h - 5g + 7h$ (n) $v + 3w - v + 3w$ (o) $a^2 + b^2 - a^2 + b^2$

2. Simplify by multiplying :-

(a) $6 \times t$ (b) $p \times 4$ (c) $a \times a$ (d) $w \times w$

(e) $7 \times f$ (f) $15 \times r$ (g) $m \times 12$ (h) $s \times 8$

(i) $x \times x \times 3$ (j) $x \times 4 \times 5$ (k) $3 \times d \times 5$ (l) $8m \times 3$

(m) $a \times a \times 9$ (n) $g \times 7 \times g$ (o) $p \times q$ (p) $m \times n \times 11$

(q) $p \times 5 \times q$ (r) $8 \times k \times k$ (s) $5a \times 3b$ (t) $9x \times 3x$

(u) $2d \times 3d \times 4$ (v) $5a \times a \times a$ (w) $3w \times 2w \times w$

(x) $(4a)^2$ i.e. $(4a \times 4a)$ (y) $(6x)^2$ (z) $(2ab)^2$

3. Simplify the following expressions :- :-

(a) $x^2 + 3x^2$ (b) $x \times 2x$ (c) $3p \times p$ (d) $8v - 3v$

(e) $6m \times 2m$ (f) $3n \times 8n$ (g) $3n + 8n$ (h) $4x \times 5y$

(i) $5y \times 4x$ (j) $7a + a$ (k) $3y \times 2x \times y$ (l) $20b - 18b + 2b$

(m) $14t - t$ (n) $15x - 14x + y$ (o) $5a + 3a + 1$ (p) $a^2 + 4a^2$

(q) $3y^2 - 2y^2$ (r) $3a + b + a$ (s) $8p + 1 - p$ (t) $9x^2 + 2x^2 - 10x^2$

(u) $3d + 9 - 2d$ (v) $7 + 3h + 5h$ (w) $8 - 2x + 7x$

(x) $5a^2 - 4 + a^2$ (y) $7v^2 - 6v^2 + 10v^2$ (z) $2a^2 + 5b^2 + a^2 - b^2$

4. In the following examples, simplify the expression then find its value when $p = 2$ and $q = 3$.

(a) $3p + 4p$
$= 7p$
$= 7 \times 2$
$=$ ....

(b) $3q + q$ (c) $4p + q - p$

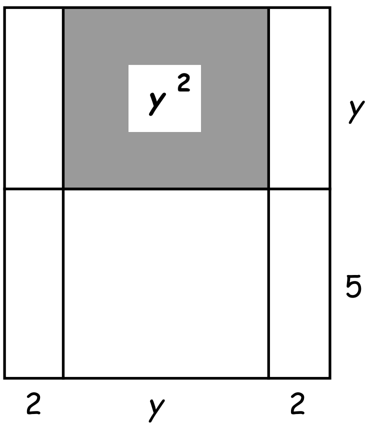
(d) $5p + 2q - p$ (e) $2p + 2q - p - q$ (f) $p - 3q + 4p + 6q$

(g) $4p + 3q + p + 4q$ (h) $q + q + 3p - q$ (i) $8q - p - 2q$

(j) $6p + 4q - 2p + q$ (k) $p^2 + q^2$ (l) $(p + q)^2$

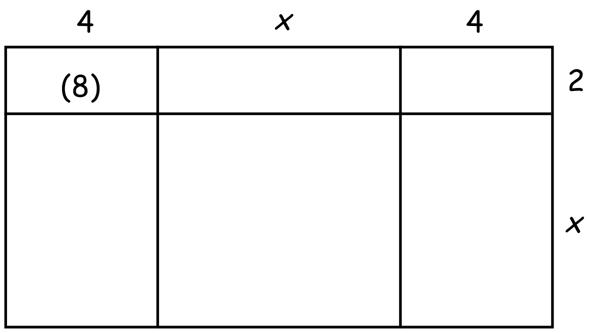
5. For each rectangular design, find an expression for the area of each small rectangle then find the total area of the design in its simplest form. (Ignore units !)

(a)



Area = 10 + ... + $5y$ + ... + ... + ...

(b)



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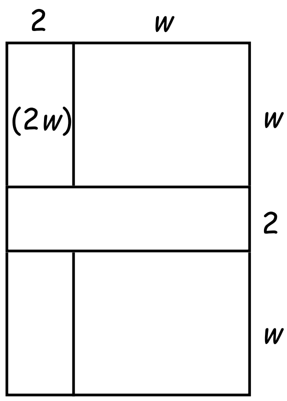
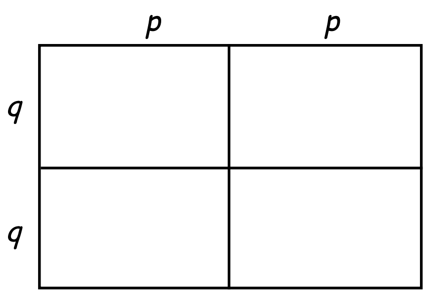
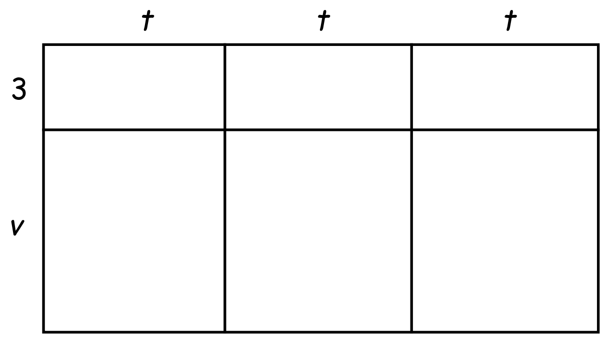


(d)



(e)



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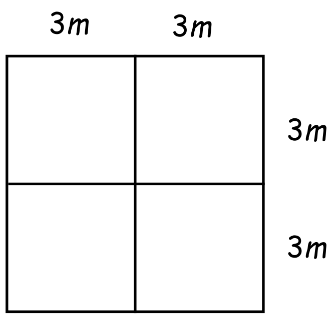
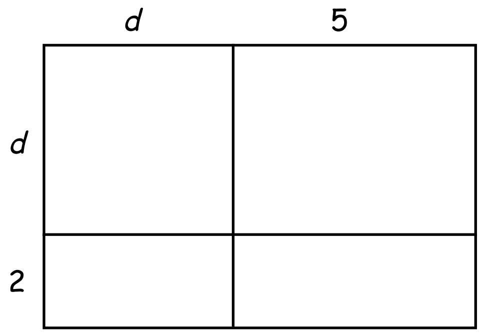


(g)



Removing Brackets

Examples

1. $2(x+4)$

   $= \underline{\underline{2x+8}}$

2. $3(2a-5b)$

   $= \underline{\underline{6a-15b}}$

3. $p(p+3)$

   $= \underline{\underline{p^2+3p}}$

4. $3x(4x-8y)$

   $= \underline{\underline{12x^2-24xy}}$

5. $-2(x+4)$

   $= \underline{\underline{-2x-8}}$

6. $-3(d-1)$

   $= -3d - 3 \times -1$

   $= \underline{\underline{-3d+3}}$

(Note the double negative = +ve)

Exercise 2

1. Remove the brackets :-

(a) $2(d+4)$	(b) $3(c+1)$	(c) $4(e+5)$	(d) $5(g+8)$
(e) $6(h+7)$	(f) $2(n-3)$	(g) $4(a-2)$	(h) $5(t-1)$
(i) $2(b-3)$	(j) $7(k-6)$	(k) $5(n-9)$	(l) $9(1+x)$
(m) $4(3+y)$	(n) $8(1+m)$	(o) $2(1-p)$	(p) $4(4-q)$
(q) $6(a+b)$	(r) $2(f+g)$	(s) $7(g-k)$	(t) $10(d-100)$
(u) $12(3+b)$	(v) $15(r+2)$	(w) $20(a-5)$	(x) $50(g-4)$

2. Multiply out the brackets :-

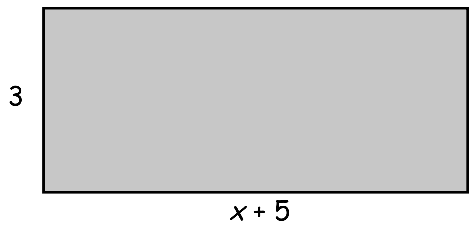
(a) $2(2g + 3)$	(b) $3(4a + 1)$	(c) $5(1 + 2d)$	(d) $2(3 - 4k)$
(e) $6(6h - 1)$	(f) $10(3 - 7n)$	(g) $4(2a + 3y)$	(h) $5(3t + x)$
(i) $2(4b - 3c)$	(j) $8(10k - 3p)$	(k) $7(11n - 9x)$	(l) $6(3ab - d)$
(m) $x(y + 5)$	(n) $a(p + 8)$	(o) $w(t - 1)$	(p) $g(g - 2)$
(q) $a(n + 9)$	(r) $w(m - a)$	(s) $e(f - 10)$	(t) $x(2 + x)$
(u) $a(2n + g)$	(v) $x(4y + 3u)$	(w) $6a(2 - 4a)$	(x) $3u(10u - w)$

3. Rewrite the following without brackets :-

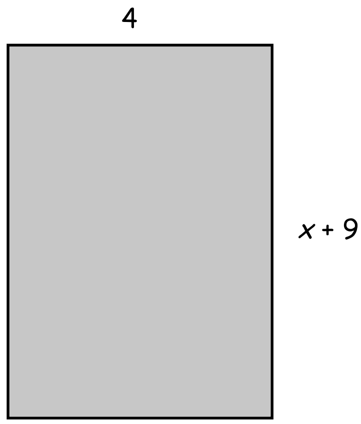
(a) $2(3a + 4b + 1)$	(b) $3(5x + 2y + 3)$	(c) $5(7c + 2d + 6)$	(d) $4(2k + 3j + 4f)$
(e) $6(2v + 4w + 5z)$	(f) $10(p + q - 2r)$	(g) $2(5a - 2b - 4c)$	(h) $6(3p - 5q - 7)$
(i) $5(2x - 3y - 5t)$	(j) $8(a - 2b - 5c)$	(k) $9(5 - 4f - 3g)$	(l) $a(a - b - 4c)$

4. Write the areas of these rectangles :- (i) with brackets (ii) without brackets :-
(All units are in centimetres)

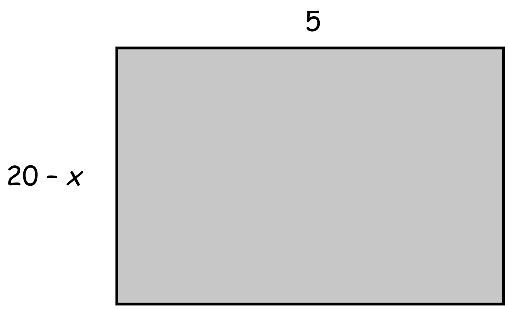
(a)



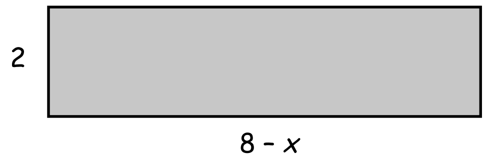
(b)



(c)



(d)



5. Remove these brackets :-

(a) $-2(a + 1)$	(b) $-3(x - 2)$	(c) $-5(3 + d)$	(d) $-4(5 - c)$
(e) $-(p + q)$	(f) $-(p - q)$	(g) $-6(d + e)$	(h) $-5(d - e)$
(i) $-p(p + 4)$	(j) $-h(h - 1)$	(k) $-x(1 + x)$	(l) $-2m(m + 3)$
(m) $-a(4a - 1)$	(n) $-h(5h + 4k)$	(o) $-x(5y - 4x)$	(p) $-2x(x - 3k)$

Removing Brackets and Simplifying

Examples

1. $2(x + 5) - 7$
$= 2x + 10 - 7$
$= \underline{\underline{2x + 3}}$

2. $4(3p + 2q) - 7q$
$= 12p + 8q - 7q$
$= \underline{\underline{12p + q}}$

3. $3(a + 4) + 2(a - 1)$
$= 3a + 12 + 2a - 2$
$= \underline{\underline{5a + 10}}$

4. $3(2x + 5) - 2(x - 1)$ careful
$= 6x + 15 - 2x + 2$
$= \underline{\underline{4x + 17}}$

5. $5 - 2(x + 1)$
$= 5 - 2x - 2$ [NOT $3(x + 1)$]
$= \underline{\underline{3 - 2x}}$

Exercise 3

A L G E B R A

1. Multiply out the brackets and collect like terms :-

(a) $3(x + 3) + 5$	(b) $2(a + 1) + 4$	(c) $4(b + 6) + 1$
(d) $2(c + 2) - 3$	(e) $6(w + 1) - 5$	(f) $4(r + 5) - 20$
(g) $2(y + 3) + 3y$	(h) $8(x + 1) + 7x$	(i) $3(m + 4) - 2m$
(j) $8(n + 3) - 3n$	(k) $10(4 + h) - 7h$	(l) $3x + 3(x + 1)$
(m) $4a + 5(a - 2)$	(n) $5p + 2(4p + 1)$	(o) $6q + 3(5q - 2)$
(p) $3v + (v - 1)$	(q) $2a + 3(a + 2b)$	(r) $x + 2(5x + 4y)$
(s) $6x + 2(5x - 14y)$	(t) $90p + 10(10p + q)$	(u) $7 + 3(h + 1)$
(v) $6(4x - 2y) - 24x$	(w) $8w + 6(3w + 2v)$	(x) $8 + 2(p - 4)$

2.

(a) $2(x + 1) + 2(x + 2)$	(b) $3(a + 3) + 4(a + 1)$	(c) $5(d + 1) + 6(d + 3)$
(d) $2(m - 1) + 4(m + 1)$	(e) $4(c - 3) + 3(c + 4)$	(f) $3(n - 5) + 6(n + 3)$
(g) $6(2 + v) + 5(1 - v)$	(h) $3(1 - x) + 4(1 + x)$	(i) $8(2q + 1) + (3q - 8)$
(j) $2(4d - 3) + 2(3d + 5)$	(k) $6(2 - 3h) + 7(1 + 3h)$	(l) $2(5v + 2w) + 2(4w - v)$

3.

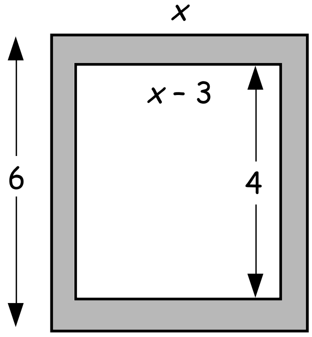
(a) $4(x + 1) - 2(x + 2)$	(b) $5(a + 2) - 4(a + 2)$	(c) $3(b + 5) - 2(b + 7)$
(d) $3(2c + 4) - 2(c + 5)$	(e) $6(3p + 2) - 4(p + 3)$	(f) $4(x + 3) - 2(x - 3)$
(g) $5(x + 1) - 3(x - 2)$	(h) $6(1 + 2e) - 2(1 - e)$	(i) $10(2 - v) - 12(1 - v)$
(j) $x(x + 1) + 2(x - 1)$	(k) $n(n + 6) - 4(n + 1)$	(l) $w(3w - 1) - 2(3w - 8)$

4.

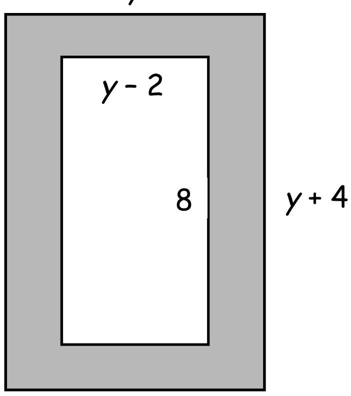
(a) $7 - 2(y + 3)$	(b) $5 - 2(p - 1)$	(c) $3 - 3(d - 1)$
(d) $4 + 3(h + 1)$	(e) $2 + 8(2 - c)$	(f) $4 - 2(1 - u)$
(g) $9(b - 2) - 8$	(h) $-2(n - 1) + 3$	(i) $m + 3(m - 4)$
(j) $x - (3 - x)$	(k) $9k - 3(k + 6)$	(l) $3w - 2(2 - 3w)$

5. By calculating the area of the large rectangle, then the area of the small rectangle, find the area in terms of x or y which is shaded. (answer in sq. units)

(a)



(b)



Evaluating Expressions - A Number for a Letter

Examples

If $a = 3$, $b = 4$ and $c = -1$, find the values of :-

1. $5a$
$= 5 \times 3$
$= \underline{\underline{15}}$

2. $2a + c$
$= 2 \times 3 + (-1)$
$= \underline{\underline{5}}$

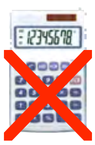
3. $b^2 - a^2$
$= (4 \times 4) - (3 \times 3)$
$= 16 - 9$
$= \underline{\underline{7}}$

4. $2b^2$
$= 2 \times b \times b$
$= 2 \times 4 \times 4$
$= \underline{\underline{32}}$

5. $5c^2$
$= 5 \times c \times c$
$= 5 \times (-1) \times (-1)$
$= \underline{\underline{5}}$

6. $2a^2 + 3b + 30c$
$= (2 \times a \times a) + (3 \times b) + (30 \times c)$
$= (2 \times 3 \times 3) + (3 \times 4) + (30 \times (-1))$
$= 18 + 12 - 30$
$= 0$

Exercise 4



1. Find the value of each of the following when $p = 3$.

(a) $p + 4$ (b) $p - 2$ (c) $6p$ (d) $4p - 11$

(e) $2 + 5p$ (f) $20 - 6p$ (g) p^2 (h) p^3

(i) $p^2 - 8$ (j) $2p^2$ (k) $p^3 + p$ (l) $p^2 - 3p$

2. Find the value of each of the following when $a = 5$.

(a) $3a$ (b) $6a$ (c) a^2 (d) $4a^2$

(e) a^3 (f) $2a^2$ (g) $2a^3$ (h) $30 - a^2$

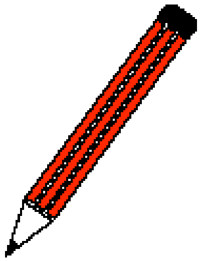
3. Find the values of each of the following :-

(a) $y + 7$ when $y = 6$ (b) $3b + 4$ when $b = 8$

(c) $m - 9$ when $m = 17$ (d) $11n - 20$ when $n = 3$

(e) $15 - w$ when $w = -2$ (f) $v + w$ when $v = -8$ and $w = 6$

(g) $5ab$ when $a = 3$ and $b = -1$ (h) $12 - 4pq$ when $p = 1$ and $q = -2$



4. Given $v = 1$, $w = 3$ and $x = 6$, calculate the values of :-

(a) $v + w + x$ (b) $4v + 3w + 2x$ (c) $w + x - 8v$

(d) vwx (e) $10v + 10w + 10x$ (f) $vw + wx + xv$

(g) $3x + 2v - w$ (h) $10vw - 5x$ (i) $2vwx - 36$

5. If $e = 3$ and $g = 4$, find the values of :-

(a) e^2 (b) $e^2 + g^2$ (c) $(g - e)^2$ (d) $g^2 - e^2$

(e) $(g + e)^2$ (f) $2g^2$ (g) $6e^3$ (h) $(g - 2)^2$

(i) $20 - 2e^2$ (j) $50 - 3g^2$ (k) $\sqrt{g}$ (l) $\sqrt{(e^2 + g^2)}$

6. If $a = 3$, $b = -2$ and $c = 1$, find :-

(a) $2a + b$ (b) $3b + 7c$ (c) b^2

(d) $10 + 2c^2$ (e) $2a^2 + 2b^2$ (f) $c^2 + b^2$

(g) $2a^2 + 3b - c$ (h) $4b^2 - 5a - c$ (i) $3c^2 + 3b^2 - 5a$

7. Given $x = 4$, $y = 6$, $z = 10$ and $w = -2$, find :-

(a) $\frac{1}{2}x$ (b) $\frac{1}{3}y$ (c) $\frac{3}{5}z$

(d) $\frac{1}{2}(x + y)$ (e) $\frac{1}{4}(z + w)$ (f) $\frac{1}{3}(z - w)$

(g) $\frac{1}{2}(y - w)$ (h) $\dfrac{y + z}{8}$ (i) $\frac{3}{4}(y + z)$

(j) $\dfrac{x + y + z}{5}$ (k) $\dfrac{x + y + z + w}{9}$ (l) $\dfrac{x + 2y + z - 4w}{17}$

8. In this question, $x = 3$, $y = 5$, $z = -6$ and $w = -2$. Find :-

(a) $2x^2$ (b) $xy + w$ (c) $z^2 - 2xy$

(d) $y^2 + w^2$ (e) $2z^2 - 25x$ (f) $\dfrac{y^2 - x^2}{w}$

(g) $\dfrac{9w^2}{xz}$ (h) $\dfrac{2x^2 + 2y + w^2}{(z + 8)^2}$ (i) $\sqrt{\dfrac{(x - w)^2}{z + 7}}$

Factorising - "The Common Factor"

Remember :-

when we multiplied out the brackets, $4(x + 5)$, we obtained :- $4x + 20$

If we start with the expression $4x + 20$,

we can **reverse** the process

=> and we can see, from above, that we obtain $4(x + 5)$.

When you are given the expression, $4x + 20$ and you are asked :–

"What is the HIGHEST factor of the two terms, $4x$ **and** 20 ?" => Answer is 4.

Now take the 4 outside a set of brackets => 4(.........)

and decide what goes in the bracket so that when multiplied, you obtain $4x + 20$.

Example

$4x + 20$
$= \underline{\underline{4(x + 5)}}$

This is called "**FACTORISING**" the expression.

* Note - In the above example, the "4" is the **highest common factor**".

You must always use the h.c.f. if the expression is to be factorised FULLY !

Further Examples

Factorise fully :-

1. $6x - 12$
$= \underline{\underline{6(x - 2)}}$

[6 is h.c.f.] Check !

2. $8a - 12b$
$= \underline{\underline{4(2a - 3b)}}$

[4 is h.c.f.] Check !

3. $am + an$
$= \underline{\underline{a(m + n)}}$

["a" is h.c.f.]

4. $9x - 6x^2$
$= \underline{\underline{3x(3 - 2x)}}$

[$3x$ is h.c.f.] Check !

Exercise 5

1. COPY and complete :-

(a) $2p + 2q = 2(\text{..........})$ (b) $3a + 6b = 3(\text{..........})$ (c) $6x + 4y = 2(\text{..........})$

(d) $ax + ay = a(\text{..........})$ (e) $mn + m = m(\text{..........})$ (f) $vw + w^2 = w(\text{..........})$

(g) $pr^2 + p = p(\text{..........})$ (h) $2hx + 2hy = 2h(\text{..........})$ (i) $8v - 12g = 4(\text{..........})$

(j) $24m - 16n = 8(\text{..........})$ (k) $4gk - 8k = 4k(\text{..........})$ (l) $6a - 15a^2 = 3a(\text{..........})$

2. Factorise the following, by considering the highest common factor in each case :-

(a) $5x + 10$ (b) $4a + 12$ (c) $8x - 40$ (d) $7x + 7y$

(e) $8a - 8b$ (f) $20f - 20g$ (g) $7n - 21$ (h) $12v + 48$

(i) $4p + 6q$ (j) $14h - 28k$ (k) $10u - 15w$ (l) $4a - 24b$

(m) $9y + 15$ (n) $24n - 8$ (o) $32e - 80d$ (p) $24x + 32y$

(q) $6u + 9v$ (r) $2a + 6b + 8c$ (s) $12x + 30y + 36z$ (t) $60r + 40s - 80t$

3. Factorise fully :-

(a) $4a + ac$ (b) $6v - gv$ (c) $xy + xz$ (d) $p^2 + 9p$

(e) $3g - g^2$ (f) $n^2 - 4n$ (g) $7xr + 7xs$ (h) $3jk - 6jh$

(i) $12vw - 12w$ (j) $3d^2 + 8d$ (k) $9g^2 - 15ge$ (l) $2n^2 - n$

(m) $4a + 14a^2$ (n) $p - 2p^2$ (o) $3c^2 - 12dc$ (p) $16ab + 24b^2$

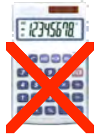
4. Completely factorise :-

(a) $x^2 + 3xw - 5x$ (b) $9ab - 9ac + a$ (c) $w^3 - w^2$ (d) $3m^3 - 15m$

(e) $x^2y + xy^2$ (f) $18de^2 - 24de$ (g) $4p^2 - 6pq$ (h) $\frac{1}{2}bc + \frac{1}{2}dc$

(i) $xy + \frac{1}{3}my^2$ (j) $20j^2z + 8jz^2$ (k) $ab - ac + a^2$ (l) $3g^2 - 12gh + 3g$

Algebra



Topic in a Nutshell

1. Simplify the following expressions :-

(a) $x + x$ (b) $a \times a$ (c) $9p \times p$ (d) $6p \times 7q$

(e) $6e + 3 - 5e$ (f) $x^2 + 7x^2$ (g) $x^2 \times 7x^2$ (h) $6 - 5m + 9m$

2. Simplify the following expressions then find their value when $a = 4$ and $b = 5$.

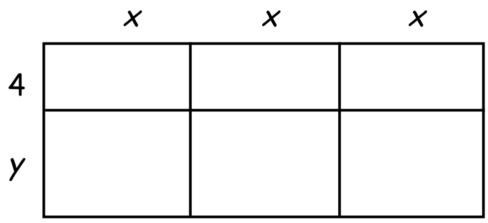
(a) $6a - 5b$ (b) $7a + b - a$ (c) $5b - 2a - 2b$

(d) $a^2 + b^2$ (e) $3b^2 - 10a - b^2$ (f) $(b - a)^2$

3. For the rectangular shape, write down an expression for the :-

(a) total perimeter

(b) total area



4. Remove the brackets :-

(a) $3(6 + 2x)$ (b) $8(5 - 6n)$ (c) $6v(4 + 8v)$ (d) $m(m - n + 4p)$

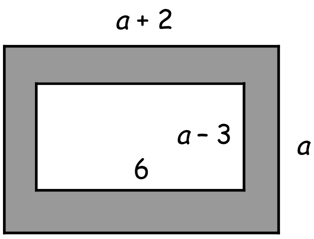
(e) $-3(2 + w)$ (f) $-u(1 + u)$ (g) $-a(2a - 3b)$ (h) $-2k(5k - 2p)$

5. Remove the brackets and simplify :-

(a) $2(x + 1) + 4$ (b) $6a + 4(a - 5)$ (c) $12 + 5(p - 2)$

(d) $3(c - 1) + 2(c + 4)$ (e) $9(x + 1) - 4(x - 1)$ (f) $7q - 2(2 - 2q)$

6. Find the shaded area in terms of a. (answer in sq. units)



7. Given $x = 2$, $y = 3$, $z = 5$ and $w = -3$, find :-

(a) $6y + 5w$ (b) $1 + 2y^2$ (c) $4x^2 + y^2 - 4w$

(d) $\frac{3}{4}(9y + z)$ (e) $\dfrac{y^2 - x^2}{z}$ (f) $\dfrac{7w^2 + 7}{xz}$

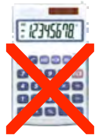
8. Factorise fully :-

(a) $6x - 24$ (b) $35w - 5$ (c) $20a + 25b$ (d) $12m + 18n$

(e) $12h - h^2$ (f) $6kp - 8pd$ (g) $4q^2 - 12qr$ (h) $6m^3 - 18m$

Chapter 13

Decimals



until
Question
15

Revision of Level E

Exercise 1

1. What does the 5 stand for in each of the following numbers :-

   (a) 159·496 (b) 2·057 (c) 0·513 (d) 215·729 (e) 6·075 ?

2. What is the number that is :-

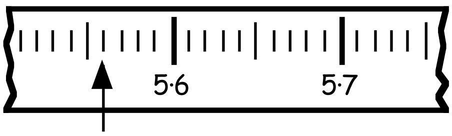
   (a) $\frac{3}{10}$ **up** from 5·8 (b) $\frac{7}{100}$ **down** from 8·23 (c) $\frac{9}{1000}$ **up** from 4·745 ?

3. What number lies **half way** between :-

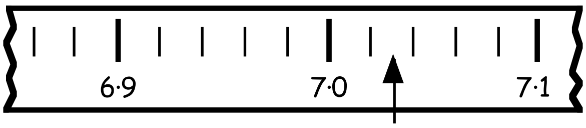
   (a) 0·13 and 0·25 (b) 0·1 and 0·04 (c) 0·03 and 0·005 ?

4. To which numbers are each of the following arrows pointing :-

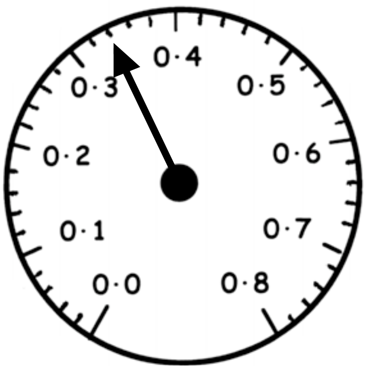
   (a)



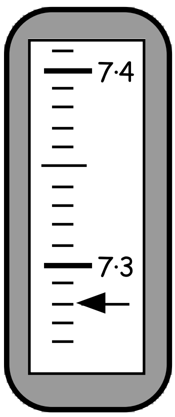
   (b)



   (c)

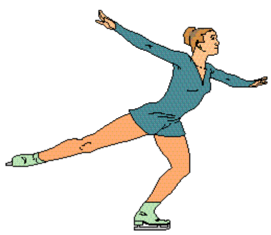


   (d)



5. Round these numbers to 1 decimal place :-

   (a) 4·218 (b) 7·384 (c) 30·192 (d) 0·752 (e) 33·974278.

6. 

   Three judges in a figure skating competition gave Narnia scores of 9·5, 8·9 and 9·7 out of 10.

   What was Narnia's **total** score from them ?

7. Gran Pollock lives 2·8 kilometres away from her local post office. A bus can take her to within a 0·19 km walking distance from it.

   What is the length of her bus journey ?

8. Set down and find :-

(a) 24·78 + 19·87 (b) 145·76 + 247·68 (c) 16·7 – 3·86 (d) 9 – 0·23.

9. Joyce weighs 62·84 kilograms and Shelley weighs 57·39 kilograms.

(a) What is their combined weight ?

(b) By how much is Shelley lighter than Joyce ?

10. Write the answers to :–

(a) 2·574 × 10 (b) 9·076 × 100 (c) 6·92 × 1000 (d) 2000 × 3·5

(e) 2·7 ÷ 10 (f) 1·2473 ÷ 100 (g) 364·5 ÷ 1000 (h) 2500 ÷ 5000.

11. (a) When 1000 paper clips are weighed, their total weight is 23·4 grams.

What is the weight of 1 paper clip ?

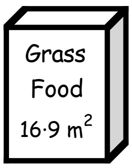
(b) One hundred people each win £35·62 in the lottery. What was the total pay out ?

12. Set down and find :-

(a) 8·7 × 6 (b) 19·3 × 7 (c) 3·96 × 8 (d) 12·65 × 9

(e) 10·2 ÷ 6 (f) 101·5 ÷ 7 (g) 261·6 ÷ 8 (h) 673·2 ÷ 9.

13. It says on the packet that the grass food will cover 16·9 m^2 of lawn.



How much lawn can be treated with 7 packets ?

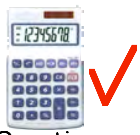
14. 

Rita is paid £54·96 for working 8 hours as a barmaid.

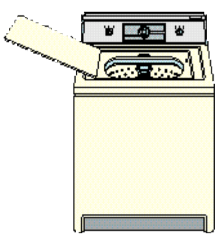
How does she earn per hour ?

15. William has an annual salary of £12 771·20 and is paid weekly.

How much does he earn per week ?



Questions 15, 16 & 17

16. 

Mrs Broadfoot paid a deposit of £65 when she bought a washing machine.

She then paid £15·20 per month for 24 months.

How much did the washing machine cost her altogether ?

17. MHV Music are selling packs of 25 blank CD's for £20. CD World are selling the same blank CD's in packs of 40 for £31·60.

Which shop is cheaper for the CD's and by how much per CD ?

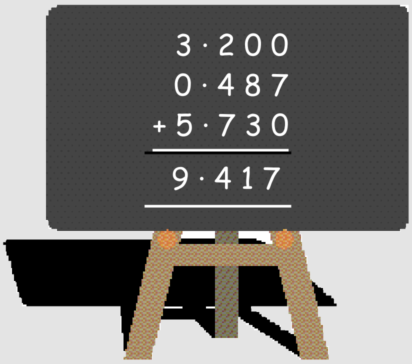
Addition & Subtraction of Decimals

When adding and subtracting decimals :-

- **Always keep the decimal points in line.**
- **Make use of "trailing 0's" when required.** e.g. for 3·5, use 3·500 when needed.

Example 1

3·2 + 0·487 + 5·73



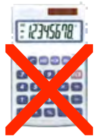
note that the decimal points stay in a line

Example 2

6·2 – 3·19 + 1·783

Add first	6 · 2 0 0
	+ 1 · 7 8 3
	7 · 9 8 3
then subtract	– 3 · 1 9 0
	4 · 7 9 3

Exercise 2



1. Set down and find :-

   (a) 23·42 + 35·89 (b) 42·63 – 5·94 (c) 24·327 + 1·38 (d) 36·2 – 7·195

2. Calculate :-

   (a) 1·7 + 2·43 + 4·587 (b) 0·68 + 0·976 + 5 (c) 11 + 9·65 + 0·098

   (d) 6·4 – 5·32 + 7·237 (e) 5 + 2·574 – 6·209 (f) 6·3 – 1·08 – 3·715

   (g) 27·2 – 8·67 + 15·9 (h) 35·24 + 4·219 – 9·337 (i) 20·3 – 7·428 – 8·06

3. When a length of iron rod is heated for a few minutes it expands by 0·004 metres.

   (a) If the length was 2·38 m to begin with, what was its new length after it was heated ?

   (b) A hot iron bar is 1·16 m long. When it cools it contracts by 0·003 m. What will its new size be ?

4. A pack of 3 Rostlers fast food products weighs 0·6 kg. The Barby-Q-Rib weighs 0·175 kg and the Chicken Burger weighs 0·243 kg.

   The third product is the Rostlers Hot Dog. What must it weigh ?

5. Three bottles of sauce are shown.

875·3 ml 620·85 ml 400·375 ml

   (a) What is the total volume in the 3 bottles ?

   (b) Compared with the medium bottle :-

      (i) How much more sauce does the large one hold ?

      (ii) How much less sauce does the small one hold ?

6. The temperature at the top of Ben Aldon was recorded as –23·7°C.
At the foot of the mountain the temperature was 4·5°C.

What was the difference in temperature from top to bottom ?

7. On Jan. 30th 2004, the temperature at noon in Aberdeen was –11·1°C.
On the same day, further south in Gretna the temperature at noon was –5·4°C.

(a) What was the difference in temperature at noon between both places ?

By 8 pm that day, the temperature in Aberdeen had risen by 1·8 degrees, whereas in Gretna the temperature had fallen by 0·5 degrees.

(b) What was the difference in temperature at 8 pm between Aberdeen and Gretna ?

8. Young Brian always keeps a note of what he saves and spends.

Shown is a page from his note book.

	Money IN	Money OUT	What I've Got
Week 1	£21·35		£21·35
Week 2		£19·29	....
Week 3	£5·16		....
Week 4		£10·35	....

(a) COPY and complete the table to show what Brian had at the end of each week.

(b) He found that he owed money at the end of week 4 !

How much money did his father give Brian to clear his debt **and** also pay for his £6·50 haircut ?

9. COPY the following and fill in the correct numbers to replace the ink blots :-

(a)
```
   6 · 8 ■ ■
 + ■ · ■ 7 3
   8 · 0 9 0
```

(b)
```
   9 · ■ 0 ■
 – ■ · 3 ■ 9
   5 · 0 8 7
```

(c)
```
    ■ · 1 7 ■
    3 · ■ 9 5
  + 2 · 6 ■ 9
  ■ 0 · 4 8 7
```

10. Try the following :-

(a) (-2·4) + 4·6
(b) (-5·6) + 2·7
(c) (-1·4) – 3·6
(d) 5·2 + 1·38 – 7
(e) (-3·25) + 6·155
(f) (-8·1) + 9·055
(g) 8·7 + 1·2 – 9
(h) (-2·25) + 6·55 – 12·15
(i) 3·75 – (-1·25)
(j) 8·26 – (-1·74)
(k) (-3·14) + (-0·66)
(l) (-1·07) – (-2·1)

Simple Multiplication of Decimals

When carrying out simple multiplications of decimals without the use of a calculator, there are many shortcuts which can be taken. Some of these are illustrated below.

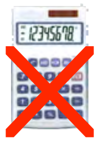
Example 1 $6 \times 0{\cdot}4 \Rightarrow 6 \times 4 = 24$ (& put in the decimal point <u>one</u> place in from the right) = <u>2·4</u>

Example 2 $50 \times 0{\cdot}3 \Rightarrow 100 \times 0{\cdot}3 = 30$ (& halve the answer) = <u>15</u>

OR $50 \times 0{\cdot}3 \Rightarrow 0{\cdot}3 \times 10 \times 5 = 3 \times 5 =$ <u>15</u>

Example 3 $0{\cdot}28 \times 5000 \Rightarrow 0{\cdot}28 \times 1000 = 280$ (& mult. the answer $\times$ 5) = <u>1400</u>

Exercise 3



In this exercise try to find the quickest way to do the multiplication.

Possibly, some of the questions could be done mentally !

1. Calculate :-

(a) $5 \times 0{\cdot}3$ (b) $9 \times 0{\cdot}6$ (c) $0{\cdot}4 \times 8$ (d) $0{\cdot}7 \times 7$

(e) $15 \times 0{\cdot}3$ (f) $22 \times 0{\cdot}6$ (g) $0{\cdot}4 \times 36$ (h) $0{\cdot}7 \times 51$

(i) $62 \times 0{\cdot}3$ (j) $110 \times 0{\cdot}5$ (k) $0{\cdot}8 \times 250$ (l) $0{\cdot}9 \times 530$

(m) $4 \times 0{\cdot}21$ (n) $8 \times 0{\cdot}54$ (o) $0{\cdot}23 \times 6$ (p) $0{\cdot}75 \times 9$

2. Calculate :-

(a) $60 \times 0{\cdot}3$ (b) $80 \times 0{\cdot}6$ (c) $0{\cdot}8 \times 90$ (d) $0{\cdot}6 \times 50$

(e) $300 \times 0{\cdot}4$ (f) $600 \times 0{\cdot}8$ (g) $0{\cdot}5 \times 400$ (h) $0{\cdot}9 \times 700$

(i) $4000 \times 0{\cdot}3$ (j) $8000 \times 0{\cdot}7$ (k) $0{\cdot}9 \times 5000$ (l) $0{\cdot}6 \times 9000$

3. A catering size jar of honey weighs 1·2 kg.

What would the following weigh :-

(a) 1000 jars (b) 30 jars

(c) 400 jars (d) 5000 jars ?

4. 

A mini-lollipop costs £0·08.

What is the cost of :-

(a) 5 lollipops (b) 60 lollipops

(c) 300 lollipops (d) 7000 lollipops ?

5. Try these :-

(a) $5 \times (-0{\cdot}4)$ (b) $9 \times (-0{\cdot}8)$ (c) $0{\cdot}4 \times (-5)$ (d) $0{\cdot}6 \times (-7)$

(e) $14 \times (-0{\cdot}3)$ (f) $(-20) \times (-0{\cdot}6)$ (g) $(-0{\cdot}4) \times (-80)$ (h) $(-0{\cdot}9) \times (-50)$

Simple Division of Decimals

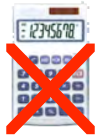
Similar methods can be found when dividing decimals without the use of a calculator.
There are many shortcuts which can be taken when dividing decimals. Here are a few :-

Example 1 6·3 ÷ 7 => 63 ÷ 7 = 9 (& put the decimal point <u>one</u> place in from the right) = <u>0·9</u>

Example 2 32 ÷ 40 => 32 ÷ 4 ÷ 10 = 8 ÷ 10 = <u>0·8</u>

Example 3 4·2 ÷ 600 => 4·2 ÷ 6 ÷ 100 = 0·7 ÷ 100 = <u>0·007</u>

Exercise 4



In this exercise try to find the quickest way to do the division.

Possibly even do some of it mentally !

1. Calculate :-

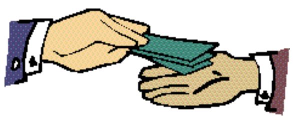
(a) 2·8 ÷ 2	(b) 4·5 ÷ 5	(c) 5·6 ÷ 8	(d) 8·1 ÷ 9
(e) 24 ÷ 30	(f) 54 ÷ 60	(g) 72 ÷ 80	(h) 36 ÷ 90
(i) 240 ÷ 600	(j) 450 ÷ 500	(k) 540 ÷ 900	(l) 280 ÷ 700
(m) 2·7 ÷ 300	(n) 3·5 ÷ 500	(o) 4·8 ÷ 800	(p) 1·8 ÷ 900

2. Calculate :-

(a) 3·6 ÷ 1000	(b) 3·6 ÷ 2000	(c) 3·6 ÷ 3000	(d) 3·6 ÷ 4000
(e) 7·5 ÷ 5000	(f) 6·4 ÷ 8000	(g) 8·1 ÷ 9000	(h) 4·9 ÷ 7000

3. What a disappointment ! Trevor won £120 in a prize draw, but it had to be shared amongst 300 people, including himself.

   How much did they each get ?



4. 

   When 400 drawing pins are weighed, their total weight is 140 grams.

   What is the weight of 1 drawing pin ?

5. 291 millilitres of a chemical is poured equally into 300 small phials.

   How much chemical should go into each phial ?

6. I walked 25·2 kilometres in 9 hours.

   How far had I travelled, on average, each hour ?

7. Try these :-

(a) (-9·6) ÷ 2	(b) 3·5 ÷ (-5)	(c) 6·4 ÷ (-8)	(d) (-4·5) ÷ 9
(e) 36 ÷ (-40)	(f) 54 ÷ (-90)	(g) (-42) ÷ 60	(h) (-5·6) ÷ (-800)

Multiplication of Decimals by a Single Decimal Digit

Again, watch out for shortcuts. Some of these are illustrated below.

Example 1 0·6 × 0·4

We know that 6 × 4 = 24. There are **2** digits after the decimal point in the question so there has to be **2** digits after the decimal point in the answer.

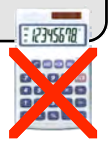
0·6 × 0·4 = 0·24

Example 2 0·09 × 0·3 (9 × 3 = 27; **3** digits after point in Qu; **3** digits after point in Ans.)

0·09 × 0·3 = 0·027

Example 3 0·071 × 0·5 (71 × 5 = 355; **4** digits after point in Qu; **4** digits after point in Ans.)

0·071 × 0·5 = 0·0355



Exercise 5

In this exercise try to find the quickest way to do the multiplication.

Possibly even do bits of it mentally !

1. Calculate :-

(a) 0·7 × 4 (b) 0·7 × 40 (c) 0·7 × 400 (d) 0·7 × 4000

(e) 0·7 × 0·4 (f) 0·07 × 0·4 (g) 0·007 × 0·4 (h) 0·0007 × 0·4

2. Calculate :-

(a) 0·7 × 0·6 (b) 0·9 × 0·2 (c) 0·4 × 0·4 (d) $(0·5)^2$

(e) 0·07 × 0·3 (f) 0·08 × 0·9 (g) 0·06 × 0·8 (h) 0·01 × 0·1

(i) 0·068 × 0·2 (j) 0·045 × 0·3 (k) 0·047 × 0·4 (l) 0·098 × 0·5

(m) 0·017 × 0·6 (n) 0·054 × 0·7 (o) 0·096 × 0·8 (p) 0·053 × 0·9

3. Calculate :-

(a) 0·09 × 40 000 (b) 300 × 0·000 000 4 (c) 0·2 × 0·3 × 0·4 (d) 0·5 × 0·6 × 0·7

(e) 20 × 0·9 × 0·3 (f) 40 × 0·1 × 600 (g) 0·2 × 50 × 0·3 (h) 0·6 × 500 × 0·1

4. James buys 300 chews at £0·09 each. What does this cost him ?

5. The forecast expected 2·05 centimetres of rain to fall every hour.

What depth of rain fell during the 30 minutes the storm actually lasted ?

6. Try these :-

(a) 0·03 × 0·02 (b) 0·07 × 0·04 (c) 0·05 × 0·09 (d) 0·04 × 0·08

(e) (–0·7) × 0·6 (f) 0·03 × (–0·02) (g) (–0·06) × (–0·01) (h) 0·007 × 0·002

Division of Decimals by a Single Decimal Digit

Do not attempt to divide by a decimal. Multiply to make the divisor a whole number.

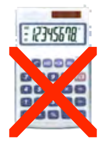
Example 1 3·5 ÷ 0·7 (**multiply the 3·5 and the 0·7 by 10**) = 35 ÷ 7 = 5

Example 2 0·8 ÷ 0·2 (**multiply the 0·8 and the 0·2 by 10**) = 8 ÷ 2 = 4

Example 3 0·036 ÷ 0·04 (**multiply the 0·036 and the 0·04 by 100**) = 3·6 ÷ 4 = 0·9

Example 4 Reminder ... 24 ÷ 3000 = 24 ÷ 3 ÷ 1000 = 8 ÷ 1000 = 0·008

Exercise 6



1. Find :-

(a) 8 ÷ 0·2 (b) 16 ÷ 0·4 (c) 25 ÷ 0·5 (d) 48 ÷ 0·6

(e) 56 ÷ 0·7 (f) 81 ÷ 0·9 (g) 100 ÷ 0·1 (h) 99 ÷ 0·9

2. Find :-

(a) 1·4 ÷ 0·7 (b) 2·6 ÷ 0·2 (c) 5·6 ÷ 0·8 (d) 5·4 ÷ 0·6

(e) 2·55 ÷ 0·5 (f) 9·24 ÷ 0·6 (g) 22·26 ÷ 0·7 (h) 37·36 ÷ 0·8

3. Calculate :-

(a) 8 ÷ 0·02 (b) 40 ÷ 0·08 (c) 4·2 ÷ 0·03 (d) 6·3 ÷ 0·07

(e) 0·024 ÷ 0·08 (f) 0·081 ÷ 0·09 (g) 0·005 ÷ 0·01 (h) 0·015 ÷ 0·05

4. Calculate :-

(a) 0·27 ÷ 0·003 (b) 0·64 ÷ 0·004 (c) 0·48 ÷ 0·006 (d) 0·035 ÷ 0·007

(e) 0·065 ÷ 0·005 (f) 0·008 ÷ 0·002 (g) 0·0153 ÷ 0·003 (h) 0·906 ÷ 0·006

5. Calculate :-

(a) 42 ÷ 60 (b) 18 ÷ 20 (c) 15 ÷ 500 (d) 12 ÷ 400

(e) 54 ÷ 900 (f) 32 ÷ 8000 (g) 210 ÷ 7000 (h) 350 ÷ 5000

6. 4000 floppy disks can store 6160 megabytes.

How many megabytes can be stored on one such disk ?

7. A small paint pen for colour testing holds 0·08 litres of paint.

How many pens can be filled from a drum which contains :-

(a) 1·6 litres (b) 40 litres (c) 100 litres (d) 0·72 litres ?

8. A box of 2000 large envelopes weighs 1·4 kg, not including the weight of the box itself.

Work out the weight of one envelope, (a) in kg's. (b) in grams.

9. Try these :-

(a) 10 ÷ 0·0002 (b) 50 ÷ 0·0005 (c) 3·33 ÷ 0·0003 (d) (–0·42) ÷ 0·7

(e) 0·18 ÷ (–0·6) (f) (–0·24) ÷ (–0·4) (g) 0·0001 ÷ 0·001 (h) 0·0005 ÷ 0·005

Rounding to Any Number of Decimal Places

When rounding to :-

- **1 Decimal Place** ..... look at the **2nd** decimal figure. e.g. $4\cdot3\underline{5}8$
- **2 Decimal Places** ..... look at the **3rd** decimal figure. e.g. $2\cdot46\underline{3}8$
- **3 Decimal Places** ..... look at the **4th** decimal figure. e.g. $9\cdot471\underline{2}3$
- **4 Decimal Places** ..... look at the **5th** decimal figure. e.g. $5\cdot3547\underline{9}13$

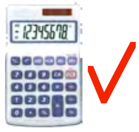
if the decimal figure is a 5, 6, 7, 8 or 9 => round the digit before that figure UP by 1.

if the decimal figure is a 0, 1, 2, 3 or 4 => leave the digit before it as it is.

Example :- Examine the decimal number 4·2615937

Rounded to **1** dec. pl.	$4\cdot2\underline{6}15937 = 4\cdot3$	(a "6", so round the "2" UP)
Rounded to **2** dec. pl.	$4\cdot26\underline{1}5937 = 4\cdot26$	(a "1", so leave the "6" alone)
Rounded to **3** dec. pl.	$4\cdot261\underline{5}937 = 4\cdot262$	(a "5", so round the "1" UP)
Rounded to **4** dec. pl.	$4\cdot2615\underline{9}37 = 4\cdot2616$	(a "9", so round the "5" UP)

Exercise 7



1. Round these numbers to 1 decimal place :-

   (a) 4·24 (b) 3·48 (c) 6·451 (d) 7·9923

2. Round these numbers to 2 decimal places :-

   (a) 7·583 (b) 9·627 (c) 3·985 12 (d) 5·3991

3. Round these numbers to 3 decimal places :-

   (a) 2·7923 (b) 7·5047 (c) 8·2987 (d) 25·4025

   (e) 31·457 12 (f) 6·080 82 (g) 34·999 12 (h) 2·999 99

4. Use your calculator to carry out these calculations correct to 2 decimal places :-

   (a) 3·157 + 7·998 (b) 26·9 × 37·84 (c) 14 ÷ 9 (d) 34·3 ÷ 72·4

5. Do these calculations and round your answer to the number of decimal places shown in the brackets :-

   (a) 4·67 × 0·358 (**3**) (b) 0·254 × 9·777 (**2**) (c) 8·847 × 2·584 (**3**)

   (d) 0·29 ÷ 4·145 (**4**) (e) 17·35 ÷ 19·887 (**1**) (f) 0·3 × 0·24 × 0·99 (**4**)

6. Quite often, taken in context, the rules of rounding do not apply.

   (a) Seven people share £6. How much does each receive ? (600 ÷ 7 = 85·71....p)

   By rule of rounding, each should get 86p ..... but 86p × 7 = £6·02 (more than £6 !)

   Taken in context, they cannot get 86p .... what is the most each can get ?

   (b) Share £8·20 amongst 6 people. How much can each get ?

Significant Figures

In mathematics, a figure or digit in a number is "significant" if it gives some sense of

Quantity & Accuracy.

"Zeros" can be complicated - when do we count them ? – when do we ignore them ?

If zeros are used only to show where the position of the decimal point is, then they are NOT significant.

Example 1

503 has 3 significant figures

50·3 has 3 significant figures

5·03 has 3 significant figures

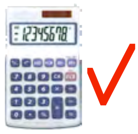
0·503 has 3 significant figures

0·05030 has 4 significant figures
(The front zero positions the decimal point, BUT trailing zero shows accuracy)

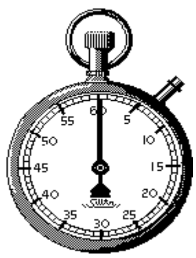
Example 2

6275 rounded to 1 significant figure is	=>	6000
28 432 rounded to 3 significant figures	=>	28400
3·419 rounded to 3 significant figures is	=>	3·42
0·005 387 rounded to 2 significant figures is	=>	0·0054

Exercise 8



1. How many significant figures does each number have in the following context :-
   (a) There are **300** pennies in £3.
   (b) The official attendance at the Renfrew v Linlithgow Rose match was **5000**.
   (c) The cost of a computer magazine is exactly £**3·90**.
   (d) There are **360°** in a complete turn.
   (e) The weight of a packet of sweets is about **1·3** kg.
   (f) There are **172 800** seconds in 2 days. (Need to check !)
   (g) The volume of a medium-size bottle of water is **500** ml.



2. Write down how many significant figures there are in each of these numbers :-

(a) 25·0	(b) 7·00	(c) 2·003	(d) 324
(e) 90·4	(f) 21·3320	(g) 19·20	(h) 0·214
(i) 0·027	(j) 6·000 003	(k) 0·010 10	(l) 0·000 50
(m) 20·000 20	(n) 178·000	(o) 0·000 003	(p) 0·000 000 30

3. Round each number to 1 significant figure :-

(a) 42 (b) 684 (c) 7249 (d) 87216

(e) 1299 (f) 1599 (g) 5·84 (h) 0·147

(i) 0·675 (j) 0·0034 (k) 0·000 684 (l) 39·99

4. Round each number to 2 significant figures :-

(a) 507 (b) 8129 (c) 40 800 (d) 481 124

(e) 27·46 (f) 37·55 (g) 6·371 (h) 0·132

(i) 0·449 (j) 0·006 594 (k) 0·043 32 (l) 29·712

5. Round each number to 3 significant figures :-

(a) 4872 (b) 63 094 (c) 84 961 (d) 781 399

(e) 7·213 (f) 12·817 (g) 0·287 45 (h) 0·287 51

(i) 0·005 687 (j) 0·010 693 (k) 0·046 93 (l) 0·039 999

6. Find the weight of a box of 250 blank C.D.s if each C.D. weighs 47 grams.
(Give your answer in grams to 2 significant figures)

7. What is the total volume, in millilitres, of 75 bottles of juice each containing 375 ml ?
(Give your answer in millilitres to 3 significant figures)

8. My bank interest for a year worked out at 2·8% of £23 450.

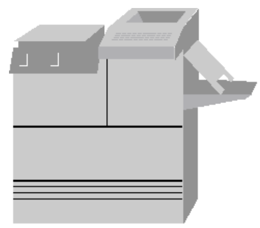
Calculate my interest, correct to 2 sig. figs.

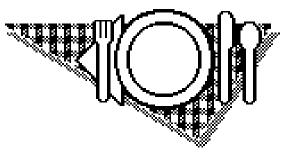
9. Jemma's garage bill came to £247·11 + VAT at 17·5%. (17·5 ÷ 100 x £247·11)

Calculate the VAT, correct to 4 sig. figs.

10. Calculate, correct to 3 sig. figs., the total price on this photocopier, priced at £1979 + VAT.

£1979
(+ VAT)



11. 

The total annual wage bill for the 17 employees at Roy's Cafe came to £364 786.

Calculate the average wage of each employee, to 2 sig. figs.

12. A jar of mustard weighs 0·487 kg.

Round the weight to 1 sig. fig. and **estimate** the total weight of 800 jars.

Everyday Applications with Money

We carry out decimal calculations every day when we use money.

Let us consider a few of these :-

Profit & Loss **Hire Purchase** **Salaries/Overtime** **Foreign Exchange**

Profit & Loss

If you buy a CD for £20 and sell it for £18·50 you are said to have "made a **LOSS** of £1·50".

If you buy a watch for £43 and sell it for £47 you are said to have "made a **PROFIT** of £4".

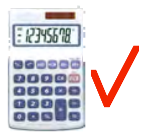
PROFIT	= Selling Price – Buying Price	(if selling price > buying price)
LOSS	= Buying Price – Selling Price	(if buying price > selling price)

Example

I bought a pair of boxing gloves for £87·55 and sold them to a friend for £79.

How much of a **loss** did I make ?

=> Loss = £87·55 – £79 = <u>£8·55</u>

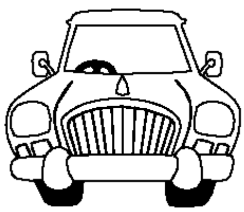


Exercise 9

1. Mr Scott bought a treadmill for £350, and a rowing machine for £195.

   One year later he sold the treadmill for £298·50 and the rowing machine for £125·75.

   How much of a loss did he make altogether ?

2. 

   I bought an old car for £1725.

   It cost me £36·50 for a new tyre and £21·75 for a new fan belt.

   (a) How much did I pay altogether ?

   (b) If I then sold the car for £1820, how much profit did I make ?

3. Simon bought 360 blank C.D.'s for £108.

   He packed them in boxes holding 30 discs each and sold each box for £10·60.

   (a) How many packs of 30 C.D.'s did he make up ?

   (b) How much money did he make if he sold all the packs ?

   (c) How much profit did he make altogether ?

Hire Purchase

Example

Hoover Cash Price £365·50 or by H.P. £75 deposit + 12 Payments at £28·80.

Calculate (a) the total H.P. price of the Hoover.

(b) the difference between the cash price and the H.P. price.

(a) Total H.P. price = £75 + 12 × £28·80 = <u>£420·60</u>

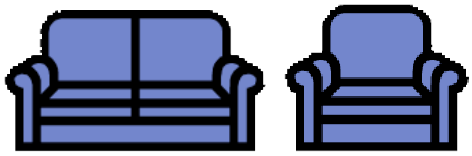
(b) Difference = £420·60 – £365·50 = <u>£55·10</u>

4. I bought a new suite from "SFD".

I paid a deposit of £475 and followed this with 24 monthly payments of £140·50.

(a) Calculate how much I paid in total using the Hire Purchase method.

(b) How much cheaper would it have been if I had paid cash ?



£3250 Cash

5. 

£1599·99 Cash

A greenkeeper bought a bunker-raker from "Golf Machines".
He took out a Hire Purchase agreement.

The deposit was £245 followed by 36 monthly payments of £50·75.

(a) How much did it cost altogether for the machine using H.P. ?

(b) How much more was this than the cash price ?

6. Joe and Gail bought a Coolpoint tumble drier for £284·90 from "Drier Electrics".

(a) How much of a deposit had they to pay ?

(b) What were their monthly repayments ? (careful !)

DRIER ELECTRICS
deposit – £30·00
pay balance
back in 15 months
– no extra cost

7. 

Price — £12 200

Eric recently bought a new Rover 75.
He paid a deposit of £2750 and signed an agreement to pay the car up over 48 months

His first monthly payment was £320·50.
The next 46 payments were for £285·75 each
The final (48th) payment was to be £315·50.

(a) How much will it cost Eric altogether using the above Hire Purchase scheme ?

(b) If he had decided to pay cash, the car dealer would have offered a 5% discount.

How much would Eric have saved paying cash rather than taking out the hire purchase deal ?

Salaries/Overtime

Example 1

Henry the mechanic gets paid £1725·20 per month.

What is his annual salary ?

Salary = 12 × £1725·20
= <u>£20 702·40</u>

Example 2

Sandra, a clerkess receives an annual salary of £19 003·40.

Calculate her weekly pay.

Weekly Pay = £19 003·40 ÷ 52
= <u>£365·45</u>

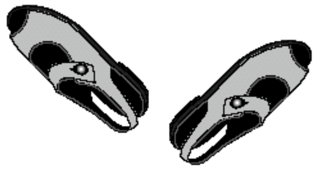
Example 3

Percy works for £12·90 per hour. His overtime rate is "time & a half".
What does he get paid for 4 hours overtime ?

Overtime = 4 × £12·90 × 1·5
= <u>£77·40</u>

8. David works as a shoe salesman. He gets paid £6·85 per hour.
Last week he kept the shop open later, so worked a total of 52 hours.

What was his pay for that week ?



9. 

Joan and Mandy are paid £8·15 an hour at the call centre.

(a) How much would Joan earn if she worked 48 hours ?

(b) Mandy only worked 41 hours. How much did she earn ?

(c) How much **MORE** did Joan earn than Mandy ?

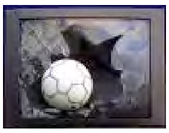
10. Jan is a dental assistant and earned £235·50 last week for working 25 hours.

Don is a joiner and earned £395·85 for working 39 hours last week.

Calculate the hourly rate of both and say who has the better <u>rate of pay</u>.

11. Alan is a sales assistant for an electrical company. He is paid £845·99 per month.

Calculate Alan's annual pay.

12. 

Jodie is a T.V. repair lady and earns a fixed salary of £18 436·60 per year.

Calculate Jodie's weekly wage.

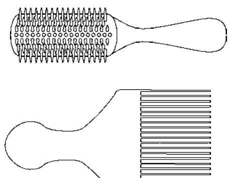
13. George is a labourer who is paid a basic rate of £7·30 per hour.

On Thursday night, he worked 6 hours overtime
for which he was paid <u>**double**</u> time.

(a) Calculate George's overtime hourly rate.

(b) How much did George earn in total for his overtime hours ?

14. Rashelle is a junior hairdresser and is paid £4·90 per hour.
Last month she worked a total of 10 hours overtime at <u>**time and a half**</u>.



(a) Calculate Rashelle's overtime rate of pay.

(b) Calculate how much she earned altogether for her 10 hours overtime.

Foreign Exchange

(Euros (€) are now widely used throughout Europe)

Examples :-

(a) To change £80 into euros
simply **MULTIPLY** :- 80 × 1·54
= <u>123·20€</u>

(b) To change 616€ into £'s
simply **DIVIDE** :- 616 ÷ 1·54
= <u>£400</u>

Exchange Rates (Jan 2003)
£1 = 1·54€ (All Europe)
£1 = $1·40 (America)
£1 = 2·42 (Australian Dollars)

15. Change :-

(a) £300 to euros
(b) £42 to euros
(c) £480 to American dollars
(d) £250 to Australian dollars
(e) £850 to euros
(f) £1240 to American dollars

16. How much would be given when the following amounts were exchanged for £'s :-

(a) 924 euros
(b) 115·50 euros
(c) $672
(d) $7·28
(e) 726 Australian dollars
(f) 2904 Australian dollars ?

17. I brought 847 Australian dollars back from holiday.
How many £'s will I receive for them ?

18. 

The Emery's flew to New York.
and changed £1280 into dollars.
How many dollars did they get ?

19. In the UK, a certain laptop costs £1570.
In Italy, the same computer costs 2002 euros .

(a) If I bought the laptop in Italy, how much would the equivalent cost be in £'s ?
(b) How much would I have saved in £'s if I had bought the laptop in Italy ?

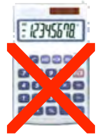
20. The Lawson family flew to Venice for a two week stay.
They exchanged £1500 to euros at the above rate.

(a) How many euros did they receive ?

Altogether, the Lawsons spent 1950 euros in Venice.

(b) If the exchange rate had changed to £1 = 1·62 euros, how much would they expect to receive in £'s when they changed it back ?

Decimals



(until qu. 16 - 19)

Topic in a Nutshell

1. Work out :-

(a) 28·92 + 37·57 (b) 54·35 – 9·87 (c) 14 + 9·78 + 0·047

(d) 11·4 – 8·32 + 24·274 (e) 9 + 2·278 – 10·509 (f) 33·1 – 7·247 – 15·08

2. A pack of 3 sirloin steaks weighs 2·1 kg.
One steak weighs 0·775 kg, another weighs 0·768 kg.

The third steak is a small one. What does it weigh ?

3. The temperature at the top of the "Rest and Be Thankful" was recorded as –18·2°C.
In the village at the foot of the mountain the temperature was 1·9°C.

What was the difference in temperature from the top of the hill to the village ?

4. Find :-

(a) (-3·4) + 7·1 (b) (-5·4) – 2·9 (c) 6·55 – (-4·45) (d) (-4·1) + (-3·26)

5. Calculate :-

(a) 0·4 × 26 (b) 80 × 0·7 (c) 0·8 × 300 (d) 0·6 × 5000

(e) 9 × (-0·3) (f) 0·5 × (-8) (g) (-0·6) × (-40) (h) (-0·7) × (-90)

6.

A colouring pencil costs £0·07.

What is the cost of :-

(a) 5 pencils (b) 70 pencils

(c) 400 pencils (d) 9000 pencils ?

7. Find :-

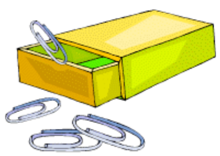
(a) 5·5 ÷ 5 (b) 63 ÷ 90 (c) 1·4 ÷ 700 (d) 4·8 ÷ 6000

(e) (-3·6) ÷ 2 (f) 5·6 ÷ (-7) (g) (-45) ÷ (-50) (h) (-4·5) ÷ (-300)

(i) 0·9 × 20 (j) 0·007 × 0·6 (k) 0·005 × 0·9 (l) 0·065 × 0·5

(m) 40 × 0·8 × 0·2 (n) 30 × 0·1 × 800 (o) 0·4 × 50 × 0·9 (p) $0{\cdot}1 \times (0{\cdot}4)^2$

8.



When 600 paper clips are weighed, their total weight is 150 grams.

What is the weight of 1 paper clip ?

9. Joan buys 400 caramels at £0·06 each. What does this cost her ?

10. Calculate :-

(a) $72 \div 0{\cdot}8$ (b) $5{\cdot}8 \div 0{\cdot}2$ (c) $5{\cdot}1 \div 0{\cdot}03$ (d) $0{\cdot}008 \div 0{\cdot}01$

(e) $0{\cdot}24 \div 0{\cdot}006$ (f) $0{\cdot}003 \div 0{\cdot}01$ (g) $12 \div 20$ (h) $480 \div 6000$

(i) $15 \div 0{\cdot}0003$ (j) $0{\cdot}0008 \div 0{\cdot}008$ (k) $0{\cdot}56 \div (-0{\cdot}7)$ (l) $(-0{\cdot}49) \div (-0{\cdot}7)$

11. A box of 5000 sticky labels weighs 1·5 kg, not including the weight of the box itself.
Work out the weight of one sticky label, (a) in kg's. (b) in grams.

12. Round :-

(a) 7·58 to 1 decimal place. (b) 3·9816 to 2 decimal places.

(c) 54·2287 to 3 decimal places. (d) 2·145 434 to 4 decimal places.

13. Share £5·40 amongst 8 people. How much can each get ?

14. Write down how many significant figures there are in each of these numbers :-

(a) 37·0 (b) 0·045 (c) 4·000 009 (d) 0·000 10

15. Round :-

(a) 1720 to 1 significant figure. (b) 0·001 43 to 2 significant figures.

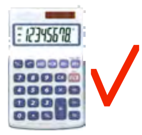
(c) 57 962 to 3 significant figures. (d) 0·049 999 to 3 significant figures.

---

16. 

£35 700 + VAT

Correct to 3 sig. figs., work out the VAT (17·5%) on this truck, priced £35 700.



(qu. 16 - 19)

17. When I bought new carpets for my house
I paid a deposit of £475 and followed this with
36 monthly payments of £124·75.

(a) Calculate how much I paid in total using the Hire Purchase method.

(b) How much cheaper would it have been if I had paid the cash price of £3999 ?

18. Butch is a milkman and is paid £6·85 per hour.
Last month he worked a total of 20 hours overtime
at **time and a half**.

(a) Calculate Butch's overtime rate of pay.

(b) Calculate how much he earned altogether for his 20 hours overtime.

19. I brought 119·32 euros back from holiday.
How many £'s will I receive for them with the exchange rate at 1·52 euros to the £ ?

Chapter 17

Statistics

Averages

Remember :

The **Mean** = $\dfrac{\text{total of all the scores}}{\text{number of scores}}$

If we look at this set of data : 1, 1, 1, 1, 2, 3, 26

=> The **mean** would be $\dfrac{1+1+1+1+2+3+26}{7} = \dfrac{35}{7} = 5$

Can you see that this is not the most suitable of averages since five out of the six numbers are all below the mean of 5 ?

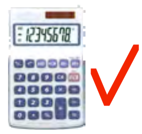
Any **average** should indicate a "measure of central tendency" but should also indicate what the distribution of data looks like.

This is why we have **three** different types of averages to consider.

1. The **Mean** (total of all the scores ÷ number of scores).
2. The **Median** (put the data in order then use the middle number).
3. The **Mode** (the number that appears most often).

Can you see that for the above data the median (= 1) or mode (= 1) are better averages ?

Exercise 1



1. Calculate the **mean** for each set of data :-

(a) 1, 2, 3, 4, 5, 6, 7, 8, 9

(b) 3, 4, 7, 8, 8, 13, 17, 20

(c) 11, 12, 14, 17, 17, 19

(d) 0·1, 0·2, 0·4, 0·5, 0·7, 0·7, 0·9

(e) 21, 23, 23, 26, 36, 81

(f) 12, 17, 9, 16, 22, 8, 17, 11, 12, 3

2. Find the **median** for each set of data :- (Remember to put the numbers in order first)

(a) 5, 8, 4, 2, 1, 6, 3, 9, 7

(b) 11, 21, 14, 16, 27, 9, 15

(c) 1·6, 2·2, 1·3, 2·4, 1, 1·7, 2·2

(d) 142, 153, 96, 204, 175, 150, 188

If there is not a single middle number, take the **mean of the middle two numbers**.

Example : 1, 1, 3, 4, 5, 6, 7, 9

The **median** is (4 + 5) ÷ 2 = **4·5**

3. Find the **median** for the following :-

(a) 4, 11, 7, 8, 12, 7

(b) 4, 6, 7, 15, 3, 17, 12, 8, 10, 9

(c) 11, 7, 8, 6, 4, 7, 3, 10

(d) 1·3, 1·4, 0·8, 1·7, 2·3, 1·6, 0·9, 1

4. Find the **mode** for each set of data :-

(a) 1, 2, 3, 4, 5, 6, 7, 7, 8

(b) 11, 22, 13, 54, 11, 13, 31, 10, 13

(c) 1·7, 2·1, 2·3, 1·4, 2·1, 6·0, 2·8

(d) 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1

(e) 131, 210, 113, 124, 21, 120, 124

(f) $\frac{3}{4}, \frac{1}{4}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{1}{4}, \frac{3}{4}$

The **RANGE** is a mathematical tool used to measure how widely spread a set of numbers are.

=> **Range = highest score – lowest score**

Example :- For the data set, 2, 2, 3, 5, 7, 7, 8, 10, 12, 12 => range = 12 – 2 = **10**.

5. Calculate the **range** for each set of data in :- (a) question 3 (b) question 4.

6. Look at this data set :-

5, 6, 1, 7, 9, 1, 2, 3, 56

(a) Find the **range**.

(b) Find the **mean**, **median** and **mode**.

(c) Which average is best suited to this data set.

(d) Explain why you think the other two averages are less suitable.

7. Calculate the **mean**, **median**, **mode** and **range** of each data set below : –

(a) 1, 2, 2, 2, 4, 8, 16

(b) 5·6, 2·2, 4·3, 4·3, 5·0, 4·3, 3·7

(c) 107, 106, 93, 114, 106, 98

(d) 30, 32, 23, 41, 55, 36, 27, 30

(e) 15, 15, 13, 14, 17, 16, 17, 17

(f) 15 000, 12 000, 17 000, 12 000, 21 000.

8. The weights of six children are shown :-

40 kg 50 kg 63 kg 40 kg 47 kg 49 kg.

(a) Find the **range** of their weights.

(b) Calculate the **mode** and **median** weights.

(c) Choose which is the better average of the two and explain why.

9. Rory buys 10 Easter Eggs. The number of chocolates in each is listed below :-

8, 7, 9, 6, 8, 7, 8, 11, 5, 9

(a) Calculate the **mean**, **median** and **mode**.

(b) How many eggs have **less than** the mean number of chocolates ?

10. (a) Calculate the **mean** of the first 10 **prime** numbers.

(b) Calculate the **mean** of the first ten **square** numbers.

11. The weights of six children are shown opposite.

24 kg,	24 kg,	33 kg,
35 kg,	40 kg,	42 kg

Bob says, " the average weight is 24 kg."
Bill says, " the average weight is 33 kg."
Ben says, " the average weight is 34 kg"

(a) Explain why, technically, all three could be correct.

(b) Which of the three would be least likely to be used ?

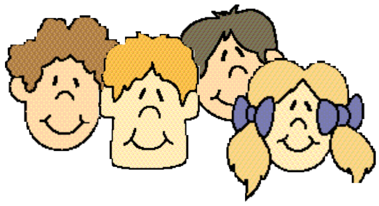
12. 

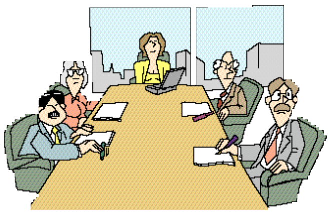
The mean weight of two crates is 26 kilograms.

If one of the crates weighs 19 kg, what must the weight of the other crate be ?

13. The mean age of four children is 13 years old.
Three of the childrens' ages are 9, 12 and 16.

What is the age of the fourth child ?



14. 

A group of five people at a meeting have a mean age of 32 years.

When a sixth person joins the meeting, the mean age then increases to 35 years.

What is the age of that sixth person ?

15. The contents of ten boxes of marbles are examined.

The boxes contain the following marbles : -

15, 17, 13, 16, 14, 15, 14, 14, 17, 15.


mean number - 16 per box

(a) Why is the manufacturer's claim incorrect ?

(b) An eleventh box is examined. How many marbles would need to be in that box in order for the manufacturer's claim to **then** be considered to be correct ?

16. 

At an archery contest the mean score for the first nine contestants was 27.

Contestant number ten pushed the mean score up to 29.

What must contestant number ten have scored ?

17. At a bowling alley the mean score of the six children was 127.
The mean score of the five adults with them was 139.

Calculate the mean score of the whole eleven in the group.

18. 

Megan's dad will give her £50 if she can get a mean score of at least 75% for her five Maths tests this year.
In her first four tests Megan scored : 71%, 66% 82% and 54%.

Can Megan possibly do well enough to get the £50 ? (Explain !!)

Class Intervals

When a set of data is large, the numbers have to be grouped into "**class intervals**."

- Each interval must have the same **number** of values.
- Ideally, there should be between 6 and 10 intervals.

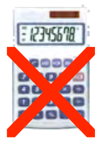
Example : – The test scores of a group are to be entered into a frequency table. (The first 6 have been done)

~~12~~	~~23~~	~~41~~	~~55~~	~~77~~	~~15~~	32	40
51	69	21	12	16	43	56	71
32	75	34	42	55	76	21	73
22	56	41	19	20	47	78	17

Can you see that there are 7 **class intervals** and each interval contains 10 numbers ?

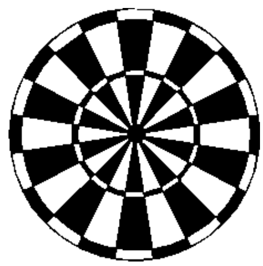
Class Intervals	Tally	Frequency
10 - 19	II	
20 - 29	I	
30 - 39		
40 - 49	I	
50 - 59	I	
60 - 69		
70 - 79	I	

Exercise 2



1. (a) Copy and complete the frequency table above.
   (b) How many students scored over 49 ?
   (c) Draw a neat labelled **bar graph** to show this information.

2. Each number below shows the score of 3 darts thrown by each member of class $1A_3$.



15	13	31	42	64	34	32	20	11	8	21
55	19	51	45	64	35	75	50	46	55	67
21	33	12	6	40	79	76	47	29	10	15

   (a) How many numbers are in each interval ?
   (b) How many intervals will there be in the table ?
   (c) Copy and complete the table.
   (d) How many pupils are in class $1A_3$?
   (e) How many pupils scored under 30 ?
   (f) Draw a neat labelled bar graph showing this information.

Class Intervals	Tally	Frequency
0 - 9		
10 - 19		
20 - 29		
30 - 39		
40 -		

3. The number of pets in each class in a school is shown below.

1	14	8	27	16	7	12	15	21	20	17	0	11	15	10
12	14	4	5	10	14	11	9	19	15	21	13	4	11	16

   Show this information on a frequency table. (*Use class intervals of 0 - 4, 5 - 9, 10 - 14, etc*)

4. A class were asked to tidy their bedrooms and say how many coins they found !

The number of coins found by each pupil is shown.

4	3	18	15	31	9	0	2
11	6	27	15	12	11	15	4
22	15	16	26	25	17	13	3
9	7	1	9	16	7	21	10
12	20	1	14	19	3	0	12

(a) Find the range.

(b) Which of these would be the best class interval to start with : –

(0 - 9) or (0 - 3) or (0 - 4) or (0 - 2) ?

(c) Construct a frequency table using your chosen class interval.

(d) Draw a neat labelled bar graph to show this information.

5. A list of waiting times (in minutes) in a doctors surgery are shown.

0	4	22	11	11	19	10	12
5	8	26	25	15	17	18	2
20	13	19	21	22	13	23	13
8	9	1	6	26	8	18	10
14	10	3	24	17	5	3	22

(a) Find the range.

(b) Which of these would be the best class interval to use : –

(0 - 9) or (0 - 1) or (0 - 4) or (0 - 3) ?

(c) Construct a frequency table showing this information.

6. For each table below, construct a frequency table using an appropriate class interval.

(a)

14	13	18	15	11	9	4	1
15	34	32	25	12	16	15	14
9	15	18	25	25	19	14	3
9	8	2	7	16	27	23	20
22	20	11	13	16	30	4	22

(b)

10	35	28	45	71	69	50	42
11	36	27	15	62	72	65	54
42	35	26	16	25	37	43	53
69	52	47	31	29	19	47	31
20	12	60	51	24	49	43	40

(c)

127	152	163	174	101	133	167	155	171	110	117	129
111	134	125	164	115	122	150	160	129	144	141	153
130	128	166	154	122	169	140	151	163	162	100	174

(d)

3·6	2·3	4·6	1·7	5·6	4·2	1·1	4·0	5·2	6·3	6·9	4·1
2·5	2·8	1·3	2·5	6·6	5·1	1·4	4·6	2·2	3·3	5·1	0·4
5·0	2·9	4·3	2·1	5·4	4·6	5·3	6·1	2·2	5·7	5·8	1·3

Mean from a Frequency Table

When given a frequency table, adding a third column will help us find the total number of items and the **mean**.

This table shows the number of coins in the pockets of some children.

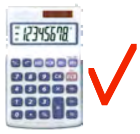
=> Mean number of coins $= \frac{40}{16} = \mathbf{2 \cdot 5}$

Each pupil has an "average" of **2·5** coins.

No. of coins (x)	Freq (f)	f × x
1	5	1 × 5 = 5
2	5	2 × 5 = 10
3	1	3 × 1 = 3
4	3	4 × 3 = 12
5	2	5 × 2 = 10
TOTALS	**16**	**40**

Total pupils Total coins

Exercise 3



1. This table shows the results from a group of students who were asked how many pens they carried to college.

(a) Copy and complete the table.

(b) How many students were asked ?

(c) How many pens in total were there ?

(d) Calculate the mean number of pens.

No. of pens (x)	Freq (f)	f × x
0	1	0 × 1 = 0
1	7	1 × 7 = ...
2	12	2 × .. = ...
3	5	.. × .. = ...
4	5	.. × .. = ...
	...	...

2. The table shows the number of goals scored by a school football team each week.

(a) Copy and complete the frequency table.

(b) Find the total number of games.

(c) Find the total number of goals scored.

(d) Calculate the mean number of goals.

No. of goals (x)	Freq (f)	f × x
0	4	
1	6	
2	10	
3	3	
4	2	
	...	...

3. Copy and complete each of the following tables, add a third column and calculate the mean.

(a)

No. of cars (x)	Freq (f)
1	7
2	10
3	5
4	2
5	6

(b)

No. of sides (x)	Freq (f)
3	2
4	8
5	3
6	5
7	1
8	1

(c)

No. of runs (x)	Freq (f)
5	2
6	7
7	3
8	5
9	1
10	1
11	0
12	1

4. Look at the tables in question 3.

Question (a) has range (5 – 1) = 4. Question (b) has range (8 – 3) = 5.

Find the range for 3 (c).

5. Shown are the test scores of class $1A_2$.

(a) How many pupils are in class $1A_2$?

(b) Find the range of scores.

(c) Find the mean score for class $1A_2$.

(d) Can you find the median from this table ?
(Hint : it is the middle number from 10, 12, 12, 12, 12, 14...).

Test score (x)	Freq (f)
10	1
12	4
14	10
16	5
18	5

6. A group of 18 year old girls were asked how old they were when they went out on their first "date".

The results are shown in this bar graph.

(a) Form a frequency table from the information in the bar graph.

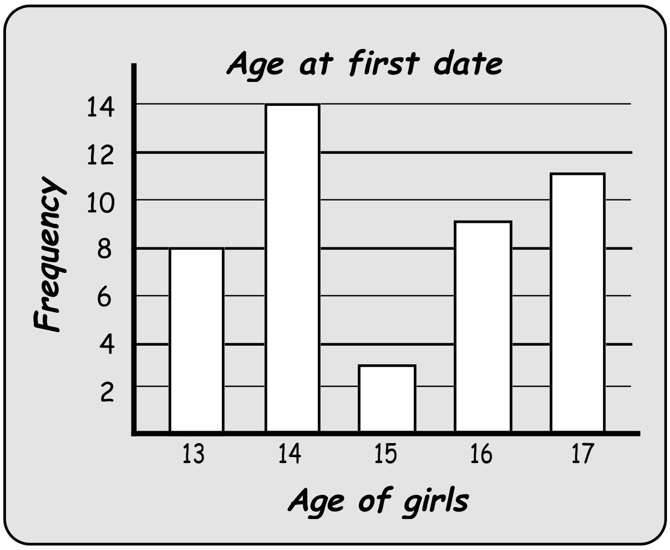
(b) Calculate the : –

(i) mode

(ii) range

(iii) mean

(iv) median.



Cumulative Frequency Tables

This frequency table shows the number of eggs laid by a clutch of chickens each day over a seven day period.

A third column has been added to keep a running total.

This makes it easier to get the total number of items.

Other information can be more easily obtained from this column.

e.g. 12 eggs had been collected by day four.
(The **cumulative frequency** on day 4 is 12).

Day	Frequency (no. collected)	Cumulative freq. (total so
far)		
1	2	2
2	3	5
3	1	6
4	6	12
5	5	17
6	8	25

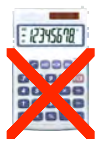
(2 + 3)

(5 + 1)

Median :- If 29 eggs were collected altogether, then the 15th egg must be the **median**.
(14 eggs either side of this 15th egg).

=> The 15th egg (median) was collected on day 5.

Exercise 4



1. A hospital noted the number of cases of a specific viral infection.

   The results are shown in the frequency table.

   (a) Copy and complete the table.

   (b) How many patients in total were there ?

   (c) How many patients had been infected by the end of week 5 ?

   (d) Which week was the infection at its worst ?

   (e) Find the median.

Week	Frequency (no.of cases)	Cumulative freq. (total so far)
1	4	4
2	9	13
3	11	...
4	24	...
5	16	...
6	7	...
7	2	...

2. For each of the frequency tables below :-

   (i) add a cumulative frequency column (ii) find the median.

(a)

Pets	Frequency
0	2
1	11
2	17
3	8
4	4
5	2
6	1

(b)

Grade	Frequency
0	1
1	3
2	4
3	10
4	21
5	7
6	4

(c)

No.	Frequency
10	7
11	7
12	10
13	20
14	15
15	20
16	5

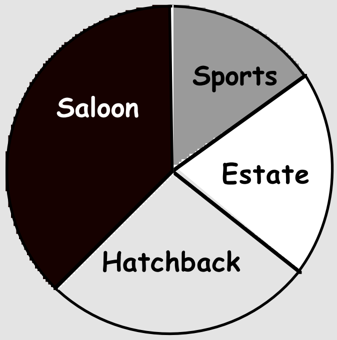
Pie Charts

The table of data shows the number of different vehicles bought from a car showroom.

Type of Car	Number
Saloon	34
Hatchback	24
Estate	18
Sports	14

When drawing a pie chart, it is sometimes easier to add columns to the table for calculations.

Type of Car	Number	Fraction	Angle
Saloon	34	$\frac{34}{90}$	$\frac{34}{90} \times 360 = 136°$
Hatchback	24	$\frac{24}{90}$	$\frac{24}{90} \times 360 = 96°$
Estate	18	$\frac{18}{90}$	$\frac{18}{90} \times 360 = 72°$
Sports	14	$\frac{14}{90}$	$\frac{14}{90} \times 360 = 56°$
TOTAL	**<u>90</u>**	**1**	**360°**



- step 1 is to add all the "numbers" together to get a total (90).
- step 2 is to express each "number" as a fraction of this total. (e.g. $\frac{34}{90}$).
- step 3 is to find that fraction of 360° each time (e.g. $\frac{34}{90} \times 360 = 136°$).
- step 4 is now to draw the pie chart using the angles in the table and a protractor.

Exercise 5

1. (a) Copy and complete the table showing a group of 180 people's favourite season.

   (b) Construct a pie chart using a compass, a protractor and the table information.

Season	Number	Fraction	Angle
Spring	20	$\frac{20}{180}$	$\frac{20}{180} \times 360 = 40°$
Summer	90	$\frac{90}{180}$	$\frac{90}{180} \times 360 = ...°$
Autumn	10	$\frac{....}{180}$	$\frac{....}{180} \times 360 = ...°$
Winter	60	$\frac{....}{180}$	$\frac{....}{180} \times 360 = ...°$
TOTAL	**<u>180</u>**	**1**	**360°**

2. (a) Copy and complete the table showing the number of grades a class obtained in their last test.

   (b) Construct an accurate pie chart showing this information.

Grades	Number	Fraction	Angle
A	5	$\frac{5}{45}$	$\frac{5}{45} \times 360 = 40°$
B	20	$\frac{20}{45}$	$\frac{20}{45} \times 360 = ...°$
C	18	$\frac{....}{45}$	$\frac{....}{45} \times 360 = ...°$
D	2	$\frac{....}{45}$	$\frac{....}{45} \times 360 = ...°$
TOTAL	**<u>45</u>**	**1**	**360°**

3. (a) Copy and complete the table showing a class's eye colour.

Eye colour	Number	Fraction	Angle
Brown	10	$\frac{10}{30}$	$\frac{10}{30} \times 360 = ...°$
Blue	12		$\times 360 = ...°$
Green	7		$\times 360 = ...°$
Grey	1		$\times 360 = ...°$
TOTAL	**30**		**360°**

(b) Construct an accurate pie chart showing this information.

4. For each table below, copy each table (add new columns to show your working) then construct an accurate pie chart to show the information.

(a)

Favourite sport	Number
Football	36
Tennis	20
snooker	4
Netball	12
TOTAL	....

(b)

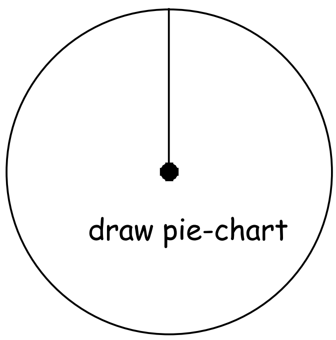
People's ages	Number
10 - 19	400
20 - 29	240
30 - 39	70
40 - 49	10
TOTAL	....

5. The table shows the results of a survey asking people's favourite holiday destination.

America	France	Italy	Spain	Italy	Spain	France	America
Spain	Italy	France	Spain	France	Spain	Italy	Spain
France	Spain	U.K	America	France	U.K	Spain	Spain
Italy	France	U.K	Spain	Spain	America	U.K	Italy
France	Spain	Spain	Spain	France	Spain	France	America

(a) Copy and complete the table below :-
(add any columns you might need to help you make a pie chart).

Destination	*Tally Mark*	*Number*
America		
France		
Italy		
Spain		
U.K		



(b) Construct an accurate pie chart for this information.

Stem-and-leaf Graphs

A stem and leaf graph is another way of displaying information.

This stem and leaf graph shows the ages of people waiting in a queue at a post office.

The key explains what each number in the graph represents.

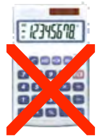
The first line reads 24, 26 and 28 years of age.

Age in Years

stem	leaves
2	4 6 8
3	0 1 3
4	4 4 5 6 7 9
5	0 3 4 9
6	1 4 5 6

Key :-
2 | 4 means 24

Exercise 6



1. The 2nd line of the above graph reads 30, 31 and 33 years of age.
   (a) Write the ages given by each line in the graph above.
   (b) (i) What age was the youngest person in the queue ?
   (ii) What age was the oldest person in the queue ?
   (c) How many people were in the queue ?

2. The ages of a group of people waiting in a queue at a bank were recorded and put into the stem and leaf graph shown.
   (a) The first line (level 2) reads 21 years, 22 years, 24 years and 27 years.
   Write out the ages in level 3.
   (b) Write out the ages of level 4.
   (c) What age was the :-
   (i) youngest person (ii) oldest person ?
   (d) Were most of the people in their 20's, 30's or 40's ?

Key :
2 | 4 means 24

Age in years

2	1 2 4 7
3	1 7 9
4	2 2 3 6 8 8
5	3 3 3 5
6	0 8

3. Some pupils were asked how much money they had. The results are shown in the stem and leaf graph.
   (a) List the amount of money each pupil had.
   (b) Which level has the most data ?
   (c) Which amount of money appears the most often (mode) ?
   (d) How many pupils were asked in the survey ?

Key :
1 | 2 means £1·20

Pupil's money

1	1 2 5 7 8
2	0 1 2 6
3	4 4 4 5 8 8
4	9
5	0 3

4. The table shows the time it took in seconds for a puzzle to be solved by some students.

(a) Write a key for this stem and leaf graph.

(b) State what was the :-

(i) fastest time (ii) slowest time,

taken for the puzzle.

(c) How many pupils tried the puzzle.

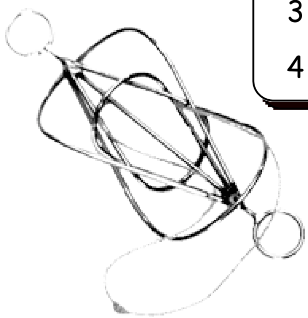
(d) How many pupils took more than 22 seconds to complete the puzzle ?

(e) Find the modal time (mode).

(f) Work out the median (middle) time.

Puzzle Time

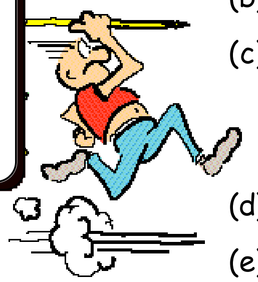
0	9					
1	6	8	9			
2	0	1	1	1	1	4
3						
4	1	7				



5. This stem and leaf graph has not been put in order.

Javelin throw

0	9					
1						
2	7	2	6	8	2	
3	9	6	1	9	2	9
4	2	1	5	0		



The graph shows the lengths (in metres) thrown in a javelin competition.

(a) Copy the graph, but this time show the distances in order.

(b) Write a key for this graph.

(c) What was the :-

(i) greatest distance thrown ?
(ii) least distance thrown ?

(d) What does the empty space at "1" mean ?

(e) Find the :- (i) mode
(ii) median.

6. For each set of data :- (i) Construct an ordered stem and leaf graph with a key.
(ii) Find the mode and median.

(a)

14	13	18	15	11	9	4	1
15	34	32	25	12	16	15	14
9	15	18	25	25	19	14	3
9	8	2	7	16	27	23	20
22	20	11	13	16	30	4	22

(b)

11	22	27	49	61	68	60	52
45	34	47	25	52	62	65	45
24	52	62	61	52	31	63	33
59	42	37	21	29	19	47	34
30	22	60	41	34	59	53	10

(c)

137	142	153	164	111	123	157	165	161	104	107	119
101	124	135	154	125	132	140	160	139	154	151	123
140	138	156	164	132	159	160	111	143	152	110	164

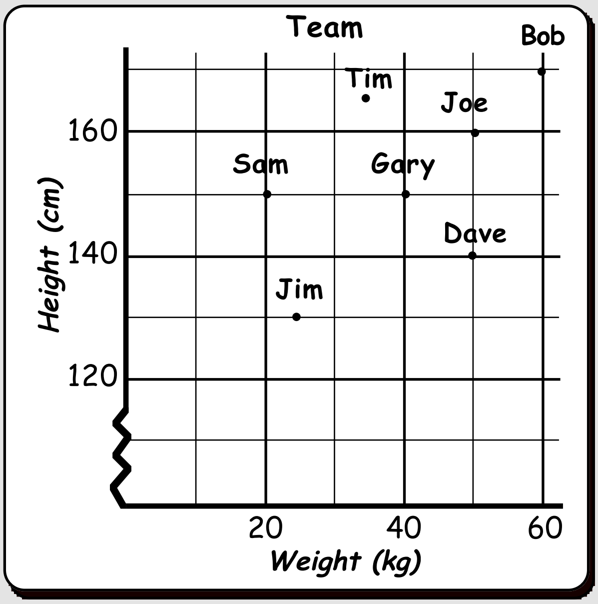
Scattergraphs

This scattergraph displays the **heights** and **weights** of a sevens football team.

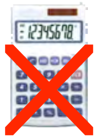
Gary weighs 40 kg.

Joe is 160 cm tall.

Jim is 130 cm tall and weighs approximately 25 kg.



Exercise 7

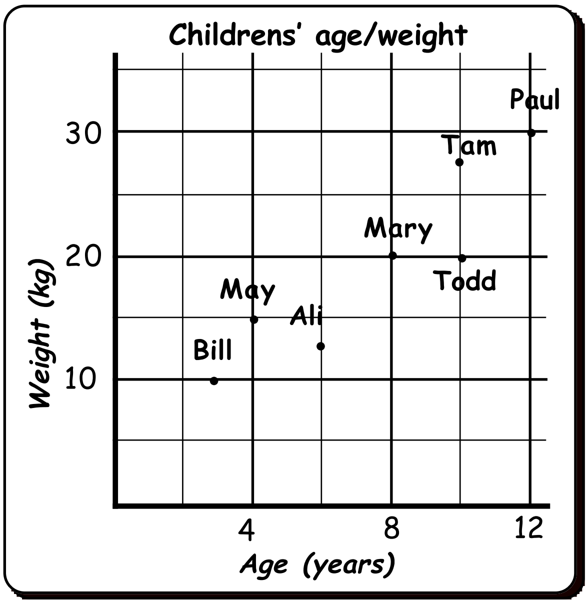


1. For the scattergraph above, write down the height and weight of each player.

2. This scattergraph shows the ages and weights of several children.

   (a) Who is :- (i) the youngest
   (ii) the lightest
   (iii) the oldest
   (iv) the heaviest child ?

   (b) Write down the age and weight of each child.



When two quantities are strongly connected, we say there is a strong **correlation** between them.

3. Say whether you think there will be a **correlation** between :-

   (a) the temperature and the sales of ice-cream.

   (b) the temperature and the amount of people on a beach.

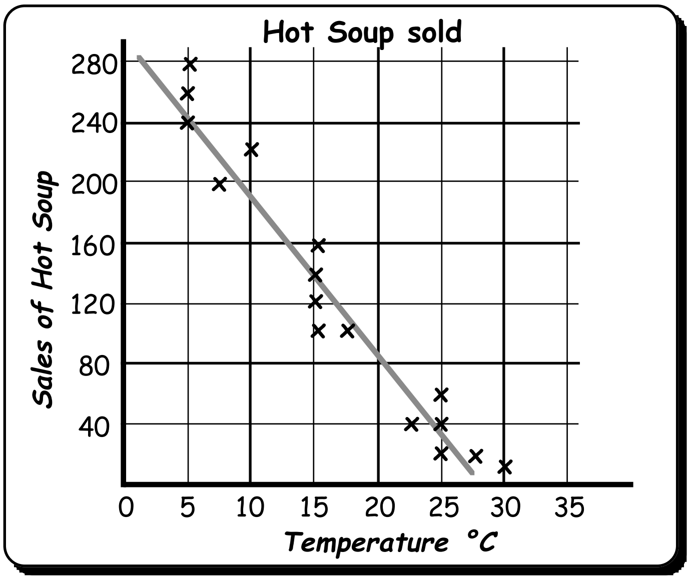
   (c) the amount of rain and the sales of umbrellas.

   (d) the distance a taxi travels and the fare.

   (e) the temperature and the sales of gloves.

   (f) the number of workmen and the time taken to build a wall.

4. This scattergraph shows the sales of cups of hot soup at a football ground.



This would be called a strong **negative correlation** since all the points lie roughly on a straight line going downwards from left to right.

The line is called a **line of best fit**.

Use the line of best fit to estimate :-

(a) the sales at 20°C.

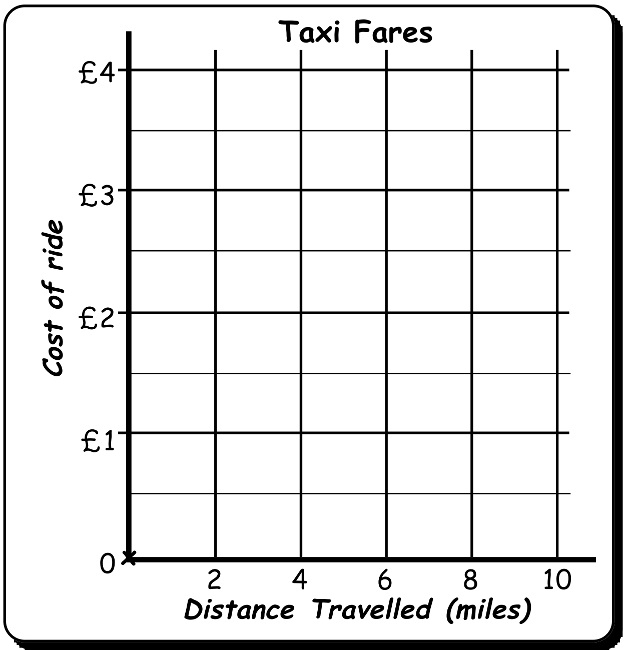
(b) the temperature when the sales were approximately 240 cups.

5. This graph represents the cost of different taxi fares and the distances travelled.

(a) Copy the graph.

(b) Use the table below to plot the points on the graph.

Distance (km)	Cost (£)
2	1·50
3	2·50
2	1·75
5	3·25
5	3·50
6	4·00



(c) Does this graph show a strong negative or positive correlation ?

(d) Draw a best line of fit on your graph.

(e) Estimate how much a 4 kilometre journey would cost.

6. For each data set below, construct a scattergraph and show a best line of fit.

(a)

Age (years)	0	1	1	2	3	3	3	4	4	5	6	7	8	8	9	9	10	10
Car price (£1000)	10	9	8	8	7	6	5	5	4	2	3	3	3	2	2	1	2	1

(b)

Temp. (°C)	0	5	5	5	10	15	20	20	20	25	25	30	30	25	20
No. of People in the park	1	3	5	5	10	15	25	35	20	40	50	60	55	35	30

Probability

Probability is the likelihood or chance of something happening.

Examples What is the probability that :-

if today is Tuesday, then tomorrow will Sunday ?	(impossible)
it will be sunny **everyday** in December ?	(unlikely)
if I toss a coin, it will land tails ?	(Even chance)
if I toss 10 coins, at least one will be heads ?	(likely)
if I jump into a river, I will get wet ?	(certain)

Exercise 8 (Oral exercise)

For each statement below, say whether the probability of it happening is :-

Impossible - unlikely - evens (50-50) - likely - certain.

1. If today is Monday, yesterday was Thursday.
2. The next person I see will be male.
3. No trains will be on time tomorrow.
4. There will be snow in January.
5. I will win the jackpot lottery this week.
6. I will have a birthday this year.
7. Christmas will be in November next year.
8. I will blink my eyes today.

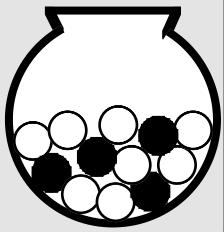
Calculating Probability

Probability can be thought of as a simple fraction.

$$\text{Probability of an event happening} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes.}}$$

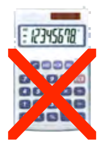
Example A bowl contains 4 black balls and 8 white balls.

If a ball is picked at random what is the probability that it will be black ?



$$\textbf{P(black)} = \frac{\textbf{4 (black balls)}}{\textbf{12 (balls altogether)}} = \frac{4}{12} = \frac{1}{3}$$

Exercise 9



1. A bag contains 6 black balls and 12 white balls.

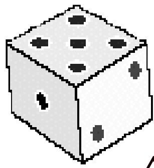
   If a ball is picked at random, what is the probability that it will be black ? (*Use the notation :-* P(black) = ...)

2. A bag has 3 red sweets, 6 green sweets and 9 blue sweets.

   If a sweet is picked at random, what is the probability that the sweet will be :-

   (a) red (b) green (c) blue (d) orange ?

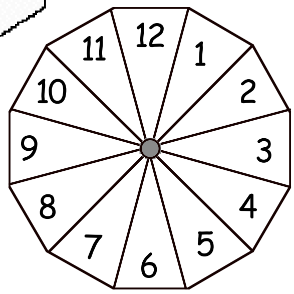
3. A dice numbered from 1 to 6, is rolled.



(a) What is the probability that it will show a 2 ? (P(2) = ...)

(b) Find :- (i) P(3) (ii) P(odd) (iii) P(8)

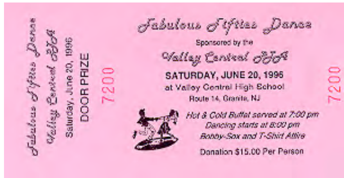
4. A duo-decagon (12 sides) spinner is spun and its number is noted.



Find :- (a) P(less than 4) (b) P(multiple of 3)

(c) P(prime) (d) P(factor of 12)

5. A bag contains 20 raffle tickets.

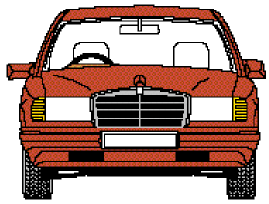


Four tickets win a cuddly toy, two tickets win £10 and the rest are losing tickets.

Find :- (a) P(win a toy) (b) P(losing ticket)

(c) P(win £10) (d) P(**not** win £10).

6. A garage forecourt has the following coloured cars :-



12 blue, 8 green, 6 silver, 4 white, 3 black, 2 red, 1 yellow.

Find:- (a) P(blue) (b) P(green) (c) P(silver)

(d) P(white) (e) P(black) (f) P(red)

(g) P(yellow) (h) P(red or blue) (i) P(not red or blue)

7. In a word game, letters are chosen at random from the word :-

A B R A C A D A B R A

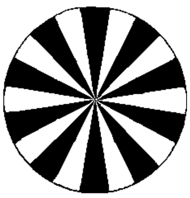
Find :- (a) P(A) (b) P(R) (c) P(vowel) (d) P(consonant)

8. The probability of an event happening is said to be $\frac{3}{7}$.
What is the probability of the event **not** happening ?

9. Three coins are tossed at the same time.

(a) List all the possible outcomes. (HHH, HHT, etc....).

(b) Find:- (i) P(all heads) (ii) P(2 tails).

10. 

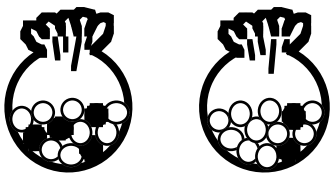
One dart is thrown at this dart board, numbered 1 – 20.

If the dart actually lands on the board, find :-

(a) P(16) (b) P(over 12) (c) P(even) (d) P(prime).

11. Look at the two bags shown.

How many **more** black balls do I have to put into bag 2 so that each bag has the same probability of picking, at random, a black ball ?



Bag 1 **Bag 2**

Surveys – Your teacher will discuss this with you !!

When setting up a survey, preparation is very important.

Have you avoided bias in your survey ? (What is bias ?)

Will you use a tally box or a questionnaire ?

Will you use discrete (countable) or continuous (measurable) data.

What form will the final information take?

Exercise 10

1. Jason is to conduct a survey asking whether or not a local weekend disco should be closed down.

   Explain why he should **not** ask the following groups :-

   (a) The staff of the disco.

   (b) People leaving the disco at 3 a.m. on Saturday morning.

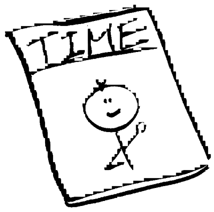
   (c) The old folks home across the road.

2. Construct a questionnaire to allow several responses to the following surveys :-

   (a) How much would you spend each week on magazines ?

   (b) On average, how many hours sleep do you get each weekend ?

   (c) Approximately how many kilometres do you travel each day ?



3. Describe each sentence below using either the words discrete or continuous.

   (a) The number of pets each person has in a class.

   (b) The distances pupils walk to school.

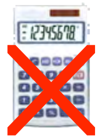
   (c) The temperatures at noon everyday for a week.

   (d) Time taken by runners in a 100 metre race.

4. Conduct a survey by asking the class how many pets they have.
   Use in your final results three separate methods of displaying the information.

5. Conduct a survey of your choosing, using a group of 50 people.
   Use three separate methods of displaying your information.
   Explain why you chose your subject matter and any other relevant details.

Statistics



Topic in a Nutshell

1. Find the mean, median, mode and range of each data set below :-

   (a) 5, 6, 2, 2, 1, 7, 8, 3, 4 (b) 2·3, 2·7, 2·5, 1·9, 2·5, 3, 2·9, 2

2. The mean age of a group of eight girls was 15.
   One more girl joined the group and the mean age became 14.

   How old was the ninth girl ?

3. The number of pupils in each classroom in a school is shown below.

   5 14 28 21 16 17 32 35 27 30 7 0 12 18 10

   22 34 24 15 19 14 11 9 29 25 29 33 4 12 16

   Show this information on a frequency table.
   (Use class intervals of 0 - 4, 5 - 9, 10 - 14, etc).

4. Construct a frequency table using the data below :-

137	142	153	164	111	123	157	145	161	120	127	139
131	164	145	124	105	112	160	160	149	114	161	123
140	108	166	164	152	159	100	131	153	122	160	164

5. The frequency table shows the maximum temperature each day in February.

   (a) Copy the frequency table and add a third column to help you find the mean temperature.

   (b) Copy the table again, but this time add a cumulative frequency column and use it to determine the median.

Max Temp (°C) (x)	Freq (f)
5	7
6	8
7	5
8	2
9	6

6. (a) Copy and complete the table showing a class's favourite pet.

Type of Pet	Number	Fraction	Angle
Dog	10	$\frac{10}{30}$	$\frac{10}{30} \times 360 =$ ....°
Cat	15		.... x 360 = ...°
Fish	4		.... x 360 = ...°
Other	1		.... x 360 = ...°
TOTAL	**<u>30</u>**		**360°**

   (b) Construct an accurate pie chart showing this information.

7. The stem and leaf graph show the ages of people at a family 50th birthday party.

(a) Write a key for this stem and leaf graph.

(b) How many people were at the party ?

(c) List the ages of the people at the party.

(d) Find the modal age.

(e) Find the median.

People's Ages

1	9
2	0 2 7
3	2 2 4 7 8
4	0 0 0 4 7 9
5	0 3

8. Construct a stem and leaf graph from the data set below.

27	32	43	54	61	12	57	65	63	14	37	19
11	24	35	54	25	32	40	60	39	54	51	23

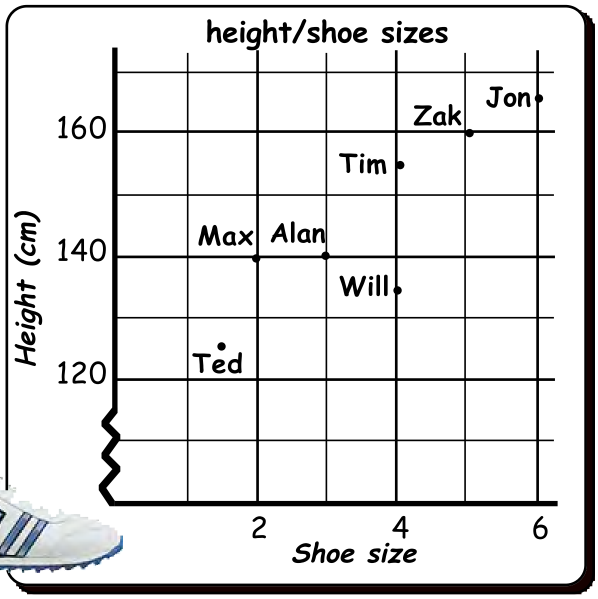
9. The scattergraph shows the shoe size and heights of several pupils.

(a) Write down the shoe size and height of :-

(i) Max (ii) Alan
(iii) Zak (iv) Tim
(v) Will (iv) Jon

(b) Is there a positive or negative correlation.

(c) Estimate the shoe size of Tom who is 150 centimetres tall.



10. (a) Construct a scattergraph for the set of data below :-

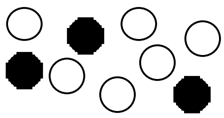
Height (cm)	140	150	160	110	155	170	160	155	130	120
Weight (kg)	30	45	60	25	45	60	55	50	35	15

(b) Draw a line of best fit for your scattergraph.

(c) From your line of best fit, estimate the height of a girl who weighs 20 kilograms.

11. A bag contains 3 black balls and 6 white balls.

What is the probability of randomly picking a white ball from the bag ?



12. A lucky dip contains ticket numbers from 1 up to 50. Calculate the following probabilities :-

(a) P(23) (b) P(odd) (c) P(single digit number)

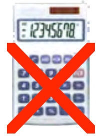
(d) P(multiple of 4) (e) P(factor of 64) (f) P(square number)

(g) P(prime) (h) P(prime factor of 30)

Chapter 19

Percentages

Revision of Level E



(until qu. 7)

Exercise 1

1. Make a COPY of this table and complete it :-

percentage	50%	25% 75%	$33\frac{1}{3}$% $66\frac{2}{3}$%	20% 40% 60% 80%	10% 30% 70% 90%
fraction	$\frac{1}{2}$			$\frac{1}{5}$	$\frac{7}{10}$

Learn the above percentage —> fraction conversions. You will need them here :-

2. Find the following without a calculator :-

(a) 10% of £25 (b) 70% of £60 (c) 20% of £3·50

(d) 80% of 40p (e) 25% of £1260 (f) $33\frac{1}{3}$% of £36

(g) 75% of £4·80 (h) 1% of £120 (i) 60% of £12 000

(j) 50% of £$\frac{1}{2}$ million (k) $66\frac{2}{3}$% of £18·60 (l) 10% of 70p

(m) 40% of £550 (n) 90% of 10p (o) 5% of £25

(p) 2% of £800 (q) 3% of £50 (r) 2·5% of £2

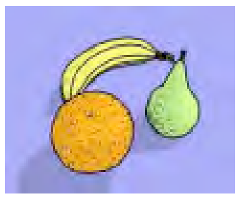
3. 30% of the S2 pupils in Lochee Academy wear glasses.

If there are 140 pupils in S2 in the school, how many do **not** wear glasses ?

4. $33\frac{1}{3}$% of the trees in an orchard are pear trees, 15% are banana trees and the rest are orange trees.

If there are 660 trees in total in this orchard, how many are :-

(a) pear (b) banana (c) orange ?



5. Write each of the following as a fraction **AND** as a decimal :-

(a) 28% (b) 35% (c) 61% (d) 23% (e) 58%

(f) 4% (g) 12% (h) 7% (i) 12·5% (j) 2·5%

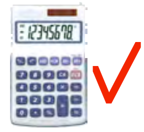
6. Write these percentages as fractions and simplify :-

(a) 15% = $\frac{15}{100}$ = $\frac{?}{20}$ (b) 40% = $\frac{40}{100}$ = ...

(c) 45% (d) 70% (e) 15% (f) 75% (g) 2%

(h) 35% (i) 5% (j) 48% (k) 60% (l) 95%

7. Copy the following and use your calculator to change each fraction to a percentage :-



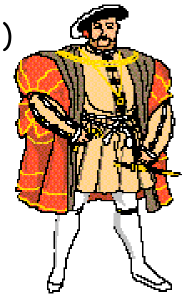
this page

(a) $\frac{9}{50}$ = 9 ÷ 50 = 0·...... = (0·...... x 100%) = ... %

(b) $\frac{1}{5}$ = 1 ÷ 5 = 0·...... = (0·...... x 100%) = ...%

(c) $\frac{8}{50}$	(d) $\frac{10}{25}$	(e) $\frac{7}{20}$	(f) $\frac{13}{20}$	(g) $\frac{24}{25}$	(h) $\frac{1}{8}$
(i) $\frac{7}{25}$	(j) $\frac{5}{8}$	(k) $\frac{30}{40}$	(l) $\frac{7}{8}$	(m) $\frac{19}{20}$	(n) $\frac{49}{98}$

8. Change each of these History marks to percentages :-



(a) Julie scored 28 out of 40 ($\frac{28}{40}$ = 28 ÷ 40 = 0· ..... x 100% = ... %)

(b) Francis scored 23 out of 50

(c) Ricky scored 20 out of 80

(d) Chic scored 3 out of 10

9. Use your calculator to find the following :-

(a) 12% of £40 = (12 ÷ 100) x 40 = £......

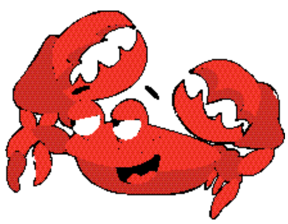
(b) 15% of £80	(c) 28% of £60	(d) 64% of £3500
(e) 36% of £8·50	(f) 75% of £38·40	(g) 95% of £2
(h) 7% of £40	(i) 3% of £12	(j) $17\frac{1}{2}$% of £250

10. During a storm the level of rain which fell outside my front door before dark was 140 millimetres.

During the night the water level rose by another 45%.

What level of rain water was outside my house when I woke in the morning ?

11.

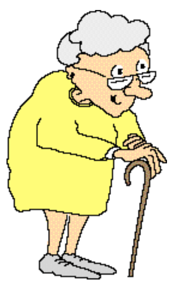


Only 65% of young crabs are expected to survive the first few weeks of their young lives.

At Rocky Swamp last year 2·5 million crabs were born.

How many were expected to survive the early stages of their lives ?

12. Glennifer Town Council are expected to increase council tax by 4·5% for the oncoming year.
Mrs McGlinchie of Glennifer paid council tax totalling £850 last year.



What should she expect to be paying in total this year ?

% Increase/Decrease

Example 1

Moira works for Holmes the Painter & Decorator firm. Her weekly wage last year was £240. This year Mr Holmes gave her a 5% increase in her pay.

What is Moira's new weekly wage ?

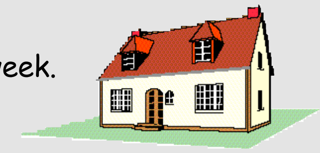
Increase	=	5% of £240	=	5 ÷ 100 × £240	=	£12.
New Wage	=	£240 + £12	=	£252		

Appreciation/Depreciation

The value of a house usually increases with time. Its value "APPRECIATES".

A car's value usually falls each year. Its value "DEPRECIATES".

Example 2



A cottage, bought for £80 000 in 2001, was put on the market last week. Its value had **appreciated** by 40% since 2001.

What is the cottage now worth ?

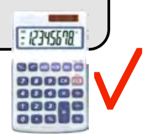
Appreciation	=	40% of £80 000	=	40 ÷ 100 × £80 000	=	£32 000
Cottage now worth	=	£80 000 + £32 000	=	£112 000		

Example 3

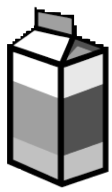
Mrs Greig bought a new car for £8500. In the first year its value **depreciated** by 20% and in the second year its value fell by a further 15%.

What was her car worth at the end of year 2 ?

Initial Value	At start	=	£8500
Yr. 1 Depreciation	20% of £8500 = 20 ÷ 100 × £8500	=	£1700
Value at end of Yr. 1	£8500 – £1700	=	£6800
Yr. 2 Depreciation	15% of £6800 = 15 ÷ 100 × £6800	=	£1020
Value at end of Yr. 2	£6800 – £1020	=	£5780



Exercise 2



1. Milk costs 80p per litre, but the price is expected to rise shortly by 5%.

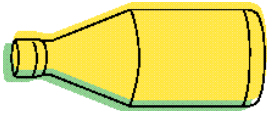
   What will the new cost of a litre of milk be ?

2. Mark takes the local service bus to school. His return fare is £1·50 per day. The bus company decide to increase all fares by 8%.

   What is Mark's new return fare ?

3. In a sale, a pair of gloves, normally priced at £12·30, is reduced by $33\frac{1}{3}\%$.

What is the sale price of the gloves ?

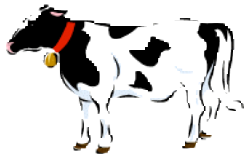
4. 

A bottle of diluting juice normally contains 400 ml of liquid. In a special offer, you can get 12·5% extra free.

How many millilitres does this special offer bottle hold ?

5. Farmer Baines had 60 cattle, but due to an outbreak of sickness he lost 85% of his herd.

How many cattle does he still have left ?



6. 

The average attendance at Glebe Football Stadium last season was 48 000.

This season, it has dropped by 11·25%.

What is the average attendance at the Stadium this season ?

7. (a) Increase £60 by 2%.
   (b) Decrease £400 by 7%.
   (c) Increase £280 by 12·5%.
   (d) Decrease £3200 by 32·5%.
   (e) Increase £810 by 17·5%.
   (f) Decrease £40 000 by 1·28%.

8. A plot of land, worth 28 000 euros in 2003 in Spain, is now worth 15% more.

What is the new value of the land ?

9. A tractor bought for £25 000 in 2002 has depreciated in value over the past few years.
It is now worth 32% less than the original value.

What is the tractor worth today ?

10. 

The Faulds bought a retirement villa in Florida for $125000.

After one year its value had increased by 15% and by the end of a second year it had gone up by a further 20%.

What was the value of their villa :-

(a) after 1 year
(b) at the end of the 2nd year ?

11. Iqbal bought a second-hand car for £3600. It value depreciated by 12% over the first year and 8·5% over the second year.

How much was Iqbal's car worth after 2 years ?

12. 

A unique violin was purchased in 2002 for £1500. Since then its value has risen by 15% each year.

If this trend continues, what is the first year when the violin will be worth at least double its original value ?

13. A PC bought for £2000 depreciated in value each year by 12%.

After how many years will its value be **less than half the original price** ?

14. Christina deposits £500 in the Bank where just now the interest rate is 3% per annum.

How much will Christina have in the bank after :-

(a) 1 year (b) 2 years ?

15. George borrows £400 from a dodgy Finance Company.

They add on interest of 25% in the first month, 30% in 2nd month and 35% in the third month.

Including the amount he borrowed, how much will George owe after 3 months ?

Expressing "A" as a Percentage of "B"

Example 1

Dorothy scored 13 out of 20 in a Geography test. What was her percentage mark ?

=> % Mark => $\frac{13}{20} = (\frac{13}{20} \times 100\%) = (13 \div 20) \times 100\% = \underline{\underline{65\%}}$

Example 2

Angus delivers papers for Andy's Newsagents Store. Last year he was paid £20 per week. This year he gets £22·50 per week.

Calculate his **percentage increase** in pay.

Actual Increase = £22·50 – £20 = £2·50

% Increase = $\frac{\text{increase}}{\text{original pay}} = \frac{£2\cdot50}{£20} \times 100\% = \underline{\underline{12\cdot5\%}}$

Example 3

A cottage bought for £80 000 in 2001 was on the market last week valued at £110 000.

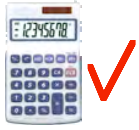
Calculate the percentage appreciation in value. (i.e. the % increase)

Actual Increase (appreciation) = £110 000 – £80 000 = £30 000

% Increase (% appreciation) = $\frac{£30000}{£80000} \times 100\% = \underline{\underline{37\cdot5\%}}$

To Express A as a % of B ..... begin with $\frac{A}{B}$ then x by 100%

Exercise 3



1. Express £10 as a percentage of :-

(a) £20	(b) £40	(c) £50	(d) £100
(e) £80	(f) £200	(g) £500	(h) £1000

2. Express :-

(a) £48 as a percentage of £96	(b) £20 as a percentage of £50
(c) £80 as a percentage of £320	(d) £4·30 as a percentage of £430
(e) 70p as a percentage of £1·05	(f) 150 cm as a percentage of 3000 cm

3. Of the 30 guests at a dinner party, only 6 were men.

   What percentage were :- (a) men (b) women ?

4. From a weekly wage of £280, I pay £67·20 in rent.

   What percentage of my wage goes on rent ?

5. 

   Jamie works in Telesales on a Saturday.
   When she first started she got paid £4·80 per hour,
   but that was soon increased to £5·20 per hour.

   What percentage pay rise did she get ? (to 1 dec. pl.)

6. Bill got 56 out of 80 for Chemistry and 54 out of 75 for Physics.

   (a) By changing both marks into a percentage, find in which subject he fared better.

   (b) When the marks were graded, it was found that in Chemistry 70% and over was awarded a grade A, but in Physics a mark of 75% was required for an A pass.

   Did Bill get any A passes ? ......If so, in which subject(s) ?

7. A trumpet, bought for £120, was later sold for £135.

   Calculate the profit as a percentage of the cost price.

8. 

   The Cairns family moved to a house with a smaller garden.
   Their old house had a garden with an area of 200 square metres
   compared with the new house's garden of 80 square metres.

   Express the reduction in area as a percentage of the area of their old garden.

9. A van bought new for £8250 was worth only £2500 a few years later.

   Calculate the depreciation, and express it as a percentage of the cost when new. (*Round to nearest whole %*)

10. A pair of cufflinks bought for £50 some years ago has now been valued at £3000 !

    Calculate the appreciation as a percentage of the buying price.

Finding the Initial Value after a % Increase/Decrease

Example 1

Tony's train fare has just gone up by 10%. His new fare is £4·95.

What was Tony's train fare before the price increase ?

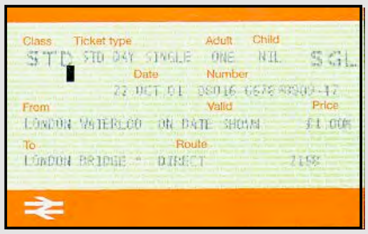
Original Price 100%

New Price = 100% + 10% = 110% of original price

=> 110% = £4·95

=> 1% = £4·95 ÷ 110 = £0·045

=> 100% = £0·045 x 100 = £4·50



Example 2

The value of a caravan depreciated by 20%. It is now worth £14000.

How much was the caravan originally worth ?

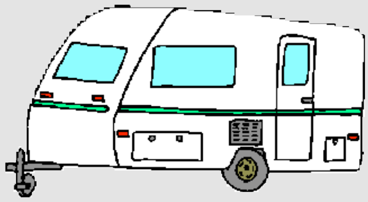
Original Value 100%

New Value = 100% – 20% = 80% of original value

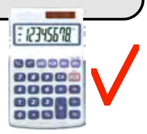
=> 80% = £14 000

=> 1% = £14 000 ÷ 80 = £175

=> 100% = £175 x 100 = £17 500



Exercise 4



1. After a 10% pay rise Sally's wage for her part-time job went up to £44.
   Calculate Sally's pay before her pay rise.

2. now only £60

   A skirt is on sale for £60 after its price had dropped by 20%.
   What was the original price of the skirt ?

3. The Wok Wong restaurant increased its prices by 6% across the whole range of meals.
   What was the original cost of a main course which now appears on the menu priced £7·95 ?

4. 

   Due to wet weather, this year's fruit yield was down by 20% on last year's.
   This year my garden produced 40 kg Blackberries, 56 kg Gooseberries and 72 kg Raspberries.

   What was my yield last year for :- (a) Blackberries (b) Gooseberries (c) Raspberries ?

5. Colin's annual golf fees have increased by 18% this year.
He now has to pay £590·00

Calculate Colin's golf fees for last year.

6. 

Over the past 3 years the cost of a colour printer has fallen by 23% to a price just now of £61·60.

What did this colour printer cost 3 years ago ?

7. A cottage has appreciated in value by 12·5%.

It is now valued at £148 500.

Calculate the original value of the cottage.

8. Lisa's garage bill is shown.

The total bill came to £176·25.

Calculate :-

(a) the Sub-Total.
(the total bill before VAT)

(b) the VAT.

(c) the cost for parts.

AL's GARAGE

Lisa McMillan

Labour	£85
Parts	£.....
Sub-Total	£.....
VAT @ 17·5%	£.....
TOTAL BILL	**£176·25**

9. Temoc Electrical Store have a sale on.
Calculate the original prices of these electrical goods.

(a)

HOOVER
15% OFF
Now **£199·75**

(b)

WASHING MACHINE
22% OFF
Now **£355·68**

(c)

COMPUTER
18·5% OFF
Now **£994·30**

(d)

HOME ENTERTAINMENT SYSTEM
12·5% OFF
Now **£2056·25**

10. George bought a lawnmower from Macco Discount warehouse.
He was given a 20% discount, but V.A.T. at 17·5% was then added on.
George ended up paying £75·20.

Calculate the original price, before the discount and the V.A.T.

Percentages

Topic in a Nutshell

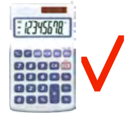
1. Work out the following **without a calculator** :-

(a) 10% of £25 (b) $33\frac{1}{3}$% of £24·06 (c) 75% of £6·80

(d) 5% of £15 (e) 2% of £325 (f) 2·5% of £3·20

2. Freda scored a mark of 39 out of 60 in her History exam.

What was her percentage mark ?



Q2 - end

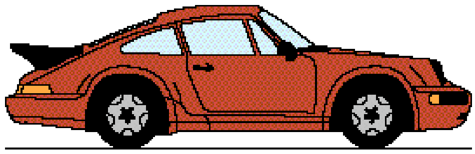
3. 

Toni increased his snack prices by 12·5 % in his restaurant.

What is the price now for fish and chips, which originally cost £3·60 ?

4. A car bought for £17 820 last year has depreciated in value throughout the year.
It is now worth 24% less than the original value.

What is the car worth today ?



5. 

A coffee maker is on offer at £44·50 + 17·5% VAT.

How much will it cost to buy it including the VAT ?

6. Mabel borrows £2000 from a Finance Company.
Interest is added on as follows – 2% in year 1, 5% in year 2 and 8% in year 3.

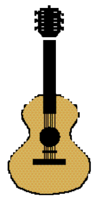
Including the amount she borrowed, how much will Mabel owe after 3 years ?

7. Express :-

(a) £1·35 as a percentage of £1·80 (b) £0·80 as a percentage of £1·20.

8. A guitar bought new for £350 was worth only £275 a few years later.

Calculate the depreciation and express it as a percentage of the cost when new.
(*Round to nearest whole %*)



9. 

Over the past year the price of a certain laptop has fallen by 28% to a price just now of £756.

What did this laptop cost last year ?

10. A house has appreciated in value by 15% since last year. It is now valued at £109 250.

Calculate the value of the house last year.

Chapter 23

Areas

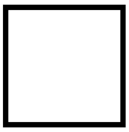
The Area of a Rectangle, Square and Right-Angled Triangle

Reminders

Area of Rectangle

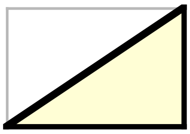
Area = L x B

Area of Square



Area = L x L

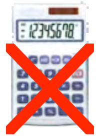
Area of Right-Angled Triangle



Area = Half the Area of Surrounding Rectangle

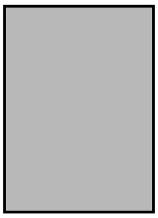
[Answer in square units e.g. cm^2]

Exercise 1



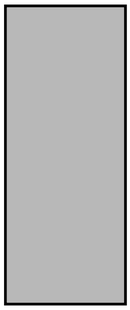
1. Calculate the area of each of the following rectangles, using the formula $A = L \times B$:-

(a)

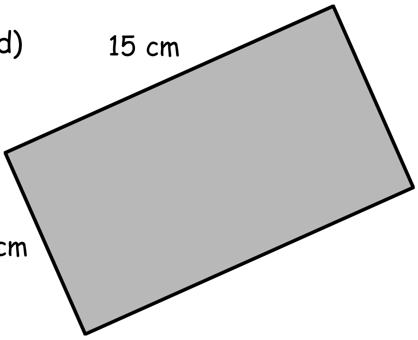


(b)

(c)

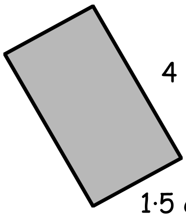


(d)



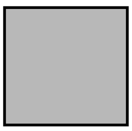
(e)

(f)

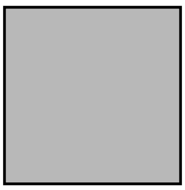


2. Calculate the areas of these squares, using the correct formula :-

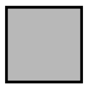
(a)



(b)

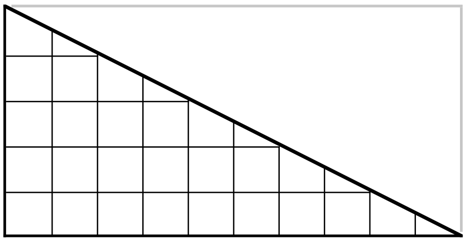


(c)



3. Calculate the areas of the triangles by first finding the area of the surrounding rectangle :-

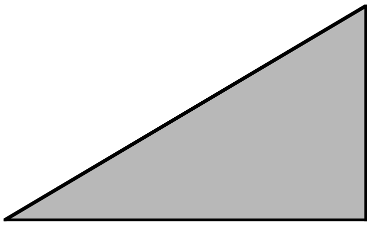
(a)



5 cm

10 cm

(b)

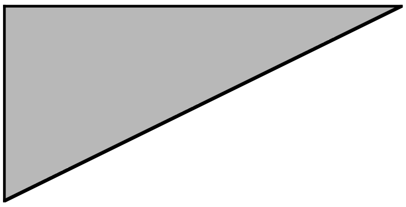


4 cm

8 cm

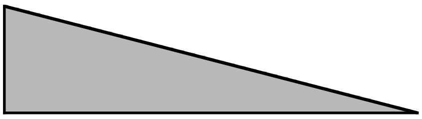
(c)

10 cm



6 cm

(d)

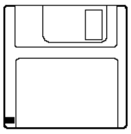


1·5 cm

8 cm

4. Calculate the areas of the objects shown below, using an appropriate formula :-

(a)



square disk
of side 8 cm

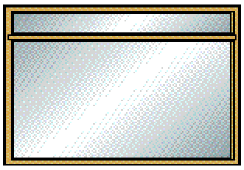
(b)

Scottish Flag

30 cm

80 cm

(c)

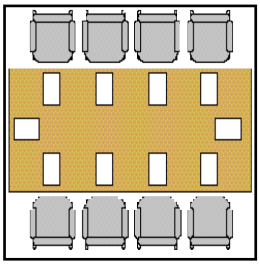


double-glazed
window

2 m

2·5 m

(d)

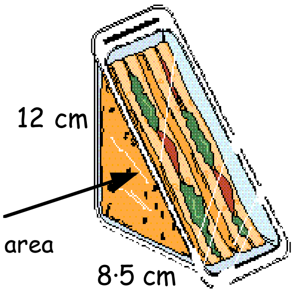


square shaped
conference
room

15 m

(e)

12 cm

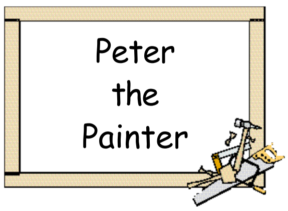


triangular
sandwich

Calculate the area
of this face

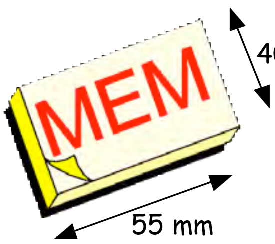
8·5 cm

(f)



advertisement
board
6·5 m by 4 m

(g)



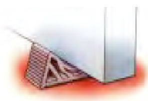
40 mm

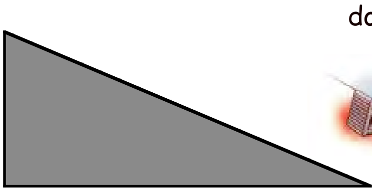
top sheep of
memo pad

55 mm

(h)

door stop

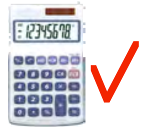




4·5 cm

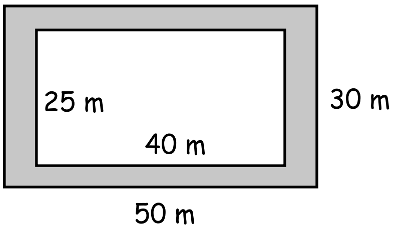
18 cm

5. The designs below are constructed from more than one shape. These are called **Composite Shapes**. (You will meet more later.)

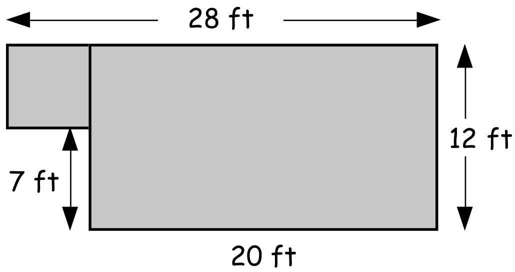


Try to calculate the shaded area each time :-

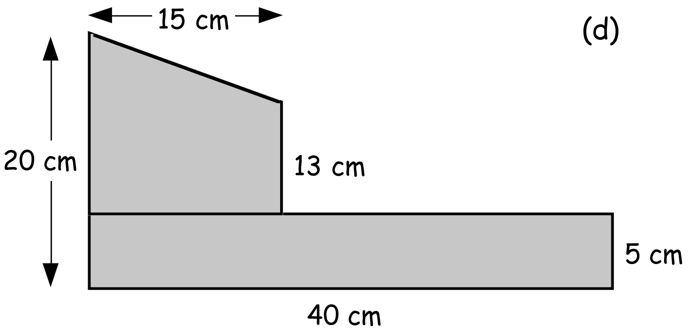
(a)



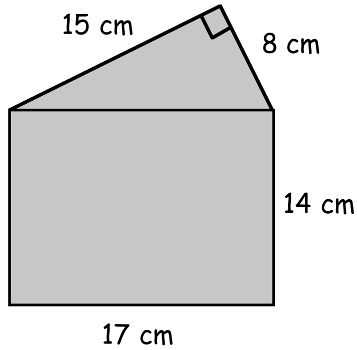
(b)



(c)

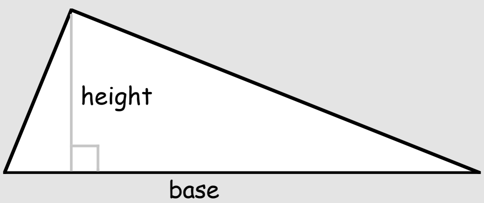


(d)



The Area of a Triangle (any triangle)

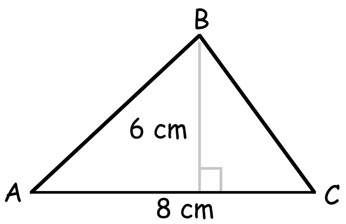
Formula for the Area of a Triangle, given its base and height.



Note - the base and the height (altitude) of a triangle must meet at right angles.

$$\textbf{AREA of TRIANGLE} = \tfrac{1}{2} \times \textbf{BASE} \times \textbf{HEIGHT}$$

Example 1

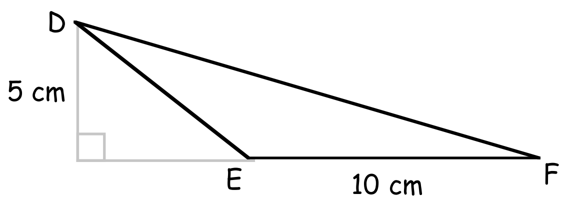


Area ΔABC $= \frac{1}{2} \times B \times H$

$= \frac{1}{2} \times 8 \times 6$

$= \underline{\underline{24}}\ \text{cm}^2$

Example 2



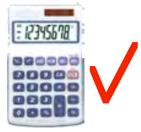
Area ΔDEF $= \frac{1}{2} \times B \times H$

$= \frac{1}{2} \times 10 \times 5$

$= \underline{\underline{25}}\ \text{cm}^2$

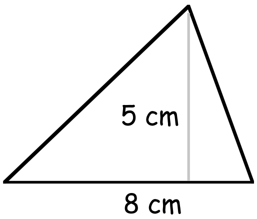
> **Remember :-** If the length and breadth are in cm => Area is in **cm^2**.
> If the length and breadth are in mm => Area is in **mm^2**.
> If the length and breadth are in m => Area is in **m^2**.

Exercise 2

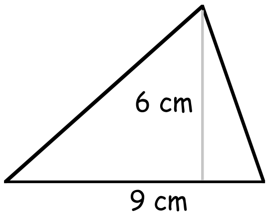


1. Use the formula $Area = \frac{1}{2} \times B \times H$ each time to calculate the areas of the following triangles :-

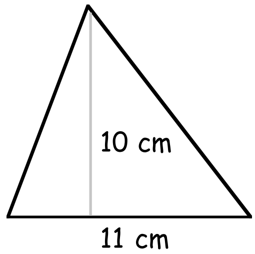
(a)



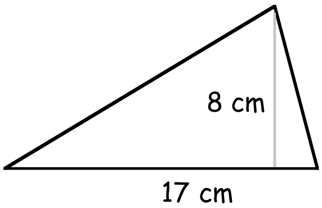
(b)



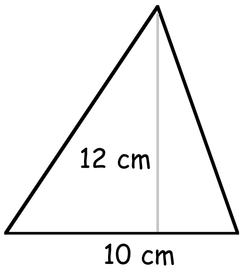
(c)



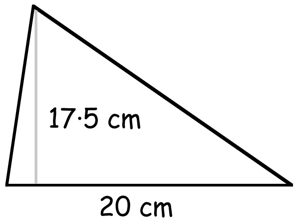
(d)



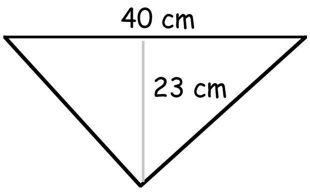
(e)



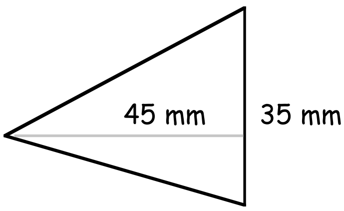
(f)



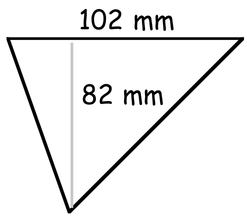
(g)



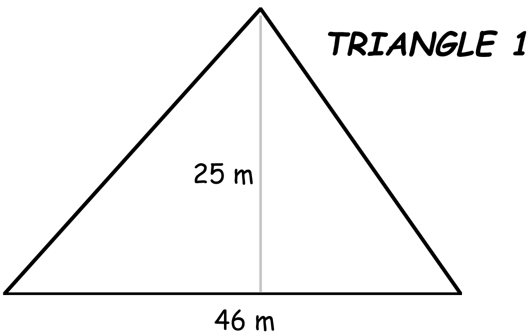
(h)

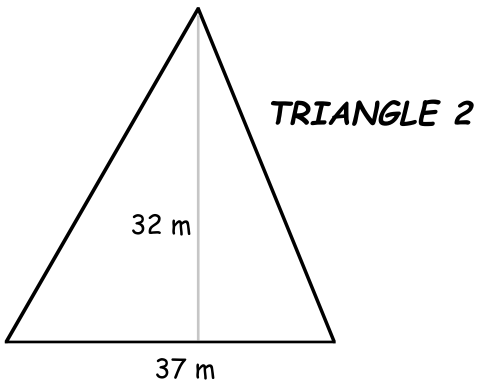


(i)



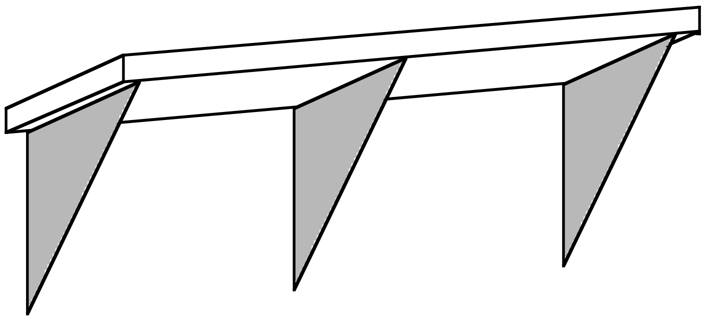
2. (a) Which of these triangles has the bigger area ?

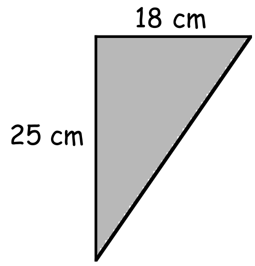




(b) By how many square metres is one bigger than the other ?

3. Three identical wooden brackets are used to support a shelf.



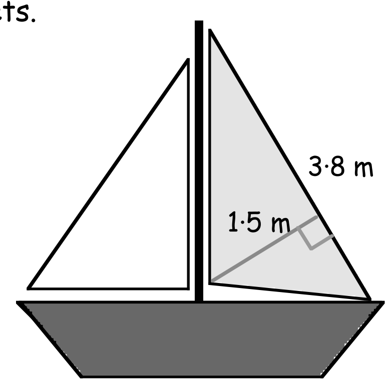


Each bracket is a right-angled triangle, as shown opposite.

Calculate the total area of wood needed to make ALL 3 brackets.

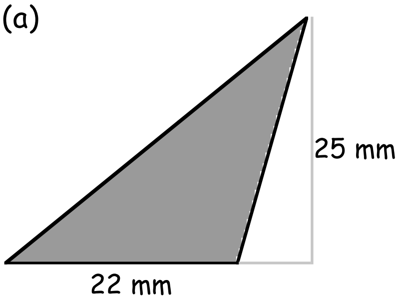
4. The coloured sail of this yacht is in the shape of an obtuse angled triangle with base 3·8 metres and height 1·5 metres.

Calculate its area in m^2.

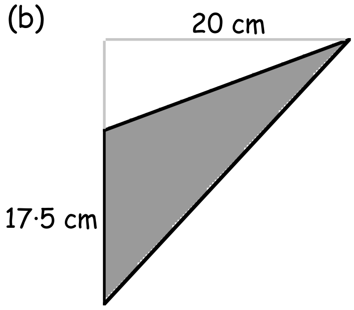


5. Calculate the area of each of these triangles :-

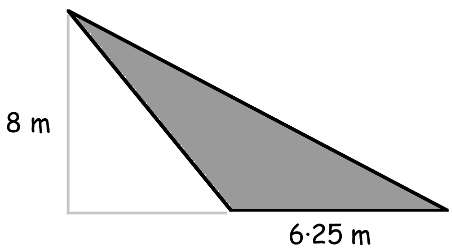
(a)



(b)



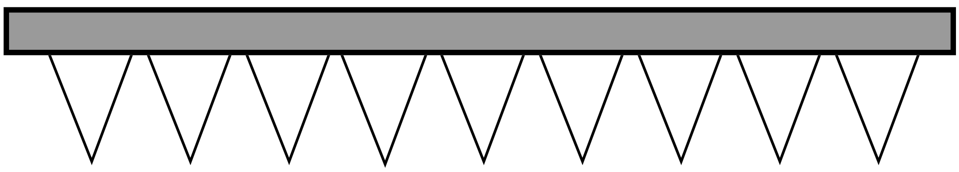
(c)

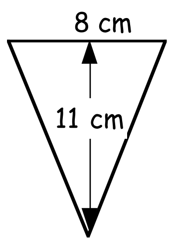


6. A joinery company used the painted logo below to advertise the sharpness of their saws.

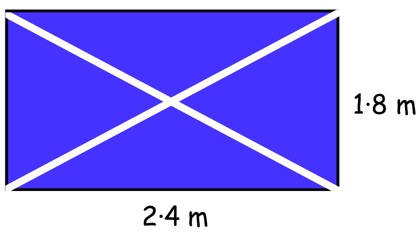
Each triangle measured 8 cm wide with an 11 cm drop.

Calculate the total area of the 9 pointed triangular teeth.





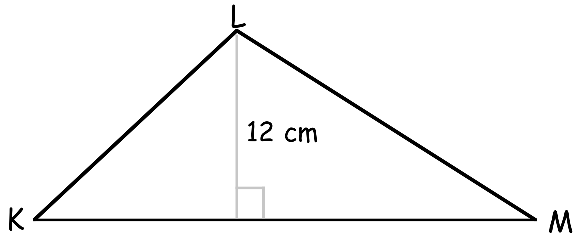
7.

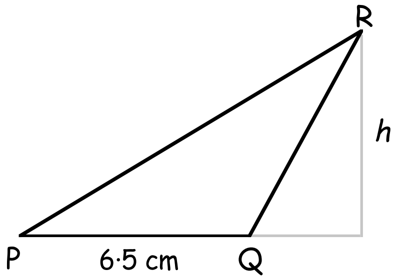


This is a picture of the large Scottish banner which the "Tartan Army" displayed during Scotland's 1 - 0 defeat of Holland at Hampden in 2003.

Calculate the area of one of the obtuse angled triangles and the area of one of the acute angled triangles.

8. The areas of triangle KLM and PQR are given.





Area ΔKLM = 108 cm^2 Area ΔPQR = 26 cm^2

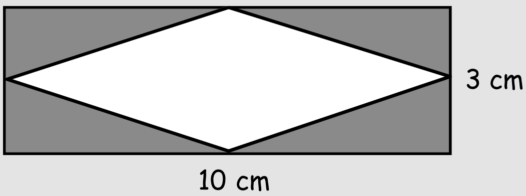
Calculate :- (a) the length of the base of ΔKLM (KM).

(b) the height (h cm) of ΔPQR. (Show all working)

The Area of a Rhombus and a Kite

Area of a Rhombus

Draw (or imagine) the rectangle that just surrounds the rhombus.



The rhombus shown above has length 10 cm and height 3 cm.

Its AREA is calculated by finding the area of the surrounding rectangle and **halving** the answer found.

Area Rect. = $L \times B$ = 10×3 = 30 cm^2.
Area Rhombus = $\frac{1}{2}$ of 30 cm^2 = 15 cm^2.

Note that the length and breadth of a rhombus are actually the measurements of its **diagonals**.

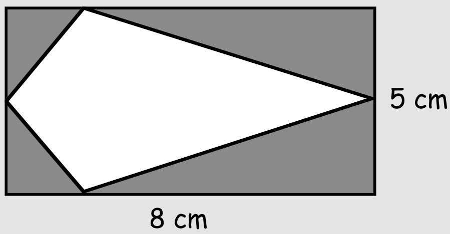
So Area of Rhombus = $\frac{1}{2}$ diagonal $\times$ diagonal

or $\boxed{\text{Area} = \frac{1}{2}(D \times d)}$

(where D and d are lengths of big and small diagonals)

Area of a Kite

Found in the same way as the Rhombus.



Area Rect = $L \times B$ = 8×5 = 40 cm^2.
Area Kite = $\frac{1}{2}$ of 40 cm^2 = 20 cm^2.

OR

Area Kite = $\frac{1}{2}$ diagonal $\times$ diagonal
= $\frac{1}{2} \times 8 \times 5$
= 20 cm^2.

So Area of Kite = $\frac{1}{2}$ diagonal $\times$ diagonal

or $\boxed{\text{Area} = \frac{1}{2}(D \times d)}$

(where D and d are lengths of big and small diagonals)

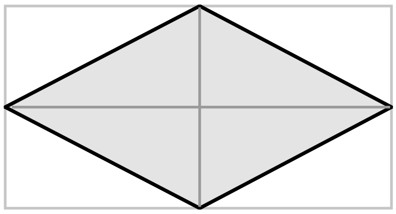
Exercise 3

1. (a) Make an accurate drawing of a rhombus with diagonals measuring 8 cm and 6 cm.
(*Draw the 2 diagonals 8 cm by 6 cm meeting at right angles in the middle.*)

(b) On your diagram, draw a rectangle round the rhombus.

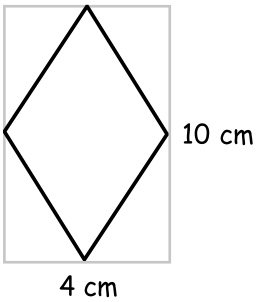
(c) Calculate the area of the rectangle.

(d) Now calculate the area of the rhombus.

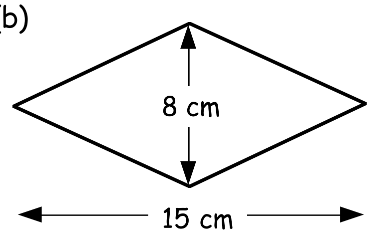


2. For each rhombus below :- (i) sketch it.
(ii) surround it with a rectangle.
(iii) calculate the area of the rectangle.
(iv) calculate the area of the rhombus.

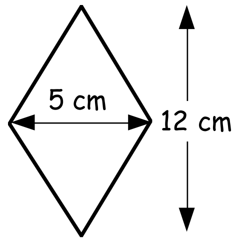
(a)



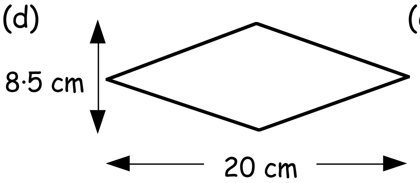
(b)



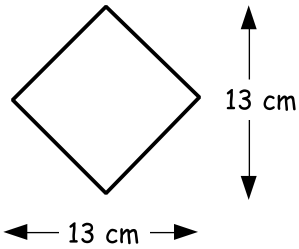
(c)



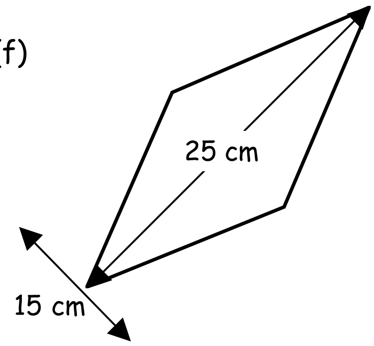
(d)



(e)

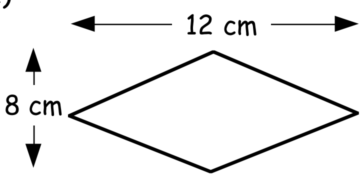


(f)

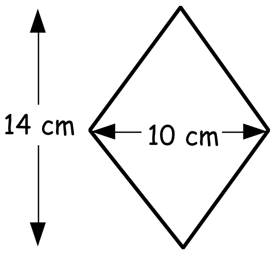


3. Use the formula "Area of Rhombus = $\frac{1}{2}(D \times d)$" to find the areas of these rhombi :-

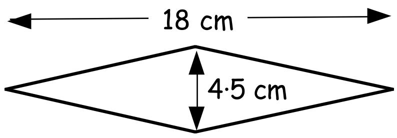
(a)



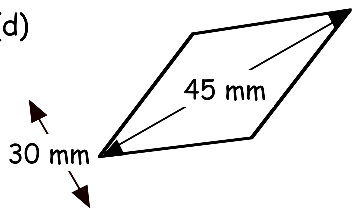
(b)



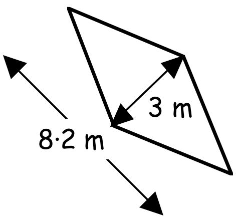
(c)



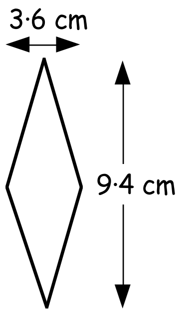
(d)



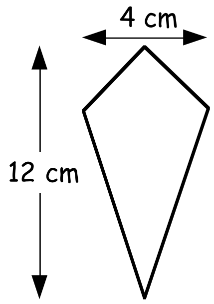
(e)

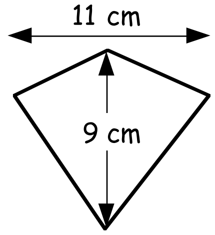


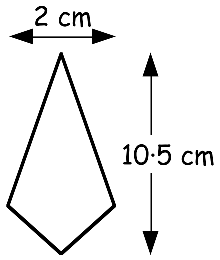
(f)

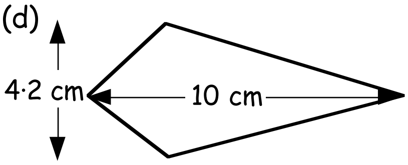


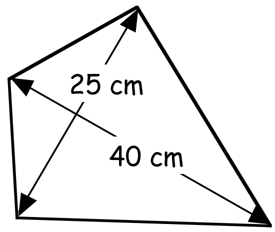
4. Use the formula "Area of Kite = $\frac{1}{2}(D \times d)$" to find the areas of these kites :-

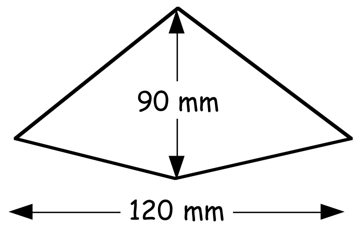
(a)


(b)


(c)


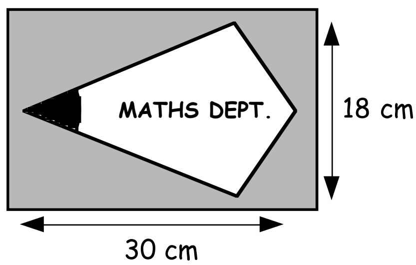
(d)


(e)


(f)


5. On parents' evenings, the maths department put up this wooden sign on the first floor of the school to direct parents to their rooms.

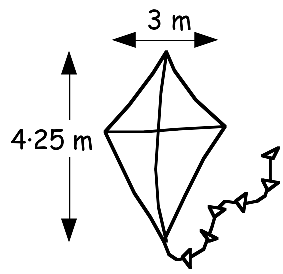
Calculate the area of the wooden kite-shape.



6. A giant polythene kite flew above the marquee at the wedding reception of the managing director of "Kites-R-4-U".

The kite was strengthened by 2 plastic poles measuring 4·2 metres and 3 metres which were fitted as diagonals of the feature.

Calculate the area of the giant kite.



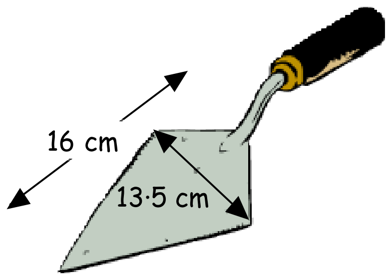
7.

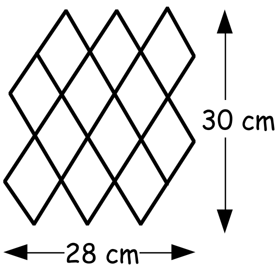

Local fishermen used to nickname this fish "The Rhombus".

Find the approximate area of its body if its measurements are 25 cm long and 9 cm in height.

8. The base of the trowel shown is in the shape of a kite.

Find its area.



9.


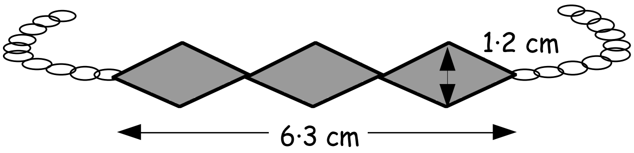
A tiling company glued 12 similar rhombus-shaped tiles onto a plywood board and used this to illustrate how their tiles gelled together to make ideal designs.

Calculate the area covered by ALL the tiles.
(*Hint - calculate the dimensions of one of the rhombi first*)

10. The main design on the pair of cufflinks shown is in the shape of rhombus.
The diagonals of each rhombus are 0·8 centimetres and 1·2 centimetres.

Calculate the total area taken up by the faces of rhombi.

11. Marjorie's necklace was made up with 3 identical golden rhombi on a chain.



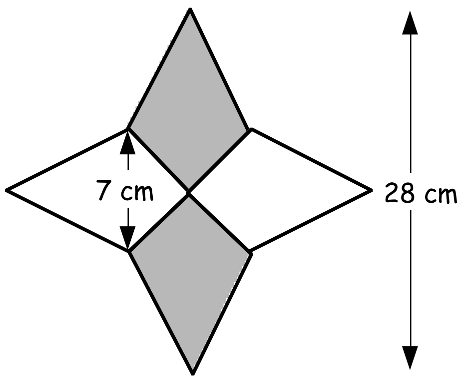
The 3 rhombi together measure 6·3 centimetres long and each has a height of 1·2 centimetres.

Calculate :-

(a) the length of the diagonal of one of the rhombi.

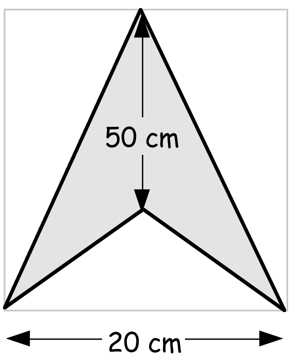
(b) the total area of the 3 golden rhombi.

12. Calculate the area of the star-shape, constructed from 4 identical kites.

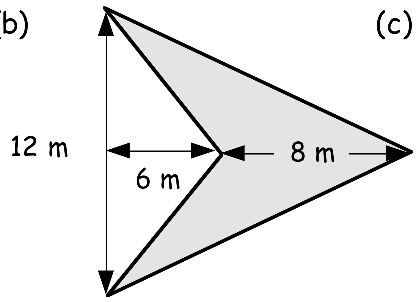


13. Calculate the area of each V-kite.

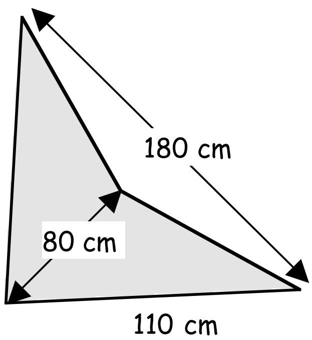
(a)



(b)



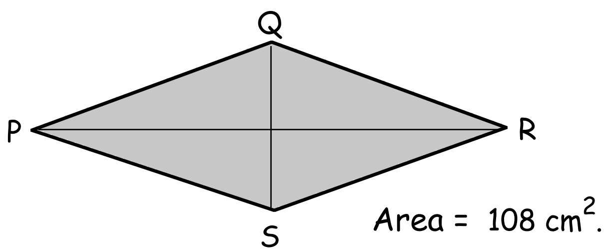
(c)



14. The area of rhombus PQRS is 108 cm^2.

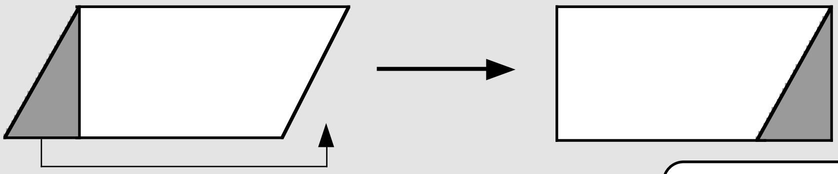
The length of diagonal PR is 24 cm.

Find the length of diagonal QS.



The Area of a Parallelogram

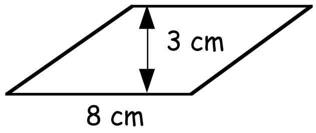
It is easy to see why **the area of a parallelogram = the area of a rectangle.**



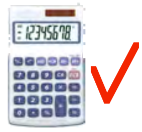
A difference in **notation**:-
(Area of Rectangle = Length x Breadth)

AREA of Parallelogram = Base x Height

Example



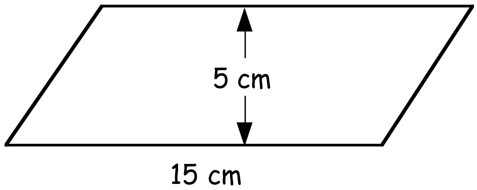
Area $= B \times H$
$= 8 \times 3$
$= \underline{\underline{24 \text{ cm}^2}}$



Exercise 4

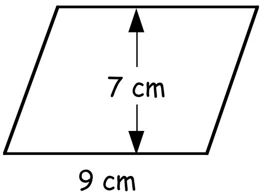
1. This is a sketch of a parallelogram.

   Use the formula $A = B \times H$ to find its area.

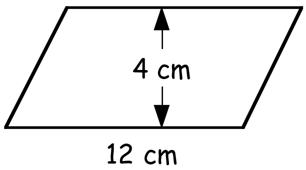


2. Calculate the areas of these parallelograms :-

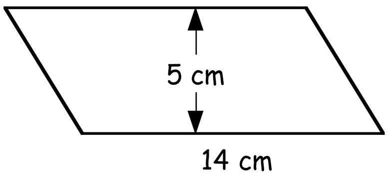
(a)



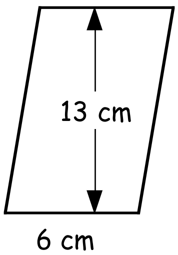
(b)



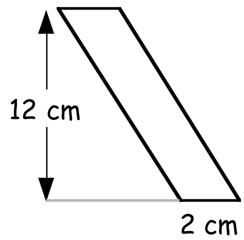
(c)



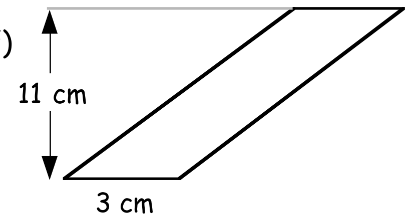
(d)



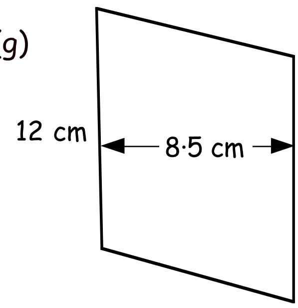
(e)



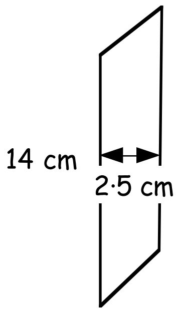
(f)



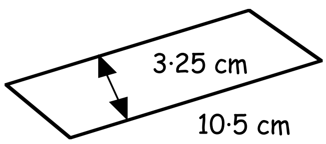
(g)



(h)

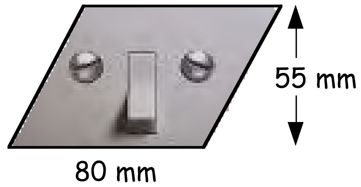


(i)



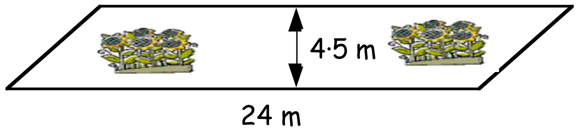
3. This light switch is in the shape of a parallelogram.

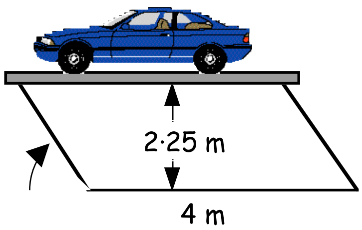
Calculate its area.



4. Mrs Galbraith made her front garden into a parallelogram shape.

Calculate the area of her garden.



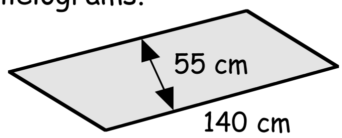
5. 

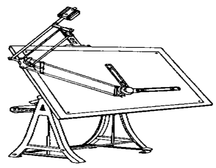
The ramp in the garage is the form of a parallelogram.

Calculate the area of the gap shown.

6. Council workers use this machine when mending roads.
Many of its moving parts are parallelograms.

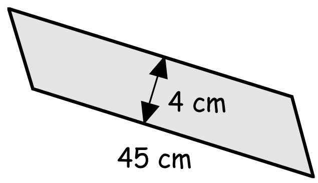
Find the area of the one shown.



7. 

Fraser, an architect, often uses parts of parallelograms when drawing up plans.

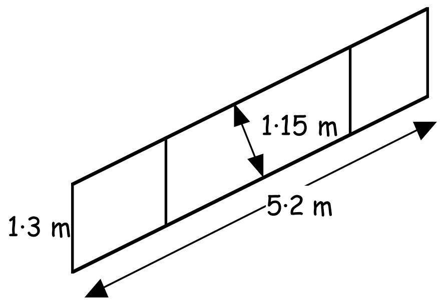
Calculate the area of this part.



8. The movable stairway is used at many older airports to allow passengers to disemb
from aircraft.

Again, a parallelogram shape is noticeable.

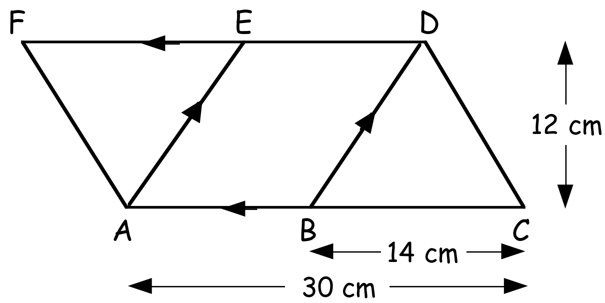
Find the area of the large parallelogram.

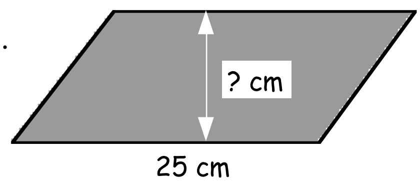


9. Look at the diagram shown and :-

(a) name 2 parallelograms.

(b) calculate the area of each one.



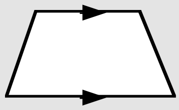
10. 

The area of the shaded parallelogram is 350 cm^2.

What is its height ?

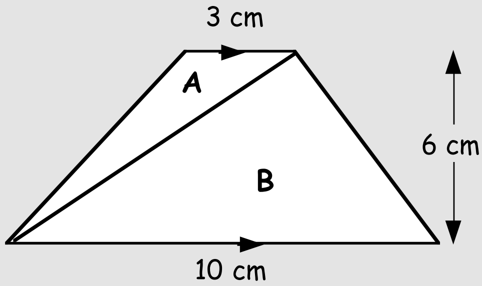
The Area of a Trapezium

A **TRAPEZIUM** is a 4-sided figure (Quadrilateral) with 2 sides parallel.



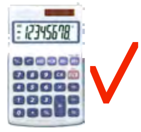
The Area of a Trapezium is found by :-

- drawing in one of its diagonal lines, splitting the figure into 2 triangles,
- working out the area of each triangle,
- adding the 2 triangular areas together.



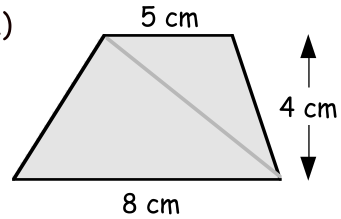
Area ΔA	=	$\frac{1}{2} \times B \times H = \frac{1}{2} \times 3 \times 6 = 9\ cm^2$
Area ΔB	=	$\frac{1}{2} \times B \times H = \frac{1}{2} \times 10 \times 6 = 30\ cm^2$
Total Area	=	$9\ cm^2 + 30\ cm^2 = \underline{\underline{39}}\ cm^2$

Exercise 5

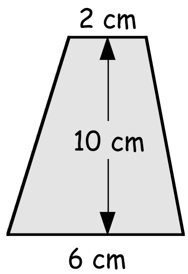


1. For each of the following, sketch and split each trapezium into 2 triangles, and calculate the area :-

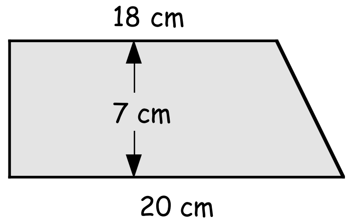
(a)



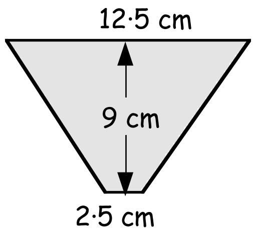
(b)



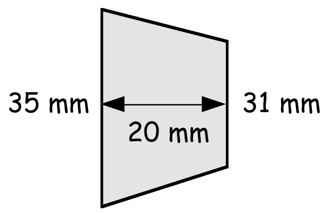
(c)



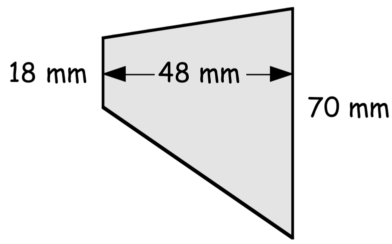
(d)



(e)



(f)



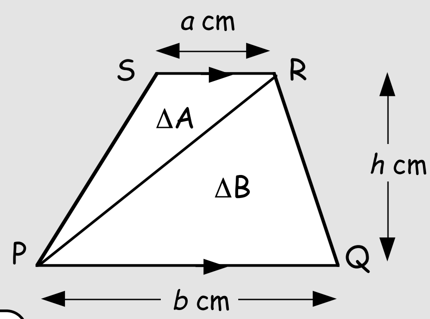
The Area of a Trapezium - A FORMULA

Area of trapezium PQRS

$= $ Area of ΔA + Area of ΔB

$= \frac{1}{2}ah + \frac{1}{2}bh$

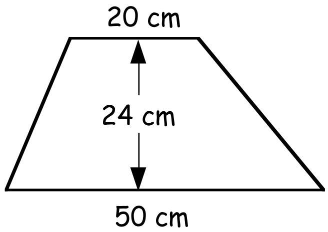
$= \frac{1}{2}(a + b)h$



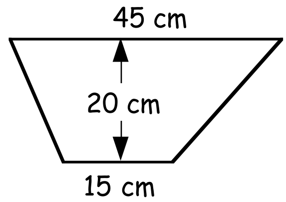
Area of a Trapezium $= \frac{1}{2}$ (the sum of parallel sides) $\times$ height

2. Use the formula Area $= \frac{1}{2}(a+b)h$ to calculate the area of each of the trapezia :-

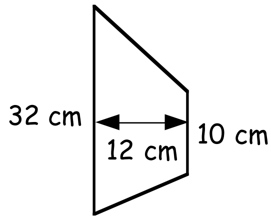
(a)



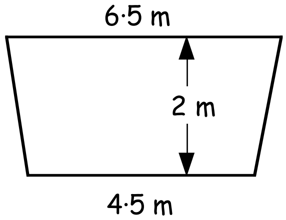
(b)



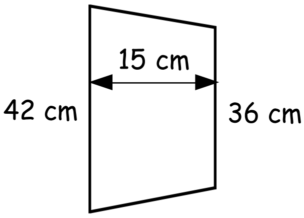
(c)



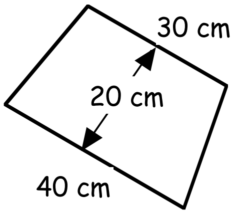
(d)

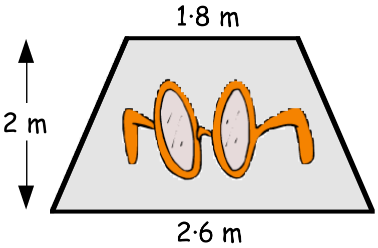


(e)



(f)



3. 

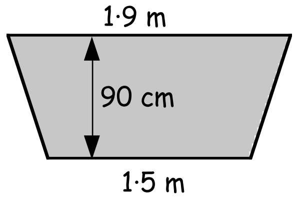
The sign outside "Trapezia Spectacles" is similar to the one shown.

Calculate the area of the trapezium.

4. Calculate out the area of this Malaysian stamp.

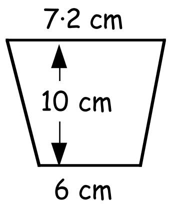
5. The top of this office table is in the shape of a trapezium.

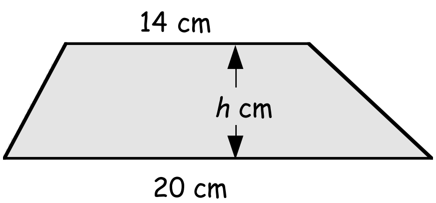
Find its area in cm^2.



6. The gaps in the alloy wheels are trapezium shaped.

Calculate the total area taken up by the 8 gaps in this wheel.



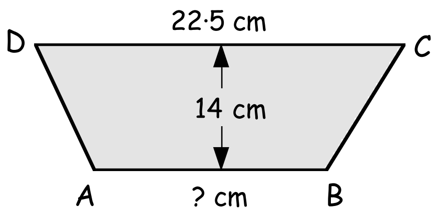
7. 

The area of this trapezium is 136 cm^2.

Calculate its height (h cm).

8. The area of the trapezium ABCD is 273 cm^2.

Calculate the length of the line AB.

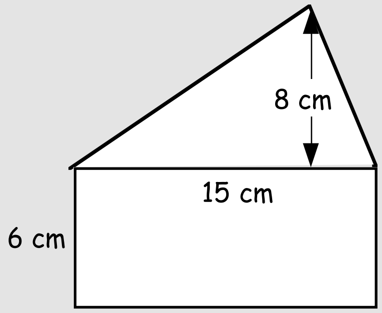


Composite Shapes

In this chapter, we have studied the area of a Rectangle, Square, Triangle, Kite, Rhombus Parallelogram and Trapezium. We now look at examples where we will combine these areas.

Example

Find the area of the shape below, consisting of a triangle on top of a rectangle.

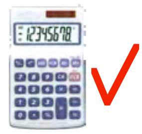


Area Rectangle $= L \times B$
$= 15 \times 6$
$= \underline{\underline{90\ \text{cm}^2}}$

Area Triangle $= \frac{1}{2}\, B \times H$
$= \frac{1}{2}$ of 15×8
$= \underline{\underline{60\ \text{cm}^2}}$

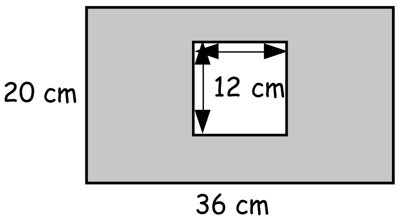
TOTAL $= \underline{\underline{150\ \text{cm}^2}}$

Exercise 6

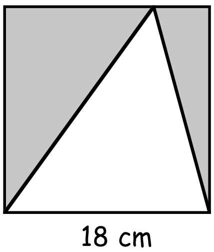


1. Calculate the shaded areas :-

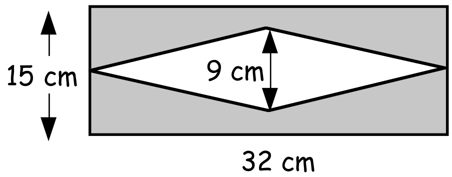
(a) Rectangle & Square



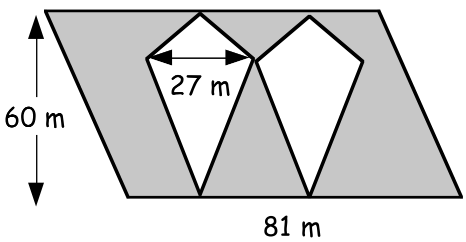
(b) Square & Triangle



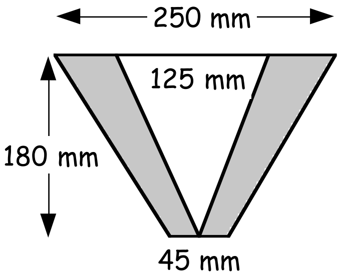
(c) Rectangle & Rhombus



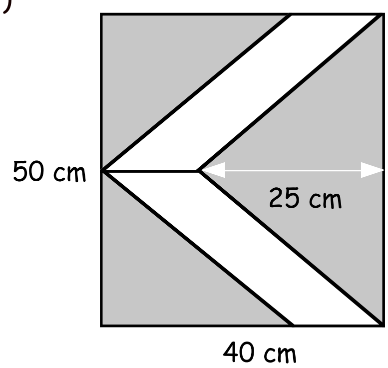
(d) Parallelogram & Identical Kites



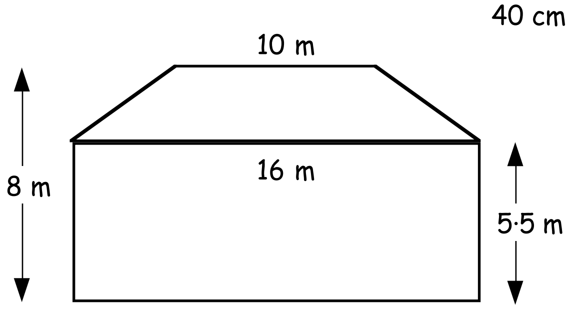
(e) Trapezium & Triangle



(f) Rectangle & Identical Parallelograms

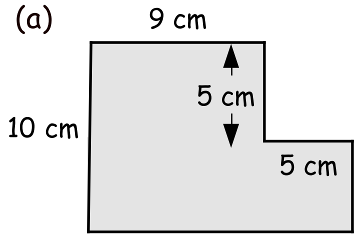


2. Find the area of this house-shape, made up of a rectangle and a trapezium.

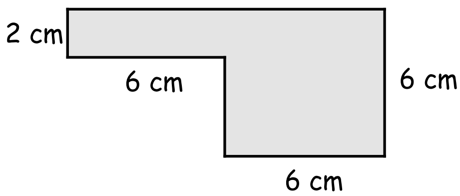


3. Calculate these areas :-

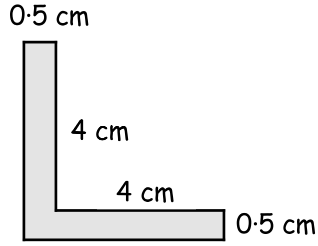
(a)



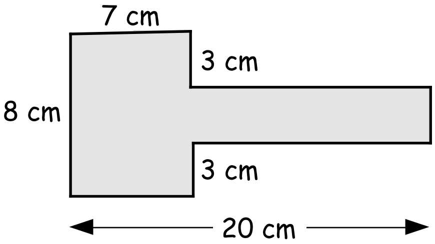
(b)



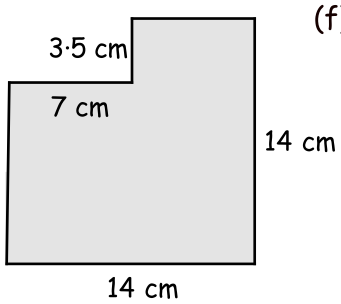
(c)



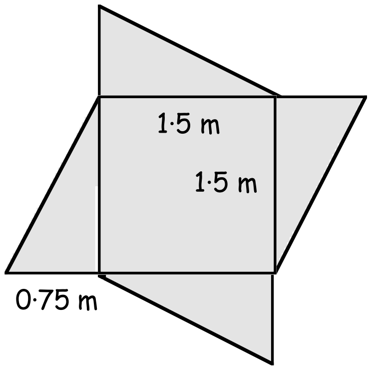
(d)



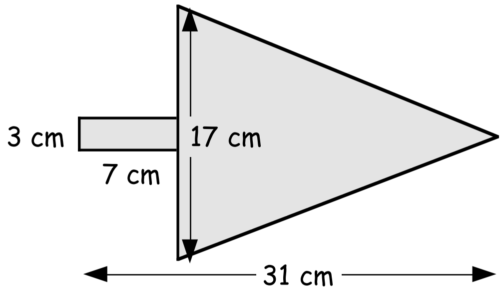
(e)



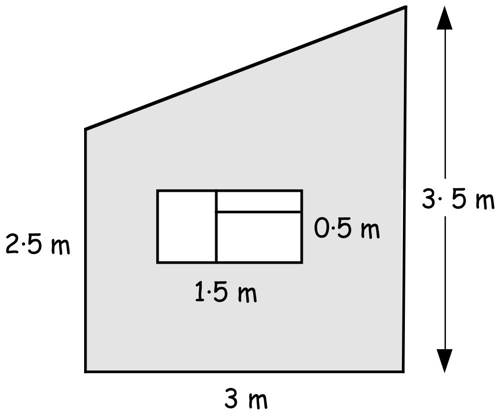
(f)



4. Calculate the area of this arrow, consisting of a triangle and a rectangle :-



5.

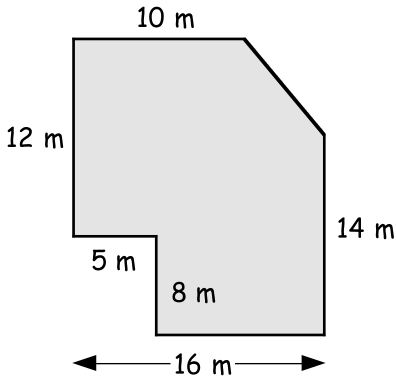


Calculate the painted area of the side of a shed.

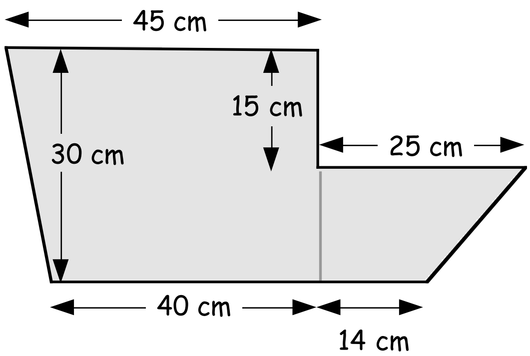
The dimensions of the window are 0·5 m by 1·5 m.

6. Calculate the area of the composite shapes :-

(a)



(b)

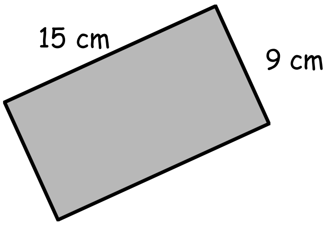


Areas

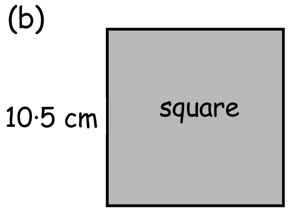
Topic in a Nutshell

1. Calculate the area of each of the following figures, using an appropriate formula. Make sure your answers are given in the correct units.

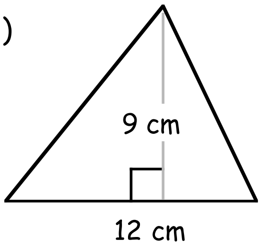
(a)



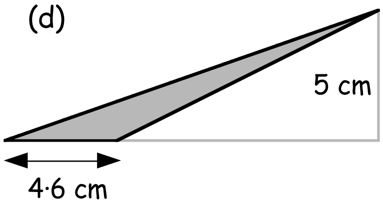
(b)



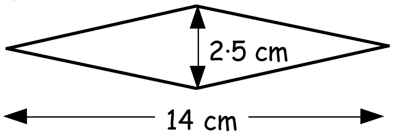
(c)



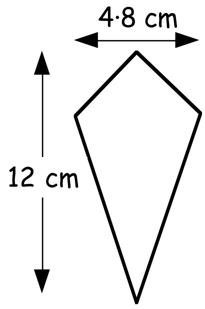
(d)



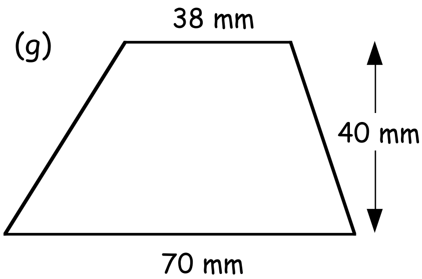
(e)



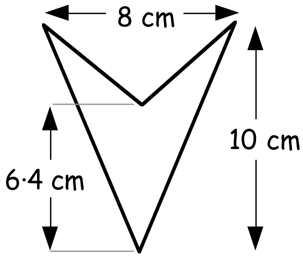
(f)



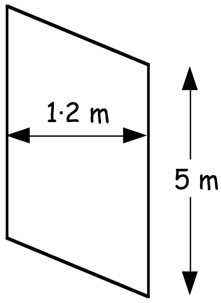
(g)



(h)

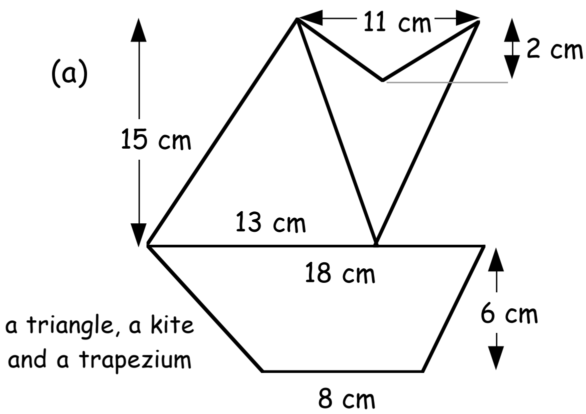


(i)

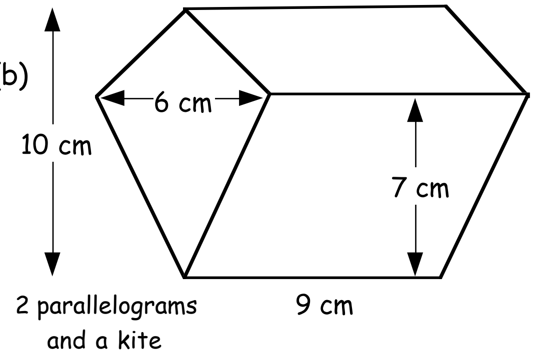


2. Calculate the area of each of the following shapes :-

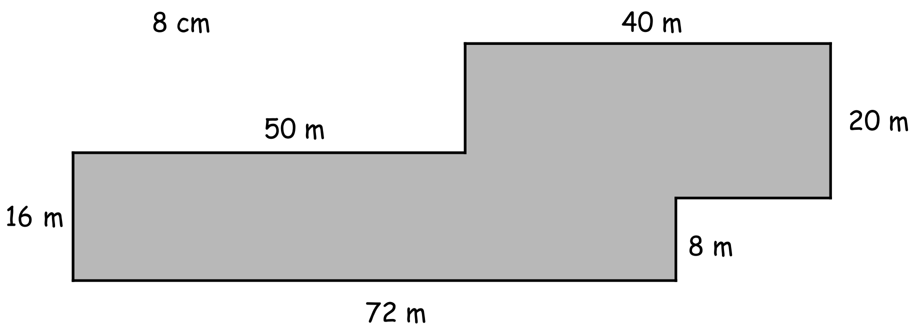
(a)



(b)

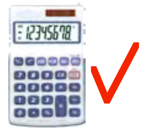


(c)

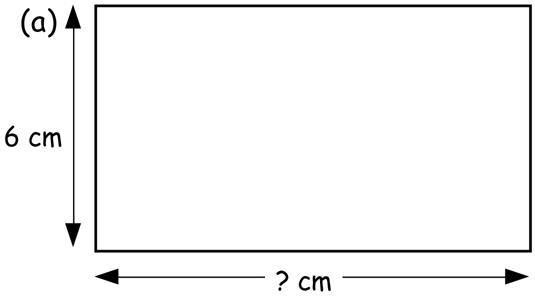


3. The area of each shape shown below is given.

Calculate the length of the particular side you are asked for.



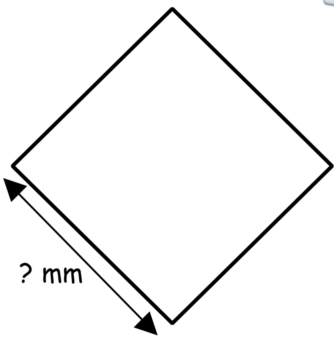
(a)



Rectangle

Area = 57 cm^2

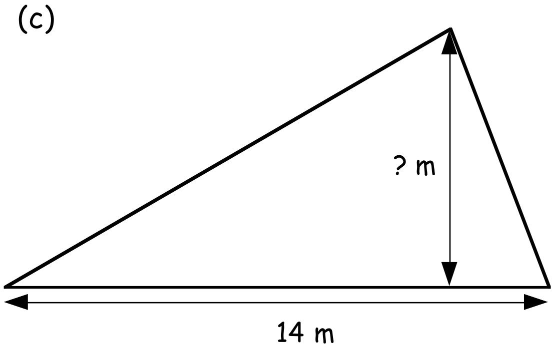
(b)



Square

Area = 64 mm^2

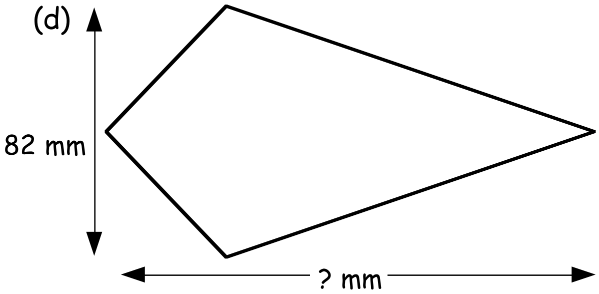
(c)



Triangle

Area = 63 m^2

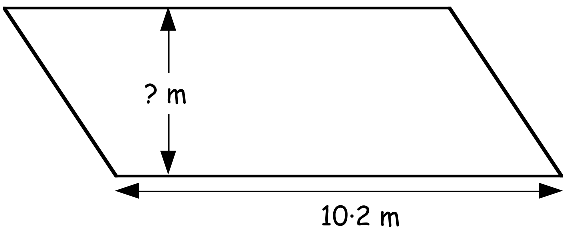
(d)



Kite

Area = 5125 mm^2

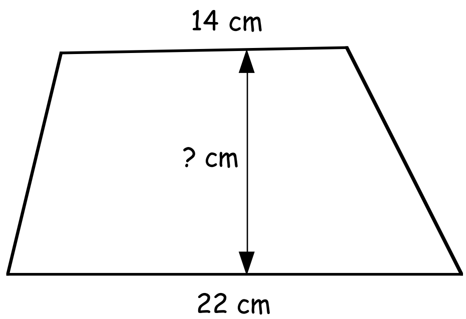
(e)



Parallelogram

Area = 65·28 m^2

(f)



Trapezium

Area = 180 cm^2

Chapter 29

Circle Work

Circumference

Remember that the perimeter or
the **circumference** of a circle
can be measured by the formula : -

$C = \pi D$

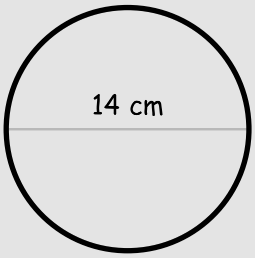
Circumference = 3·14 x Diameter

Example :- Calculate the circumference of this circle which has a diameter of 14 centimetres :-

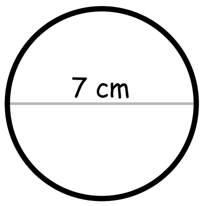
=> $C = \pi D$

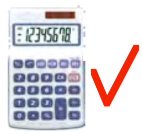
=> $C = 3{\cdot}14 \times 14$

=> $C = 43{\cdot}96$ cm



Exercise 1





1. Calculate the circumference of a circle with diameter 7 cm.
   Copy and complete : -

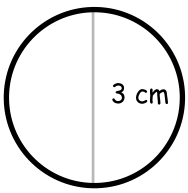
   => $C = \pi D$

   => $C = 3{\cdot}14 \times 7$

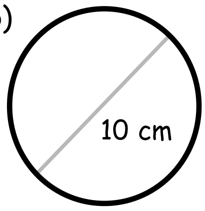
   => C = ... cm.

2. Calculate the circumference of each circle below : -
   (You **must** set down 3 lines of working for each)

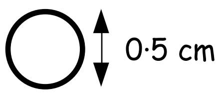
   (a)



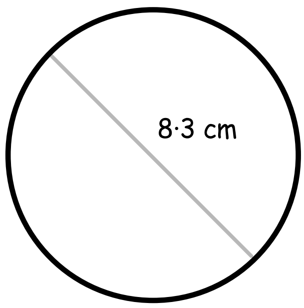
   (b)



   (c)

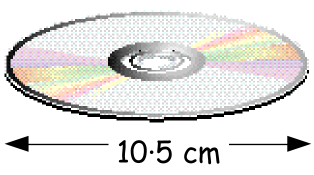


   (d)

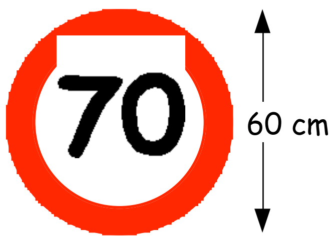


3. Find the circumference of each object below :-

   (a)



   (b)



   (c)

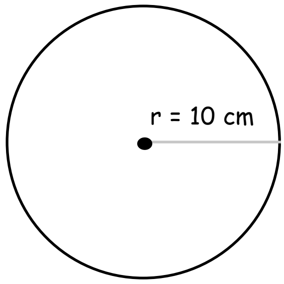
diameter = 4·8 cm

Remember if you are given a radius you need to **double** it to find the diameter.

4. Calculate the circumference of a circle with radius 10 cm.

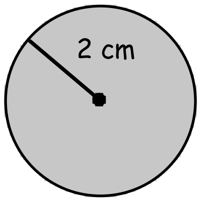
   Copy and complete : –

   radius = 10 cm
   => diameter = 20 cm
   => $C = \pi D$
   => $C = 3{\cdot}14 \times 20$
   => C = ... cm.

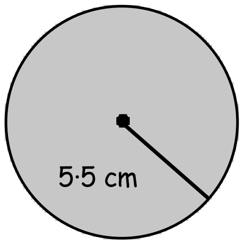


5. Calculate the circumference of each circle below : –

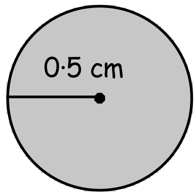
(a)



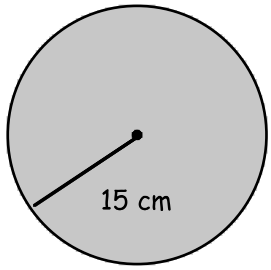
(b)



(c)



(d)



6. Find the perimeter of each object below :-

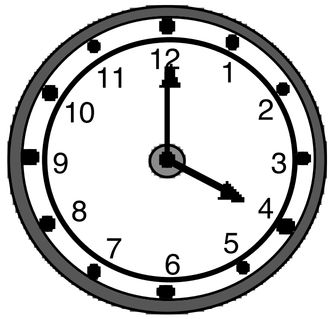
(a)

radius = 10 mm

(b)

radius = 16 cm

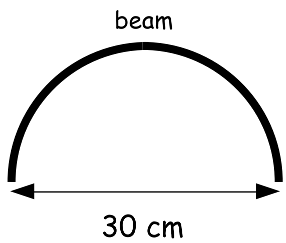
(c)



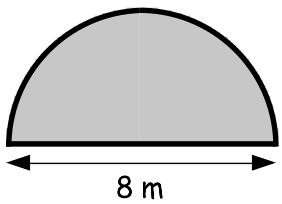
radius = 25·5 cm

7. A wooden beam, in the shape of a semi-circle, has diameter 30 centimetres.

   Calculate the length of the wooden beam.



8.

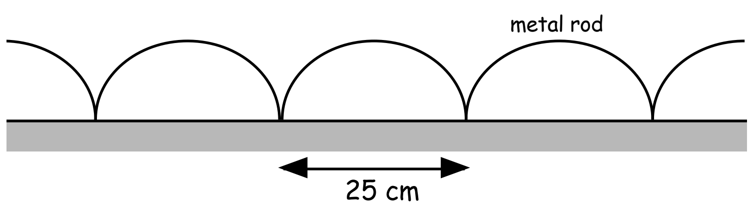


A semi-circular garden has a diameter of 8 metres.

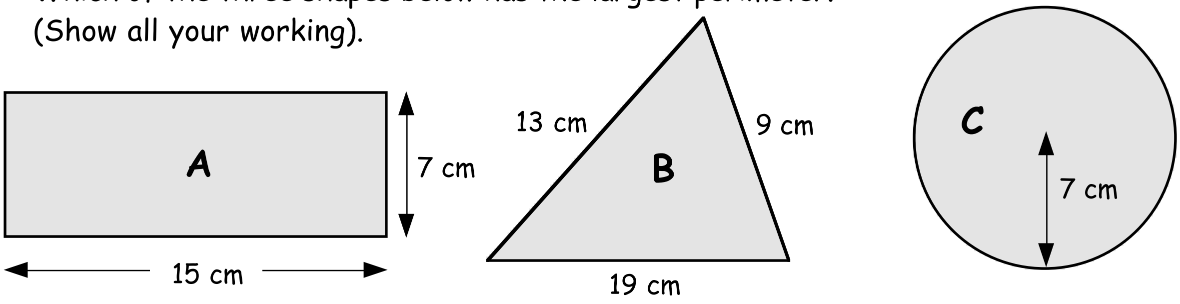
Calculate the perimeter of the garden.

9. A garden path has a fence made from strips of metal rod bent into semi-circles. Each semi-circle has a diameter of 25 centimetres.

   Find the length of metal rod needed to make the fence which has to be 4 metres long.

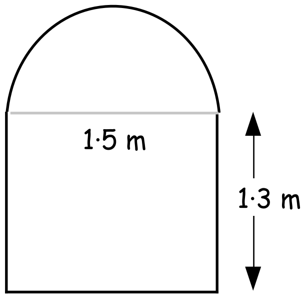


10. Which of the three shapes below has the largest perimeter. (Show all your working).

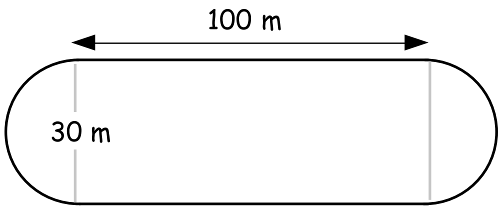


11. Calculate the perimeter of each shape below : –

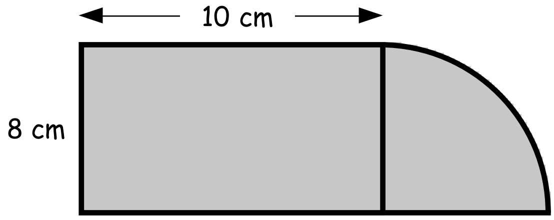
(a)



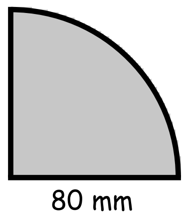
(b)



(c)



(d)



(Hint: length of a quarter circle....use $C = \pi D$ then ÷ 4)

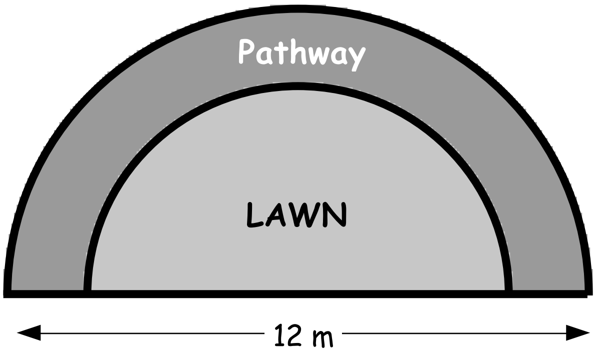
12. The semi-circular garden shown has a diameter of 12 metres.

A semi-circular brick pathway **one metre wide** partly surrounds the grass lawn.

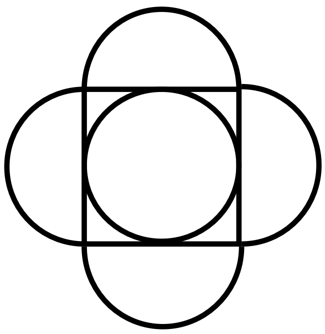
Find the perimeter of : –

(a) the grass lawn

(b) the brick path.



13.



A florist uses this flower pattern as his logo.

The pattern is formed from a square, four semi-circles and a circle.

The front of the shop has this design, made from wrought iron, bolted onto it.

If the square has each side 90 cm, what length of iron bar was needed to make the whole logo ?

Finding the Diameter

Can you see that you can use the formula :-

$$C = \pi D$$

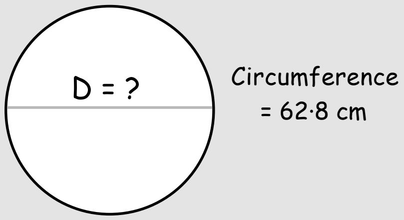
to calculate the **diameter** of a circle if you have been given its circumference ?

Example :- Find the diameter of a circle with circumference 62·8 cm.

$C = \pi D$

$62{\cdot}8 = 3{\cdot}14 \times D$

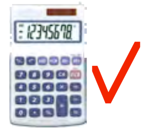
$D = \dfrac{62{\cdot}8}{3{\cdot}14} = \underline{\underline{20\text{ cm}}}$



Can you see that the formula needed to calculate the diameter is :-

Diameter → $\boxed{D = \dfrac{C}{\pi}}$ ← Circumference, ← 3·14

Exercise 2



1. Find the diameter of a circle with circumference 21·98 cm. Copy and complete :-

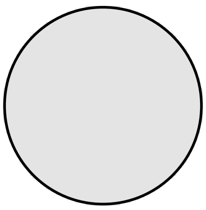
$D = \dfrac{C}{\pi}$

=> $D = \dfrac{21{\cdot}98}{3{\cdot}14}$

=> D = ... cm

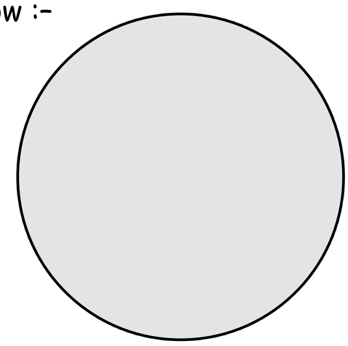
2. Calculate the diameter of each circle below :-
(You **must** set down 3 lines of working)

(a)



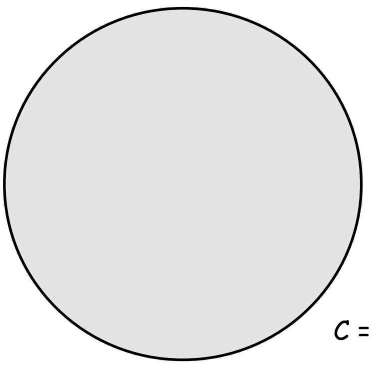
C = 28·26 cm

(b)



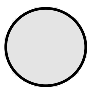
C = 56·52 cm

(c)



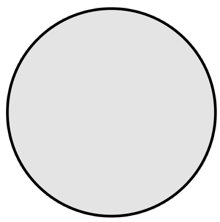
C = 10 346·3 mm

(d)



C = 3·14 mm

(e)



C = 15·7 m

3. Find the diameter of a circle with circumference :-

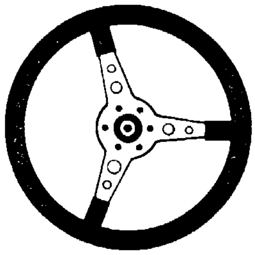
(a) 12·56 cm (b) 188·4 mm (c) 226·08 m

4. Write down the radius of each of the circles in question 3.

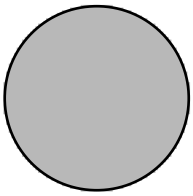
5. Find the **radius** of a circle with circumference 157 centimetres.

6. Find the radius of a circle with perimeter 471 metres.

7. (Give all answers to one decimal place).

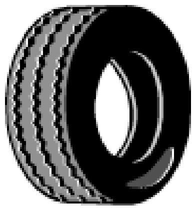


(a) Determine the diameter of the steering wheel given that its circumference is 60 centimetres.



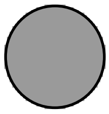
C = 3 m

(b) Find the diameter of the circular rug shown if the circumference is 3 metres in length.



(c) The circumference of a tyre is 1 metre.

Determine the radius of the tyre in cm.



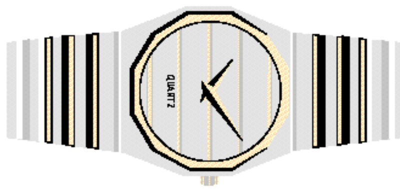
C = 15 cm

(d) A sausage slice has a circumference of 15 cm.

Find the radius of the sausage.

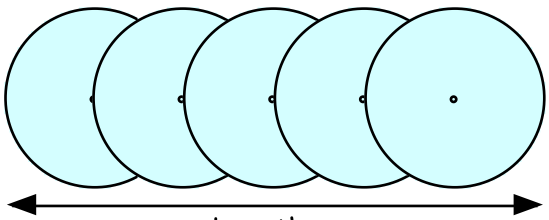
(e) A circular cog in a watch mechanism has a circumference of 1 millimetre.

Find the radius of the cog to 2 decimal places.



8. A company logo uses five circles, each with a **circumference** of 100 cm, which overlap as shown.

Find the total length of the company logo.



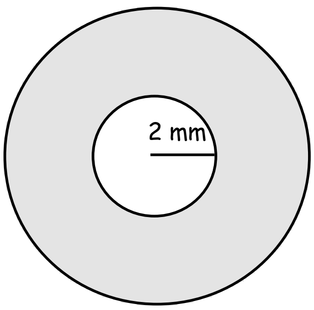
Length = ...

9. A machine washer has an outer circumference of 36 millimetres. The hole has a 2 millimetre radius.

Calculate :-

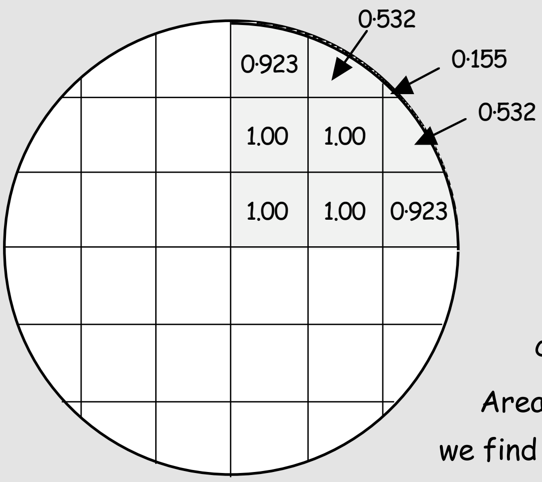
(a) the outer diameter of the washer.

(b) the circumference of the hole.



Circ = 36 mm

Area of a circle



The shaded area (quarter circle) has been put onto a square centimetre grid and the area for each part has been measured and given.

The total shaded area (quarter circle) is 7·065
This means the total area of the circle is

$$7{\cdot}065 \times 4 = \mathbf{28{\cdot}26\ cm^2}$$

There is an formula (or rule) we can use to calculate the area of a circle.

Area generally uses two measurements (... cm x ... cm) we find that if we find $r \times r$ (or r^2) and multiply it by π, we get an answer of **$28{\cdot}26$ cm^2**

(which is the same value as we found by measuring !)

To find the area of a circle we can use $\pi \times r \times r$ **or** $\boxed{A = \pi r^2}$

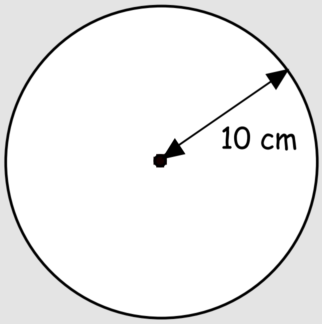
Area = 3·14 x radius x radius

Example :- Calculate the area of a circle with radius 10 cm.

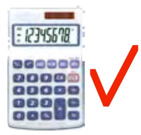
$A = \pi r^2$

=> $A = 3{\cdot}14 \times 10 \times 10$

=> $A = \underline{\underline{314\ cm^2}}$ (square centimetres)



Exercise 3

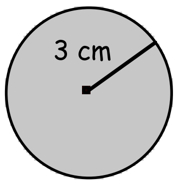


1. Find the area of a circle with radius 3 cm.

Copy and complete :- $A = \pi r^2$

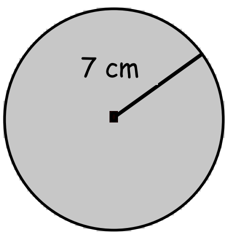
=> $A = 3{\cdot}14 \times 3 \times 3$

=> $A = ...\ cm^2$

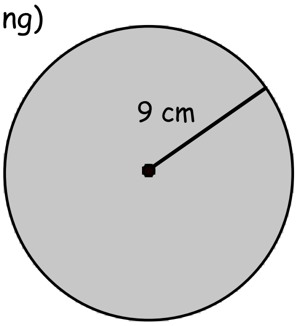


2. Calculate the area of each circle below :-
(You **must** set down 3 lines of working)

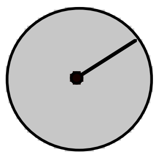
(a)



(b)



(c)



radius = 7·5 mm

Give each answer to 2 decimal places where necessary.

3. Find the area of each object below : -

(a) 

Button Radius = 10·5 mm

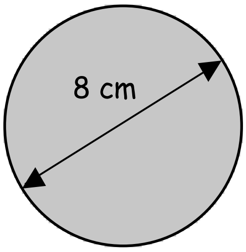
(b) 

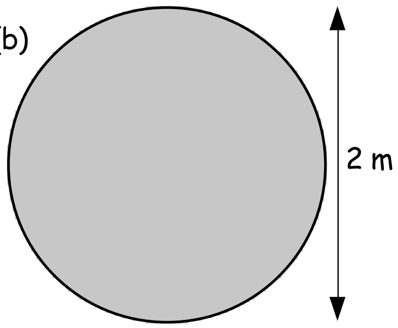
biscuit radius = 3·5 cm

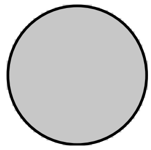
(c) 

manhole cover radius = 30·2 cm

4. Find the area of each circle below : –
(Remember you must use the **radius**)

(a) 

(b) 

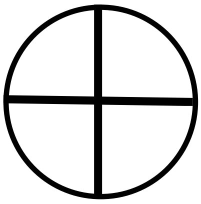
(c) 

Diameter = 20 mm

5. (a) Find the area of a circular painting with diameter 70 centimetres.

(b) Find the area of a circular place-mat whose radius is 13 centimetres.

(c) Find the area of a circular rug with diameter 1·2 metres.



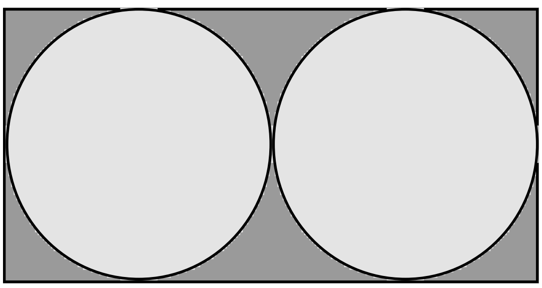
(d) The circular glass window shown has a radius of 0·75 metres.
Find the area of the window.

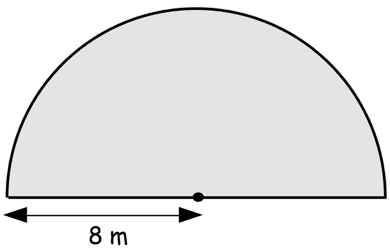
6. Two circular mirrors of radius 60 centimetres are placed side by side on a frame as shown.

(a) Find the area of each circle.

(b) Find the total area of the frame.

(c) Find the area of frame **not** covered by **both** mirrors.



7. 

A garden is in the shape of a semi-circle.

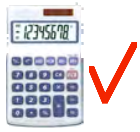
Find the area of the garden.

Mixed Problems

Remember, to find : -

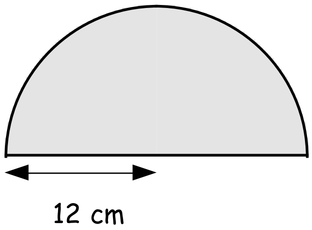
(i)	the area of a semicircle	find the area of the whole circle and half it.
(ii)	the area of a quarter circle ...	find the area of the whole circle and quarter it.
(iii)	a composite area	find the area of each part and add them.

Exercise 4

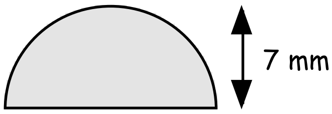


1. Find the area of each semi-circle : -

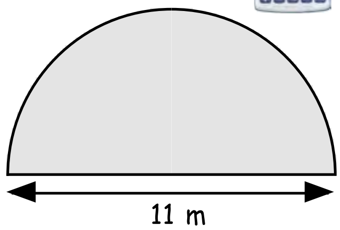
(a)



(b)



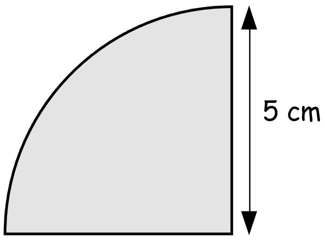
(c)



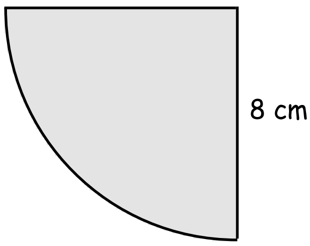
2. Find the perimeter of each shape in question 1.

3. Find the **area** of each of these shapes : -

(a)



(b)



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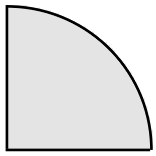
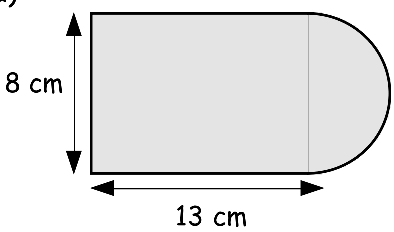


Quarter circle with 1 metre diameter

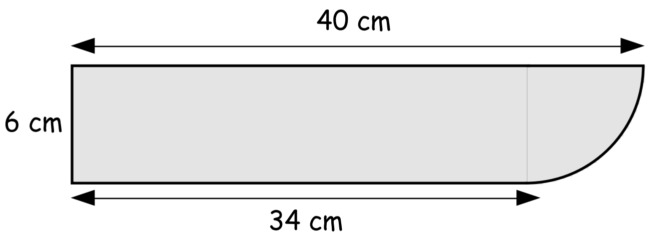
4. Find the **perimeter** of each shape in question 3.

5. For each of the shapes below find : (i) the area (ii) the perimeter.

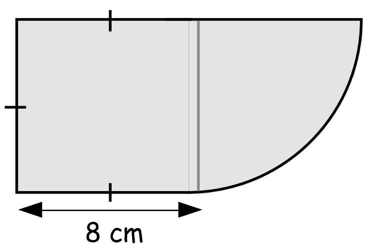
(a)



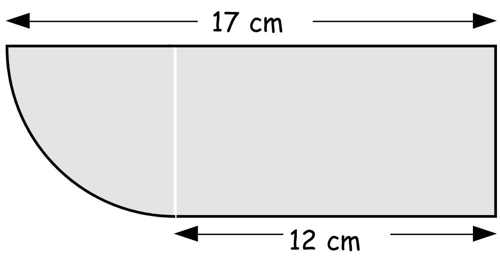
(b)



(c)

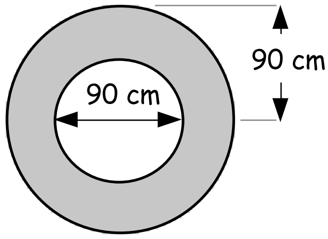


(d)



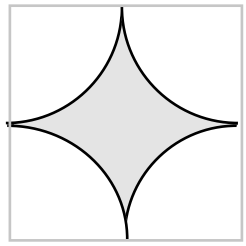
6. A hole (90 cm in diameter) has been cut from a flat circular sheet of metal with a 90 cm radius.

Find the area of the metal remaining.



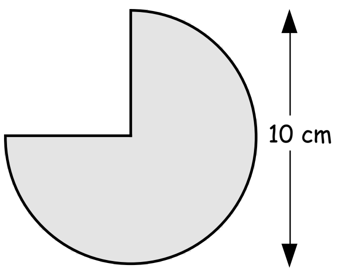
7. A square with side 8 centimetres has four identical quarter circles cut out from each corner as shown.

Determine the shaded area.

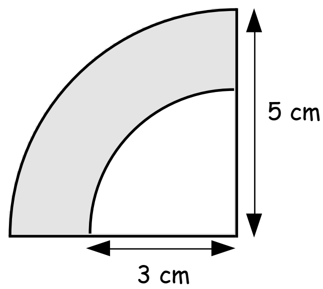


8. For each of the following, find the shaded area : –

(a)

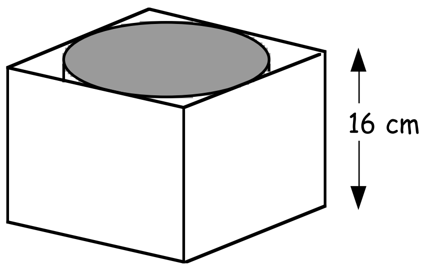


(b)



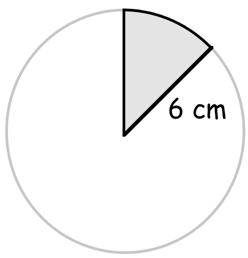
9. A circular tin with circumference 78·5 centimetres fits exactly into a box in the shape of a cuboid.

Find the volume of the box.
(*Volume = length × breadth × height*)



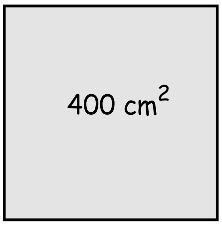
10. The shape shown opposite is one **eighth** of a circle which has a radius of 6 centimetres.

Calculate the area of this shape.



11. A circular grass lawn has an area of 314 square metres.
Find the **circumference** of the lawn.

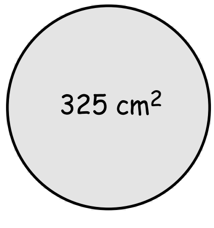
12.



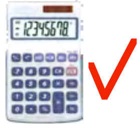
Mandy has a square piece of material with an area of 400 cm^2.

She is trying to make a circular speaker cover which has to have an area of 325 cm^2.

Explain why Mandy cannot make it from the square piece of cloth.



Circles



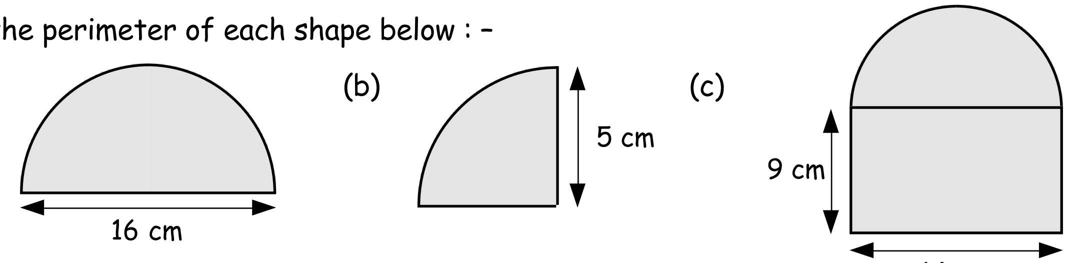
Topic in a Nutshell

1. Find the circumference of a circle with : –

   (a) a diameter of 20 cm   (b) a radius of 4 m.

2. Find the perimeter of each shape below : –

   (a) (b) (c)



3. Answer each question below (to one decimal place).

   (a) Find the diameter of a circle with circumference of 30 centimetres.

   (b) Find the radius of a circle with a perimeter of 120 metres.

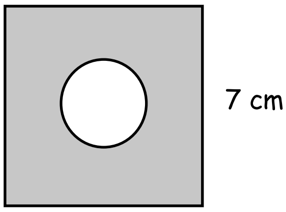
4. Find the area of a circle with :-

   (a) a radius of 20 cm   (b) a diameter of 42 mm.

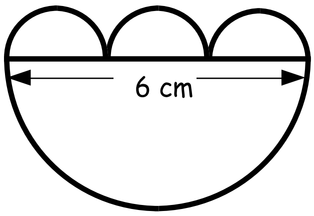
5. Find the area of each shape in question 2.

6. A metal square, with side 7 centimetre, has a 2 centimetre diameter hole punched through its middle.

   Find the area of metal remaining.



7.

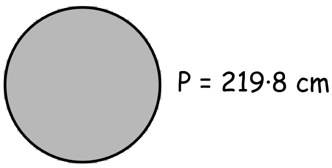


An earring is made of silver wire using three small semicircles of equal size, one large semi-circle and a 6 centimetre straight piece of silver wire.

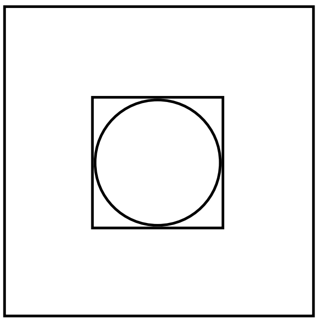
Find the total length of silver wire needed.

8. A large circular pane of glass has a perimeter of 219·8 cm.

   Calculate the **area** of the glass.



9.



The large square has side 20 cm.

The area of the smaller square is <u>one quarter</u> that of the large square.

(a) If the circle fits perfectly inside the smaller square, find the circumference of the circle.

(b) Find the area of the circle.

Chapter 31

Fraction Calculations

Reminder

A fraction consists of 2 parts. $\frac{3}{5}$ ← **the numerator** ← **the denominator**

It may be possible to "simplify" a fraction by dividing top and bottom by a number.

$\frac{9}{12}$ becomes $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

$\frac{18}{27}$ becomes $\frac{18 \div 9}{27 \div 9} = \frac{2}{3}$

Mixed Fractions

A fraction, like, $\frac{19}{4}$, where the **numerator** is bigger than the **denominator** is called a "**top-heavy**" fraction.

A number consisting of "whole" part and a "fraction" part is called a **mixed** fraction.

Example 1 Changing a top-heavy fraction to a mixed fraction :-

=> $\frac{23}{4}$ really means $23 \div 4$ => $4\overline{)23}$ with 5 above (remainder 3) => $5\frac{3}{4}$

note :- the 3 is divided by the 4

$\frac{25}{7}$ really means $25 \div 7$ => $7\overline{)25}$ with 3 above (remainder 4) => $3\frac{4}{7}$

Example 2 Changing a mixed fraction to a top-heavy fraction :-

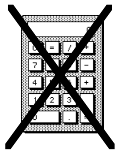
To change $6\frac{2}{3}$ into "thirds"

- Step 1 - multiply the 6 by the 3
- Step 2 - now add on the 2 (thirds).

=> $6\frac{2}{3}$ = ((6 x 3) + 2) thirds = 20 "thirds" = $\frac{20}{3}$

=> $2\frac{5}{8}$ = ((2 x 8) + 5) eights = 21 "eights" = $\frac{21}{8}$

Exercise 1



1. Copy and complete the following :-

(a) $\frac{15}{2}$ really means $15 \div 2$ => $2\overset{\dots}{\overline{)15}}$ (remainder ....) => $7\frac{...}{2}$.

(b) $\frac{23}{6}$ really means $23 \div 6$ => $6\overset{\dots}{\overline{)23}}$ (remainder ....) => $3\frac{...}{6}$.

(c) $\frac{13}{9}$ really means $13 \div \dots$ => $\dots\overset{\dots}{\overline{)13}}$ (remainder ....) => $\dots\frac{...}{9}$.

(d) $\frac{17}{5}$ really means $\dots \div \dots$ => $5\overset{\dots}{\overline{)\dots}}$ (remainder ....) => $\dots\frac{...}{5}$.

2. In a similar way, change the following top-heavy fractions to mixed numbers :-

(a) $\frac{11}{3}$ (b) $\frac{21}{4}$ (c) $\frac{29}{6}$ (d) $\frac{9}{2}$

(e) $\frac{32}{5}$ (f) $\frac{65}{8}$ (g) $\frac{73}{10}$ (h) $\frac{31}{20}$

3. (a) Four boys decide to share 11 bars of chocolate evenly.
What will each boy receive (as a mixed number) ?

(b) 23 kg of potatoes are packed evenly into 5 bags.
What weight of potatoes goes into each bag ?

(c) A container holds 17 litres of water. An equal quantity of water is poured into 6 cups such that each holds the same amount.
How much water will be in each cup ?

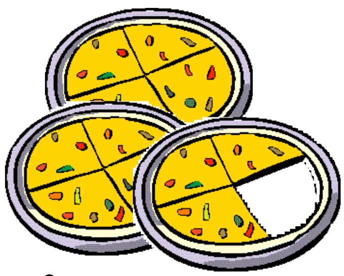
4. Copy and complete :- $\frac{20}{6} = 20 \div 6 = 3\frac{2}{6} = 3\frac{...}{3}$ (<— simplified).

5. Change each of the following to mixed numbers and simplify where possible :-

(a) $\frac{15}{6}$ (b) $\frac{18}{4}$ (c) $\frac{18}{8}$ (d) $\frac{32}{10}$

(e) $\frac{30}{9}$ (f) $\frac{38}{8}$ (g) $\frac{45}{20}$ (h) $\frac{175}{100}$

6. This diagram represents $2\frac{3}{4}$ pizzas.



(a) How many "$\frac{1}{4}$" pizza slices do you get from 1 pie ?

(b) How many "$\frac{1}{4}$" pizza slices do you get from 2 pies ?

(c) How many "$\frac{1}{4}$" pizza slices do you get from $\frac{3}{4}$ of a pie ?

(d) How many "$\frac{1}{4}$" pizza slices is this altogether from the $2\frac{3}{4}$ pizzas ?

(e) Write this as $2\frac{3}{4} = \frac{...}{4}$.

7. These "pizzas" have been cut into "thirds".

(a) From the 4 whole pizzas, you get ..... thirds?

(b) From the $\frac{2}{3}$ pizza, you get ..... thirds?

(c) How many thirds is this altogether ?

(d) Write this as $4\frac{2}{3} = \frac{...}{3}$.

8. Copy and complete :-

(a) $3\frac{2}{5}$ = ((3 x 5) + 2) "fifths" = 17 "fifths" = $\frac{...}{5}$.

(b) $1\frac{3}{10}$ = ((1 x 10) + 3) "tenths" = 13 "tenths" = $\frac{...}{10}$.

(c) $2\frac{7}{8}$ = ((2 x ...) + ...) "eighths" = ... "eighths" = $\frac{...}{...}$.

(d) $10\frac{7}{9}$ = ((... x ...) + ...) "ninths" = ... "ninths" = $\frac{...}{...}$.

9. Copy and complete :-

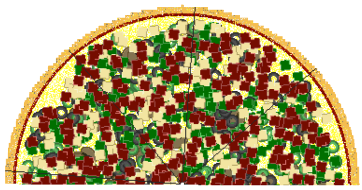
(a) $4\frac{1}{3}$ = ... (b) $2\frac{4}{7}$ = ... (c) $8\frac{4}{5}$ = ... (d) $2\frac{9}{10}$ = ...

10. Change each of the following mixed numbers to top heavy fractions :-

(a) $1\frac{1}{2}$ (b) $5\frac{3}{4}$ (c) $6\frac{2}{5}$ (d) $7\frac{1}{8}$

(e) $10\frac{2}{3}$ (f) $3\frac{9}{10}$ (g) $2\frac{5}{9}$ (h) $6\frac{6}{7}$

11. How many $\frac{1}{2}$ pizza slices can I get from :-



(a) 2 pizzas (b) 5 pizzas

(c) $3\frac{1}{2}$ pizzas (d) $10\frac{1}{2}$ pizzas ?

12. How many $\frac{1}{3}$ litre glasses can be filled from :-

(a) 3 litres (b) $1\frac{2}{3}$ litres (c) $2\frac{1}{3}$ litres (d) $6\frac{2}{3}$ litres ?

13. How many $\frac{1}{4}$ kg bags of salt can be filled from :-

(a) 2 kg (b) $1\frac{3}{4}$ kg (c) $4\frac{1}{4}$ kg (d) $3\frac{1}{2}$ kg ?

14. To add $2\frac{3}{5} + 4\frac{3}{5}$ we could change them to "$\frac{1}{5}$'s"

Copy and complete :- $2\frac{3}{5} + 4\frac{3}{5}$

$= \frac{...}{5} + \frac{...}{5}$

$= \frac{...}{5} = 7\frac{...}{5}$.

Adding and Subtracting Fractions (basic)

Simple Rule :- You can only add (or subtract) two fractions if

THEY HAVE THE SAME DENOMINATOR.

Example 1

$\frac{3}{7} + \frac{2}{7}$

$= \frac{5}{7}$

Example 2

$\frac{7}{8} - \frac{1}{8}$

$= \frac{6}{8}$ $(= \frac{3}{4})$

Example 3

$2\frac{3}{5} + 1\frac{4}{5}$

$= 3\frac{7}{5}$

$= 4\frac{2}{5}$

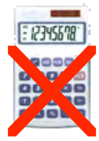
Example 4

$5\frac{5}{6} - 1\frac{1}{6}$

$= 4\frac{4}{6}$

$= 4\frac{2}{3}$

Exercise 2



1. Copy and complete the following :-

(a) $\frac{3}{5} + \frac{1}{5}$ $= \frac{\dots}{5}$

(b) $\frac{7}{9} - \frac{5}{9}$ $= \frac{\dots}{9}$

(c) $\frac{7}{10} - \frac{3}{10}$ $= \frac{\dots}{10} = \frac{\dots}{5}$

(d) $\frac{3}{8} + \frac{3}{8}$ $= \frac{\dots}{8} = \frac{\dots}{4}$

2. Copy the following and simplify :-

(a) $\frac{2}{7} + \frac{4}{7}$ (b) $\frac{1}{9} + \frac{5}{9}$ (c) $\frac{7}{8} - \frac{3}{8}$ (d) $\frac{4}{5} + \frac{4}{5}$

(e) $\frac{7}{11} - \frac{3}{11}$ (f) $\frac{2}{3} + \frac{2}{3}$ (g) $\frac{5}{6} + \frac{1}{6}$ (h) $\frac{7}{12} - \frac{1}{12}$

3. Copy the following and simplify :-

(a) $2\frac{1}{2} + 3\frac{1}{2}$ (b) $5\frac{3}{4} - 1\frac{1}{4}$ (c) $4\frac{1}{3} + 3\frac{1}{3}$ (d) $2\frac{7}{9} + 1\frac{4}{9}$

(e) $6\frac{3}{4} - 1\frac{1}{4}$ (f) $4\frac{2}{7} + 3\frac{3}{7}$ (g) $5\frac{7}{9} - 1\frac{2}{9}$ (h) $10\frac{4}{5} + 3\frac{3}{5}$

4. Of the $\frac{5}{8}$ kilometre to his school, David had walked $\frac{1}{8}$ km. How much further had he to go ?

5. Hat sizes go up in $\frac{1}{8}$'s of an inch at a time.
Billy wears a hat size $6\frac{7}{8}$. Alex is 3 sizes bigger than this.
What is Alex's hat size ?

6. John mixes $3\frac{3}{5}$ kg sand with $4\frac{4}{5}$ kg of cement.
What is the total weight of the mixture ?

7. (a) A piece of rope was $6\frac{4}{5}$ metres long.

A piece measuring $3\frac{1}{5}$ was cut off.

What length of rope remained ?

(b) 2 jugs of water were poured into an empty basin.
The first jug held $2\frac{3}{4}$ litres and the second held $1\frac{3}{4}$ litres.

How much water was in the basin in total ?

(c) Of the $10\frac{5}{6}$ kilometres from her house to the shops,
Lucy had cycled $7\frac{1}{6}$ kilometres.

How much further has Lucy to cycle to reach the shops ?

(d) George ate $\frac{3}{5}$ of a pizza, Billy ate $\frac{4}{5}$ and Amanda ate $\frac{2}{5}$.

How much had they eaten altogether ?

(e) Bunty weighed $42\frac{6}{7}$ kilograms.

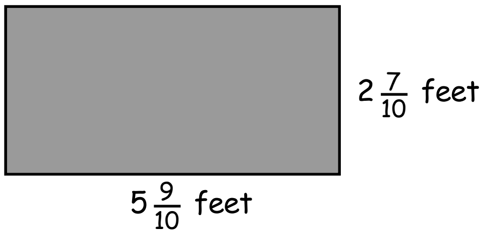
She went on a diet and lost $4\frac{3}{7}$ kilograms.

What is Bunty's new weight ?

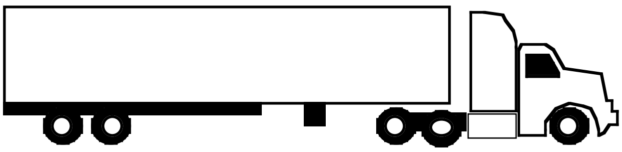
8. A table measures $5\frac{9}{10}$ feet long by $2\frac{7}{10}$ feet wide.

(a) By how much is the length bigger than the breadth ?

(b) Calculate the **perimeter** of the table top.



9. A lorry weighs $3\frac{7}{8}$ of a tonne. Crates are loaded onto the lorry.



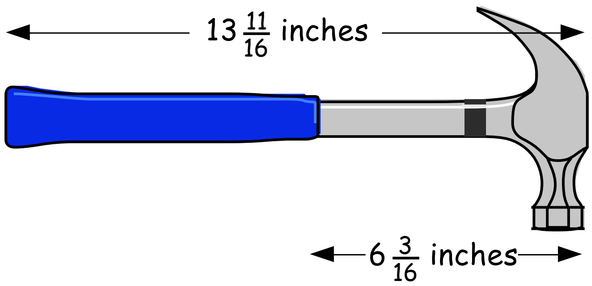
Each crate weighs $\frac{5}{8}$ tonnes.

What is the total weight of the lorry carrying a load of :-

(a) 1 crate (b) 2 crates (c) 3 crates ?

10. Look at the picture of the hammer.

Calculate the length of the rubber handle.



Adding and Subtracting Harder Fractions

<u>Remember the Golden Rule</u> :- The denominators **MUST** be the same if you wish to add or subtract.

What do we do if the denominators are not the same ?

Change each fraction so that they **DO HAVE THE SAME** denominator.

Example 1 Find $\frac{2}{3}+\frac{1}{2}$. [they do **not** add to give $\frac{3}{5}$ X]

- the denominators $\underline{3}$ and $\underline{2}$ are not the same.
- what is the l.c.m. (lowest common multiple) of 3 and 2 —> $\underline{\underline{6}}$.
- we must change $\frac{2}{3}$ and $\frac{1}{2}$ to $\frac{1}{6}$'s.

$$\frac{2}{3}+\frac{1}{2}$$

note $\quad \frac{4}{6}+\frac{3}{6} \quad$ note

$$=\frac{7}{6}=1\frac{1}{6}$$

$(\frac{2}{3}=\frac{?}{6}) \longrightarrow ? = 4$

$(\frac{1}{2}=\frac{?}{6}) \longrightarrow ? = 3$

Example 2

(8 and 5 go into 40)

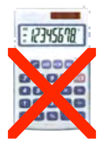
$$\frac{5}{8}-\frac{1}{5}$$
$$\frac{?}{40}-\frac{?}{40}$$
$$=\frac{25}{40}-\frac{8}{40}$$
$$=\frac{17}{40}$$

Example 3

(6 and 4 go into 12)

$$\frac{5}{6}+\frac{3}{4}$$
$$\frac{?}{12}+\frac{?}{12}$$
$$=\frac{10}{12}+\frac{9}{12}$$
$$=\frac{19}{12}=1\frac{7}{12}$$

Exercise 3



1. Copy each of the following and complete :-

(a) $\frac{3}{4}+\frac{1}{3}$ $=\frac{9}{12}+\frac{?}{12}$ $=\frac{?}{12}=1\frac{?}{12}$

(b) $\frac{4}{5}-\frac{2}{3}$ $=\frac{?}{15}-\frac{?}{15}$ $=\frac{?}{15}$

(c) $\frac{7}{8}-\frac{3}{4}$ $=\frac{?}{8}-\frac{?}{8}$ $=\frac{?}{8}$

(d) $\frac{6}{7}+\frac{2}{3}$ $=\frac{?}{21}+\frac{?}{21}$ $=\frac{?}{21}$

2. Show how to simplify the following :-

(a) $\frac{2}{3}+\frac{1}{5}$ (b) $\frac{3}{4}-\frac{1}{2}$ (c) $\frac{5}{8}+\frac{2}{3}$ (d) $\frac{4}{5}+\frac{1}{2}$

(e) $\frac{5}{6}-\frac{1}{3}$ (f) $\frac{3}{4}-\frac{2}{3}$ (g) $\frac{7}{10}+\frac{2}{5}$ (h) $\frac{7}{9}-\frac{1}{2}$

3. Show your working here :-

(a) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ (b) $\frac{5}{6} - \frac{1}{2} - \frac{1}{3}$ (c) $\frac{2}{3} + \frac{3}{5} - \frac{1}{2}$

<u>Mixed Fractions</u> :- Deal with the **whole numbers first**, then the fractions.

Example 4

$$2\tfrac{1}{2} + 3\tfrac{2}{3}$$
$$= 5(\tfrac{1}{2} + \tfrac{2}{3})$$
$$= 5(\tfrac{3}{6} + \tfrac{4}{6})$$
$$= 5\tfrac{7}{6}$$
$$= 6\tfrac{1}{6}$$

Example 5

$$7\tfrac{7}{8} - 4\tfrac{2}{3}$$
$$= 3(\tfrac{7}{8} - \tfrac{2}{3})$$
$$= 3(\tfrac{21}{24} - \tfrac{16}{24})$$
$$= 3\tfrac{5}{24}$$

Example 6

$$4\tfrac{3}{4} + \tfrac{5}{6}$$
$$= 4(\tfrac{3}{4} + \tfrac{5}{6})$$
$$= 4(\tfrac{9}{12} + \tfrac{10}{12})$$
$$= 4\tfrac{19}{12}$$
$$= 5\tfrac{7}{12}$$

4. Copy and complete the following :-

(a) $5\frac{1}{3} + 2\frac{1}{2}$ (b) $3\frac{3}{4} - 1\frac{1}{3}$ (c) $7\frac{7}{8} - 1\frac{1}{4}$ (d) $4\frac{1}{2} + 3\frac{3}{5}$

(e) $2\frac{5}{6} - 1\frac{1}{3}$ (f) $3\frac{2}{3} + 1\frac{5}{8}$ (g) $7\frac{1}{5} + 1\frac{1}{4}$ (h) $5\frac{9}{10} - 1\frac{1}{2}$

(i) $6\frac{1}{3} + \frac{7}{9}$ (j) $4\frac{4}{5} + 1\frac{3}{4}$ (k) $3\frac{1}{10} + 2\frac{2}{5}$ (l) $5\frac{1}{4} - 5\frac{1}{6}$

5. Copy and complete the following :-

(a) $6 - 2\frac{1}{3}$
$= 4 - \frac{1}{3}$
(6 - 2)
$= 3\frac{\dots}{3}$

(b) $5 - 3\frac{2}{5}$
$= 2 - \frac{2}{5}$
$= 1\frac{\dots}{5}$

(c) $10 - 7\frac{5}{8}$
$= 3 - \frac{\dots}{8}$
$= 2\frac{\dots}{8}$

(d) $4 - 3\frac{3}{5}$
$= 1 - \frac{\dots}{5}$
$= \frac{\dots}{5}$

6. Use the above method to find :-

(a) $4 - 1\frac{1}{5}$ (b) $6 - 3\frac{4}{7}$ (c) $10 - 5\frac{5}{6}$ (d) $6 - 4\frac{3}{5}$

(e) $5 - 4\frac{7}{10}$ (f) $35 - 29\frac{3}{8}$ (g) $12 - 6\frac{5}{7}$ (h) $8 - 3\frac{1}{3}$

7. From a 6 metre length of cable, the Telewest engineer cut off a piece which was $3\frac{3}{8}$ metres long. What was the length of the piece of cable remaining ?

8. It is exactly 12 miles from Bromley to Cardrew.

Davie and Bob left Bromley and jogged for $7\frac{3}{5}$ kilometres before stopping for a rest.

How much further had they still to jog to get to Cardrew ?

A Problem with Subtraction :- $4\frac{1}{3} - 1\frac{3}{5}$?

- **Step 1** - Subtract whole numbers first —> $3(\frac{1}{3} - \frac{3}{5})$
- **Step 2** - Change both fractions to $\frac{1}{15}$'s => $3(\frac{5}{15} - \frac{9}{15})$

  (* you cannot take $\frac{9}{15}$ from $\frac{5}{15}$!!!! *)
- **Step 3** - Take 1 whole number from the 3 and write it as $\frac{15}{15}$ (= 1)

  —> $3(\frac{5}{15} - \frac{9}{15})$

  becomes $2 + \frac{15}{15} + (\frac{5}{15} - \frac{9}{15}) = 2 + \frac{20}{15} - \frac{9}{15}$

  $= 2\frac{11}{15}$

Example 7

$6\frac{1}{4} - 1\frac{2}{3}$

$= 5(\frac{1}{4} - \frac{2}{3})$

$= 5(\frac{3}{12} - \frac{8}{12})$

change 1 (of the 5) to $\frac{12}{12}$

$= 4 + \frac{12}{12} + (\frac{3}{12} - \frac{8}{12})$

$= 4\frac{7}{12}$

Example 8

$5\frac{3}{5} - 1\frac{5}{6}$

$= 4(\frac{3}{5} - \frac{5}{6})$

$= 4(\frac{18}{30} - \frac{25}{30})$

$= 3 + \frac{30}{30} + (\frac{18}{30} - \frac{25}{30})$

$= 3\frac{23}{30}$

9. Copy and complete the following :-

(a)

$5\frac{2}{5} - 1\frac{1}{2}$

$= 4(\frac{2}{5} - \frac{1}{2})$

$= 4(\frac{4}{10} - \frac{5}{10})$

$= 3 + \frac{10}{10} + (\frac{4}{10} - \frac{5}{10})$

$= 3\frac{\dots}{10}$

(b)

$4\frac{3}{8} - 2\frac{3}{5}$

$= 2(\frac{3}{8} - \frac{3}{5})$

$= 2(\frac{15}{40} - \frac{24}{40})$

$= 1 + \frac{40}{40} + (\frac{15}{40} - \frac{24}{40})$

$= 1\frac{\dots}{40}$

(c)

$6\frac{1}{5} - \frac{3}{4}$

$= 6(\frac{1}{5} - \frac{3}{4})$

$= 6(\frac{4}{20} - \frac{15}{20})$

$= 5 + \frac{\dots}{20} + (\frac{4}{20} - \frac{15}{20})$

$= 5\frac{\dots}{20}$

10. Show all your working here :-

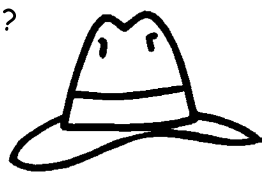
(a) $4\frac{1}{5} - 1\frac{1}{2}$ (b) $6\frac{3}{5} - 1\frac{5}{6}$ (c) $4\frac{1}{4} - 2\frac{1}{2}$ (d) $6\frac{3}{8} - 4\frac{3}{4}$

(e) $10\frac{1}{3} - 7\frac{1}{2}$ (f) $6\frac{1}{7} - 1\frac{1}{2}$ (g) $8\frac{1}{3} - 3\frac{7}{10}$ (h) $8\frac{1}{6} - 5\frac{2}{5}$

11. 

A fruit shop owner had $7\frac{1}{4}$ kg of strawberries.

If he sold $1\frac{1}{2}$ kg to a customer, what weight of strawberries did he have left ?



12. What is the difference in the two hat sizes, $7\frac{1}{8}$ and $6\frac{3}{4}$?

13. Of the $7\frac{1}{2}$ hours my flight takes to New York, I had flown $4\frac{3}{4}$ hours of it. How much longer will my journey take ?

14. Morag mixed the raw ingredients weighing $2\frac{1}{3}$ kg, when she baked a large loaf.

When it was finished the cake weighed $1\frac{3}{4}$ kg.

What was the weight loss in baking the cake ?

15. 

From a 7 metre length of cable, $2\frac{3}{5}$ metres was cut off.

What length of cable remained ?

Multiplying Fractions

The rule for multiplying two basic fractions is very simple.

To multiply $\frac{3}{5} \times \frac{4}{7}$ —> multiply the numerators —> multiply the denominators —> $\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$

Example 1

$\frac{3}{4} \times \frac{3}{5}$

$= \frac{3 \times 3}{4 \times 5}$

$= \frac{9}{20}$

Example 2

$\frac{4}{5} \times \frac{5}{6}$

$= \frac{20}{30} \left(\frac{\div 10}{\div 10}\right)$

$= \frac{2}{3}$ (simplified)

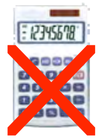
Example 3

$\frac{8}{9} \times \frac{3}{4}$

$= \frac{24}{36} \left(\frac{\div 12}{\div 12}\right)$

$= \frac{2}{3}$

Exercise 4



1. Copy each of the following and complete :-

(a) $\frac{2}{3} \times \frac{4}{5}$

$= \frac{2 \times 4}{3 \times 5}$

$= \frac{?}{15}$

(b) $\frac{5}{6} \times \frac{1}{3}$

$= \frac{5 \times 1}{6 \times 3}$

$= \frac{?}{?}$

(c) $\frac{3}{4} \times \frac{5}{6}$

$= \frac{3 \times 5}{4 \times 6}$

$= \frac{15}{24} = \frac{?}{8}$ (simplified)

2. Multiply the following fractions and simplify (where possible) :-

(a) $\frac{2}{5} \times \frac{2}{3}$ (b) $\frac{5}{6} \times \frac{3}{5}$ (c) $\frac{3}{7} \times \frac{4}{9}$ (d) $\frac{3}{10} \times \frac{5}{6}$

(e) $\frac{3}{8} \times \frac{4}{5}$ (f) $\frac{7}{12} \times \frac{4}{7}$ (g) $\frac{11}{16} \times \frac{2}{5}$ (h) $\frac{2}{9} \times \frac{9}{10}$

3. Calculate the **area** of a rectangular sheet of metal measuring $\frac{5}{6}$ metre by $\frac{3}{8}$ metre.

4. I spent $\frac{3}{4}$ of my pocket money in a shop. Of that, $\frac{2}{5}$ of it went on comics.
What fraction of my money was spent on comics ? (i.e $\frac{2}{5} \times \frac{3}{4}$)

Dealing with Mixed Fractions :- $\quad (3\frac{1}{2} \times 2\frac{1}{3})$

Simple Rule :- You **MUST CHANGE** mixed fractions to be top-heavy fractions.

Example 4

$$3\frac{1}{2} \times 2\frac{1}{3} = \frac{7}{2} \times \frac{7}{3} = \frac{49}{6} = 8\frac{1}{6}$$

Example 5

$$4\frac{3}{4} \times 1\frac{1}{3} = \frac{19}{4} \times \frac{4}{3} = \frac{76}{12} = \frac{19}{3} = 6\frac{1}{3}$$

5. Copy and complete the following :-

(a) $1\frac{1}{2} \times 2\frac{1}{3} = \frac{3}{2} \times \frac{7}{3} = \frac{21}{6} = 3\frac{\dots}{6} = 3\frac{\dots}{\dots}$

(b) $5\frac{2}{3} \times 1\frac{1}{4} = \frac{17}{3} \times \frac{5}{4} = \frac{85}{12} = 7\frac{\dots}{\dots}$

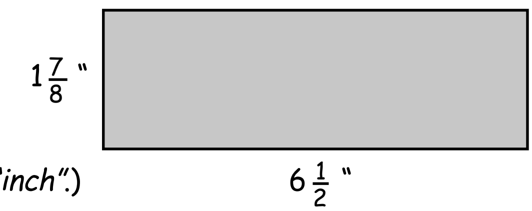
(c) $2\frac{2}{3} \times 1\frac{3}{4} = \frac{\dots}{3} \times \frac{\dots}{4} = \frac{\dots}{12} = 4\frac{\dots}{\dots} = 4\frac{\dots}{\dots}$

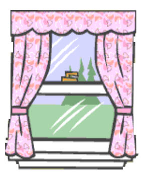
6. Do the following fractions in the same way (simplify if possible) :-

(a) $2\frac{1}{3} \times 2\frac{1}{2}$	(b) $4\frac{1}{5} \times 2\frac{1}{2}$	(c) $5\frac{1}{3} \times 3\frac{3}{4}$	(d) $1\frac{2}{7} \times 4\frac{2}{3}$
(e) $6\frac{1}{4} \times 1\frac{3}{5}$	(f) $2\frac{5}{6} \times 5\frac{1}{2}$	(g) $1\frac{3}{10} \times 4\frac{1}{3}$	(h) $1\frac{1}{2} \times 7\frac{2}{5}$
(i) $3\frac{2}{3} \times 1\frac{3}{4}$	(j) $5\frac{1}{2} \times 4\frac{4}{5}$	(k) $10\frac{1}{2} \times \frac{6}{7}$	(l) $6\frac{1}{2} \times \frac{4}{5}$

7. A rectangular piece of metal measures $1\frac{7}{8}$ inches wide by $6\frac{1}{2}$ inches long. Calculate its area.

(*note* " - *is the old symbol used to stand for "inch".*)

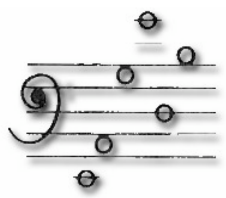


8. 

One metre length of curtain fabric weighs $3\frac{3}{4}$ kg.
What would a $2\frac{1}{2}$ metre length of the fabric weigh ?

9. David's dad found that he weighed $1\frac{2}{3}$ times as much as David did.
If David weighed $31\frac{1}{2}$ kilograms, what did his dad weigh ?

10. A music "jingle" on the radio lasted $12\frac{1}{4}$ seconds.
The new replacement jingle lasts $1\frac{3}{4}$ times as long as this.
For how long does the new jingle last ?



Division of Fractions

tricky !!

It is almost impossible to divide fractions like $(\frac{2}{3} \div \frac{3}{5})$ by actually dividing.

=> What we do instead, is change a "**division**" to a "**multiplication**" (which is easier).

—> $\frac{2}{3} \div \frac{3}{5} = \boxed{\dfrac{\frac{2}{3}}{\frac{3}{5}}}$ - we can simplify the denominator by multiplying it, <u>and</u> the numerator, by $\frac{5}{3}$.

$$\frac{\frac{2}{3}}{\frac{3}{5}} = \frac{\frac{2}{3} \times (\frac{5}{3})}{\frac{3}{5} \times (\frac{5}{3})}$$

$$= \frac{\frac{10}{9}}{\frac{15}{15}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

[This is O.K., since $\frac{\frac{5}{3}}{\frac{5}{3}} = 1$]

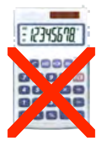
This becomes easier if we miss out the bottom line which **always becomes 1**.

$\frac{4}{5} \div \frac{2}{3}$ becomes $\frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$

* **A simple rule is** :- instead of **dividing** by $\frac{a}{b}$, => **multiply** by $\frac{b}{a}$ instead.

$\frac{5}{8} \div \frac{2}{3}$ becomes $\frac{5}{8} \times \frac{3}{2} = \frac{15}{16}$

Exercise 5



1. Copy each of the following and complete :-

(a) $\frac{3}{4} \div \frac{3}{5}$
$= \frac{3}{4} \times \frac{5}{3}$
$= \frac{?}{12} = \frac{?}{4} = 1\frac{?}{4}$

(b) $\frac{5}{6} \div \frac{2}{3}$
$= \frac{5}{6} \times \frac{3}{2}$
$= \frac{?}{12} = 1\frac{?}{?}$

(c) $\frac{3}{4} \div \frac{5}{6}$
$= \frac{3}{4} \times \frac{6}{5}$
$= \frac{?}{20} = \frac{?}{?}$

2. Divide the following fractions and simplify (where possible) :-

(a) $\frac{2}{5} \div \frac{2}{3}$ (b) $\frac{5}{6} \div \frac{7}{12}$ (c) $\frac{3}{7} \div \frac{6}{7}$ (d) $\frac{3}{10} \div \frac{4}{5}$

(e) $\frac{3}{8} \div \frac{5}{6}$ (f) $\frac{7}{12} \div \frac{7}{8}$ (g) $\frac{11}{16} \div \frac{5}{8}$ (h) $\frac{2}{9} \div \frac{1}{6}$

(i) $\frac{7}{10} \div \frac{3}{5}$ (j) $\frac{7}{16} \div \frac{3}{10}$ (k) $\frac{8}{9} \div \frac{3}{4}$ (l) $\frac{1}{5} \div \frac{1}{7}$

3. How many $\frac{2}{5}$'s are there in $\frac{3}{10}$'s ?

4. How many pieces of cloth $\frac{1}{8}$ metre long, can I cut from a piece $\frac{2}{3}$ metre long ?

Division of Mixed Fractions :- $\left(3\frac{1}{2} \div 2\frac{1}{3}\right)$

Simple Rules :-

- You **MUST CHANGE** mixed fractions to be top-heavy fractions first,
- then use the rule "turn the 2nd fraction upside down and multiply".

Example 1

$$3\tfrac{1}{2} \div 2\tfrac{1}{3}$$
$$= \frac{7}{2} \div \frac{7}{3}$$
$$= \frac{7}{2} \times \frac{3}{7}$$
$$= \frac{21}{14} = 1\tfrac{7}{14} = 1\tfrac{1}{2}$$

Example 2

$$3\tfrac{3}{4} \div 1\tfrac{2}{3}$$
$$= \frac{15}{4} \div \frac{5}{3}$$
$$= \frac{15}{4} \times \frac{3}{5}$$
$$= \frac{45}{20} = 2\tfrac{5}{20} = 2\tfrac{1}{4}$$

5. Copy and complete the following :-

(a) $2\frac{1}{4} \div 1\frac{1}{5}$
$= \frac{9}{4} \div \frac{6}{5}$
$= \frac{9}{4} \times \frac{?}{6}$
= ..... = ...

(b) $4\frac{2}{3} \div 1\frac{2}{5}$
$= \frac{14}{3} \div \frac{7}{5}$
$= \frac{14}{3} \times \frac{?}{?}$
= ..... = ...

(c) $2\frac{2}{3} \div 3\frac{1}{5}$
$= \frac{?}{3} \div \frac{?}{5}$
= ....
= ..... = ...

6. Divide the following fractions in the same way (simplify if possible) :-

(a) $3\frac{1}{3} \div 1\frac{1}{2}$ (b) $2\frac{1}{5} \div 1\frac{1}{2}$ (c) $4\frac{1}{3} \div 2\frac{3}{4}$ (d) $1\frac{2}{7} \div 2\frac{2}{3}$

(e) $4\frac{1}{4} \div 3\frac{3}{5}$ (f) $6\frac{1}{2} \div 2\frac{1}{4}$ (g) $1\frac{3}{5} \div 4\frac{2}{3}$ (h) $7\frac{1}{2} \div 1\frac{3}{7}$

(i) $5\frac{1}{3} \div 1\frac{3}{5}$ (j) $4\frac{1}{2} \div 5\frac{1}{4}$ (k) $6 \div 1\frac{1}{2}$ (l) $8 \div 2\frac{2}{3}$

7. The area of this rectangular piece of card is $7\frac{1}{2}$ square inches.
It is $1\frac{2}{3}$ inches wide. Calculate its length.

$1\frac{2}{3}$ "

Area = $7\frac{1}{2}$ sq inches

8. A $4\frac{1}{2}$ metre length of plank weighs $10\frac{1}{8}$ kilograms.

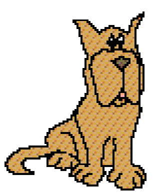
(a) What does 1 metre of the plank weigh ?

(b) What is the weight of a $3\frac{1}{4}$ metre plank of the same type of wood ?

9. Danny's mum found that she was $1\frac{1}{5}$ times as tall as Danny was.

If his mum was $1\frac{3}{4}$ metres tall, how tall was Danny ?

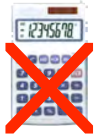
10.



$2\frac{1}{4}$ laps of the park took Tommy Muir, walking his dog, $12\frac{1}{2}$ minutes.

How long, on average, did each lap take ?

Fractions

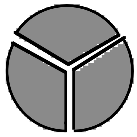


Topic in a Nutshell

1. Change to a mixed number :- (a) $\frac{24}{5}$ (b) $\frac{42}{8}$

2. Re-write as a top-heavy fraction :- (a) $4\frac{5}{6}$ (b) $10\frac{2}{7}$

3. How many $\frac{1}{3}$ pizza slices can by sold from $6\frac{2}{3}$ pizzas ?



4. Copy and complete :-

   (a) $\frac{3}{7} + \frac{2}{7}$ (b) $\frac{1}{2} + \frac{3}{4}$ (c) $\frac{5}{6} - \frac{1}{6}$

   (d) $3\frac{3}{5} + 4\frac{4}{5}$ (e) $5\frac{3}{5} - 2\frac{1}{3}$ (f) $4\frac{1}{2} - 1\frac{2}{3}$

5. Copy and complete :-

   (a) $\frac{1}{2} \times \frac{1}{3}$ (b) $\frac{8}{9} \times \frac{3}{5}$ (c) $2\frac{1}{2} \times 1\frac{1}{5}$

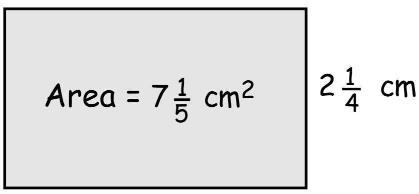
   (d) $\frac{5}{8} \div \frac{1}{4}$ (e) $\frac{11}{12} \div \frac{2}{5}$ (f) $3\frac{3}{4} \div 1\frac{2}{3}$

6. Before his diet, Tommy weighed $13\frac{1}{2}$ stones.
   He lost $2\frac{3}{4}$ stones on his diet.
   What was Tommy's new weight ?

7. 

   A hardware shop sells lengths of heavy duty chain.
   1 metre of the chain weighs $4\frac{4}{5}$ kg.
   What will the weight of a $3\frac{1}{4}$ metre chain be ?

8. A wooden box weighs $2\frac{7}{8}$ kg.
   It holds 6 large cartons of soap powder.
   Each carton weighs $3\frac{3}{4}$ kg.
   Calculate the total weight of the box and cartons.

9. 

   The area of this rectangle is $7\frac{1}{5}$ cm^2.
   Its breadth is $2\frac{1}{4}$ cm.
   Calculate its length.

10. Find :- $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$.

Chapter 37

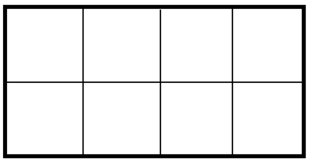
Drawing Skills

Enlargement and Reduction

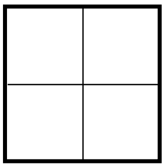
Exercise 1

1. Make a neat "two-times" enlargement of each of these shapes :- (each box = 1 cm long)

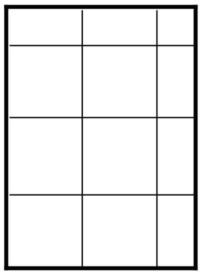
(a)



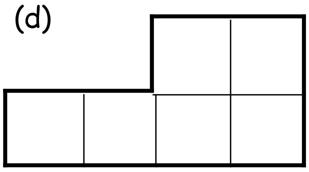
(b)



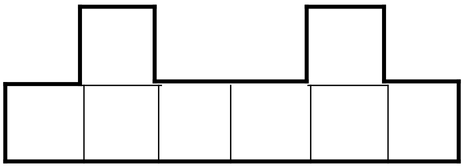
(c)



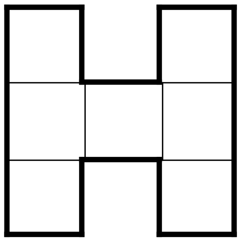
(d)



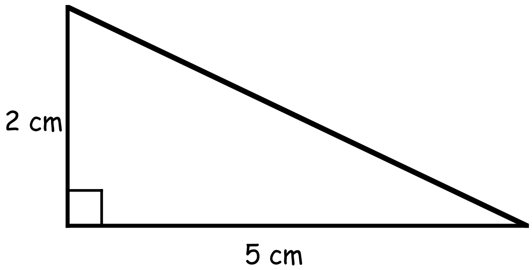
(e)



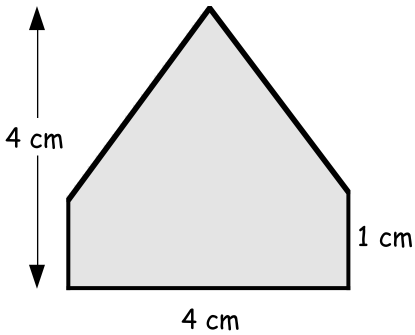
(f)



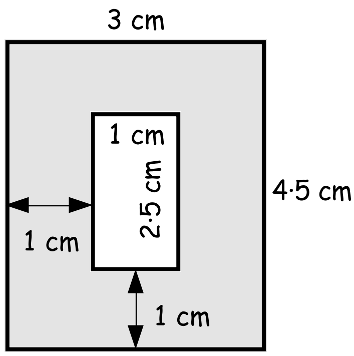
(g)



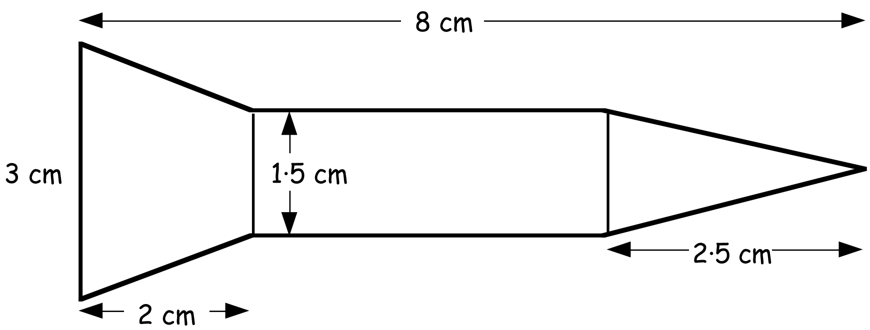
(h)



(i)



(j)

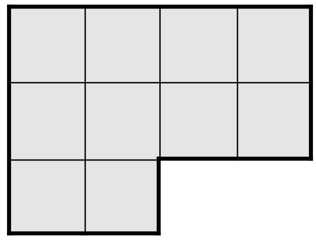


2. Make enlargements OR reductions of the following shapes using the given scales :-

(a)

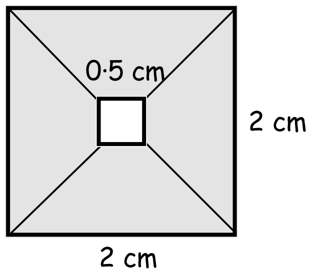
make a **three times** enlargement.

(b)



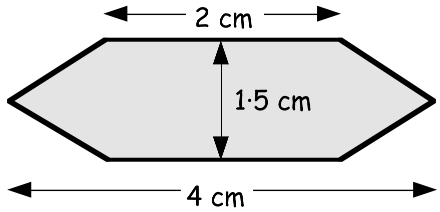
make a **four times** enlargement.

(c)



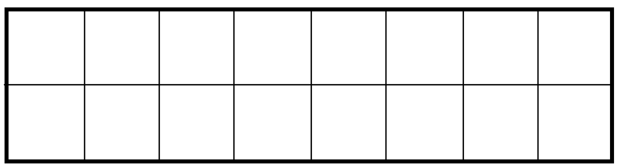
make a **six times** enlargement.

(d)



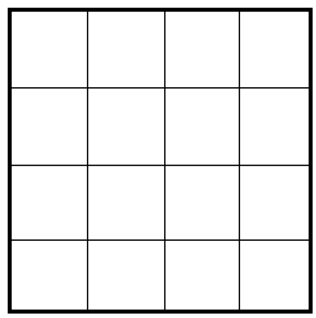
make a **four times** enlargement.

(e)



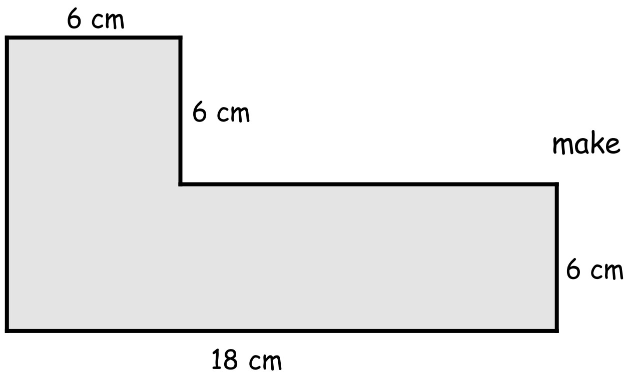
reduce this shape to **half** its size.

(f)



make this a **quarter** of its size here

(g)

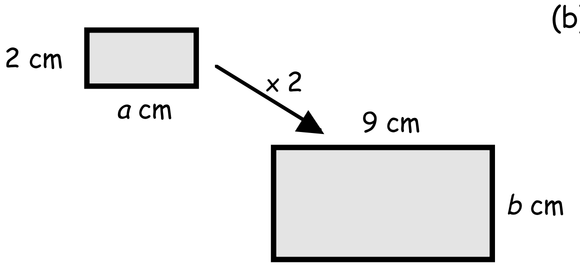


make this **one third** of its size.

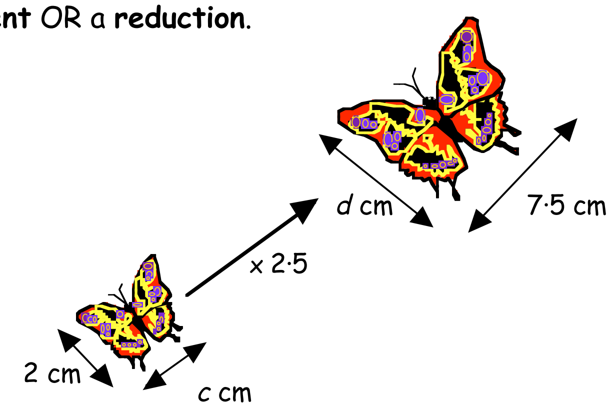
3. Each pair of pictures shows either an **enlargement** OR a **reduction**.

Calculate the unknown sizes. (Do not measure).

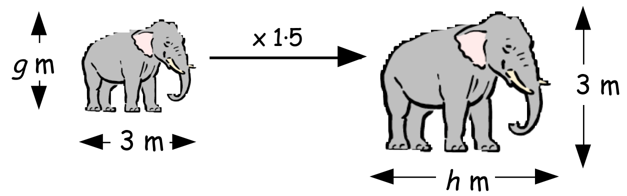
(a)



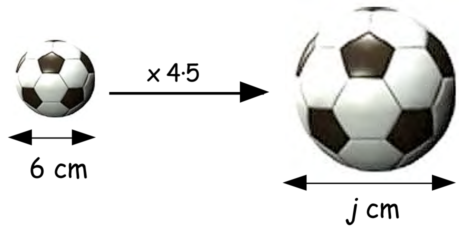
(b)



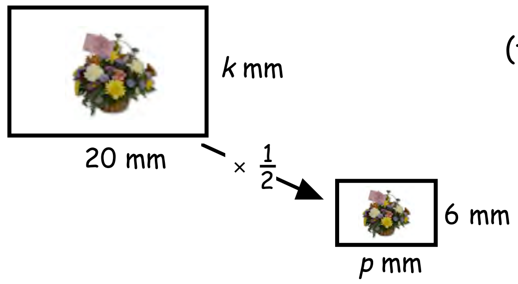
(c)



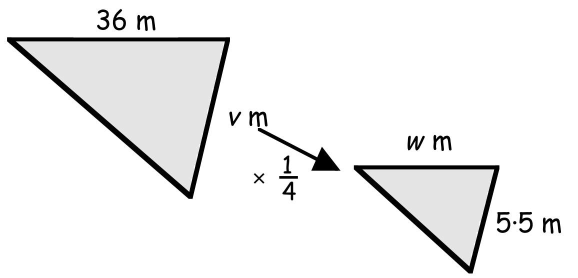
(d)



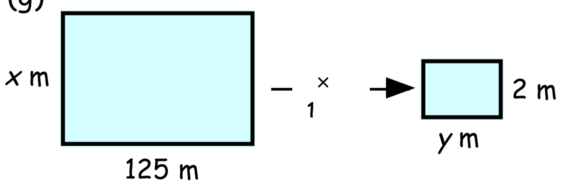
(e)



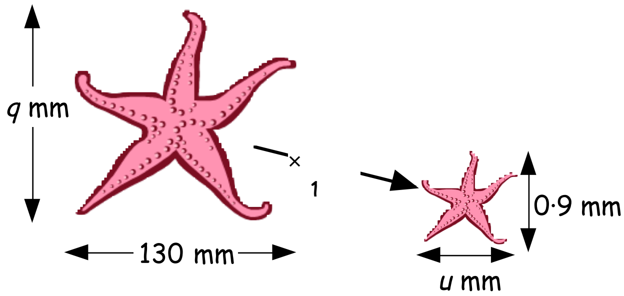
(f)



(g)



(h)

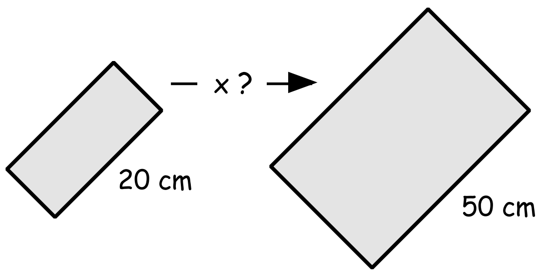


4. Each pair of pictures below shows either an **enlargement** OR a **reduction**.

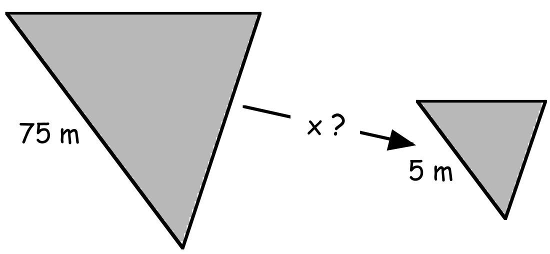
> if the picture has **doubled** in size, the enlargement factor is " x 2 ".
>
> if the picture has halved in size, the reduction factor is " x $\frac{1}{2}$ ".

In each case below, find the **enlargement factor** or the **reduction factor**.

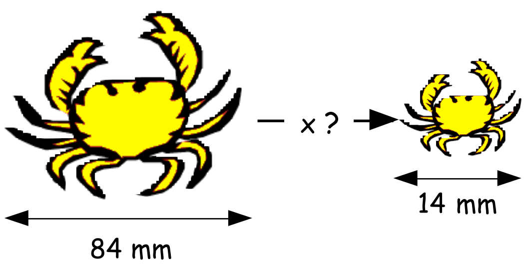
(a)



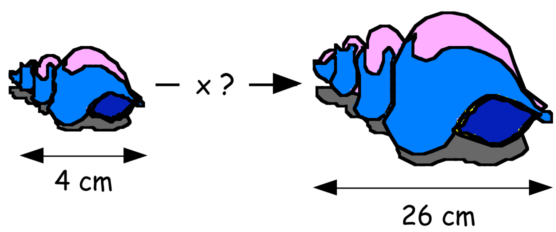
(b)



(c)



(d)



Drawing Triangles

Revision Work

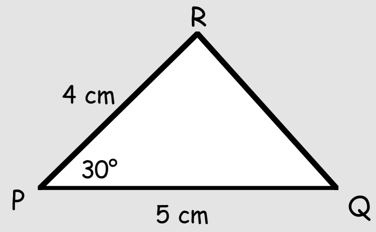
For this chapter you will require :- a RULER, a PROTRACTOR and a PAIR of COMPASSES.

* Remember - You need to be given **3 pieces of information** about a triangle before you can begin to draw it.

Two Sides and the Included Angle

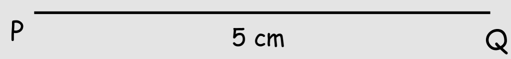
(*the angle between the 2 sides*)

Shown opposite is a <u>rough sketch</u> of ΔPQR.

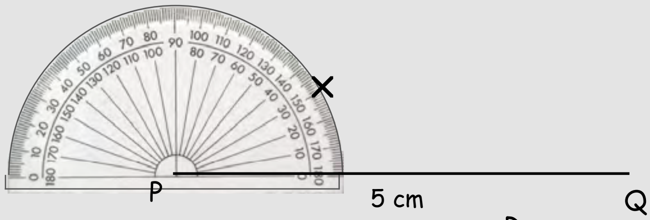


To draw it accurately :-

Step 1 :- Draw line PQ = 5 cm.

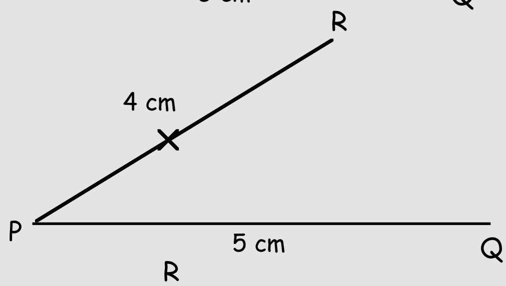


Step 2 :- Put your protractor at P and mark an angle of 30°.

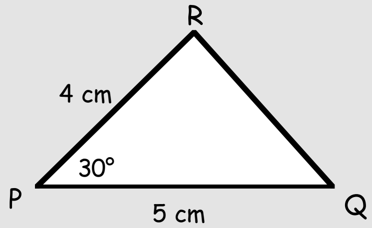


Step 3 :- Draw line PR, from P through the X, to point R.

Make sure it is 4 centimetres long.

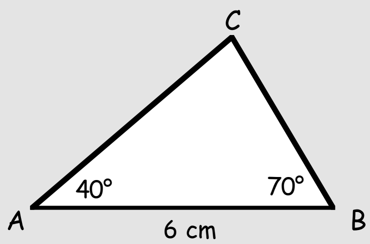


Step 4 :- Join Q to R to complete the triangle.



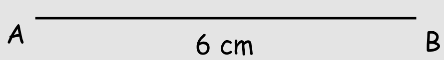
Two Angles and a Side

Shown opposite is a <u>rough sketch</u> of ΔABC.

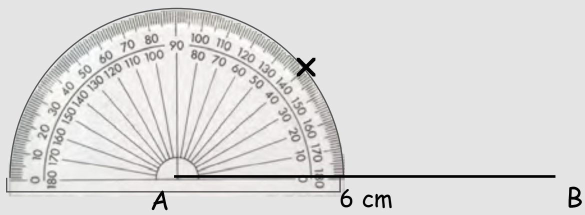


To draw it accurately :-

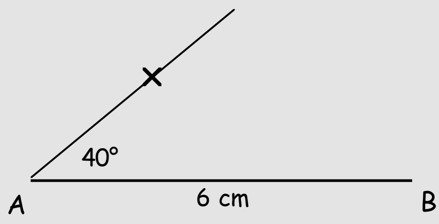
Step 1 :- Draw line AB = 6 cm



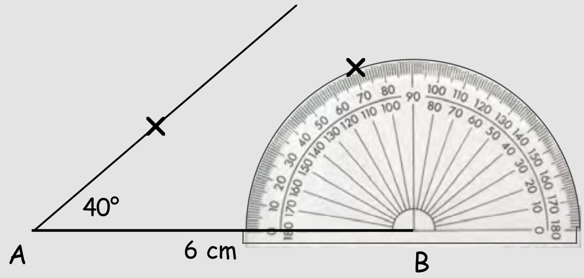
Step 2 :- Put your protractor at A and mark an angle of 40°.



Step 3 :- Draw line from A through the point X.

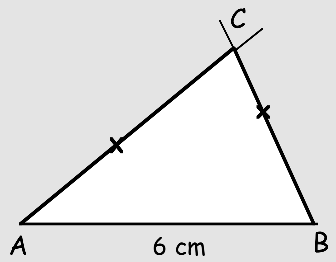


Step 4 :- Now put your protractor at B and mark an angle of 70°.



Step 5 :- Finally, draw the line from B through your new X point.

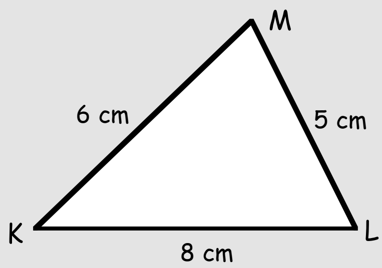
(Mark the point where the two lines meet with the letter C).



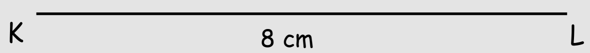
Three Sides

Shown opposite is a <u>rough sketch</u> of ΔKLM.

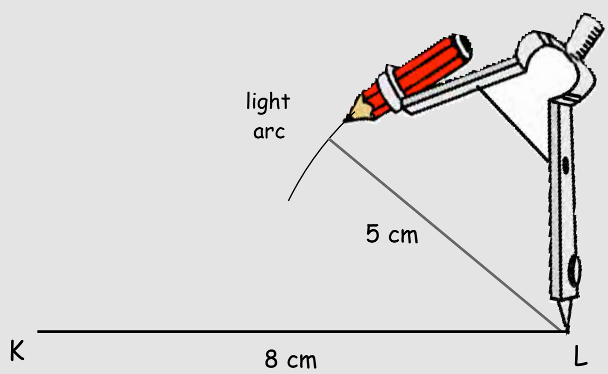
To draw it accurately :-



Step 1 :- Draw line KL = 8 cm

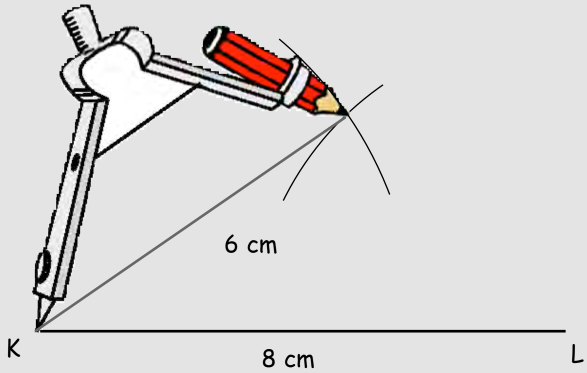


Step 2 :- Set a pair of compasses to 5 cm, place the compass point on L and draw a light arc as shown.

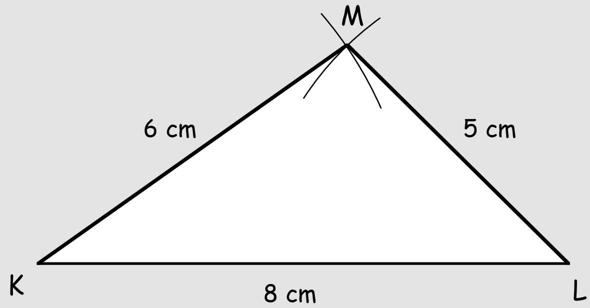


Step 3 :- Now set your compasses to 6 cm, place the compass point on K and draw a 2nd light arc.

(Call the point where the 2 arcs meet M)



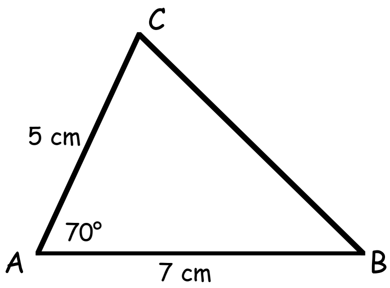
Step 4 :- Now use your ruler to join K to M and L to M.



Exercise 2

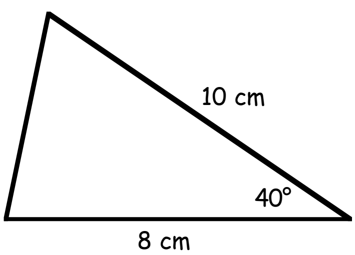
1. On the right is a sketch of ΔABC.
Follow the instructions to draw it accurately :-

Step 1 :-	Draw line AB = 7 cm
Step 2 :-	Put your protractor at A and mark (with an X) an angle of 70°.
Step 3 :-	Draw line AC, from A through the X, to point C. (*Make sure it is 5 centimetres long*).
Step 4 :-	Join C to B to complete the triangle.

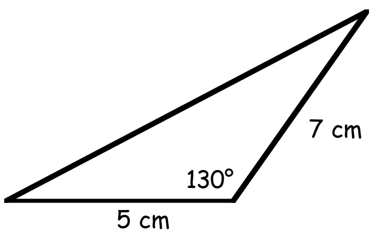


2. Make accurate drawings of the following triangles :-

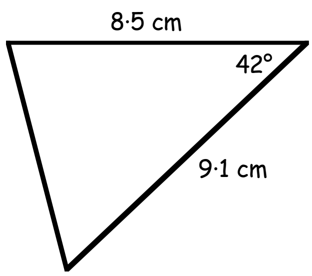
(a)



(b)



(c)



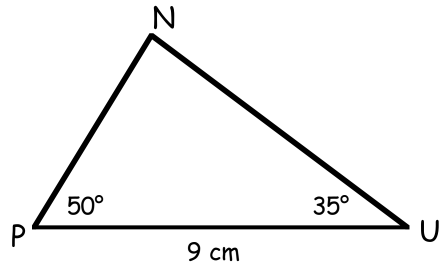
3. Make accurate drawings of the following triangles :-
(*Make rough sketches of the triangles first before drawing them accurately*).

(a) Draw ΔPQR where PQ = 11 cm, QR = 9 cm and $\angle PQR = 60°$.

(b) Draw ΔTAN where AN = 12 cm, AT = 7·5 cm and $\angle TAN = 110°$.

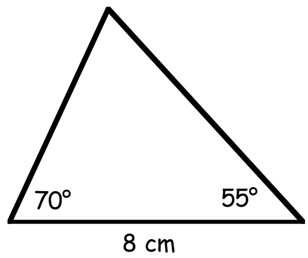
4. Shown is a rough sketch of ΔPUN.
Follow the instructions to draw it accurately :-

Step 1 :-	Draw line PU = 9 cm
Step 2 :-	Put your protractor at P and mark (with an X) an angle of 50°.
Step 3 :-	Draw a line from P through the X.
Step 4 :-	Put your protractor at U and mark (with an X) an angle of 35°.
Step 5 :-	Draw a line from U through the X, to meet your first line at point N.

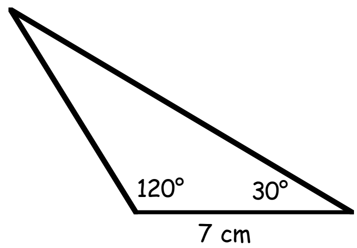


5. Make accurate drawings of the following triangles :-

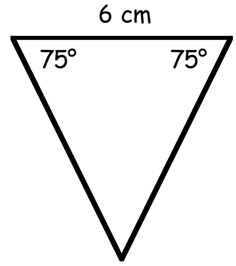
(a)



(b)



(c)

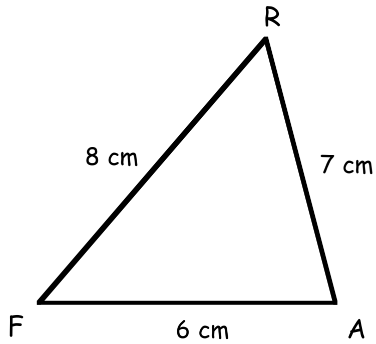


6. Make accurate drawings of the following triangles :-
(*Make rough sketches of the triangles first before drawing them accurately*).

(a) Draw ΔABC where AB = 10 cm, $\angle$CAB = 50° and $\angle$CBA = 65°.

(b) Draw ΔRYT where RY = 5 cm, $\angle$TRY = 35° and $\angle$TYR = 125°.

7.

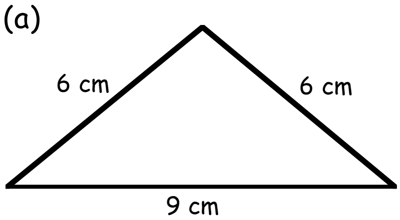


Shown is a sketch of ΔFAR.
Draw it accurately using the following instructions :-

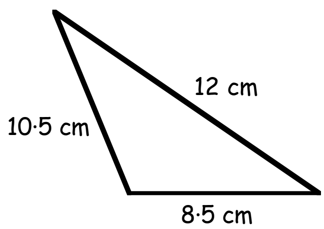
Step 1 :-	Draw line FA = 6 cm
Step 2 :-	Set your compasses to 8 cm, place the compass point on F and draw a light arc.
Step 3 :-	Now set your compasses to 7 cm, place the compass point on A and draw a 2nd arc.
Step 4 :-	Call this point where the arcs meet R and join R to F and to A.

8. Make accurate drawings of the following triangles :-

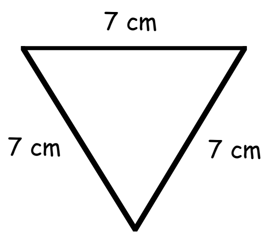
(a)



(b)

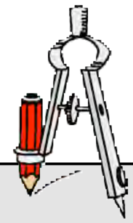


(c)



Drawing Quadrilaterals and Regular Polygons

Three examples using only a pair of compasses and a straight edge



Bisecting a line at right angles

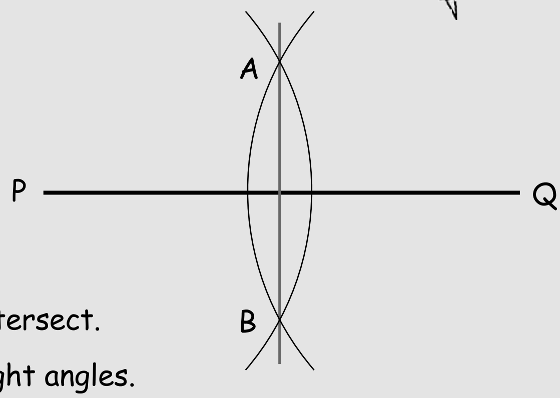
We want to find the midpoint of line PQ.

Step 1 :- Set your compasses to a size larger than half of PQ.

Step 2 :- Draw an arc, centre P and another arc, centre Q (**with same radius**).

Step 3 :- Join the 2 points (A and B) where the arcs intersect.

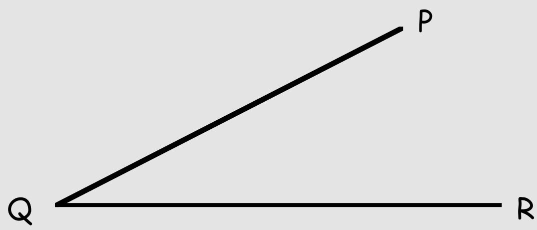
This line AB will bisect PQ, and does so at right angles.



Bisecting an angle

We want to cut ∠PQR in half (bisect it).

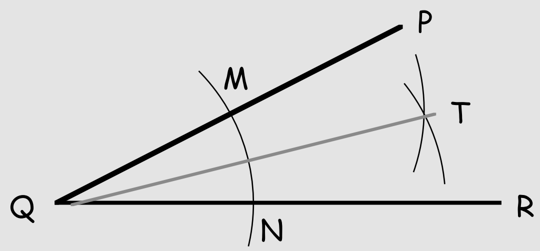
Step 1 :- With centre Q and using any radius, draw an arc, cutting PQ at M and QR at N.



Step 2 :- With the same radius as above, draw an arc centre M and another, centre N. These will meet at a point (call it T).

Step 3 :- Join Q to T. This line will cut ∠PQR in half.

Can you see that QMTN is a rhombus ?



Drawing a 60° angle.

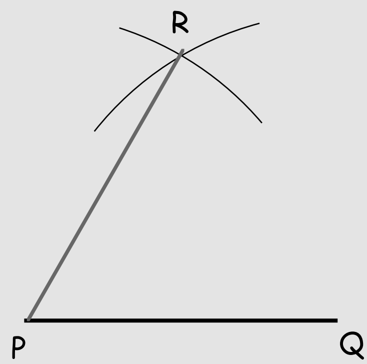
Step 1 :- Draw a line PQ.

Step 2 :- With radius PQ draw an arc centre P.

Step 3 :- Draw a 2nd arc, centre Q, with the same radius.

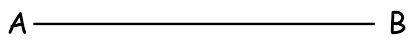
Step 4 :- The 2 arcs intersect at a point R.

Join P to R. (∠RPQ = 60°).

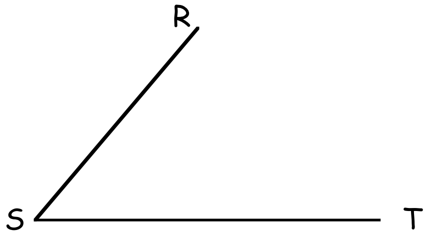


Exercise 3

1. Draw a line AB in your jotter and use the methods shown to find its mid-point. (i.e. show how to bisect the line AB)



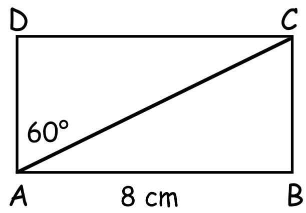
2. Draw any angle RST. Use the method shown to bisect the angle.

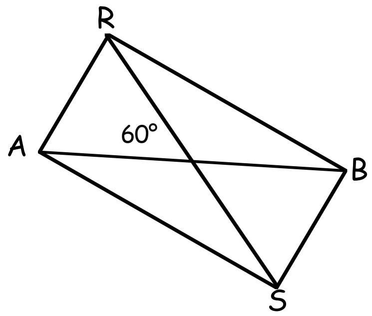


3. Draw a line MN, about 6 cm long. Show how to create an equilateral triangle MNP.

4. Draw a line AB = 8 cm and make an accurate drawing of the rectangle sketched opposite :-

   (NO PROTRACTOR ALLOWED)

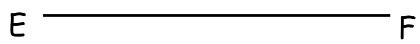


5. 

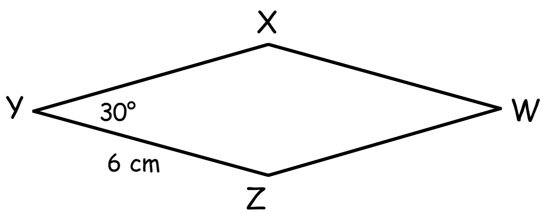
   Draw the same line AB = 8 cm, and create a rectangle, but this time the diagonal has to be 8 cm.

6. (a) Start with a line EF = 6 cm and create an angle of 60°. (call it ∠HEF).

   (b) Now show how to bisect∠HEF to create an angle of 30°.



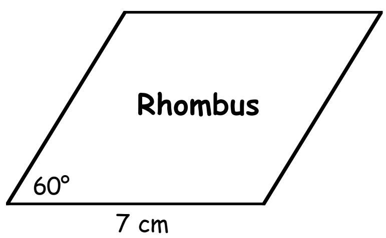
7. Show how to create the rhombus XYZW shown opposite, using only a ruler and a pair of compasses.



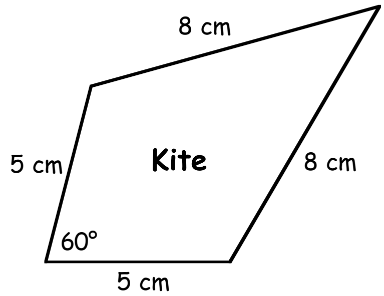
For Example 8 you may use compasses and a ruler - but a NOT a protractor !

8. Make accurate drawings of the quadrilaterals shown below.

   (a)



   (b)



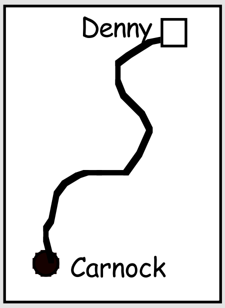
Scale, Reading Maps and Interpreting Distances

Example

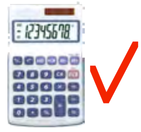
On a map, the distance between the towns of Carnock and Denny is 4·8 centimetres.
The scale of the map is 1 : 50 000.

What is the real distance from Carnock to Denny ?

Map		Real Distance
1 cm	⟶	50 000 cm
4·8 cm	⟶	50 000 × 4·8 cm
	=	240 000 cm
(÷ 100, then ÷ 1000)	=	2·4 km



Exercise 4



1. Measure the length of each line (in cm) and calculate the length it really represents.

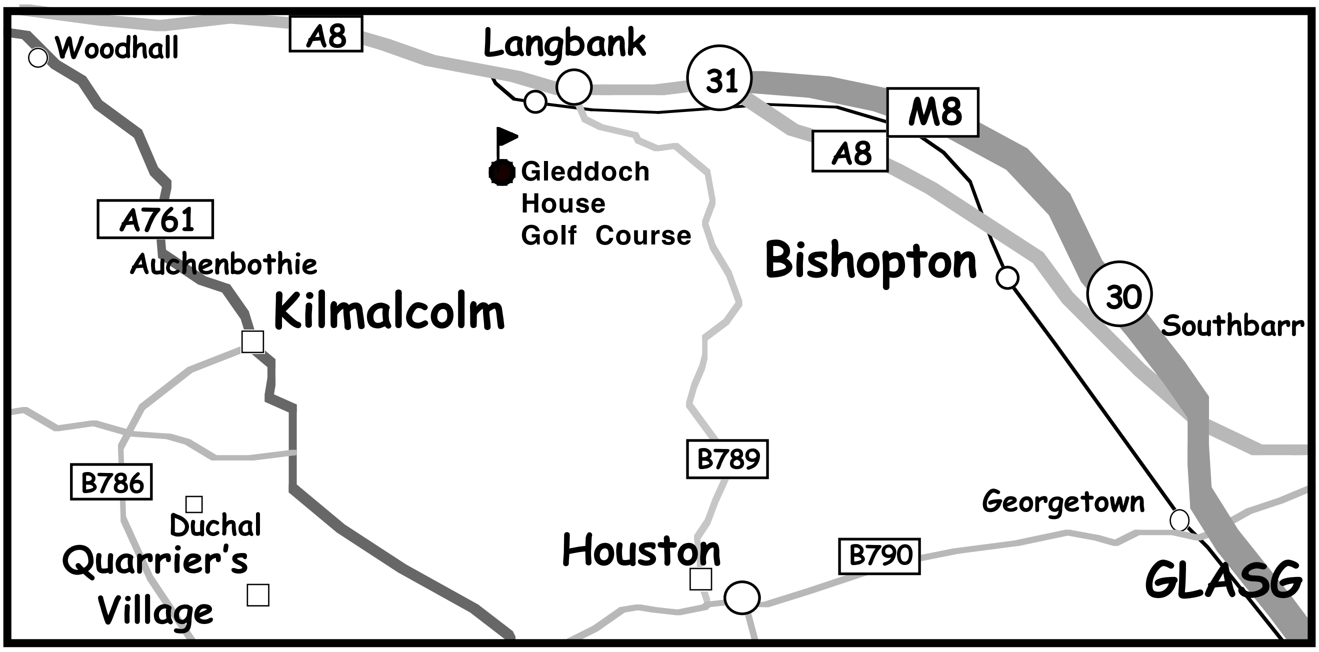
(a) scale 1 : 10

(b) scale 1 : 50

(c) scale 1 : 1000

(d) scale 1 : 50 000

2. The map below shows an area in the West of Scotland. Its scale is 1 : 50 000.



Use a ruler to measure and then write down the direct distance (in cm) from :-

(a) Kilmacolm to Woodhall. (b) Houston to Gleddoch House Golf Course.

(c) Quarrier's Village to Bishopton. (d) Woodhall to Georgetown.

3. Now calculate the real direct distance (in km) from :-

(a) Kilmacolm to Woodhall. (b) Houston to Gleddoch House Golf Course.

(c) Quarrier's Village to Bishopton. (d) Woodhall to Georgetown.

4. Jack is planning a skiing trip. On his map he measures that the distance from his home to the best ski slope is 18·3 cm.

If the scale of his map is 1 : 25 000, find how far away Jack lives from this ski slope.

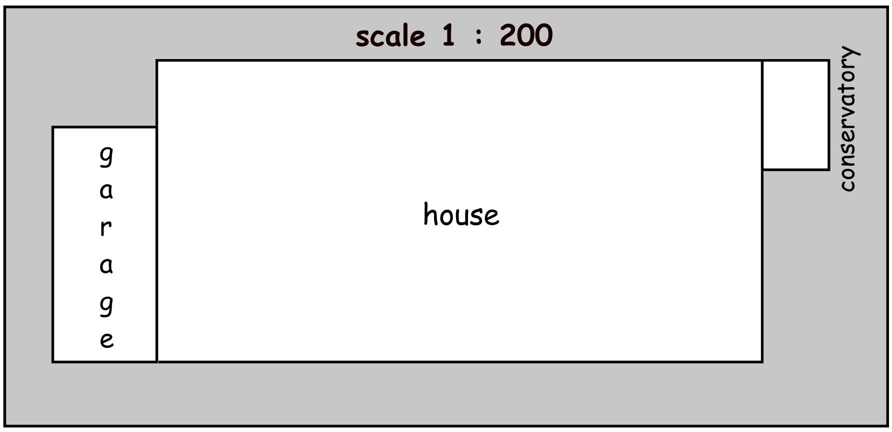
5. The distance from Golding to Beachhead is 10·7 cm on a map which has a scale of 1 : 20 000. Calculate the real distance between the two towns.

6. 

On an architect's plan the height of the lighthouse is measured as 8·5 cm.

If the scale of his plan is 1 : 500, find the real height of the lighthouse, in metres.

7. This is a plan of Jamie's house and garden.



(a) Measure and write down the dimensions (length and breadth) of :-

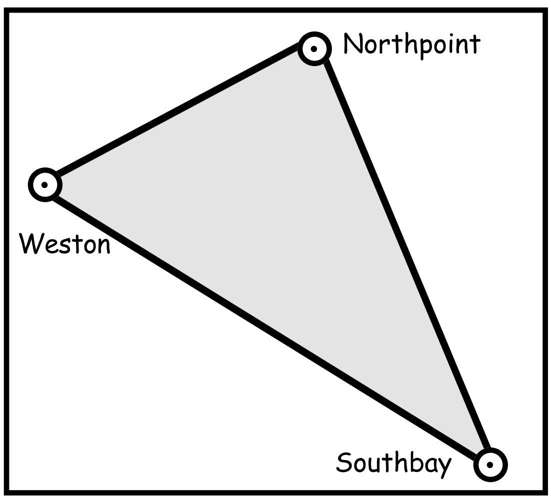
(i) the house,

(ii) the garage,

(iii) the conservatory.

(b) Calculate the real dimensions of the three buildings in metres.

8. This map shows the railway lines which link the 3 busiest towns on a holiday island. The actual distance from Northpoint to Southbay is 12 km.



(a) Measure and write down the distance from Northpoint to Southbay.

(b) Calculate the scale of the map.

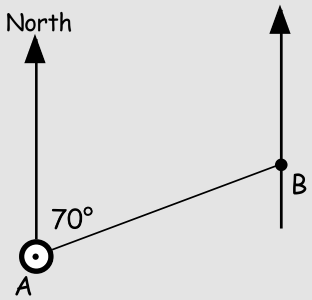
(c) Calculate the real distance from Southbay to Weston.

Bearings (Problems)

Example

The bearing from A to B is 070°.

What is the bearing from B to A ?



Answer

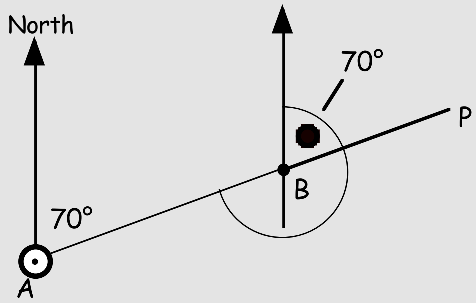
Extend line AB out to a point P.

Angle ● = 70° (Corresponding to given 70°)

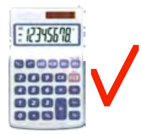
Bearing = ● + 180° (straight angle)

= 70° + 180°

= <u>250°</u>

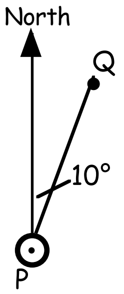


Exercise 5

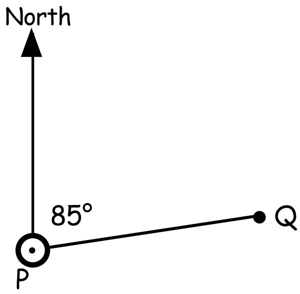


1. Calculate (do not measure) the bearing of P **from** Q in the diagrams below

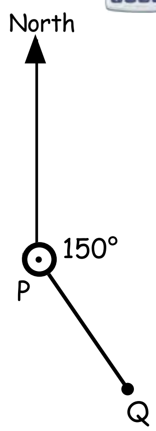
(a)



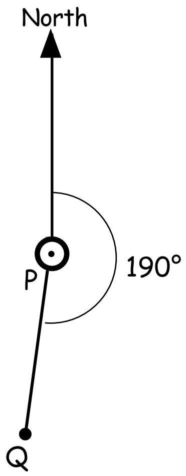
(b)



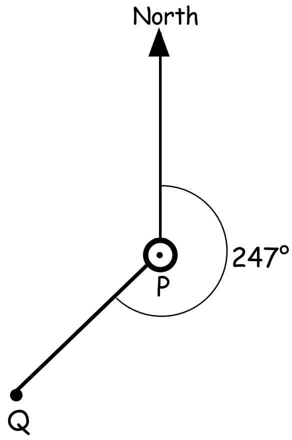
(c)



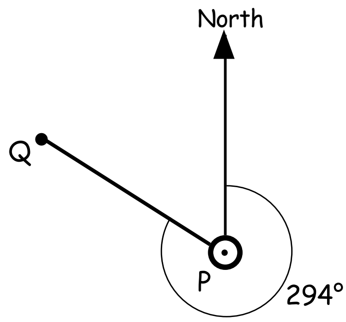
(d)



(e)



(f)



2. The scaled plan shows the position of three towns in central Dreamland.

   The scale of the plan is 1 : 50 000.

   (a) **Calculate** the **real** distances between the towns in kilometres.

   (b) Use a protractor to measure the bearing of :-

      (i) Elfton from Fairytown.

      (ii) Twinkleton from Elfton.

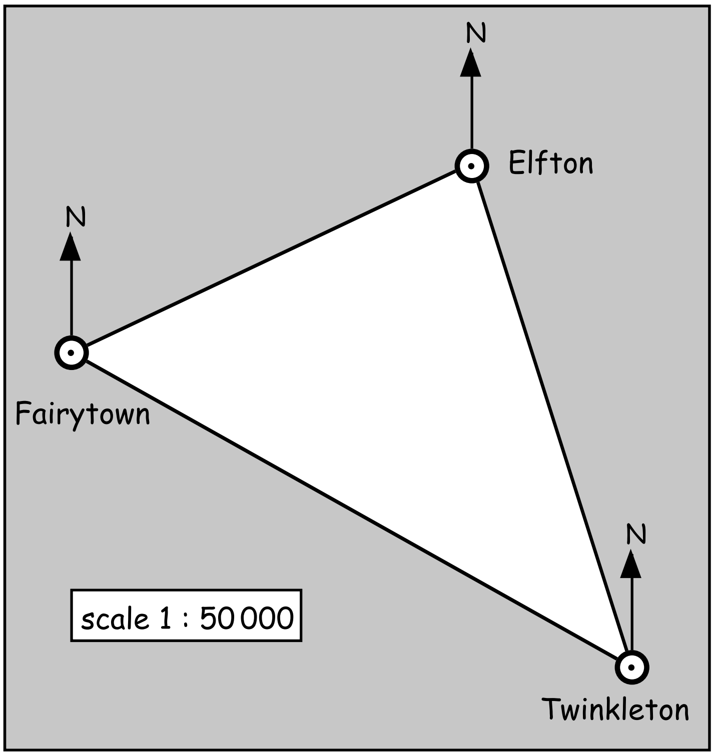
      (iii) Twinkleton from Fairytown.

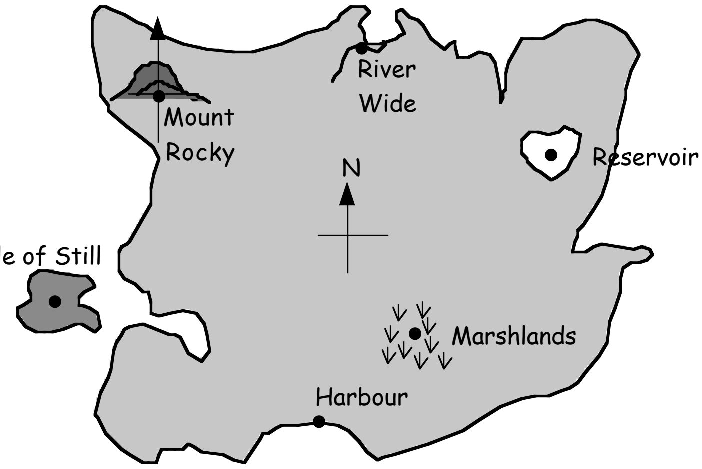
   (c) **Calculate** the bearing of :-

      (i) Fairytown from Elfton.

      (ii) Elfton from Twinkleton.

      (iii) Fairytown from Twinkleton.



3. 

scale 1 : 15 000

(a) Find the **real** distance in km from :-

   (i) River Wide to the Harbour.

   (ii) Isle of Still to the Reservoir.

(b) How far is it from Mount Rocky to the Marshlands ?

(c) Now measure the bearing from Mount Rocky to the Marshlands.

(d) **Calculate** the bearing from the Marshlands to Mount Rocky.

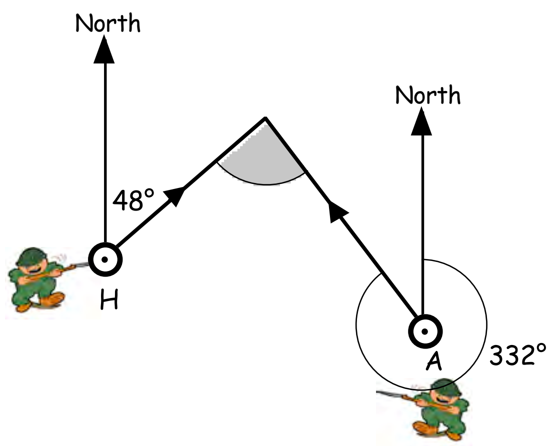
4. As part of a military training exercise, two teams of cadets are marching to a rendezvous point.

   The Highlanders (H) are travelling on a bearing of 048°.

   The Argyllans (A) are on a course of 332°.

   At what angle (shaded) will their courses meet ?

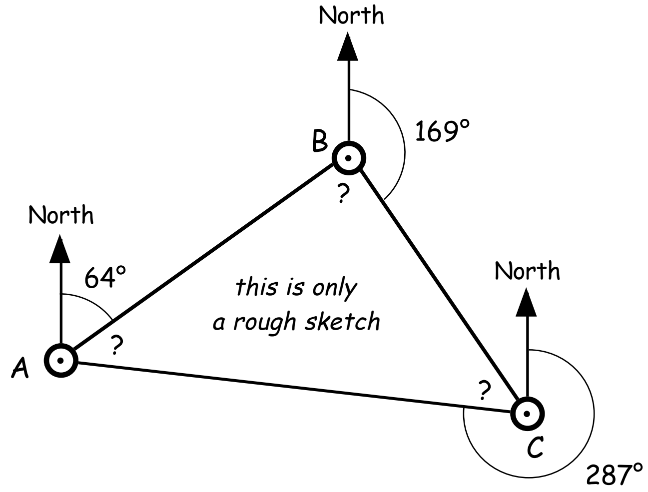
   Calculate - do NOT measure.



5. The vertices of triangle ABC are shown, together with bearings from A to B, B to C and C back to A.

Calculate the sizes of :-

$\angle ABC$, $\angle ACB$ and $\angle BAC$.



6. The small village of Adensport has one church, one garage and one school.

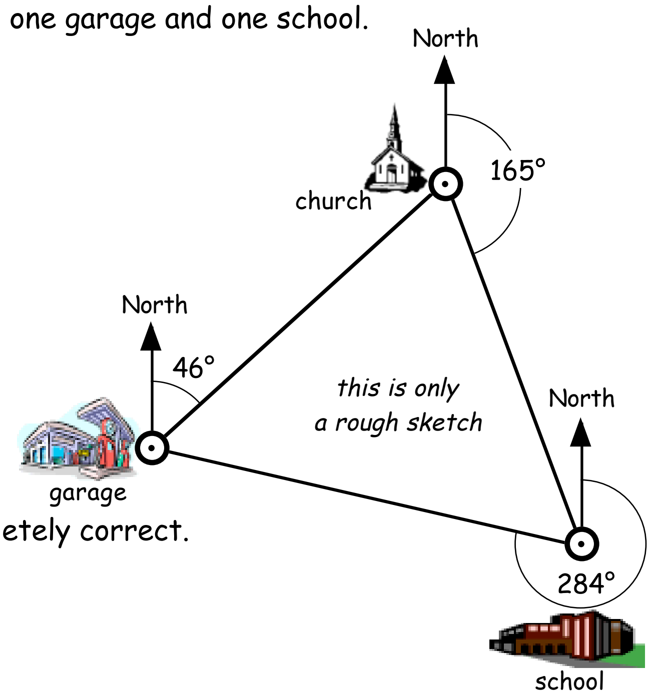
On a map of the village, the bearing of :-

- the church from the garage is 046°.
- the school from the church is 165°.
- the garage from the school is 284°.

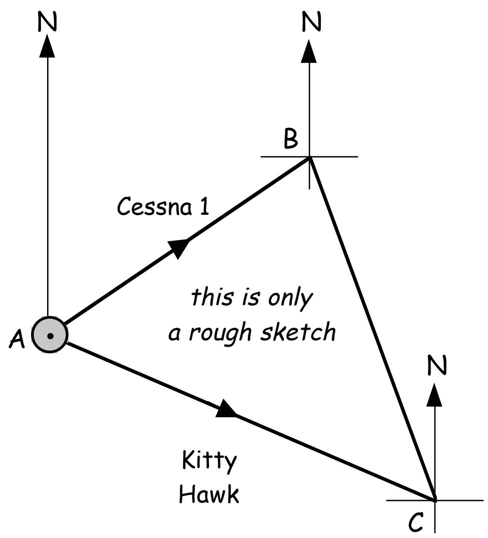
A visitor to the village looks at the map and thinks that the distance from the garage to the church looks the same as the distance from the garage to the school.

Prove that in fact, the visitor's idea is completely correct.

Calculations, no measuring !



7.



Two aeroplanes leave an airport (A) at the same time.

The Cessna 1 flies on a bearing 068° to B.

The Kitty Hawk flies on a bearing 115° to C.

From B, the bearing of the Kitty Hawk is 160°.

Make a neat **sketch** of the journeys.

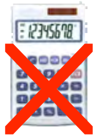
Calculate, and mark on your sketch :-

(a) the bearing of A from B.

(b) the bearing of B from C.

(c) the bearing of A from C.

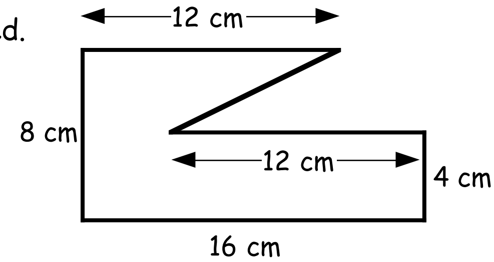
Drawing Skills



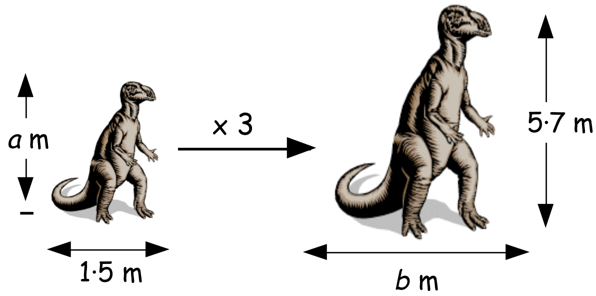
until qu. 8-11

Topic in a Nutshell

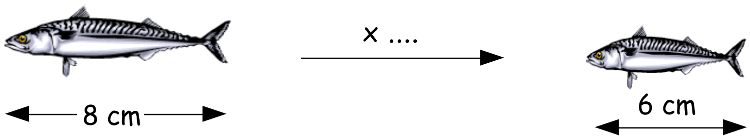
1. This sketch shows a shape with its sizes marked. Make an accurate scaled down drawing of the shape but **one quarter** of its size



2. Calculate the unknown sizes where one dinosaur is a 3 times enlargement of the other.

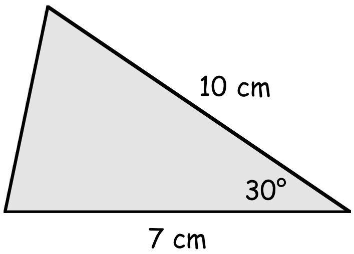


3. Find the reduction factor. from the large fish to the small one.

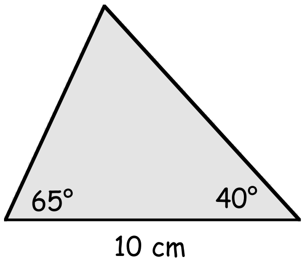


4. Make accurate drawings of the following triangles using compasses, ruler & protractor:-

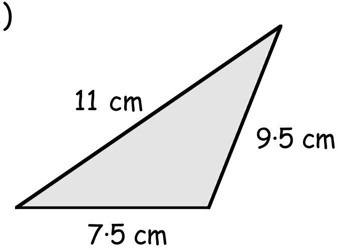
(a)



(b)

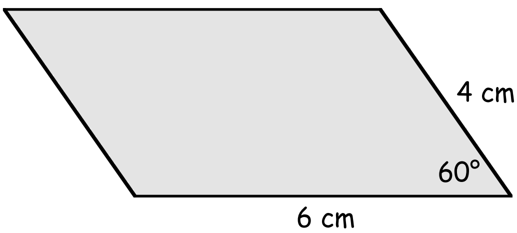


(c)



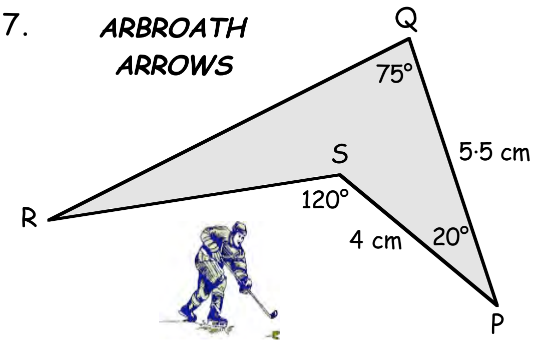
5. Use a ruler and pair of compasses to draw this parallelogram.

   (*No protractor*)



6. Use a ruler and protractor to draw a rhombus of side 5 cm which has one of its angles of 100°.

cont'd .....

7. 

The Arbroath Arrows Ice-Hockey team have this insignia sewn onto the arms of their strip.

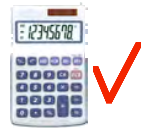
(a) Make an accurate drawing of the arrow-shape.

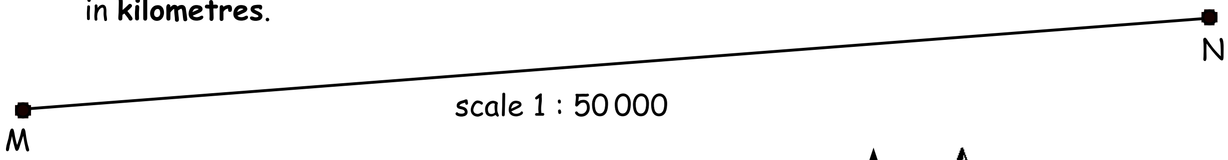
(b) Find the direct distance from P to R.

(c) Measure and write down the size of $\angle$QRS. on your drawing.

8. (a) Measure the length of line MN, below, in centimetres.

(b) If the scale is 1 : 50 000, calculate the real length it represents in **kilometres**.

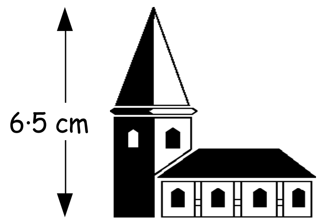




9. The real height of this tower is 32·5 metres.

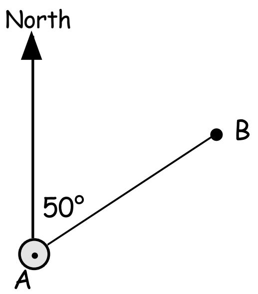
On a postcard its height appears as 6·5 cm.

Calculate the scale of the postcard in the form 1 : .....

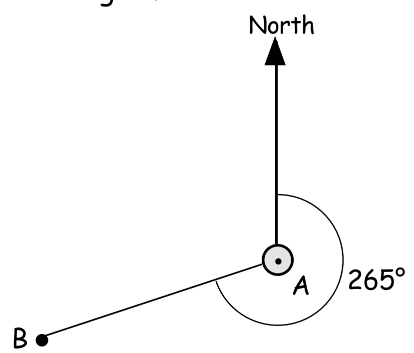


10. Calculate (do not measure) the bearing of A **from** B in the diagrams below.

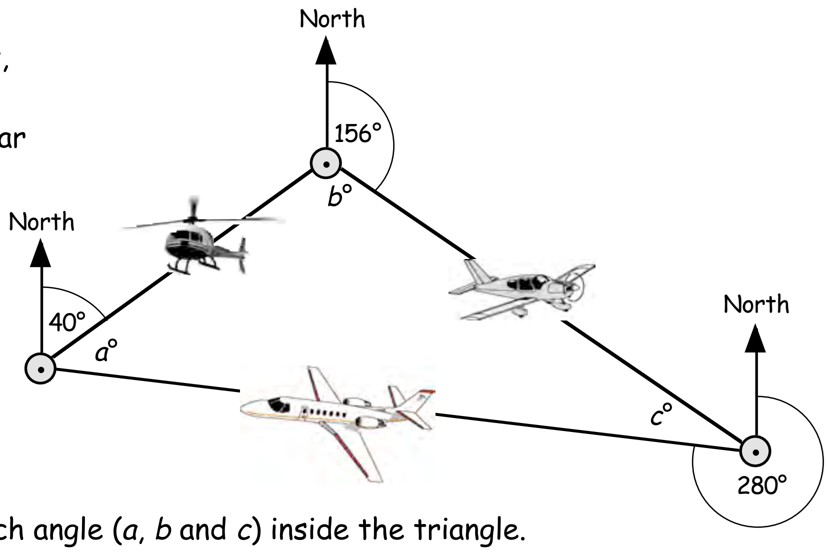
(a)



(b)



11. The flight-paths of a jet, a helicopter and a light aircraft form a triangular shape.



Calculate the size of each angle (a, b and c) inside the triangle.

Chapter 41

Ratio & Proportion

Simple Ratio

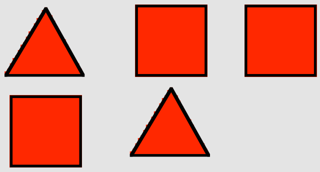
(A reminder)

Ratios can be used to compare different quantities.

<u>**Example**</u> :- There are 2 triangles and 3 squares.

The ratio of triangles to squares is 2 : 3.

Written as triangles : squares = 2 : 3.
Also squares : triangles = 3 : 2.



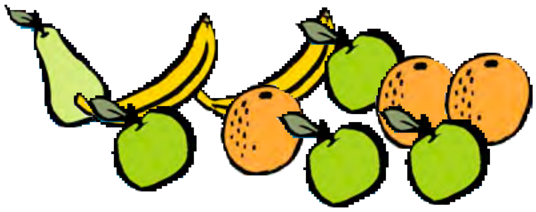
Exercise 1 (Oral Exercise)

1. Look at the picture.
   Write down the ratio of : -

   (a) cars : buses (b) buses : cars

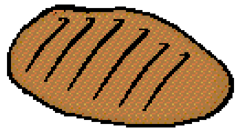
2. Look at this picture.
   Write down the ratio of : -

   (a) apples : oranges (b) apples : pears
   (c) oranges : pears (d) bananas : apples
   (e) pears : bananas (f) bananas : pears.

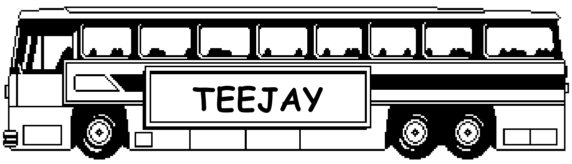


3. In a baker shop there are 122 loaves, 169 rolls and 59 baguettes.
   Write down the ratio of : -

   (a) loaves : baguettes (b) baguettes : rolls
   (c) rolls : baguettes (d) rolls : loaves.



4.



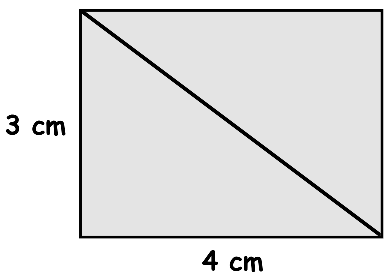
On a school trip there are 21 girls, 19 boys and 11 adults.

Write down the ratio of : -

(a) boys : girls (b) adults : girls
(c) children : adults (d) adults : people.

5. An accurately drawn rectangle has its dimension as shown. Write down the ratio of :-

   (a) length : breadth
   (b) length : perimeter
   (c) length : area (*ignore units*)
   (d) area : perimeter (*ignore units*)
   (e) length : diagonal length



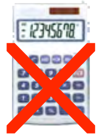
Simplifying Ratio

Simplifying a ratio is very similar to simplifying a fraction.

Fraction $\frac{6}{8}$ - (divide top and bottom by 2) gives —> $\frac{3}{4}$

Ratio Similarly **6 : 8** (divide each side by 2) gives **3 : 4**

Exercise 2 (no calculator)



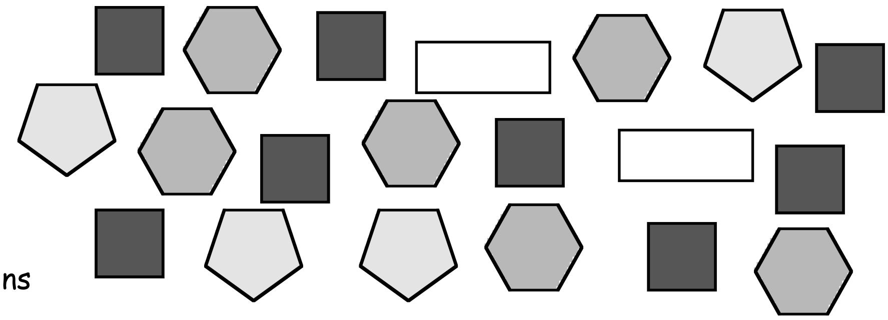
1. Simplify each ratio by dividing each value by 3 : -

   (a) 6 : 9 (b) 15 : 21 (c) 30 : 33 (d) 3 : 27 (e) 300 : 663

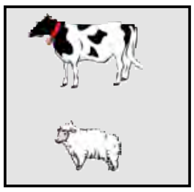
2. Copy the ratios and simplify each as far as possible : -

   (a) 3 : 36 (b) 12 : 48 (c) 30 : 180 (d) 7 : 56
   (e) 11 : 121 (f) 33 : 12 (g) 22 : 99 (h) 17 : 51
   (i) 26 : 130 (j) 57 : 171 (k) 33 : 242 (l) 15 : 615
   (m) 25 : 90 (n) 3 : 27 (o) 25 : 1250 (p) 24 : 144
   (q) 10 000 : 200 (r) 30 000 : 6000 (s) 2 : 4 : 10 (t) 14 : 84 : 21

3. Write down each ratio in its simplest from : -



   (a) pentagons : hexagons
   (b) squares : pentagons
   (c) rectangles : squares
   (d) quadrilaterals : hexagons
   (e) quadrilaterals : (pentagons + hexagons).



4. (a) A farmer has 18 sheep and 32 cows in a field.
   Write down the ratio of cows : sheep in its simplest form.
   (b) The farmer's field measures 20 metres by 35 metres.
   Write down the ratio of area : perimeter in its simplest form. (*ignore units*).

5. A large container has dimensions 4 by 3 by 2 metres.
   A small container has dimensions 2 by 2 by 1 metres.
   Write down the ratio of **volumes** (small : large) in its simplest form.

6. In a week Barry earns £250, Sharon earns £300 and Del earns £450.
   Write down the following ratios of wages in their simplest forms : -
   (a) Del : Barry (b) Sharon : total wages (c) Del : Sharon : Barry.

In some cases, when simplifying a ratio, we have to multiply rather than divide each side.

Example To simplify the ratio $\frac{1}{3} : 4$, we multiply each side by 3, giving **1 : 12.**

A ratio in which one of its values is "1", is called a **unitary ratio.**

7. Simplify the following to a unitary ratio each time : -

(a) $\frac{1}{3} : 2$ (b) $\frac{1}{3} : 5$ (c) $\frac{1}{2} : 6$ (d) $\frac{1}{2} : 2$ (e) $\frac{1}{4} : 9$ (f) $\frac{1}{4} : 12$

(g) $\frac{1}{5} : 15$ (h) $\frac{1}{8} : 8$ (i) $\frac{1}{7} : 13$ (j) $\frac{1}{15} : 20$ (k) $\frac{1}{4} : \frac{1}{2}$ (l) $\frac{1}{2} : \frac{1}{8}$

8. Write each of the following in its simplest form (not all give unitary ratios) : -

(a) $\frac{2}{3} : 4$ (b) $\frac{2}{3} : 5$ (c) $\frac{3}{4} : 15$ (d) $\frac{2}{5} : 10$ (e) $\frac{4}{7} : 2$ (f) $\frac{9}{10} : \frac{1}{2}$

(g) $\frac{7}{10} : 0{\cdot}6$ (h) $\frac{5}{6} : 5{\cdot}4$ (i) $\frac{3}{5} : 50$ (j) $\frac{3}{4} : 11$ (k) $\frac{9}{10} : 180$ (l) $\frac{2}{5} : \frac{1}{2}$

9. A recipe needs $\frac{1}{2}$ kilogram of butter, $\frac{1}{4}$ kilogram of flour and $\frac{1}{10}$ kilogram of sugar.

Write in its simplest form the ratio of : -

(a) butter : flour (b) flour : butter

(c) sugar : butter (d) flour : sugar.

When working with ratios, the two units **must** be the same.

10. Write down each ratio in its simplest form : -

(a) $\frac{1}{4}$ of an hour : 30 minutes (*hint : - change both to minutes*)

(b) $\frac{1}{4}$ kg : 150 g (c) $\frac{1}{4}$ litres : 25 ml (d) $\frac{1}{2}$ metre : 200 cm

(e) 20 kg : 200 g (f) 10 litres : 100 ml (g) 3 kilometres : 200 m

(h) 1 km : 10 cm (i) 2 tonnes : 100 g (j) 30 minutes : 1 day

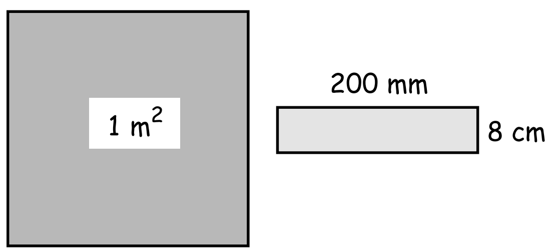
(k) 1 week : days in April (l) one million millimetres : one kilometre.

11. A rectangle has length 200 mm and breadth 8 cm.
A square has an area of 1 m^2.

Write in its simplest form the ratio of : -

(a) area of rectangle : area of square.

(b) length of rectangle : length of square.



Ratio Calculations

Tabulating a ratio calculation is usually the simplest method of obtaining an answer.

Example The ratio of men to women at a party is 2 : 3.

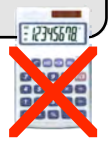
If there are 16 men at the party,
how many women are there ?

Set down like this :- since 16 = 8 x 2

=> then women = 8 x 3 = 24

	men	women	
x 8	2	3	x 8
	16	24	

There must be **24** women at the party.



Exercise 3 (no calculator)

1. (a) On a train the ratio of men to women is 3 : 4.

If there are 27 men on the train,
how many women are there ?

	men	women	
x 9	3	4	x ?
	27	. . .	

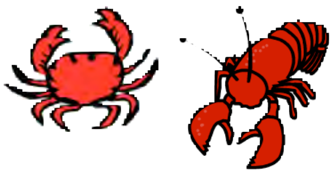
(b) In a Cat & Dog home the ratio of cats to dogs is 4 : 7.

If there are 35 dogs in the home ,
how many cats are there ?

	cats	cogs	
x ?	4	7	x 5
	. . .	35	

(c) In an orchard the ratio of apple trees to pear trees is 9 : 11.

If there are 27 apple trees, how many pear trees are there ?

2. 

In a large aquarium the ratio of crabs to lobsters is 3 : 5.

(a) If there are 12 crabs, how many lobsters are there ?

(b) If there are 30 lobsters, how many crabs are there ?

3. The ratio of Bob's weekly wage to Janet's weekly wage is 5 : 7.

(a) If Bob earns £250, how much would Janet earn ?

(b) If Janet earns £210, how much would Bob earn ?

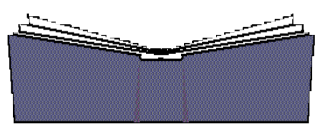
4. 

In a school the ratio of girls to boys is 8 : 7.

(a) If there are 400 girls, how many boys are there ?

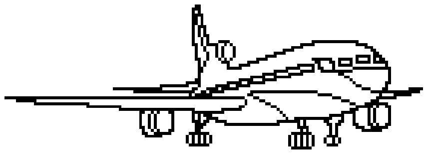
(b) If there are 651 boys, how many girls are there ?

5. The ratio of vowels to consonants in a book was 11 : 23.



(a) If there are 13 200 vowels, how many consonants are there ?

(b) If there are 690 000 consonants, how many vowels are there ?

6. 

A model aeroplane has a scale of 1 : 40.

(a) If the wing span on the model is 25 centimetres, what would be the wingspan of the real aeroplane ?

(b) If the real aeroplane has length 8 metres, what is the length of the model aeroplane ?

7. The table shows the ratios of blue and red paint for making different shades of purple.

Which shade of purple will I get if I mix : -

(a) 300 ml of blue and 500 ml of red ?

(b) 1·8 litres of blue and 200 ml of red ?

(c) 900 ml of blue and 1·5 litres of red ?

(d) 1 litre of blue and 300 ml of red ?

(e) 500 ml of red and 0·7 litres of blue ?

(f) 2·25 litres of red and 1·35 litres of blue ?

Mix in the ratio

Colour	***Blue***	***:***	***Red***
Very dark purple	9	:	1
Dark purple	10	:	3
Purple	7	:	5
Light purple	3	:	5
Very light purple	2	:	9

Proportional Division

(Sharing in a given ratio)

Example Bill and Ben share a raffle win of £400 in a ratio of 3 : 5.
How much will each receive ?

Solution :

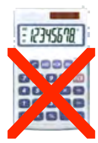
Step 1 :- Since the ratio is 3 : 5, there are (3 + 5) = **8 shares**

Step 2 :- **Each share** is worth (£400 ÷ 8) = **£50**

Step 3 :- Bill has 3 shares (3 x £50) = £150
Ben has 5 shares (5 x £50) = £250

(Check that the total is £400).

Exercise 4 (no calculator)



1. Share £200 between May and Beth in the ratio 2 : 3.

Copy and complete : -

Total number of shares = 2 + 3 = 5

Each share = £200 ÷ 5 = **£40**

May has 2 shares = 2 x £..... = £.......

Beth has 3 shares = 3 x £..... = £ ......

(check total is £200).

2. Share £18 000 between Alex and James in the ratio 2 : 7.

(*Show all your working and remember to check your total comes to £18000*).

3. Show all your working for each of the following :-

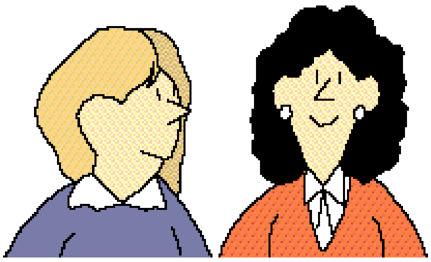
(a) Share £36 000 between Zak and Zeb in the ratio 4 : 5.

(b) Share £12 000 between Ann and Ben in the ratio 7 : 5.

(c) Share £8·60 between Caron and Denis in the ratio 1 : 3.

(d) Share 308 € between Pierre and Helena in the ratio 4 : 3.

(e) Share one million dollars between Ed and Flo in the ratio 13 : 7.

4. 

Each week Debbie and Dawn share a £12 raffle ticket cost. Debbie pays £8 and Dawn pays £4.

(a) Write down the ratio of how much Debbie and Dawn pay, in its simplest form.

(b) Last week their ticket won £240.
Use the above ratio to determine how much money each should receive.

5. Bella (age 20) and Milo (age 30) are left £200 000 in their uncle's will.
The money is to be shared between Bella and Milo **in the ratio to their ages**.

How much should Milo receive from his uncle's will ?

6. 

Samuel and Peter are in the final of a doughnut eating contest.

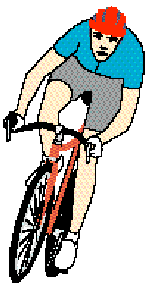
They will share the £1000 prize money in the ratio of how many doughnuts they each eat !
Samuel eats 24 doughnuts. Peter eats 26 doughnuts.

How much more prize money did the winner receive than the runner-up ?

7. (a) Share £100 amongst Ed, Ted and Zed in the ratio 2 : 3 : 5.

(b) Share $600 amongst Tammy, Sammy and Hammy in the ratio 4 : 5 : 11.

(c) Ben, Jen and Len share 75 marbles in the ratio 6 : 2 : 7. How many does each receive ?

8. 

A **two thousand kilometre** "Crazy Rally" is to be held next month.

Each contestant will walk, then cycle and then drive distances that are in the ratio 1 : 2 : 7.

How far will each contestant : -

(a) walk (b) cycle (c) drive ?

9. A drinks dispenser is programmed to give 35 litres of orange juice to three children each week in the ratio of their weights.

Sebastian is half Henry's weight.
Timmy is half Sebastian's weight.
Henry weighs 40 kilogrammes.

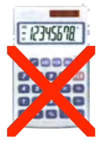
How much orange juice will each child be given in a week ?

Proportion

If you know the total cost of several items, you can easily find the cost per item.

Example	The cost of 5 cakes is £4·00.
	The cost of **1 cake** = (£4·00 ÷ 5) = **£0·80** – (simply **divide**).

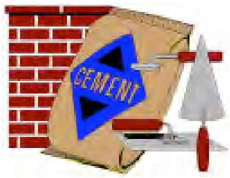
Exercise 5 (Oral Exercise)



1. The cost of 6 pies is £4·20. Find the cost of one pie.

2. Find the cost per item : -

   (a) 3 sweets costing 42p
   (b) 9 ties costing £45
   (c) 7 CD's costing £56
   (d) 11 lollies costing £1·10
   (e) 12 pastries costing £3·60
   (f) 10 mice costing £18.



3. It took a van 50 trips to move 1000 bags of cement.
   How many bags did the van move each trip ?

4. 
   A soldier can march 24 kilometres in 4 hours.
   Calculate the rate in kilometres per hour.

5. Jack exchanges £20 for 34 euros.
   Calculate the rate of € / £.

6. 
   A 3 kilogram bag of potatoes costs £2.
   What is the weight per £ ?

7. A mouse rotates a running wheel 150 times in a minute.
   Calculate the number of rotations per second.

8. 
   Jackie was "Walking Round Britain" for charity.
   During the month of June, she travelled a total distance of 480 miles.
   How many miles (on average) did she travel each day ?

9. Davie bought a set of 4 new tyres from Slow-Fit for a total of £96.
   Tim bought a set of 5 similar tyres from Tyres 'R Us for £110.
   By calculating the cost of 1 tyre each time, decide who got the better deal.

Direct Proportion

Two quantities, (for example, number of cakes and total cost), are said to be in **direct proportion**, if : -

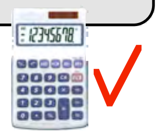
"... when you double the number of cakes you double the cost."

Example The cost of 6 cakes is £4·20. Find the cost of 5 cakes.

Set down like this : -

	Cakes		*Cost*
	6	—>	4·20
(divide)	1	—>	4·20 ÷ 6 = 0·70
(multiply)	5	—>	5 x 0·70 = **£3·50**

5 cakes cost £3·50.



Exercise 6

1. The cost of 8 books is £96·80.

   Find the cost of 7 books.

Books		Cost
8	—>	£96·80
1	—>	£96·80 ÷ 8 = £.....
7	—>	

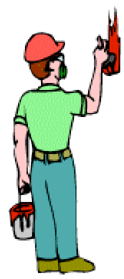
2. Nine sheets of high gloss photo paper costs £7·29.

   How much would it cost for 10 sheets ?
   (*Find the cost of 1 sheet first*)

3. 

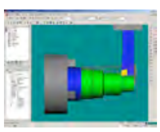
   On holiday, I exchanged £40 for $72.

   How many dollars would I have got for £45 ?
   (*Find how much for £1 first*).



4. It takes a man 2 minutes to paint a wall panel with an area of 3000 cm^2.

   What area of wall could the man paint in 9 minutes ?

5. 

   A machine wheel turns 300 times in 4 minutes.

   How many turns would it make in 5 minutes ?

6. (a) 5 air-mail letters cost £2 to post.
   How much would it cost to post 6 letters ?

   (b) Nine pies cost £8·19. How much would ten pies cost ?

7. A machine makes 300 paperclips every 6 seconds.

   How many paperclips will it make in : -

   (a) 1 second (b) 7 seconds (c) one minute (d) an hour ?

8. Which of the following definitely indicate direct proportion ?

(a) 5 pies cost £3. Six pies cost £3·50. (b) 9 sweets cost 72 p. Ten cost 81 p.

(c) 3 CD's cost £42. Four CD's cost £52. (d) 11 cakes cost £11·99. 5 cakes cost £5·45.

9. 

A bricklayer can lay 35 bricks in seven minutes.

(a) How long would it take to build a wall with 250 bricks ?

(b) How many bricks could he lay in an hour ?

10. A computer programmer writes 30 lines of computer code in an hour.

(a) How long would it take to write 26 lines of code ?

(b) A complete programme took 1 hour and 48 minutes to write.
How many lines of code were in this programme ?

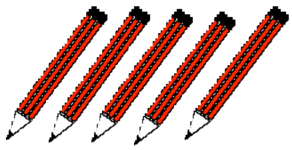
Sometimes it is easier to find the cost of 10, or 100, or 1000 items first, instead of just 1 !

Example : - 500 coloured pens cost £20.
How much would it cost for 700 pens ?

This time it would be easier to find 100 first, then multiply by 7.

Pens		Cost
500	—>	£20
100	—>	£20 ÷ 5 = £4
700	—>	£4 x 7 = **£28**

11. 400 pencils cost £8. Find the cost of 300 pencils.
(*Find the cost of 100 first*).



12. (a) 200 litres of oil costs £30. Find the cost of 150 litres.

(b) 100 tyres take 5 hours to burn, one at a time. How long would it take 70 tyres to burn ?

(c) It takes 500 bees a week to make 3·5 kg of honey.

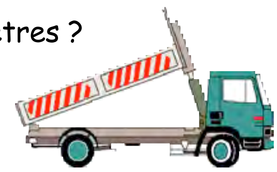
What weight of honey would you get in a week from 1200 bees ?

(d) 600 ml of orange concentrate costs £1·80.

How much would it cost for one litre ?

(e) 60 metres of chain costs £24. How much would it cost for 35 metres ?

(f) 20 trucks can remove 1300 tonnes of rubble in a day.
How much rubble could 30 trucks remove in a day ?



13. The cost of painting is directly proportional to the area being painted.

(a) A wall 12 metres by 2 metres costs £72 to paint.
How much would it cost for a wall 15 metres by 2 metres ?

(b) A factory panel (25 metres by 8 metres) costs £160 to paint.
How much would it cost to paint a 30 metres by 5 metres panel ?

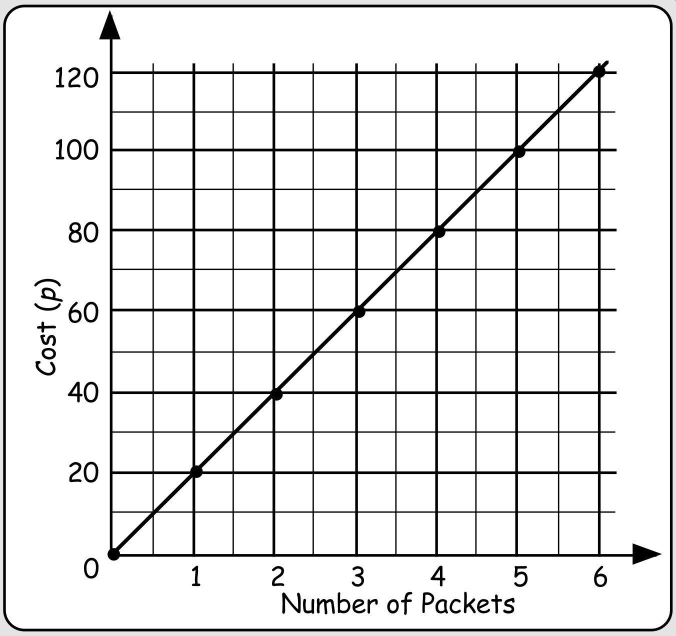
Linear Graph of Direct Proportion

The table below shows the cost of packets of "Biscuits".

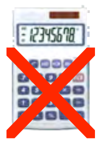
No. of Pkts	1	2	3	4	5	6
Cost (p)	20	40	60	80	100	120

THE POINTS LIE ON A STRAIGHT LINE WHICH PASSES THROUGH THE ORIGIN.

This is true for any two quantities which are in **DIRECT PROPORTION**.



Exercise 7



1. (a) Copy and complete the table.

No. of pears	1	2	3	4	5	6
Cost (p)	30	60				

(b) Using the same scales as in the above graph, plot the points (1, 30), (2, ?), ...

(c) (i) Join the points with a straight line

(ii) Does the line pass through the origin ?

(iii) Explain why the line must pass through the origin.

2. (a) Copy and complete this table.

No. of pots	1	2	3	4	5
Cost (p)	40	80			

(b) Use an appropriate scale to plot the points (1, 40), (2, ...), etc.

(c) (i) Join the points with a straight line.

(ii) Does the line pass through the origin ?

3. (a) Copy and complete this table for a cycle travelling at 10 km/hr.

Time (hrs)	1	2	3	4	5
Distance (km)	10	20			

(b) Using a scale of 2 boxes to represent 1 hour on the horizontal axis and 2 boxes to represent 10 km on the vertical axis, plot the points and draw a line through them.

(c) What distance should the cycle travel in 8 hours ?

4. (a) Draw a set of axes and plot the following points.

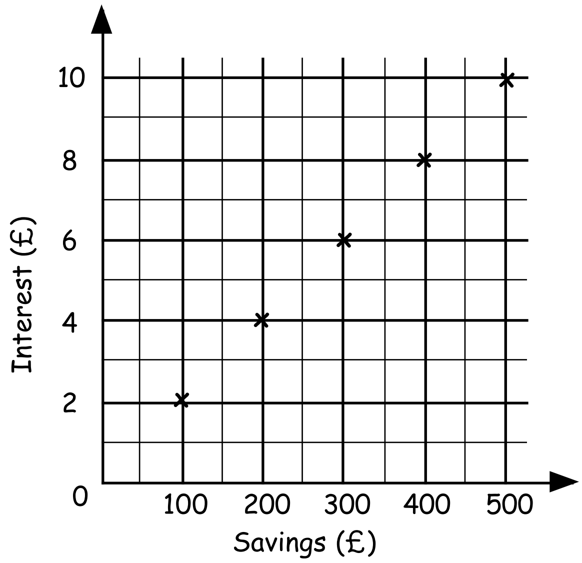
x	1	2	3	4
y	3	6	8	12

=> (1, 3), ..., etc.

A simple check for direct proportion is to divide each pair of values.
(3 ÷ 1), (6 ÷ 2), (8 ÷ 3), (12 ÷ 4)
If you always obtain the same answer, then they ARE in direct proportion.

(b) Are y and x in direct proportion here ? **Explain**.

5. This graph shows the annual interest given by the "Northern Building Society" on savings of £100, £200, £300, £400 and £500.



(a) Use the graph to copy and complete this table.

Savings (£)	100	200	300	400	500
Interest (£)	2				

(b) Are the quantities in direct proportion ? Explain.

(c) Calculate the interest gained on savings of £1000 ?

6. Which **TWO** of the following tables indicate examples of direct proportion ? (*hint - divide*)

(a)

x	1	2	3	4
y	1	4	9	16

(b)

x	1	2	3	4
y	5	10	20	40

(c)

x	1	2	3	4
y	2	4	6	8

(d)

x	1	2	3	4
y	5	4	3	2

(e)

x	1	2	3	4
y	0	2	6	10

(f)

x	2	4	6	8
y	30	60	90	120

7. For each of your two answers to question 6, verify they are in direct proportion by plotting the points and showing a line can be drawn through the points and the origin.

Assignment

Work in pairs or groups - the best graphs may be used on the wall of your classroom.

Find a currency exchange rate used somewhere in the world.

On graph paper write a report using a direct proportion graph and explain why such a graph could be used for currency conversion.

Inverse Proportion (Indirect proportion)

Direct proportion In this case, as one quantity **increases** the other also **increases**. (example :- the more cakes you buy, the more the cost).

Inverse proportion (indirect proportion) Here, as one quantity **increases** the other **decreases** (and vice-versa).

Example :- If 3 men take 8 hours to build a wall, how long will it take 4 men to build the wall ?

(Can you see that it **must** take 4 men **less** time ?)

It is still simpler to work out how long **one** man would take.

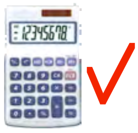
One man will take **3 times** the number of hours. (24 hours).

Four men will take a quarter of this time .

Men		Hours
3	—>	8
1	—>	3 x 8 = 24
4	—>	24 ÷ 4 = **6**

It will take 4 men 6 hours.(Check that **more** men —> **less** time for the job !)

Exercise 8



1. If it takes 5 men 12 hours to paint a fence, how long would it take 6 men ?

   Set down your answer as shown opposite.
   (*Don't forget to check : **more** men - **less** time*).

Men		Hours
5	—>	12
1	—>	5 x 12 = ...
6	—>	... ÷ 6 = ...

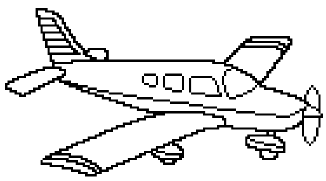
2. 

   If it takes 6 men 8 hours to erect a fence, how long would it take 5 men to erect the fence ?

   (*Check : this time that **less** men —> **more** time*).

3. An aeroplane takes 5 hours for a journey at an average speed of 240 km /hr.

   At what speed would the aeroplane have to travel to cover the same journey in 3 hours ?



4. 

   Terri reads at a rate of 250 words per minute and takes 6 hours to read a book.

   How long would it have taken her to read the book at 300 words per minute ?

5. It takes 5 girl guides 1 hour to pitch a large tent.

   How long would it take 8 girl guides working at the same rate ?

6. A squad of five soldiers have enough rations for 12 days. Ten other soldiers with no extra food join the squad.

   How many days will the rations now last the group ?

7. 

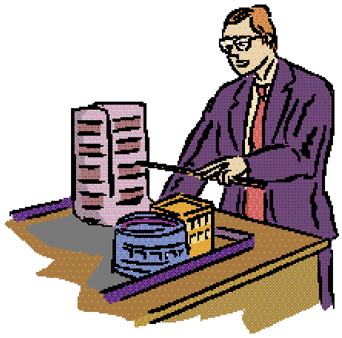
   Jason has enough fish food to feed his 20 tropical fish for 3 weeks.

   If he sells 5 of his fish, how long will the fish food last ?

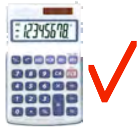
8. An architect estimated it would take 15 men 10 months to build a block of flats.

   The builder needs to do the job in 6 months.

   How many **extra** men does the builder need to complete the job on time ?



Exercise 9 (Mixed exercise)



For each question in this exercise you must decide first if it is **direct** or **inverse** proportion.

1. Bill buys 5 melons at a cost of 80p. How much would he have to pay for 6 melons ?

2. Mary can sew 240 stitches in 10 minutes. How long would it take her to sew 300 stitches ?

3. 

   Johan takes 20 minutes to walk 3 kilometres.

   How far could he walk in 45 minutes at the same speed ?

4. A car takes 4 hours to complete a journey at an average speed of 55 km/hr. What average speed is required to complete the journey in 5 hours ?

5. Ben pays £200 for 8 bottles of champagne. How much would it cost for 9 bottles ?

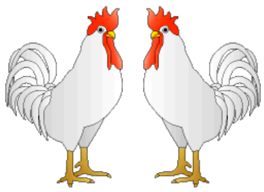
6. 

   Twelve small barrels are needed to hold 600 litres of oil.

   How many barrels are needed to hold 350 litres ?

7. It took 6 pupils 40 minutes to clear a storeroom.

   How long would it have taken 4 pupils ?

8. 

   Two dozen chickens have enough feed to last a week.

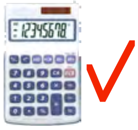
   If three of the chickens are removed, how long will the feed last those chickens which are left ?

9. A garden requires 32 edging blocks, each 1·5 metres long, to surround it completely.

   If a garden centre only sold edging blocks which were 1·2 metres long, how many would be needed to surround this same garden ?

Ratio & Proportion

Topic in a Nutshell



1. Look at the picture.
   Write down the ratio of : -

   (a) dogs : cats   (b) cats : dogs

2. Write down each ratio in its simplest form.

   (a) 3 : 6   (b) 12 : 32   (c) 42 : 18   (d) 16 : 60   (e) 12 : 18 : 33.

3. A box contains 16 CD's, 20 DVD's and 4 audio cassettes.
   Write down the ratio, in its simplest form, of : -

   (a) Cassettes : DVD's   (b) CD's : DVD's

4. Simplify each of the following to produce a **unitary** ratio : -

   (a) $\frac{1}{3}$: 12   (b) $\frac{3}{4}$: 12   (c) $\frac{4}{5}$: 12   (d) $\frac{7}{10}$: 63   (e) 0·3 : 3·9.

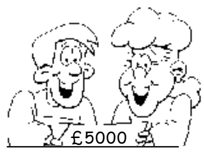
5. The ratio of managers to workers in a factory is 3 : 35.

   (a) If there are 60 managers, how many workers are there ?

   (b) If there are 385 workers, how many **people** are in the factory altogether ?

6. Alison and Colin share a £5000 lottery win in the ratio 13 : 7.

   How much money will each receive ?



7. Paul, Pat and Peter share a bag of 200 marbles in the ratio 1 : 4 : 5.

   How many marbles will each person receive ?

8. (a) Five pizza's cost £17. How much will one pizza cost ?

   (b) Seven dolls cost £24·50. How much will five dolls cost ?

   (c) 250 centimetres of computer cable costs £60. How much will 10 metres cost ?

9. (a) Draw a coordinate diagram and plot each of these points using a suitable scale.

   (b) Is this an example of direct proportion ? *Explain.*

x	1	2	3	4
y	5	10	15	20

10. (a) It takes 12 workmen 3 hours to repair a roof.
    How long would it take 9 workmen, working at the same rate?

    (b) A machine can produce 150 mouldings in an hour.
    How many mouldings can it produce in 24 minutes ?

11. To treat an area of 50 square metres of lawn with weedkiller would cost £20.

    How much would it cost to treat a lawn measuring 5 metres by 6 metres ?

Chapter 43

Equations/Inequalities

Equations - Revision

There are various ways of solving equations.

We are going to use the "**change side - change sign**" method *

* your teacher may show you an alternative method

Revision of level E Work :-

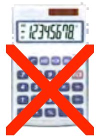
move the +3 to the other side => change it to −3

$x + 3 = 7$
$\Rightarrow \quad x = 7 - 3$
$\Rightarrow \quad x = 4$

$x - 5 = 11$
$\Rightarrow \quad x = 11 + 5$
$\Rightarrow \quad x = 16$

$x + 8 = 8$
$\Rightarrow \quad x = 8 - 8$
$\Rightarrow \quad x = 0$

Exercise 1



1. Copy each equation and solve to find the value of x, as shown above :-

(a) $x + 3 = 9$	(b) $x + 7 = 20$	(c) $x + 5 = 8$
(d) $x + 9 = 9$	(e) $x - 2 = 7$	(f) $x - 5 = 1$
(g) $x - 15 = 0$	(h) $x - 50 = 40$	(i) $x + 6 = 4$
(j) $x - 5 = 0$	(k) $x + 11 = 0$	(l) $x - 22 = 28$
(m) $8 + x = 5$	(n) $1 + x = 1$	(o) $7 + x = 0$
(p) $10 + x = 3$	(q) $6 + x = 6$	(r) $15 + x = -15$

Revision of level E Work :-

move the $\times$ 2 to the other side and change to $\div$ 2

$2x = 12$
$\Rightarrow \quad x = 12 \div 2$
$\Rightarrow \quad x = 6$

$4d = 20$
$\Rightarrow \quad d = 20 \div 4$
$\Rightarrow \quad d = 5$

$3n = 17$
$\Rightarrow \quad n = 17 \div 3$
$\Rightarrow \quad n = 5\frac{2}{3}$

2. Copy each equation and solve to find the value of the letter :-

(a) $2x = 18$	(b) $5t = 30$	(c) $3d = 12$
(d) $3f = 18$	(e) $4n = 32$	(f) $8h = 8$
(g) $6c = 15$	(h) $9d = 0$	(i) $2x = 1$
(j) $3m = 150$	(k) $10w = 180$	(l) $6m = 27$
(m) $4x = 11$	(n) $5v = 24$	(o) $7f = 20$
(p) $10c = 35$	(q) $8t = 6$	(r) $3m = 25$

Revision of level E Work :-

Move the -3 to the other side and change to $+3$

move the $\times 2$ to the other side and change to $\div 2$

$$2x - 3 = 11$$
$$\Rightarrow \quad 2x = 11 + 3$$
$$\Rightarrow \quad 2x = 14$$
$$\Rightarrow \quad x = 7 \text{ (divide)}$$

$$5x + 4 = 19$$
$$\Rightarrow \quad 5x = 19 - 4$$
$$\Rightarrow \quad 5x = 15$$
$$\Rightarrow \quad x = 3$$

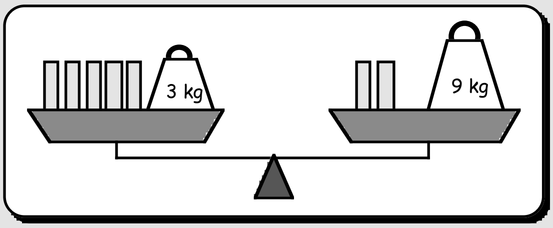
$$4x - 6 = 17$$
$$\Rightarrow \quad 4x = 17 + 6$$
$$\Rightarrow \quad 4x = 23$$
$$\Rightarrow \quad x = 23 \div 4 = 5\tfrac{3}{4}$$

3. Find the value of x in the following equations (*Show each step of working carefully*).

(a) $3x + 1 = 13$	(b) $4x + 3 = 23$	(c) $6x + 2 = 38$
(d) $2x + 5 = 9$	(e) $7x - 3 = 32$	(f) $5x - 2 = 48$
(g) $8x - 6 = 50$	(h) $4x - 8 = 0$	(i) $10x - 7 = 63$
(j) $7x - 3 = 46$	(k) $3x + 11 = 14$	(l) $8x - 1 = 79$
(m) $9x - 10 = 17$	(n) $5x + 21 = 121$	(o) $6x + 4 = 52$
(p) $2x - 3 = 55$	(q) $11x + 11 = 0$	(r) $2x - 7 = 0$
(s) $3x + 20 = 8$	(t) $6x + 5 = 20$	(u) $4x - 9 = 12$

Harder Equations

This diagram shows a set of balanced scales.



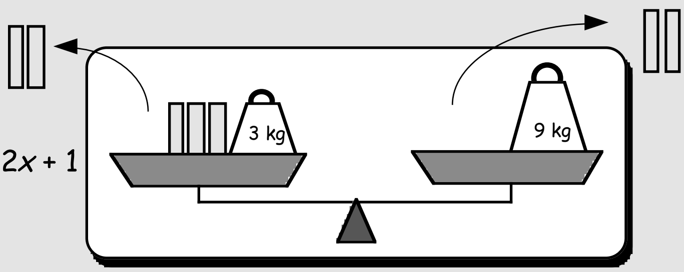
- 5 blocks and a 3 kg weight on the left
- 2 blocks and a 9 kg weight on the right.

If each block weighs x kg, then the equivalent equation for this is :-

$$\boxed{5x + 3 = 2x + 9,}$$ – to be solved.

To simplify the situation, remove 2 blocks ($2x$) from both sides.

This leaves a much simpler equation, which you already know how to solve.



$2x + 1$

remove $2x$ from both sides

$$5x + 3 = 2x + 9$$
$$\Rightarrow \quad 3x + 3 = 9$$
$$\Rightarrow \quad 3x = 9 - 3 = 6$$
$$\Rightarrow \quad x = 2$$

* your teacher may show you an alternative method

$\Rightarrow$ 1 block must weigh 2 kg.

Further Examples :-

Take $2x$ from both sides.

Move the +1 to the other side and change to −1

move the $\times 4$ to the other side and change to $\div 4$

$6x + 1 = 2x + 21$

(take "$2x$" from each side)

=> $4x + 1 = 21$

=> $4x = 21 - 1$

=> $4x = 20$

=> $x = 5$

$8x - 2 = 3x + 28$

(take "$3x$" from each side)

=> $5x - 2 = 28$

=> $5x = 28 + 2$

=> $5x = 30$

=> $x = 6$

$10x + 9 = 4x + 30$

(take "$4x$" from each side)

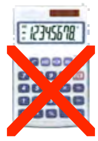
=> $6x + 9 = 30$

=> $6x = 30 - 9$

=> $6x = 21$

=> $x = 21 \div 6 = 3\frac{1}{2}$

Exercise 2



1. Copy and complete

(a) $7x + 2 = 4x + 17$
=> $3x + 2 = ...$
=> $3x = ...$
=> $x = ...$

take "$4x$" from each side

(b) $4x - 3 = x + 18$
=> $3x - ... = ...$
=> $3x = ...$
=> $x = ...$

take "$1x$" from each side

2. Solve these equations by removing an appropriate number of x's from each side first :-

(a) $4x + 1 = 2x + 7$ (b) $3x + 5 = x + 15$ (c) $6x + 7 = 5x + 13$

(d) $10x - 6 = 7x + 9$ (e) $5x - 1 = 2x + 11$ (f) $6x - 1 = x + 19$

(g) $12x - 4 = 8x + 24$ (h) $10x - 1 = 8x + 6$ (i) $4x + 4 = x + 12$

(j) $6x + 3 = 2x + 10$ (k) $9x - 2 = 4x + 19$ (l) $7x - 7 = x + 1$

3. These equations look a little "different". Solve them in the same way as shown above :-

(a) $3x = 2x + 6$ (b) $5x = x + 20$ (c) $7x = 4x + 30$

(d) $9x = 8x + 6$ (e) $3x = x + 13$ (f) $5x - 12 = 3x$

(g) $4x - 15 = x$ (h) $3x + 6 = x$ (i) $10x - 21 = 7x$

4. I bought 3 bags of marbles. My friend bought 1 bag and he also had 24 loose marbles.

We discovered that we had exactly the same number of marbles.

(a) Make up an equation to show this information.
(*let x be the number of marbles in 1 bag*)

(b) Solve the equation to determine how many marbles there are in each bag.

5. A group of people arrive at a hotel for a meeting.

The lift is used (full) 4 times and as well as this 5 people walk up the stairs.

At the end of the meeting the lift is filled 2 times and the remaining 21 people walk down the stairs.

(a) Make up an equation to show this information.
(*let x be the number in 1 full lift*)

(b) Solve the equation to determine how many people 1 full lift holds.

Equations with Brackets

Two further examples illustrating the Algebra from chapter 11:-

	$2(2x + 3) = x + 24$
Multiply out the brackets	$\Rightarrow \quad 4x + 6 = x + 24$
Take "x" from both sides	$\Rightarrow \quad 3x + 6 = 24$
Move the +6 to the other side	$\Rightarrow \quad 3x = 18$
Divide by 3	$\Rightarrow \quad x = 6$

$5(3x + 2) - 2(4x - 3) = 2x + 36$

$15x + 10 - 8x + 6 = 2x + 36$ Note :- the + 6 (not − 6)

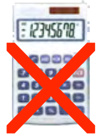
$\Rightarrow \quad 7x + 16 = 2x + 36$

$\Rightarrow \quad 5x + 16 = 36$

$\Rightarrow \quad 5x = 20$

$\Rightarrow \quad x = 4$

Exercise 3



1. Solve these equations by multiplying out the brackets first :-

(a) $2(x + 5) = 16$	(b) $3(x + 4) = 21$	(c) $5(x - 6) = 15$
(d) $4(x + 3) = 24$	(e) $6(x + 3) = 42$	(f) $2(x + 3) = 6$
(g) $10(x - 2) = 40$	(h) $9(x + 3) = 72$	(i) $2(x - 1) = 7$
(j) $5(x - 4) = 0$	(k) $4(x - 6) = 4$	(l) $3(x + 4) = 6$

2. Solve these equations :-

(a) $2(3x + 1) = 26$	(b) $3(2x - 1) = 27$	(c) $4(5x - 1) = 16$
(d) $2(4x + 1) = 18$	(e) $3(2x - 10) = 0$	(f) $2(5x - 3) = 24$
(g) $3(2x - 1) = 4x + 7$	(h) $2(3x + 2) = 3x + 19$	(i) $2(1 + 4x) = 5x + 23$
(j) $6(2x - 3) = 10x$	(k) $11(2x - 3) = 15x + 2$	(l) $10(x + 2) = 9x.$

3. Solve these equations :-

(a) $2(x + 4) - x - 6 = 10$	(b) $3(x + 1) + 2x - 3 = 25$
(c) $4(x + 4) - 2x = 22$	(d) $5(x - 1) + 2x + 3 = 40$
(e) $2x + 5 + 3(x - 1) = 32$	(f) $4x + 2(x - 5) = 8$
(g) $3(x - 3) + 2(x + 5) = 21$	(h) $5(2x + 1) + 3(1 - 2x) = 20$
(i) $4(2x + 1) + 3(x - 2) = 7x + 30$	(j) $2(3x - 2) + 4(x + 1) = 5x + 30$
(k) * $4(x + 3) - 2(x + 1) = 16$	(l) $5(x - 1) - 3(x - 4) = 11$
(m) $2(3x + 1) - 3(x - 2) = x + 20$	(n) $8(x + 2) - 2(2x + 4) = 2x - 22$

* be careful with the negative sign in front of the 2nd bracket for parts (k) to (n) !

Equations with Fractions

Fractions are a complication in equations we could well do without !!

=> Fortunately, we can **get rid of the fractions** quite easily.

Simple Rule :- We always **ELIMINATE** the fractions **right at the beginning** by **MULTIPLYING** every term by the l.c.m. of all the fractional denominators.

Example :-

Multiply both sides by 2 to eliminate the one fraction $\frac{1}{2}$

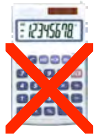
$$\frac{1}{2}x + 3 = 7$$
$$\underline{\underline{2}} \times \frac{1}{2}x + \underline{\underline{2}} \times 3 = \underline{\underline{2}} \times 7$$
$$\Rightarrow \quad x + 6 = 14$$
$$\Rightarrow \quad x = 8$$

Example :-

Multiply both sides by 12 to eliminate the two fractions, since the l.c.m. of 4 and 3 is 12

$$\frac{3}{4}x + \frac{2}{3} = 2$$
$$\underline{\underline{12}} \times \frac{3}{4}x + \underline{\underline{12}} \times \frac{2}{3} = \underline{\underline{12}} \times 2$$
$$\Rightarrow \quad 9x + 8 = 24$$
$$\Rightarrow \quad 9x = 16$$
$$\Rightarrow \quad x = 16 \div 9 = 1\tfrac{7}{9}$$

Exercise 4



1. Copy and complete the following two equations :-

(a) $\frac{1}{3}x + 4 = 10$

$\underline{\underline{3}} \times \frac{1}{3}x + \underline{\underline{3}} \times 4 = \underline{\underline{3}} \times 10$

$\Rightarrow \quad x + \ldots = \ldots$

$\Rightarrow \quad x = \ldots$

(b) $\frac{4}{5}x - 2 = \frac{1}{2}x + 1$

$\underline{\underline{10}} \times \frac{4}{5}x - \underline{\underline{10}} \times 2 = \underline{\underline{10}} \times \frac{1}{2}x + \underline{\underline{10}} \times 1$

$\Rightarrow \quad 8x - \ldots = \ldots x + \ldots$

$\Rightarrow \quad 3x - \ldots = 10$

$\Rightarrow \quad \ldots x = 30$

$\Rightarrow \quad x = \ldots.$

2. Solve each of these equations, by first of all multiplying every term by the l.c.m. of all the fractional denominators. This should eliminate all the fractions.

(a) $\frac{1}{2}x - 1 = 3$ (b) $\frac{1}{4}x + 5 = 7$ (c) $\frac{1}{8}x - 2 = 1$

(d) $\frac{2}{3}x - 4 = 2$ (e) $3 + \frac{3}{5}x = 0$ (f) $\frac{3}{8}x + 10 = 12$

(g) $\frac{3}{4}x - \frac{1}{2} = 2$ (h) $\frac{1}{2}x + \frac{1}{5} = 1$ (i) $\frac{3}{5}x - \frac{1}{3} = 0$

(j) $\frac{1}{2}x - 3 = \frac{1}{4}$ (k) $\frac{2}{3}x + 2 = \frac{1}{3}$ (l) $\frac{3}{4}x - 4 = \frac{1}{5}$

(m) $\frac{1}{2}x + 1 = \frac{1}{3}x + 3$ (n) $\frac{3}{4}x - 5 = \frac{3}{5}x - 2$ (o) $1 + \frac{5}{8}x = \frac{1}{3}x + 8$

(p) $\frac{1}{2}x - \frac{1}{3} = \frac{1}{4}$ (q) $\frac{1}{4}x + \frac{2}{5} = \frac{1}{2}$ (r) $\frac{1}{3}x + \frac{1}{2} = \frac{1}{4}x + \frac{1}{5}$

Two Final (harder) examples :-

Simple Rule :- **ELIMINATE the fractions right at the start by MULTIPLYING** every term by the l.c.m. of all the fractional denominators

Multiply both sides by 3 to eliminate the 3 in the denominator

$$\frac{x+1}{3} + 4 = 6$$

$$\underline{\cancel{3}} \times \frac{(x+1)}{\cancel{3}} + \underline{3} \times 4 = \underline{3} \times 6$$

$$\Rightarrow \quad x + 1 + 12 = 18$$

$$\Rightarrow \quad x = 18 - 1 - 12$$

$$\Rightarrow \quad x = 5$$

note how we put the $(x + 1)$ in brackets

Multiply both sides by 12 to eliminate the two denominators, 4 & 3

$$\frac{3}{4}(2x-1) + \frac{2}{3}x = 1$$

$$\underline{\cancel{12}}^{3} \times \frac{3}{\cancel{4}}(2x-1) + \underline{\cancel{12}}^{4} \times \frac{2}{\cancel{3}}x = \underline{12} \times 1$$

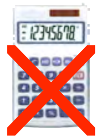
$$\Rightarrow \quad 9(2x - 1) + 8x = 12$$

$$\Rightarrow \quad 18x - 9 + 8x = 12$$

$$\Rightarrow \quad 26x = 21 \quad (12 + 9)$$

$$\Rightarrow \quad x = \frac{21}{26}$$

Exercise 5



1. Copy and complete the following two fractional equations :-

(a) $\frac{x-2}{4} - 3 = 1$

$$\underline{\cancel{4}} \times \frac{(x-2)}{\cancel{4}} - \underline{4} \times 3 = \underline{4} \times 1$$

$$\Rightarrow \quad x - \ldots - \ldots = 4$$

$$\Rightarrow \quad x = \ldots + \ldots + \ldots$$

$$\Rightarrow \quad x = \ldots$$

(b) $\frac{4}{5}(3x+2) - \frac{1}{3}x = 4$

$$\underline{\cancel{15}}^{3} \times \frac{4}{\cancel{5}}(3x+2) - \underline{\cancel{15}}^{5} \times \frac{1}{\cancel{3}}x = \underline{15} \times 4$$

$$\Rightarrow \quad 12(3x + \ldots) - \ldots x = 60$$

$$\Rightarrow \quad 36x + \ldots - \ldots x = 60$$

$$\Rightarrow \quad \ldots x = 36$$

$$\Rightarrow \quad x = \frac{\ldots}{31}$$

2. Multiply each term by the l.c.m. of the denominators to eliminate the fractions and solve :-

(a) $\frac{x+2}{5} = 3$ (b) $\frac{x+3}{4} = 5$ (c) $\frac{x-3}{2} = 4$

(d) $\frac{x+5}{3} - 2 = 3$ (e) $\frac{3x-4}{5} + 3 = 7$ (f) $4 + \frac{x-2}{4} = 0$

(g) $\frac{2}{3}(2x+3) - 10 = 0$ (h) $\frac{3}{4}(3x-1) - 10 = 5$ (i) $\frac{5}{8}(x+2) - \frac{1}{2}x = 3$

(j) $\frac{2}{5}(4x+1) - \frac{1}{3}x = 8$ (k) $\frac{5}{6}(2x+2) = \frac{3}{4}x + 9$ (l) $1 + \frac{3}{10}(3x+2) = \frac{1}{3}x + 5$

(m) $\frac{2}{3}(2x+1) + \frac{1}{2}(x-5) = 11$ (n) $\frac{3}{4}(x+2) + \frac{1}{3}(3x-9) = 9$ (o) $\frac{1}{2}(3x+1) - \frac{1}{3}(2x+2) = 4$

(p) $\frac{3}{5}(2x+3) - \frac{1}{2}(x-2) = 7$ (q) $\frac{x-4}{5} + \frac{x+1}{3} = 7$ (r) $\frac{3x-1}{4} - \frac{x+3}{3} = 0$

Solving Inequalities

$3x + 1 = 9$ and $7(x + 2) = 5x + 11$ are two examples of **equations**.

Inequalities are similar except the "=" sign is replaced with one of "<", ">", "≤" or "≥" each time.

Solving an inequality is almost identical to solving the corresponding equation.

equation	inequality
$2x - 1 = 7$	$2x - 1 < 7$
$2x = 7 + 1$	$2x < 7 + 1$
$2x = 8$	$2x < 8$
$x = 4$	$x < 4$

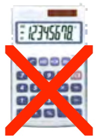
The solution this time is "x can be any number 'smaller' than 4" (not $x = 4$)

equation	inequality
$2(2x - 3) = x + 9$	$2(2x - 3) \geq x + 9$
$4x - 6 = x + 9$	$4x - 6 \geq x + 9$
$3x - 6 = 9$	$3x - 6 \geq 9$
$3x = 15$	$3x \geq 15$
$x = 5$	$x \geq 5$

The solution this time is "x can be any number 'bigger' than or equal to 5" (not just $x = 5$)

Reminder :- "<" - means "less than". ">" - means "greater than".
"≤" - means "less than or equal to".
"≥" - means "greater than or equal to".

Exercise 6



1. Solve these inequalities, leaving your answers in the form $x > 3$, etc. :-

(a) $x + 3 > 5$ (b) $x + 6 < 13$ (c) $x - 7 \leq 10$
(d) $x + 4 \geq 17$ (e) $x - 3 \leq 3$ (f) $x - 8 \geq 0$

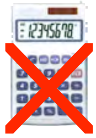
2. Solve each inequality, leaving your answers in the form $x \leq 5$, etc. :-

(a) $4x < 20$ (b) $5x > 30$ (c) $3x < 21$
(d) $8x \geq 48$ (e) $9x \leq 45$ (f) $10x > 120$

3. Solving the following inequalities :-

(a) $5x + 1 < 31$ (b) $3x + 2 > 14$ (c) $6x - 4 < 14$
(d) $2x + 5 \geq 19$ (e) $10x - 3 \leq 67$ (f) $8x - 11 > 61$
(g) $6x + 6 \leq 6$ (h) $4x - 5 < 15$ (i) $9x - 1 > 53$
(j) $8x - 16 < 0$ (k) $10x - 10 \geq 10$ (l) $2x + 7 \leq 16$
(m) $2(x + 3) < 14$ (n) $3(x + 1) > 33$ (o) $4(x - 5) \geq 40$
(p) $3(2x + 1) \leq 39$ (q) $2(5x - 1) > 8$ (r) $2(4x + 5) \leq 10$
(s) $6(x + 2) < 3x + 24$ (t) $5(2x + 4) > 6x + 36$ (u) $3(2x - 1) \geq 5x + 13$
(v) $4(3x + 11) \leq 10x + 50$ (w) $2(4x - 7) < 3x + 16$ (x) $8(2x - 1) \leq 14x$

Equations



Topic in a Nutshell

1. Copy each equation and find the value of each letter :-

(a) $x + 7 = 12$ (b) $y - 17 = 2$ (c) $w + 4 = 4$

(d) $t - 5 = 5$ (e) $2p = 12$ (f) $3g = 21$

(g) $7h = 7$ (h) $2x + 5 = 13$ (i) $3k + 19 = 10$

2. Solve each of the following equations :-

(a) $2x + 1 = x + 7$ (b) $4x + 4 = x + 19$ (c) $4x + 1 = 3x + 9$

(d) $12y - 6 = 3y + 3$ (e) $5y - 1 = 2y + 11$ (f) $4y - 10 = y + 17$

(g) $12p = 8p + 24$ (h) $10p = 8p - 6$ (i) $4w = w - 12$

(j) $5q + 2 = 3q - 10$ (k) $7m - 4 = 2m - 14$ (l) $7x + 13 = x + 1$

3. Jane had 4 bags of sweets. Bob had 2 bags and 6 loose sweets. Jane and Bob had exactly the same number of sweets.

(a) Make an equation to show this information.

(b) How many sweets does each person have ?

4. Solve each of the following equations :-

(a) $2(x + 1) = 10$ (b) $3(2x + 8) = 30$ (c) $5(5x - 1) = 20$

(d) $4(4y + 1) = 36$ (e) $9(2y - 10) = 0$ (f) $7(5y - 2) = 56$

(g) $3(k + 2) + 6 = 21$ (h) $4(2w + 1) - 3 = 17$ (i) $3(3p + 3) + 3p = -3$

(j) $5(q + 3) + 2(2q - 5) = 23$ (k) $5(3d + 2) + 3(1 - 2d) = 13$

5. Solve each of the following equations :-

(a) $\frac{1}{2}x + 3 = 9$ (b) $\frac{1}{4}x - 2 = 1$ (c) $\frac{1}{8}x + 5 = 8$

(d) $\frac{2}{3}x - 1 = 3$ (e) $\frac{3}{5}x + 11 = 0$ (f) $30 - \frac{3}{8}x = 21$

(g) $\frac{1}{2}x - 6 = \frac{1}{4}x + 2$ (h) $6 + \frac{3}{4}x = \frac{1}{8}x + 22$ (i) $\frac{1}{5}x - \frac{2}{3} = \frac{1}{2}$

(j) $\frac{x + 1}{4} - 7 = 3$ (k) $\frac{2}{3}(3x + 9) - 10 = 2$ (l) $\frac{3}{4}(3x - 2) - 10 = 20$

6. Solve each inequality, leaving your answer in the form $x > 4$, etc.

(a) $x + 7 > 12$ (b) $x - 6 < 10$ (c) $x - 1 \leq 5$

(d) $5x < 40$ (e) $3x > 12$ (f) $4x < 28$

(g) $9x - 27 < 0$ (h) $2x - 2 \geq 2$ (i) $3x + 11 \leq 35$

(j) $8(3x - 1) \leq 40$ (k) $5(2x - 1) > 5$ (l) $2(2x + 4) \leq 9$

Chapter 47

Time/Dist/Speed

Revision of Level D/E work on Time

You should be able to :- convert 12 hour time => 24 hour time

convert 24 hour time => 12 hour time

work out simple time intervals by "counting on".

8.00 am = 0800
10.45 am = 1045
2.30 pm = 1430
11.20 pm = 2320

0745 = 7.45 am
1155 = 11.55 am
1650 = 4.50 pm
2135 = 9.35 pm

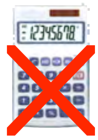
from 2.30 pm till 5.20 pm =>

$$2.30 \xrightarrow{\text{1 hour}} 3.30 \xrightarrow{\text{1 hour}} 4.30 \xrightarrow{\text{30 mins}} 5.00 \xrightarrow{\text{20 mins}} 5.20$$

total time = 1 hr + 1 hr + 30 min + 20 min

= 2 hr 50 mins

Exercise 1 (possibly completed orally)



1. Change the following 12 hour clock times to **24 hour clock times** :-

(a)	4·30 am	(b)	5·15 am	(c)	8 am
(d)	2·40 pm	(e)	5·45 pm	(f)	7 pm
(g)	6·25 am	(h)	9·55 pm	(i)	1·35 am
(j)	7·33 am	(k)	midday	(l)	12·40 am

2. Change the following 24 hour clock times to **12 hour clock times** :-

(a)	0350	(b)	1045	(c)	0705
(d)	1530	(e)	1735	(f)	2345
(g)	0110	(h)	1825	(i)	2020
(j)	1302	(k)	0000	(l)	0649

3. Calculate how long is it from :-

(a)	2·25 pm to 5·25 pm	(b)	4 am to 7·10 am
(c)	noon to 6·15 pm	(d)	5·30 pm to 8·25 pm
(e)	7·45 am to 9·25 am	(f)	1·50 am to 7·30 am
(g)	0740 to 0945	(h)	1735 to 1910
(i)	1655 to 1810	(j)	2250 to 0010 (next day ?)

Time, Distance, Speed Calculations

Calculating Distance

Imagine you were travelling in a train at a steady speed of 80 km/hr.

can you see that :- in 1 hour, you travel $1 \times 80 = 80$ km ?

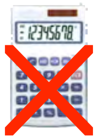
in 2 hours, you travel $2 \times 80 = 160$ km ?

in 3 hours, you travel $3 \times 80 = 240$ km ?

in other words :- **Distance (travelled) = Speed × Time**

or, using letters :-

$$\boxed{D_{istance} = S_{peed} \times T_{ime}}$$

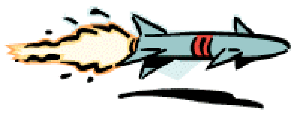


Exercise 2

1. Use the formula **D = S × T** to calculate how far the following people travel :-
   (a) jogging at 9 km/hr for 2 hours.
   (b) driving at 40 km/hr for 3 hours.
   (c) walking at 5 km/hr for 3 hours.
   (d) running at 22 km/hr for 3 hours.
   (e) flying at 210 m.p.h. for 4 hours.
   (f) on a camel at 3 m.p.h. for 8 hours.
   (g) sailing at 18 m.p.h. for 3 hours.
   (h) in a train travelling at 90 km/hr for $1\frac{1}{2}$ hours.

2. How far did the following travel :-
   (a) a train, travelling for $1\frac{1}{2}$ hours at an average speed of 80 m.p.h. ?
   (b) a $2\frac{1}{2}$ hour walk, at an average speed of 5 m.p.h. ?
   (c) a riverboat sail lasting $3\frac{1}{2}$ hours at an average speed of 20 m.p.h. ?
   (d) a helicopter flight for 30 minutes, at an average speed of 70 km/hr ?
   (e) a rocket ship journey of 10 hours 30 minutes, at an average speed of 3000 m.p.h. ?

3. What was the total distance travelled by each of the following :-
   (a) a missile, going at an average speed of 2400 m.p.h., for $\frac{1}{4}$ of an hour ?



   (b) a hydrofoil, going at an average speed of 36 m.p.h., for quarter of an hour ?
   (c) a lorry, travelling at an average speed of 60 m.p.h. for 2 hours 15 minutes ?
   (d) a racing car, travelling at an average speed of 160 km/hr for 45 minutes ($\frac{3}{4}$ hour) ?
   (e) an elephant, walking at an average speed of 8 km/hr for 1 hour 45 minutes ?
   (f) a cross country runner, running at an average speed of 16 km/hr for $1\frac{3}{4}$ hours ?

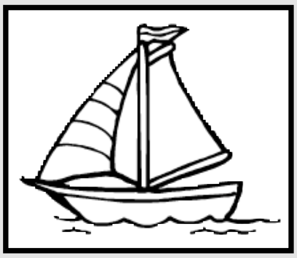
Calculating Speed

Imagine you sailed 45 miles between two islands and it took 3 hours.

can you see that :- in 3 hours, you travellled 45 miles ?

=> in 1 hour, you travelled $45 \div 3 = 15$ miles

=> this means your speed was 15 miles per hour.

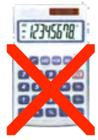


in other words :- **Speed = Distance ÷ Time**

or, using letters :-

$$S_{peed} = \frac{D_{istance}}{T_{ime}}$$

Exercise 3



1. Use the formula $S = \frac{D}{T}$ to find the average speed of these journeys :-

   (a) 20 miles in 4 hours. (b) 45 km in 9 hours.

   (c) 220 miles in 5 hours. (d) 150 km in 2 hours.

   (e) 168 miles in 3 hours. (f) 210 km in 6 hours.

2. Calculate the average speed for each of these journeys (watch the units) :-

   (a) 50 km in 2 hours. (b) 350 miles in 5 hours.

   (c) 300 metres in 10 seconds. (d) 26 km in 4 hours.

   (e) 64 000 kilometres in 8 hours. (f) 1500 metres in 30 seconds.

   (g) 75 feet in 2 seconds. (h) 560 yards in 8 minutes.

3. Calculate the average speed of the following :-

   (a) A plane flies 1380 miles in 6 hours.

   (b) A coach covers 420 kilometres in 7 hours.

   (c) A train travels 40 miles in $\frac{1}{2}$ hour.

   (d) A marathon runner covers 24 miles in 3 hours.

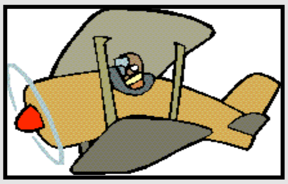
   (e) A snail travels 195 cm in 3 hours.

   (f) A 38 cm icicle melts away in 4 hours.

   (g) A van travels 378 miles in 9 hours.

   (h) A bus travels 549 miles in 9 hours.

Calculating Time



Imagine you flew 800 miles to Paris and the average speed of the plane was 200 miles/hour.

Can you see that :- to travel 200 miles takes 1 hour

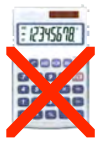
=> to travel 800 miles takes 800 ÷ 200 = 4 hours

In other words :- **Time = Distance ÷ Speed**

or, using letters :-

$$T_{ime} = \frac{D_{istance}}{S_{peed}}$$

Exercise 4

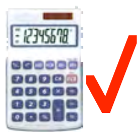


1. Use the formula $\mathbf{T} = \frac{\mathbf{D}}{\mathbf{S}}$ to calculate the time taken for each of these :-
   - (a) driving, 40 km at 40 km/hr.
   - (b) on a train, 360 miles at 60 m.p.h.
   - (c) racing, 1800 m at 20 m/sec.
   - (d) cycling, 180 km at 30 km/hr.
   - (e) swimming, 180 m at 3 m/sec.
   - (f) sliding, 45 metres at 15 m/sec.
   - (g) flying at 250 km/hr for 1000 km.
   - (h) sailing at 15 m.p.h. for 75 miles.

2. Change these times into hours and minutes :-
   - (a) $2\frac{1}{2}$ hours
   - (b) $5\frac{1}{4}$ hours
   - (c) $3\frac{3}{4}$ hours
   - (d) 6·5 hours
   - (e) 8·25 hours
   - (f) 3·5 hours
   - (g) 1·75 hours
   - (h) 0·25 hours.

3. 1 hour 30 minutes is $1\frac{1}{2}$ or 1·5 hours; 4 hour 15 minutes is $4\frac{1}{4}$ or 4·25 hours.

   Change the following times to both fractions of an hour and decimal form :-
   - (a) 2 hours 30 minutes.
   - (b) 4 hours 15 minutes.
   - (c) 3 hours 45 minutes.
   - (d) 2 hour 15 minutes.
   - (e) 5 hours 30 minutes.
   - (f) 8 hours 45 minutes.



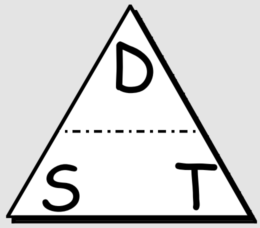
4. Use the formula $\mathbf{T} = \frac{\mathbf{D}}{\mathbf{S}}$ to calculate the time (*give answers in* ***hrs and mins***).
   - (a) driving, 90 km at 60 km/hr.
   - (b) sailing, 25 miles at 20 m.p.h.
   - (c) flying, 350 km at 200 km/hr.
   - (d) running, 6 km at 12 km/hr.
   - (e) train journey, 180 km at 80 km/hr.
   - (f) driving, 55 miles at 44 m.p.h.
   - (g) missile fired at 1200 km/hr for 300 km.
   - (h) flying at 240 m.p.h. for 660 miles.

Time, Distance, Speed Problems

In the previous 3 exercises, you learned how to use three formulae to calculate the speed, the distance or the time for a journey.

The triangle opposite shows a simple way of remembering how to use each of the three formulae. Try to memorise its shape.

$D = S \times T$

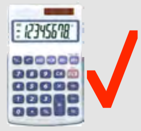


$S = \frac{D}{T}$ $T = \frac{D}{S}$

Example. David drove from his house to the coast, a distance of 135 miles.
It took him 2 hrs 15 mins to do so.
Calculate David's average **speed**.

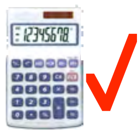
From the triangle, we can see that $S = \frac{D}{T}$

=> $\boxed{S = \frac{135}{2\text{hr } 15\text{min}} = \frac{135}{2{\cdot}25} = 60 \text{ m.p.h.}}$



Exercise 5

(Remember - time must always be entered into a calculator as a **decimal**)



1. (a)

Distance	**Speed**	**Time**
360 km	?	8 hours

(b)

Distance	**Speed**	**Time**
120 miles	30 m.p.h.	?

(c)

Distance	**Speed**	**Time**
?	37 m.p.h.	3 hrs

(d)

Distance	**Speed**	**Time**
140 km	?	$3\frac{1}{2}$ hours

(e)

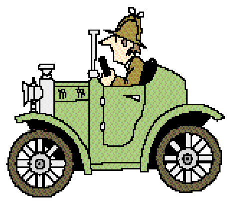
Distance	**Speed**	**Time**
?	20 m/sec	$3\frac{1}{2}$ secs

(f)

Distance	**Speed**	**Time**
225 miles	100 m.p.h.	?

2. Dougal drove for half an hour and covered a distance of 18 km.

   What was the Dougal's average speed ?



3. 

   A helicopter flew 75 km at an average speed of 60 km per hour.

   For how long was the helicopter flying ?

4. When the McPherson's towed their caravan on holiday, they maintained an average speed of 38 km/hr. The trip took $3\frac{1}{2}$ hours.

   How far was it from home to their holiday resort ?

5. A GNER train left Edinburgh Waverly at 0915 and arrived at its destination at 1145.

   If the train travelled 175 miles, what was the its average speed ?

6. 

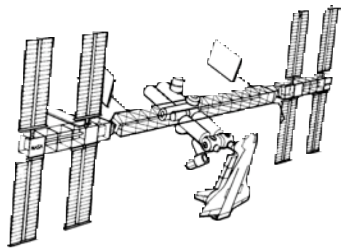
   A hill walker is crossing the valley at an average speed of 8 km/hr.

   How long will it take him to walk the whole length of the valley which is 14 km long ?

7. A space station goes round the moon at an average speed of 3200 km/hr.

   It takes $3\frac{1}{2}$ hours to complete its orbit.

   What is the length of the space station's orbit ?



8. 

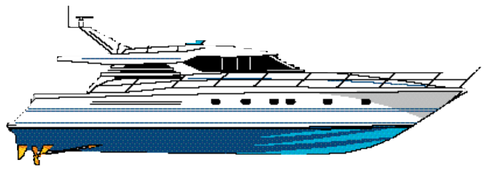
   It took old Mrs Hubbard 30 minutes to walk the $1\frac{1}{2}$ miles to the post office to collect her pension.
   Now, with the aid of her electric chair, she can do it in 15 mins.

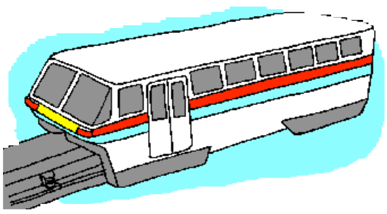
   (a) Calculate Mrs Hubbard's speed when she walked.

   (b) How much **faster** does she travel in the chair ?

9. The Halliday's took $4\frac{1}{2}$ days to sail round the islands in their cruiser.

   If they covered an average of 80 miles per day, what was the total distance they covered on their trip ?



10. 

    The monorail in Sydney travels at a speed of 250 metres per minute around its circular route.

    How long does it take to cover its route if the circuit is 2250 metres long ?

11. Of the three drivers below, who was travelling fastest ?

    - David, who covered 12 miles in 15 minutes .
    - Andy, who covered 9 miles in 10 minutes .
    - Brian, who covered 17 miles in 20 minutes .

12. A rally driver covered the first stage (105 km) in 1 hour 30 minutes, the second stage (100 km) in 1 hr 15 mins and the final stage (75 km) in three quarters of an hour.

    (a) Calculate his average speed for each of the 3 stages.

    (b) Calculate his average speed for the whole race.

Converting Hrs Mins => Decimal Times

In the last exercise you learned :- $\frac{1}{2}$ hour = 0·5 hr, $\frac{1}{4}$ hour = 0·25 hr and $\frac{3}{4}$ hour = 0·75 hr.

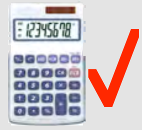
How would we enter **48 minutes** into our calculator as a decimal ?

Minutes => Decimals =>

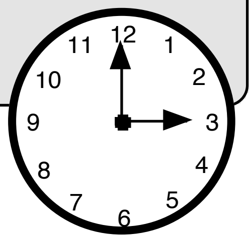
48 minutes is $\frac{48}{60}$ of an hour = 48 ÷ 60 = **0·8 hr**.

21 minutes is $\frac{21}{60}$ of an hour = 21 ÷ 60 = **0·35 hr**.

2 hr 54 mins is 2 + $\frac{54}{60}$ = 2 + (54 ÷ 60) = **2·9 hr**

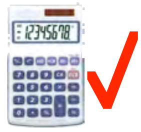


Simple rule :- "To change minutes to a decimal => divide by 60".



Exercise 6

1. You may use a calculator to change the following to decimals :-



(a) 36 minutes = $\frac{36}{60}$ hour (= 36 ÷ 60) = ... hour

(b)	24 minutes	(c)	12 minutes	(d)	42 minutes	(e)	18 minutes
(f)	54 minutes	(g)	15 minutes	(h)	9 minutes	(i)	33 minutes

2. Use your calculator to change these times to decimals giving your final answers correct to 2 decimal places :-

(a)	50 minutes	(b)	13 minutes	(c)	20 minutes	(d)	58 minutes
(e)	40 minutes	(f)	8 minutes	(g)	70 minutes	(h)	100 minutes

3. Use your calculator to change the following times to decimals :-

(a) 4 hours 12 minutes = 4 + $\frac{12}{60}$ = 4 + (12 ÷ 60) = ... hours

(b)	2 hr 36 mins	(c)	1 hrs 24 mins	(d)	3 hrs 33 mins	(e)	6 hrs 51 mins
(f)	3 hr 18 mins	(g)	5 hrs 21 mins	(h)	4 hrs 20 mins	(i)	8 hrs 3 mins

4 A plane flies at 240 km/hr for 36 minutes.
How far does it fly in that time ?

Show your working like this :-

D = S × T => D = 240 × ($\frac{36}{60}$) (not 60 × 0·24)

=> D = 240 × 0·6 (calculator)

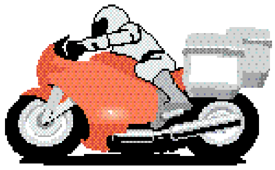
D = ... km

5. A ship sails at an average speed of 36 mph. How far will it have covered in :-

(a) 24 minutes (= 36 × $\frac{24}{60}$) ? (b) 15 minutes ?

(c)	40 minutes ?	(d)	18 minutes ?	(e)	9 minutes ?	(f)	54 minutes ?

6. Calculate the distance travelled each time here :-



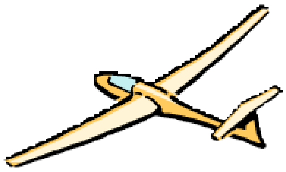
(a) A ship sailing at 25 km/hr for 36 minutes.

(b) A motor cyclist speeding at 80 miles/hr for 18 minutes.

(c) A lorry crawling along at 20 km/hr for 21 minutes.

(d) A jet plane flying at 330 mph for 10 minutes.

(e) A glider flying at 24 mph for 15 minutes.



7. George and Aileen set off at the same time :-

George drives at 64 km/hr for 45 minutes.

Aileen drives at 80 km/hr for 33 minutes.

Who travels the further, George or Aileen, and by how much ?

8. A ship sails at 24 km/hr for 2 hours 48 minutes.

How far will it have sailed ?

Show your working like this :-

$$D = S \times T \quad = 24 \times (2 + \tfrac{48}{60})$$
$$= 24 \times (2{\cdot}8)$$
$$= \ldots \text{ km}$$

calculator

(not 2·48)

9. For each of the following, calculate the distance travelled :-

(a) A jumbo jet flies at 480 m.p.h. for 3 hours 36 minutes.

(b) A van is driven at 50 mph for 1 hour 12 minutes.

(c) A marathon runner runs at 12 km/hr for 3 hours 20 minutes.

(d) A train runs at 130 km/hr for 2 hours 54 minutes.

10. A coach travels a distance of 48 kilometres in 36 minutes. Calculate its speed in km/hr.

Show your working like this :-

$$S = \frac{D}{T} \quad = 48 \div (36 \text{ mins})$$
$$= 48 \div (\tfrac{36}{60})$$
$$= 48 \div 0{\cdot}6$$
$$= \ldots \text{ km/hr}$$

(not 48 ÷ (0·36))

(calculator)

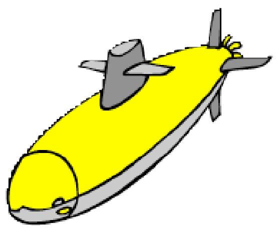
11. Find the average speed each time here :-

(a) A plane flies 175 miles in 42 minutes.

(b) A fire engine travels 8 kilometres in 6 minutes.

(c) A missile covers 480 miles in 45 minutes.

(d) A helicopter travels 72 miles in 36 minutes.



(e) A submarine covers 35 km in 1 hour 24 minutes.

(f) A truck driver travels 162 miles in 2 hours 42 minutes.

(g) A space ship flies 24 600 miles in 4 hours 6 minutes.

(h) A train travels 160 miles round the coast in 3 hours 20 minutes.

Converting Decimal Times => Hrs and Mins

In the last exercise you learned a simple rule for changing hrs & mins to decimal form.

Simple rule 1 :- "To change minutes to a decimal fraction => **divide by 60**".

If you have been using a calculator to find the time taken for a journey, it might appear as a decimal, like 0·65 hrs. There is an easy way of changing this to minutes.

Simple rule 2 :- "To change decimals back to a minutes => **multiply by 60**".

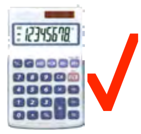
Decimals => Minutes =>

0·7 hr. = (0·7 x 60) mins = **42 minutes.**

0·15 hr. = (0·15 x 60) mins = **9 minutes.**

3·4 hr. = 3 + (0·4 x 60) mins = **3 hr 24 mins** .

Exercise 7



1. Change the following calculator display times (decimals) to minutes :-

(a) 0·65 On/Off (b) 0·4 On/Off (c) 0·3 On/Off

(0·65 x 60 = ? minutes)

(d) 0·25 On/Off (e) 0·95 On/Off (f) 0·666666666 On/Off

2. If you wish to change 4·7 hours into hours and minutes :-

— Leave the hours as they are (4 hours)

— Multiply the 0·7 by 60 => .... minutes

4 hours ... minutes.

3. Use the same technique to change the following times to hours and minutes :-

(a) 2·3 hours = 2 hour + (0·3 x 60) minutes = 2 hour ... minutes.

(b) 4·5 hours (c) 2·65 hours (d) 3·6 hours (e) 1·85 hours

(f) 6·9 hours (g) 2·66666.. hours (h) 5·8333333 hours (i) 0·125 hours.

4. Write the following calculator (decimal) times in hours and minutes :-

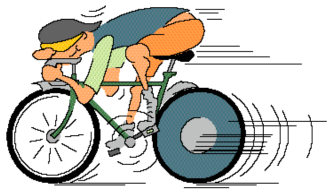
(a) 3·75 On/Off (b) 2·4 On/Off (c) 1·666666666 On/Off

5. An ocean liner covers 96 kilometres at 30 km/hr.

(a) Calculate the time taken in hours. ($T = \frac{D}{S}$) (*give answer as a decimal*).

(b) Change your answer to hours and minutes.

6. A cyclist travelled 43·2 kilometres at an average speed of 18 km/hr.



(a) Calculate how long he took. (*as a decimal*).

(b) How long did he take in hours and minutes.

7. Calculate the time taken (as a decimal) for each of the following, and then give your answer in hours and minutes :-

(a) A battleship sails 462 miles at an average speed of 140 mph.

(b) A hot air balloon flies 12 kilometres at an average speed of 36 km/hr.

(c) A police car, during a chase, travels 21 miles at an average speed of 60 m.p.h.

(d) A cross channel swimmer covers 22 miles at an average speed of 8 mph.

8. Shown is a map of a yacht race over 3 legs.

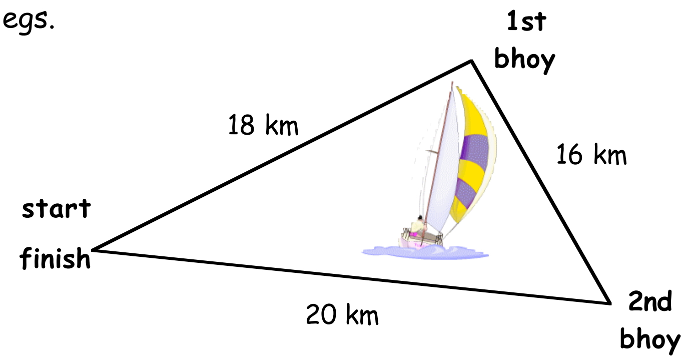
The yacht's average speed is 24 km/hr.

How long, in hours and minutes, should the yacht take to sail between the :-

(a) start and 1st bhoy ?

(b) 1st and 2nd bhoys ?

(c) start and finish ?



9. Brian covers a distance of 400 metres in 50 seconds.

(a) What is Brian's speed in metres per second ?

(b) Here is how to convert Brian's speed from metres per second to km/hr

- **step 1** change the speed to metres per minute, then metres per hour

  => 8 m/sec => 8 x 60 = 480 m/min => 480 x 60 = 28 800 m/hr

- **step 2** Change the metres to kilometres (÷ 1000)

  => 28 800 m/hr => 28 800 ÷ 1000 = ... km/hr

10. Change these speeds from metres per second to km/hr :-

(a) 10 m/sec (b) 20 m/sec (c) 300 m/sec (d) 12·5 m/sec

11. Which is faster :-

a cheetah running at 15 m/sec,

or a car travelling at 55 km/hr,

and by how much ?

Time - Distance (Speed) Graphs

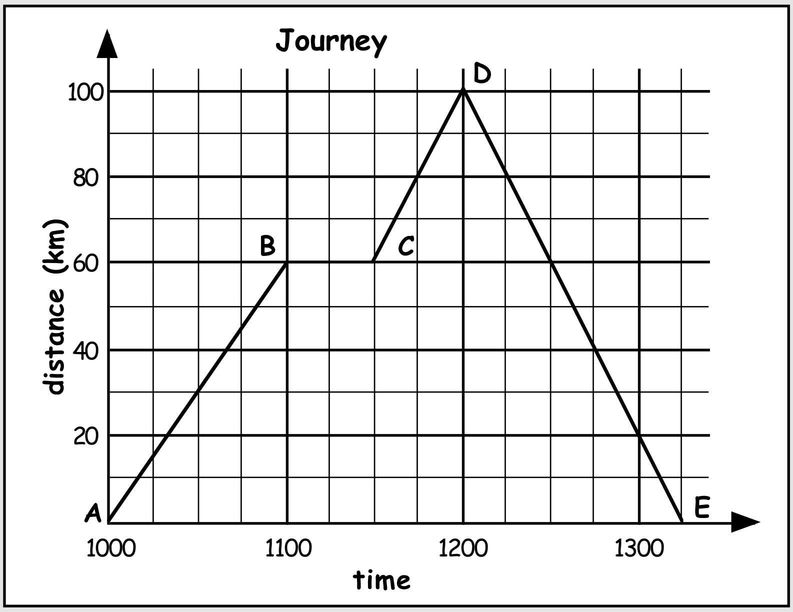
The graph opposite shows an outward journey from A -> B -> C -> D, then a return home from D -> E.

We can answer questions about the journey from the graph, including finding the speed at various stages.

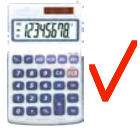
Can you see that after an hour, a stop of half an hour was made at B ?

Can you also see that since the line CD is **steeper** than the line AB, the speed was **greater** for that part.

We can calculate the **SPEED** at various stages as follows :-



A to B :-	B to C :-	C to D :-	D to E :-
Time = 1 hr	Time = $\frac{1}{2}$ hr	Time = $\frac{1}{2}$ hr	Time = $1\frac{1}{4}$ hr
Dist = 60 km	Dist = 0 km	Dist = 40 km	Dist = 100 km
$S = \frac{D}{T} = \frac{60}{1}$	$S = \frac{D}{T} = \frac{0}{0{\cdot}5}$	$S = \frac{D}{T} = \frac{40}{0{\cdot}5}$	$S = \frac{D}{T} = \frac{100}{1{\cdot}25}$
= 60 km/hr	= 0 km/hr	= 80 km/hr	= 80 km/hr



Exercise 8

1. This time-distance graph shows Lucy's journey from her home in Bower to a meeting in Crebar.

   She set out at 0800 along the motorway and stopped to do some shopping, before finishing the rest of her journey along the A17 trunk route.

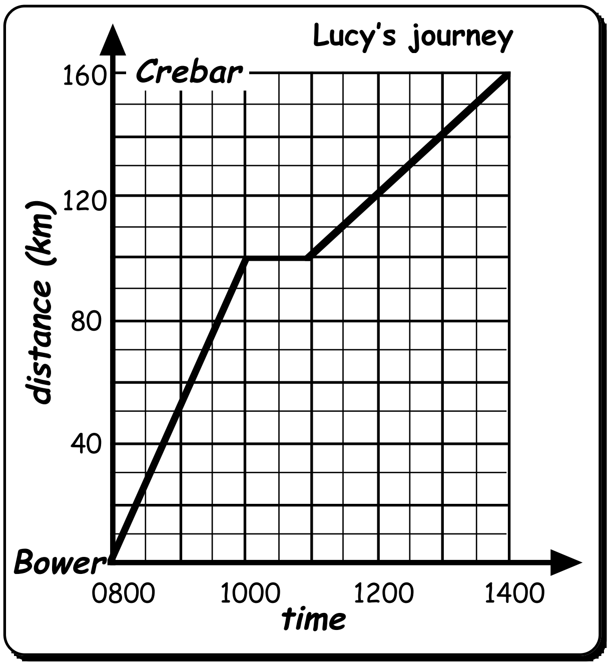
   (a) How long was the first part of her journey along the motorway ?

   (b) How long did her shopping take ?

   (c) When did she arrive in Crebar ?

   (d) Calculate Lucy's speed :-

   (i) on the motorway.
   (ii) between 1000 and 1100.
   (iii) along the A17.



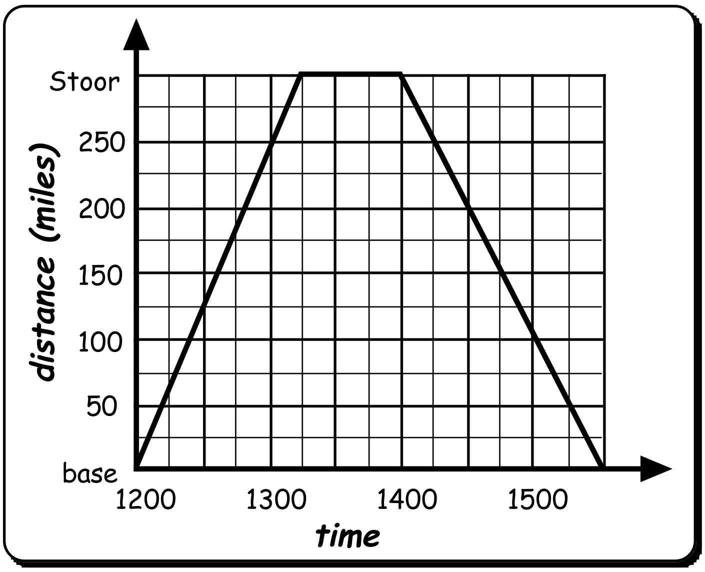
2. Biggles flew his Cessna light plane from his base to Stoor airfield, picked up 2 passengers and flew back to his base.

(a) For how long was Biggles on the ground at Stoor airfield ?

(b) Calculate his speed for the outward flight to Stoor.

(c) He hit a "head wind" on the way back. Calculate his return speed.

(d) From your answers to (b) and (c), say whether the "head wind" slowed him down or helped him go faster.



3. Louie set off in his Renault from Brie to Lyon at 8 am along the French country roads.

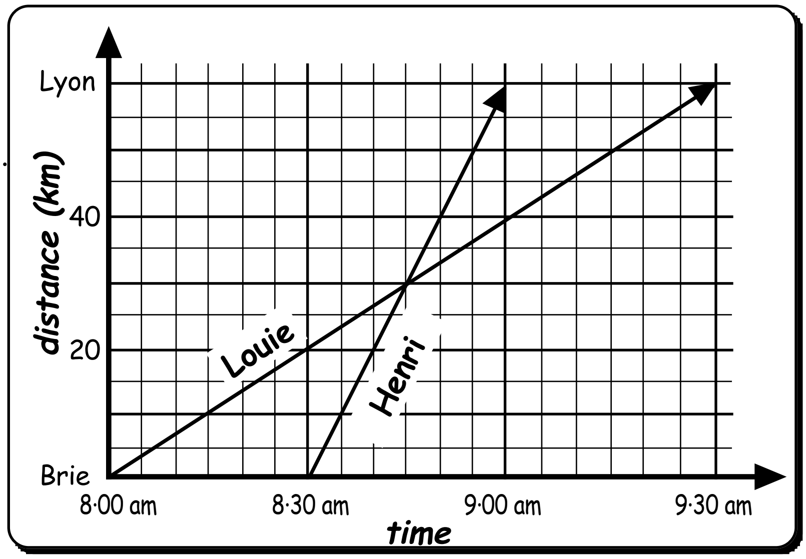
Henri caught the 8.30 train instead.

(a) Calculate Louie's speed.

(b) Calculate Henri's speed.

(c) When did Henri's train overtake Louie in his car ?

(d) How far away from **Lyon** were they when Henri overtook Louie ?



4. As part of a naval exercise, two ships set sail, a destroyer and a battleship, one from Cove and the other from Prava.

The destroyer is the **faster**.

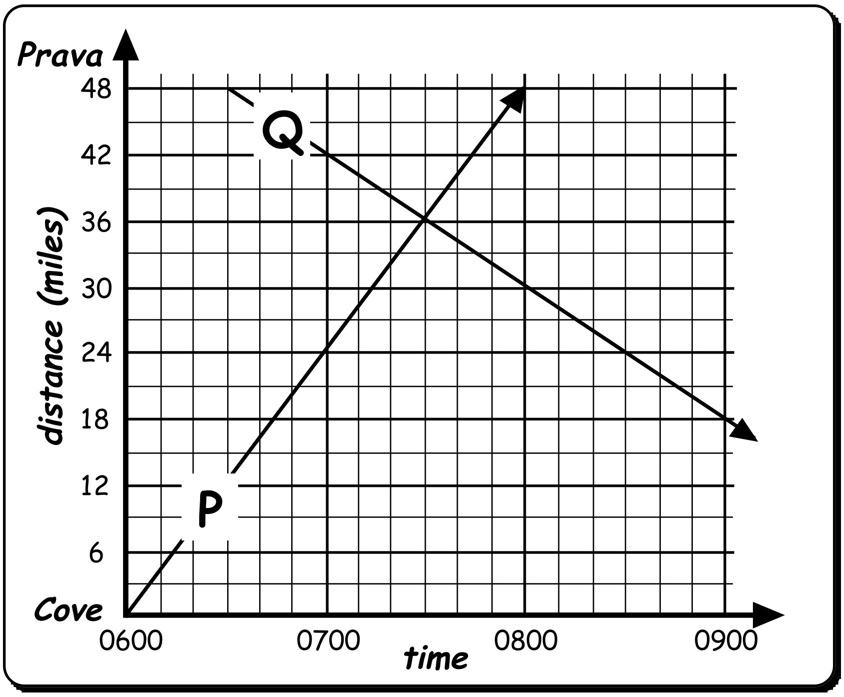
(a) Which line, P or Q, represents the destroyer's journey ? (*explain why*)

(b) Calculate the :-

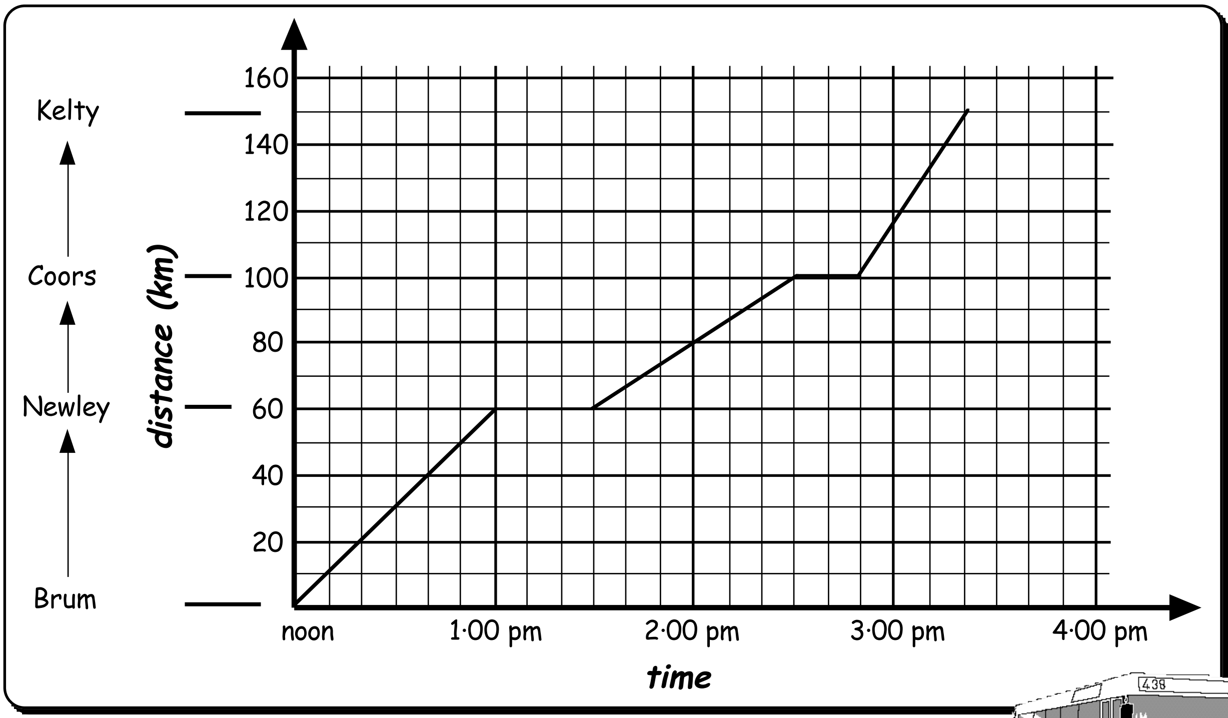
(i) destroyer's speed.
(ii) battleship's speed.

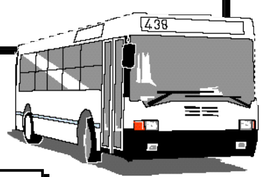
(c) At what time did the two ships pass ?

(d) At what time should the battleship reach Cove ?



5. Billy drove a coach load of passengers on a half day trip around the coast.





(a) Make a neat copy of this timetable and complete it for Billy's trip.

Brum	**Newley**		**Coors**		**Kelty**
depart	*arrive*	*leave*	*arrive*	*leave*	*arrive*
noon —>	?	?	?	?	?

(b) How many miles is it from :– (i) Brum to Newley ? (ii) Coors to Kelty ?

(c) Calculate the average speed Billy was driving at :–

(i) from Brum to Newley (ii) from Newley to Coors

(iii) from Coors to Kelty (iv) from Brum to Kelty

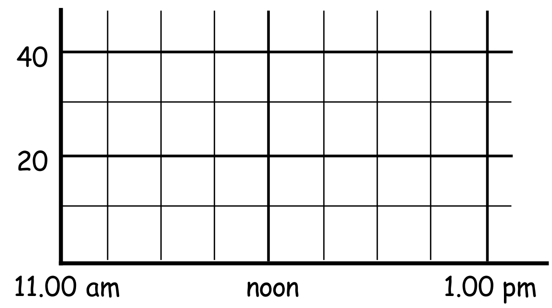
6. Davie set off driving at 11.00 am from Harcourt to Drumpton, 150 miles away.

He drove at an average speed of 40 miles per hour for the first 60 miles.
Davie then got a puncture and it took him 15 minutes to change the wheel.
He then set of again and reached Drumpton at 2.15 pm.

(a) For how long was he driving before he got the puncture ?

(b) What was his average speed after he repaired the puncture ?

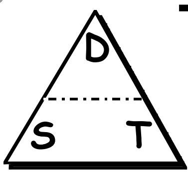
(c) Draw a graph showing all the stages of Davie's journey.

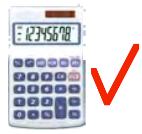


Time / Dist / Speed

Topic in a Nutshell

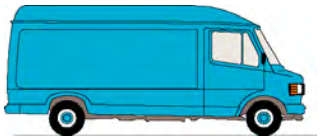
1. From the triangle shown opposite, use the most appropriate formula to answer the following :-





(a) Ron cycled at an average speed of 20 kilometres per hour for 3 hours.

How far did Ron cycle ?

(b) 

Donnie drove his van for 5 hours and travelled a distance of 200 miles.

What was Donnie's average speed ?

(c) Frank piloted a jet at a speed of 300 km per hour and travelled 450 kilometres.

What was Frank's flight time ?

2. Use a calculator to change the following times to decimals :-

(a) 12 minutes (b) 3 minutes (c) 2 hrs 48 mins (d) 3 hrs 54 mins.

3. Use a calculator to change the following to hours and minutes :-

(a) 0·6 hour (b) 0·15 hour (c) 3·7 hours (d) 1·45 hours.

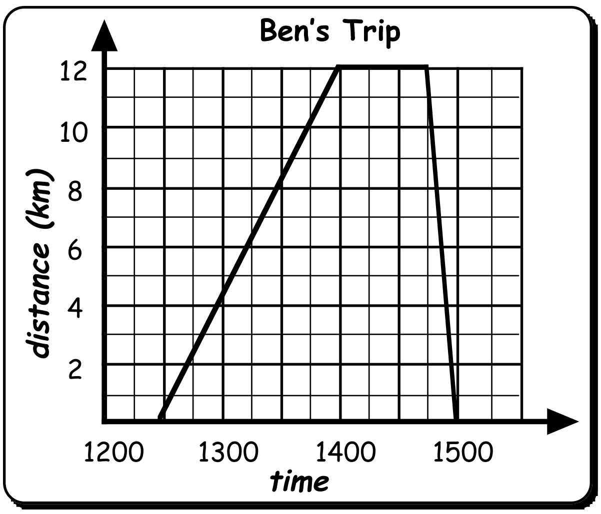
4. (a) Gina took 24 minutes each day to walk to her office 4 kilometres away.

What was Gina's average speed ?

(b) Jay drove for 120 kilometres at a speed of 42 km/hr.
Faye drove for 140 kilometres at a speed of 52 km/hr.

Which journey took longer, and by how much ?

5. Ben left his home and jogged to Sarah's house.
Later, Sarah's dad ran Ben home.



(a) At what time did Ben leave his house ?

(b) How far is Sarah's house from Ben's ?

(c) What was Ben's average speed ?

(d) How long did Ben stay at Sarah's ?

(e) How long did it take Ben to get home ?

(f) What was the average speed on the journey home ?

(g) Next day Ben cycled at one and a half times his jogging speed.

How long did it take him to cycle to Sarah's house ?

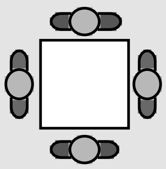
Chapter 53

Linear Patterns

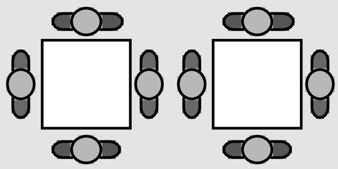
Simple Linear Patterns

Sometimes it is easier to see a NUMBER PATTERN from a diagram or a table.

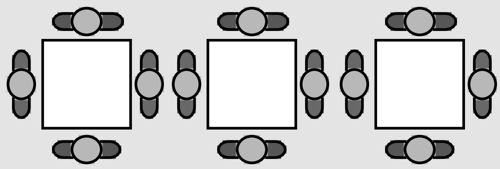
Example In a cafe, 4 people sit around each table.



1 table
4 people



2 tables
8 people



3 tables
12 people

Drawing up a table helps see the pattern : -

No. of tables (T)	1	2	3	4	5	6
No. of people (P)	4	8	12	16	?	?

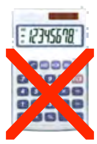
4 4 4

For every **extra** table, the number of people rises by 4.

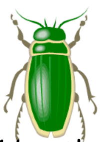
In words : - number of people = 4 times the number of tables

In symbols : - P = 4 x T, written as $\boxed{P = 4T}$

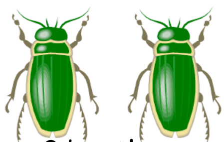
Exercise 1



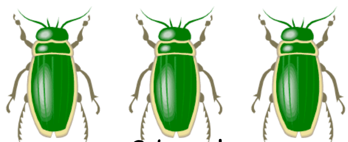
1. Look at the pattern of beetles and their legs.



1 beetle
6 legs



2 beetles
12 legs



3 beetles
18 legs

(a) Copy and complete the table :-

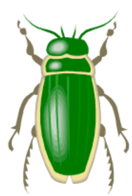
No. of beetles (B)	1	2	3	4	5	6
No. of legs (L)	6	12	18	?	?	?

6 6 6

(b) For every extra beetle, how many extra legs are there ?

(c) Copy and complete : - " number of legs = ... x number of beetles "

(d) Write down a formula using symbols to show this (L = ... x .....)



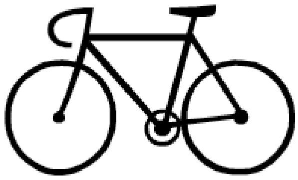
2. The cost of hiring a bike is £2 every hour.

(a) Copy and complete the table : -

No. of hours (H)	1	2	3	4	5	6
Cost in £'s (C)	2	4	6	?	?	?

? ? ?

(b) By how much does the cost rise for each extra hour ?



(c) Copy and complete : - " Cost = ... x number of hours "

(d) Write down a formula using symbols to show this (C = ... x .....)

(e) Use your formula to find the cost of hiring the bike for 12 hours.

3. A car has 5 tyres (4 on the road + 1 spare).

(a) Copy and complete the table : -

No. of cars (C)	1	2	3	4	5	6
No. of tyres (T)	5	10	15	?	?	?

? ? ?

(b) Copy and complete : - " number of tyres = ... x number of cars "

(c) Write down a formula using symbols to show this (T = ... x .....).

(d) Use your formula to find the number of tyres on 15 cars.

4. The cost of buying a silk tie was £12.

(a) Copy and complete the table : -

No. of ties (T)	1	2	3	4	5	6
Cost in £'s (C)	12	24	?	?	?	?

? ? ?

(b) Copy and complete : -

" Cost = ... x number of ties "

(c) Write down a formula using symbols to show this (C = ... x .....).

(d) Use your formula to find the cost of 20 ties.

5. Bricks are laid end to end and the table shows various lengths of different sections.

(a) What is the length of one brick (NOT 50 cm) ?

No. of bricks (B)	2	3	4	5	6	7
Length in cm (L)	50	75	100	?	?	?

? ? ?

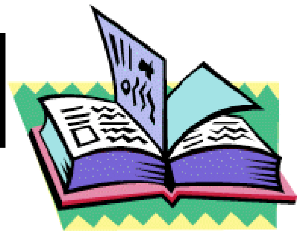
(b) Write a formula connecting the number of bricks and the total length (L = ... x .....)

(c) Use your formula to find the total length of 100 bricks.

6. The table indicates the cost of various numbers of a children's book : -

No. of books (B)	3	4	5	6	7	8
Cost in £'s (C)	3·75	5	6·25	?	?	?

? ? ?



(a) What is the cost of one book ?

(b) Write a formula connecting the number of pages and the number of books.

=> C = ...

(c) Use your formula to find the cost of 20 books.

7. Copy and complete each table and determine a formula or rule connecting the two letters :-

(a)

No. of Chairs (C)	1	2	3	4	5	6
No. of legs (L)	4	8	12	?	?	?

L = ... x ...

(b)

No. of cats (C)	1	2	3	4	5	6
No. of whiskers (W)	16	32	48	?	?	?

W = ...x...

(c)

No. of cakes (P)	3	4	5	6	7	8
Cost in pence (C)	18	24	30	36	?	?

C = ... x ...

(d)

No. of sweets (S)	2	4	6	8	10	12
Cost in pence (C)	18	36	54	?	?	?

C = ... x ... careful here *

8. For each of the following tables, determine a formula in the form y = ... x ....

(a)

x	0	1	2	3
y	0	2	4	....

(b)

x	0	1	2	3
y	0	3	6	....

(c)

x	1	2	3	4
y	5	10	15	....

(d)

x	1	2	3	4
y	0·5	1	1·5	....

(e)

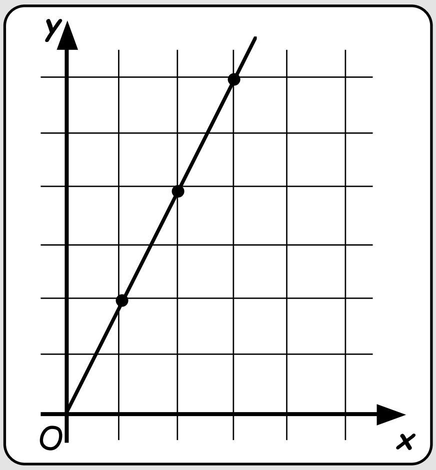
x	1	2	3	4
y	3·5	7	....	....

(f)

x	0	2	4	6
y	0	2	4	....

Look at question 8 (a).

If we take the table answers, write them as coordinates and plot them on a coordinate diagram, we get : -



x	0	1	2	3
y	0	2	4	6

(0, 0) (1, 2) (2, 4) (3, 6)

$y = 2x$

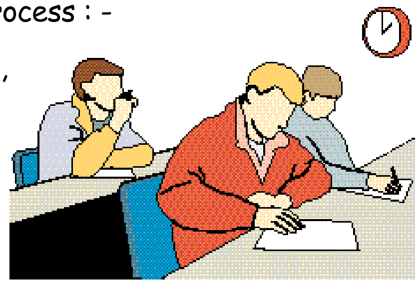
Can you see that the formula is $y = 2x$, and when the 4 points are plotted, a line can be drawn through them ?

The line can also be seen to pass through the **origin**.

9. (a) For the other 5 tables in question 8, repeat the above process : -

   (i) extract the coordinates from the table,
   (ii) plot them on a coordinate diagram,
   (iii) and join up the coordinates.

   (b) What do you notice about each of the graphs ?



10. Show for each of the following tables, that : -

   (i) a formula of the form $y = ax$ can be obtained (where a is a number)
   (ii) the coordinates lie on a line which passes through the origin.

(a)

x	0	1	2	3
y	0	6	12	18

(b)

x	0	1	2	3
y	0	8	16	24

(c)

x	1	2	3	4
y	10	20	30	....

(d)

x	3	5	7	9
y	3	5	7	....

(e)

x	2	4	6	8
y	4	8	12	....

(f)

x	0	2	4	6
y	0	1	2	....

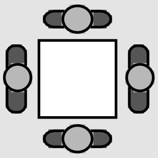
11. Compare the steepness of each line in question 9 and 10.

   Comment on the steepness of the lines $y = 1x$, $y = 2x$, $y = 3x$ etc...

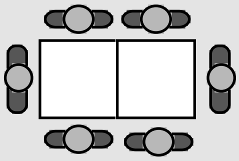
More complicated Linear Patterns

In all the patterns we have met so far, the numbers on the bottom line of the table were part of the 2 x, 3 x, 4 x, etc. tables and were easily recognisable.

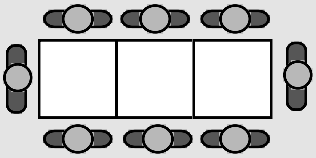
Look at this different type of pattern, showing people sitting around various numbers of tables in a restaurant.



1 table
4 customers



2 tables
6 customers



3 tables
8 customers

Drawing up a table helps to see the pattern :-

No. of tables (T)	1	2	3	4	5	6
No. of customers (*C*)	4	6	8	10	12	14

2 2 2

For every additional table the number of customers **rises by 2** !

step 1 => we can **begin** to write, in symbols :-

$C = 2 \times T$ - but this **doesn't work !**

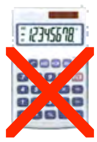
step 2 => we need a **correction number** to make the pattern work.

look at the (T=) <u>3</u> and (C=) <u>8</u> values - can you see that $2 \times \underline{3} \neq \underline{8}$

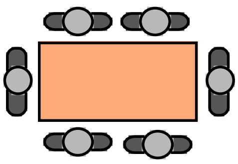
but $2 \times 3 + \mathbf{2}$ gives 8 (check that $2 \times 4 + \mathbf{2} = 10$, $2 \times 5 + \mathbf{2} = 12$)

=> so our real formula is $\boxed{C = 2 \times T + 2}$

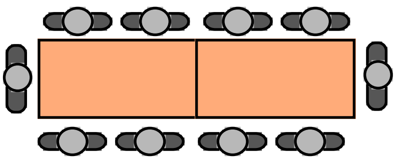
Exercise 2



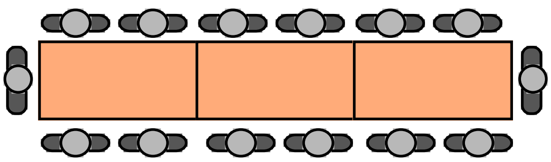
1. A cafe uses rectangular tables as shown : -



1 table
6 people



2 tables
10 people



3 tables
14 people

(a) Draw neatly the next table pattern with 4 rectangular tables.

(b) Copy and complete the following table : -

No. of tables (T)	1	2	3	4	5	6
No. of people (P)	6	10	14	?	?	?

4 4 ?

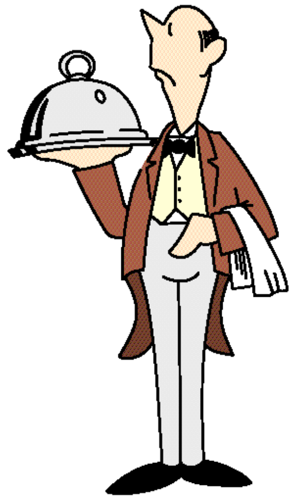
(c) For every extra table, how many extra people can be seated ?

(d) Write down the formula using **symbols** for calculating the number of people that can be seated if you know the number of tables :-

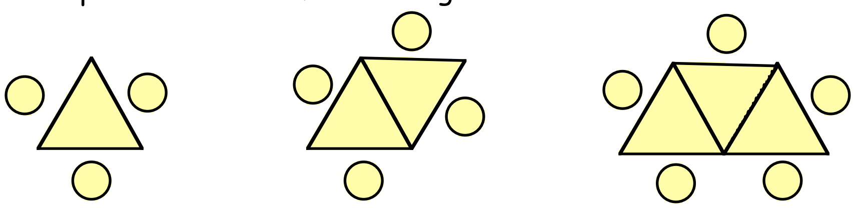
copy :- $P = ? \times T + ?$ remember the correction number

(e) Use your formula to decide how many people can be seated with 10 tables.

(f) How many people can be seated with 21 tables ?



2. This pattern is made from triangular tables.



1 table 3 people | 2 tables 4 people | 3 tables 5 people

(a) Draw neatly the next table pattern with 4 triangular tables.

(b) Copy the following table and complete it :-

No. of tables (T)	1	2	3	4	5	6
No. of people (P)	3	4	5	?	?	?

? ? ?

(c) For every extra table, how many extra people can be seated ?

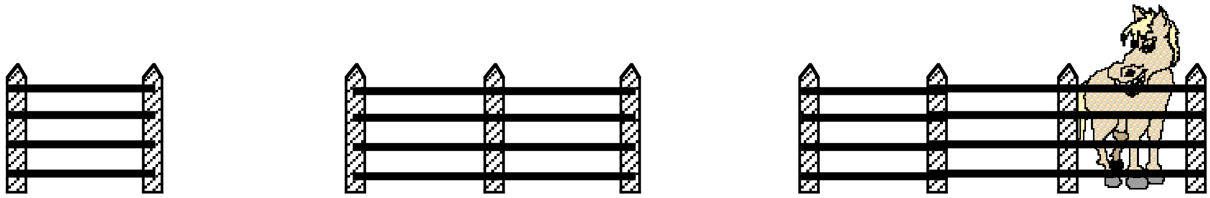
(d) Write down the formula using **symbols** for calculating the number of people that can be seated if you know the number of tables :-

copy :- $P = ? \times T + ?$ remember the correction number

(e) Use your formula to decide how many people can be seated with 9 tables.

(f) How many people can be seated with 40 tables ?

3. Four strips of wire are nailed between each fence post to make a farmers fence.



2 posts
4 strips

3 posts
8 strips

4 posts
12 strips

(a) Draw the next pattern of fence posts and wire strips.

(b) Copy and complete the following table : -

No. of posts (P)	2	3	4	5	6	7
No. of strips (S)	4	8	?	?	?	

? ? ?

This time the correction number has to be subtracted

(c) For every extra post, how many extra wire strips are needed ?

(d) Write down the formula using **symbols** $S = ? \times P - ?$

(e) Use your formula to decide how many wire strips are needed with 15 posts.

4. Copy and complete each table and determine a formula or rule connecting the two letters :-

(a)

No. of Days (D)	1	2	3	4	5	6
Hire cost (C)	12	17	22	?	?	?

C = ... x D + ...

(b)

Time in hours (T)	1	2	3	4	5	6
Depth in metres (D)	18	21	24	?	?	?

D = ... x ... + ...

(c)

Velocity (V)	3	4	5	6	7	8
Distance in metres (D)	1	5	9	13	?	?

D = ...

(d)

L	10	11	12	13	14	15
A	53	59	65	71	?	?

A = ...

(e)

x	2	4	6	8	10	12
y	4	12	20	?	?	?

y = ...

5. For each of the following tables, determine a formula in the form y = ... × .... ± .....

(a)

x	0	1	2	3
y	1	3	5	....

(b)

x	0	1	2	3
y	4	7	10	....

(c)

x	1	2	3	4
y	6	7	8	....

(d)

x	1	2	3	4
y	10	13	16	....

(e)

x	1	2	3	4
y	8	13	....	....

(f)

x	0	2	4	6
y	1	9	17	....

6. (a) Copy and complete : -
 (i) the table
 (ii) the formula
 (iii) the list of coordinates.

x	0	1	2	3
y	1	5	9	...
	(0, 1)	(1, 5)	(2, ...)	(3, ...)

$y = 4x + ...$

(b) Plot the points on a coordinate diagram.

(c) Join the points to show that $y = 4x + 1$ is a line crossing the y axis at 1.

7. Repeat question 6 for each of these tables : -

(a)

x	0	1	2	3
y	1	4	7	....

(b)

x	0	1	2	3
y	2	7	12	....

(c)

x	1	2	3	4
y	3	4	5	....

(d)

x	1	2	3	4
y	–1	1	3	....

(e)

x	1	2	3	4
y	2	6	10	....

(f)

x	0	2	4	6
y	1	7	13	....

8. (a) What do you notice about each formula and where each line cuts the y- axis ?

 (b) Comment about the steepness of each of the lines.

9. Copy and complete each sentence : -

 (a) " An equation of the form $y = ax$ makes a straight line through the o......... "

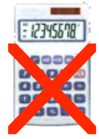
 (b) " An equation of the form $y = ax + b$ makes a straight line through the point (0, ...). "

Linear Patterns

Topic in a Nutshell

1.

1 DVD - £3   2 DVD's - £6   3 DVD's - £9



(a) Copy and complete the table : -

No. of DVD's (D)	1	2	3	4	5	6
Cost in £ (C)	3	6	9	?	?	?

(b) Find a formula connecting cost (C) and number of DVD's (D).

(c) Use your formula to find the cost of 20 DVD's.

2. (a) For each of the following tables, find a formula connecting the letters *y* and *x* : -

(i)

x	0	1	2	3
y	0	4	8	....

(ii)

x	2	4	6	8
y	6	12	18	....

(b) For each of the above tables, take the coordinates from each table, plot them on a coordinate diagram and join up the coordinates to form straight lines.

3. A steam-cleaner company asks for a £5 hire charge PLUS £2 per hour of hire.

(a) Copy and complete the table of hire charges and write down a formula connecting the letters *C* and T : -

Time in hours (T)	1	2	3	4	5	6
Cost in £'s (C)	7	9	11	?	?	?

C = ...

(b) Use your formula to find the cost of hiring a steam cleaner for 10 hours ?

4. For each of the following tables, determine a formula in the form $y = ...x \pm .....$

(a)

x	0	1	2	3
y	3	5	7	....

(b)

x	2	3	4	5
y	1	5	9	....

(c)

x	10	11	12	13
y	7	8	9	....

(d)

x	2	4	6	8
y	11	23	35	....

5. Look back at question 5(a). Use your table to write down the corresponding coordinates, plot them on a coordinate diagram and join up the points.

Chapter 59

Pythagoras

Squaring a Number

To "**SQUARE**" a number means to **multiply it by itself**.

For example "the square" of 9 is $9 \times 9 = 81$ (not 9×2)

This is written as "9 squared $= 9 \times 9 = 81$".

or even better $9^2 = 9 \times 9 = 81$.

(this is read as :- "9 squared").

9^2

Exercise 1

1. Do **NOT** use a calculator in this question. Copy and complete the following :-

(a) $7^2 = 7 \times 7 = ...$	(b) $3^2 = 3 \times 3 = ...$	(c) $4^2 = 4 \times ... = ...$
(d) $9^2 = ... \times ... = ...$	(e) 6^2	(f) 2^2
(g) 8^2	(h) 1^2	(i) 11^2
(j) 5^2	(k) 12^2	(l) $(\frac{1}{2})^2 = \frac{1}{2} \times \frac{1}{2} = ...$

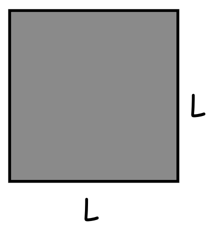
2. You **can** use a calculator this time. Find the values of :-

(a) 15^2	(b) 13^2	(c) 20^2	(d) 25^2
(e) 38^2	(f) 100^2	(g) 19^2	(h) 200^2
(i) 22^2	(j) 35^2	(k) 45^2	(l) 55^2

3. You can calculate the **AREA** of a SQUARE using the formula :-

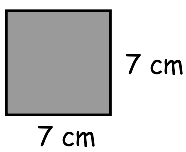
AREA = $(\text{length})^2$

or $\boxed{A = L^2}$



Use the formula to calculate the areas of the following **squares** :-

(a)

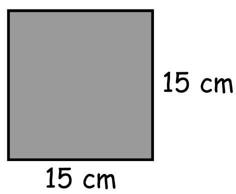


Area $= L^2$

$\Rightarrow A = 7^2$

$A = ...\ \text{cm}^2$

(b)

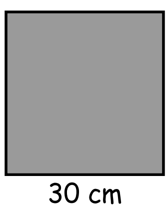


Area $= L^2$

$\Rightarrow A = 15^2$

$A = ...\ \text{cm}^2$

(c)

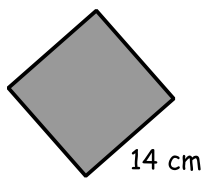


Area $= L^2$

$\Rightarrow A = ...^2$

$A = ?\ \text{cm}^2$

(d)



Area $= L^2$

$\Rightarrow A = ...^2$

$A = ?\ \text{cm}^2$

Squares Roots

You now know how to find $9^2 = 9 \times 9 = 81$

We can "undo" this by asking => "**which number, times itself, gives 81**"?

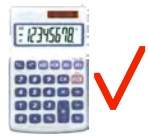
From the top line, you can see the answer is 9.

$\sqrt{81}$

This is expressed as :- "**the SQUARE ROOT of 81 is 9**".

or in symbol form :- $\sqrt{81} = 9$ (which reads as "the square root of 81 = 9").

Exercise 2



from Qu 3 onwards

1. No calculator in this question. Copy each line and complete :-

(a) since $4^2 = 16 \Rightarrow \sqrt{16} = 4$ (b) since $3^2 = 9 \Rightarrow \sqrt{9} = \ldots$

(c) since $7^2 = 49 \Rightarrow \sqrt{49} = \ldots$ (d) since $5^2 = 25 \Rightarrow \sqrt{25} = \ldots$

(e) since $10^2 = \ldots \Rightarrow \sqrt{100} = \ldots$ (f) since $6^2 = \ldots \Rightarrow \sqrt{36} = \ldots$

2. Find the following :-

(a) $\sqrt{64}$ (b) $\sqrt{4}$ (c) $\sqrt{1}$ (d) $\sqrt{121}$

3. In this question, you should use the "$\sqrt{\ }$" button on your calculator to find :-

(a) $\sqrt{144}$ (b) $\sqrt{625}$ (c) $\sqrt{169}$ (d) $\sqrt{225}$ (e) $\sqrt{324}$

(f) $\sqrt{1600}$ (g) $\sqrt{196}$ (h) $\sqrt{289}$ (i) $\sqrt{1 \cdot 69}$ (j) $\sqrt{4 \cdot 41}$

Some "square roots" are not exact :-

$\sqrt{43} = 6{\cdot}55743824 = 6{\cdot}56$ (to 2 decimal places).

4. Use your calculator to find the following **to two decimal places** :-

(a) $\sqrt{15}$ (b) $\sqrt{21}$ (c) $\sqrt{31}$ (d) $\sqrt{72}$ (e) $\sqrt{97}$

(f) $\sqrt{113}$ (g) $\sqrt{193}$ (h) $\sqrt{500}$ (i) $\sqrt{640}$ (j) $\sqrt{815}$

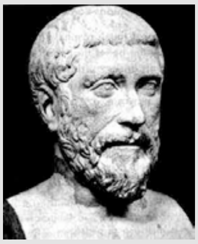
5. This square opposite has an **area** of 110 cm^2.
Calculate the length of one of its sides. ($\sqrt{110}$)

110 cm^2

6. 270 cm^2

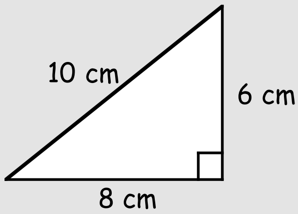
This square shown has an **area** of 270 cm^2.
Calculate the length of one of its sides.

Pythagoras Theorem



Pythagoras was a famous Greek Mathematician who discovered an amazing connection between the three sides of any **right angled triangle**. This relationship, which connects the 3 sides, means it is possible to **CALCULATE** the length of one side of a right angle triangle as long as you know the lengths of the other two.

Look at this right angled triangle with sides 6 cm, 8 cm and 10 cm.



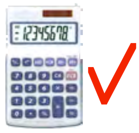
If you add the two smaller sides (6 cm and 8 cm) together, do you get the longer side (10 cm) ? – NO.

Can you see that $6^2 = 36$, $8^2 = 64$, and $10^2 = 100$?

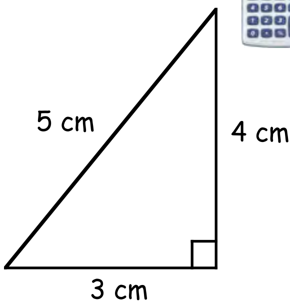
Can you also see that:- $6^2 + 8^2 = 36 + 64 = 100 = 10^2$?

Pythagoras found that this connection between the three sides of a right angled triangle was true **for every right angled triangle**.

Exercise 3 (confirmation)



1. The three sides of this right angled triangle are 3 cm, 4 cm and 5 cm.

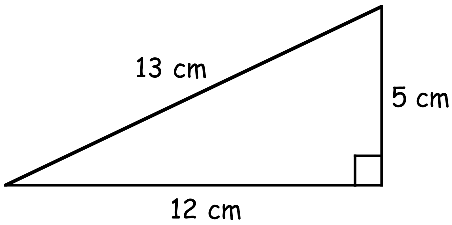


   (a) Write down the values of 3^2, 4^2 and 5^2.

   (b) Find the value of $3^2 + 4^2$.

   (c) Check that $3^2 + 4^2 = 5^2$.

2. The three sides of this right angled triangle are 5 cm, 12 cm and 13 cm.



   (a) Write down the values of 5^2, 12^2 and 13^2.

   (b) Find the value of $5^2 + 12^2$.

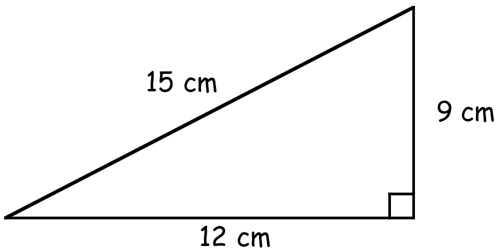
   (c) Check that $5^2 + 12^2 = 13^2$.

3. The three sides of this right angled triangle are 9 cm, 12 cm and 15 cm.

   (a) Write down the values of 9^2, 12^2 and 15^2.

   (b) Find the value of $9^2 + 12^2$.

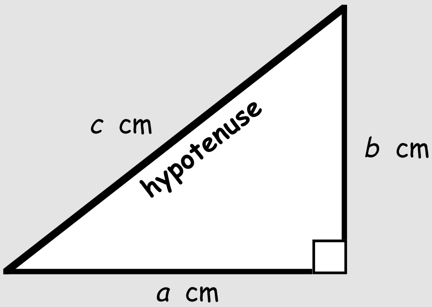
   (c) Check that $9^2 + 12^2 = 15^2$.



Pythagoras Theorem

Pythagoras came up with a simple rule which shows the connection between the three sides of any right angled triangle.

The **longest** side of a right angled triangle is called the **HYPOTENUSE**.



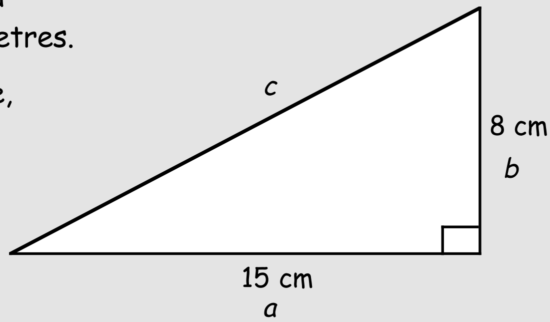
If the three sides are a cm, b cm and c cm (the hypotenuse), then Pythagoras' rule says :-

=> $\boxed{c^2 = a^2 + b^2}$

We can use this rule to calculate the length of the **hypotenuse** of a right angled triangle if we know the lengths of the two **smaller** sides.

Example 1 :- The two smaller sides of this right angled triangle are 8 centimetres and 15 centimetres.

To calculate the length of the hypotenuse, use **Pythagoras' Rule**.



=> $c^2 = a^2 + b^2$

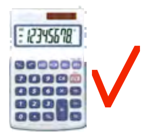
=> $c^2 = 15^2 + 8^2$

=> $c^2 = 225 + 64 = 289$

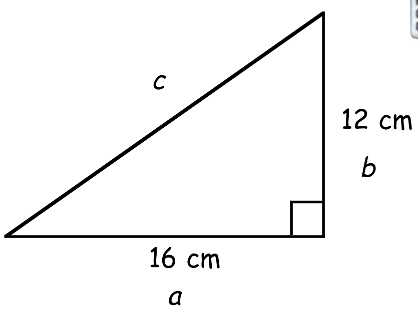
=> $c = \sqrt{289} = 17$ cm.

This is how you set down the working.

Exercise 4



1. Use **Pythagoras' Rule** to calculate the length of the hypotenuse in this triangle :-



=> $c^2 = a^2 + b^2$

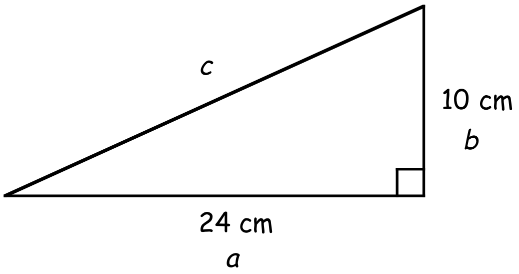
=> $c^2 = 12^2 + \ldots$

=> $c^2 = 144 + \ldots = \ldots$

=> $c = \sqrt{\ldots} = \ldots$ cm

Copy and complete the working.

2.

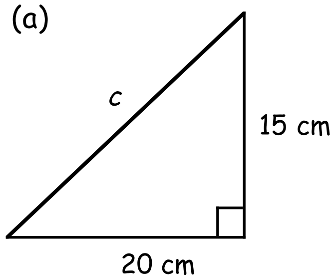


Use **Pythagoras' Rule** to calculate the length of the hypotenuse in the right angled triangle shown on the left.

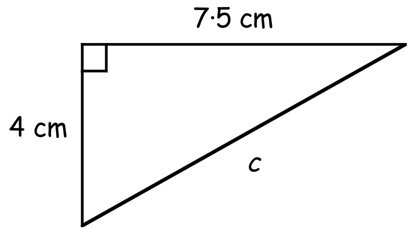
(*show clearly your 4 lines of working*)

3. Use Pythagoras' Rule (referred to as **PYTHAGORAS' THEOREM**) to calculate the length of the hypotenuse in each of these triangles :-

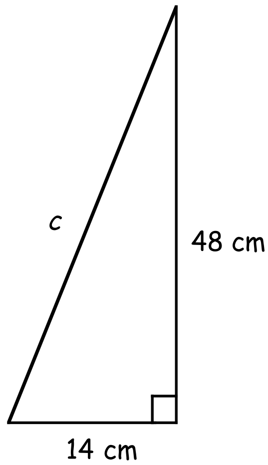
(a)



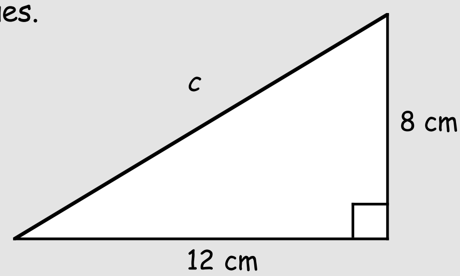
(b)



(c)



Example 2 :- In most cases, the 3 sides are not exact values.



=> $c^2 = a^2 + b^2$

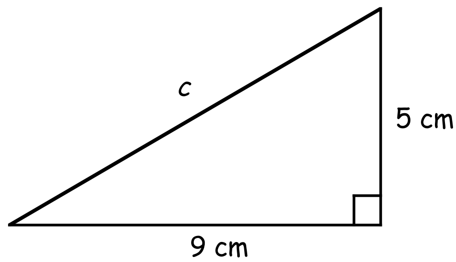
=> $c^2 = 8^2 + 12^2$

=> $c^2 = 64 + 144 = 208$

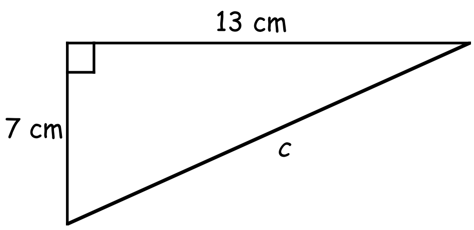
=> $c = \sqrt{208} = 14{\cdot}4222051...$

$= 14{\cdot}42$ cm (to 2 decimal places),

4. Use **Pythagoras' Theorem** to calculate the length of the hypotenuse in this triangle correct to 2 decimal places.

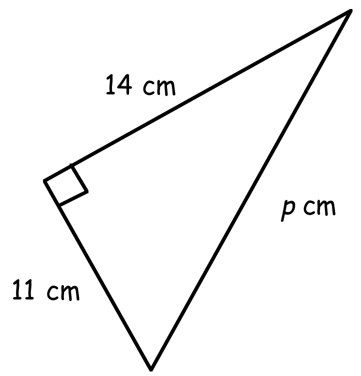


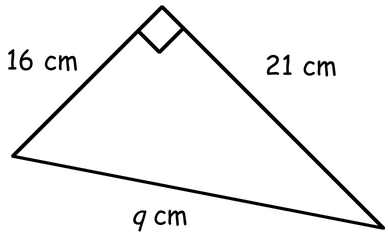
5.



Use **Pythagoras' Theorem** to calculate the length of the hypotenuse in the right angled triangle shown (2 decimal places).

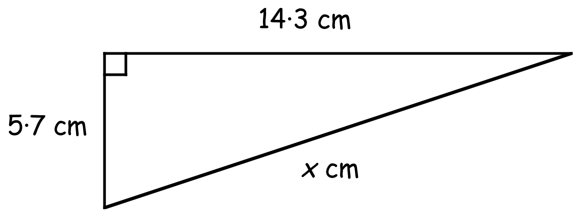
6. Calculate the length of the hypotenuse marked p cm (to 2 decimal places).



7. 

Calculate the length of the line marked q cm, to 2 decimal places.

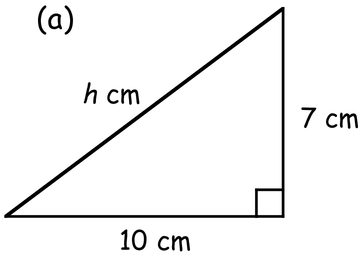
8. Calculate the length of the hypotenuse in this right angled triangle, correct to 2 decimal places.



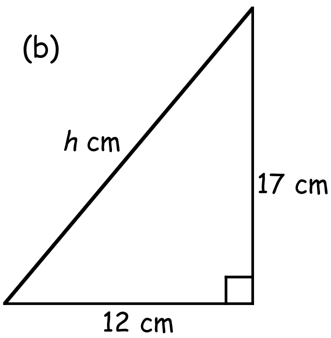
9. Sketch each of the following right angled triangles :-

Use **Pythagoras' Theorem** to calculate the length of the hypotenuse in each case, correct to two decimal places.

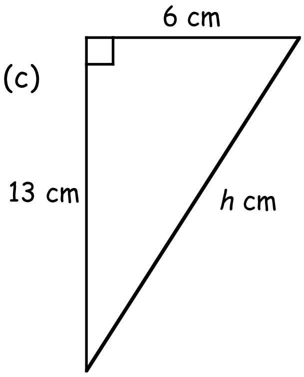
(a)



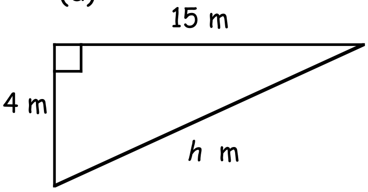
(b)



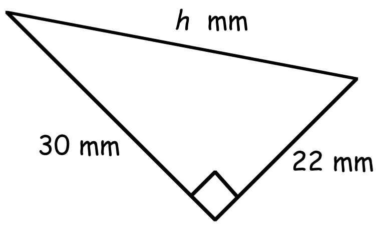
(c)



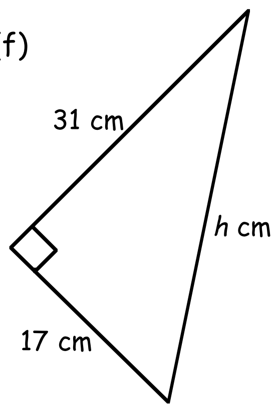
(d)



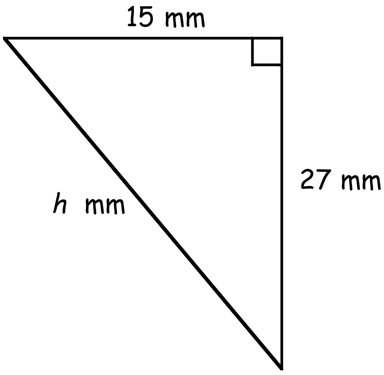
(e)



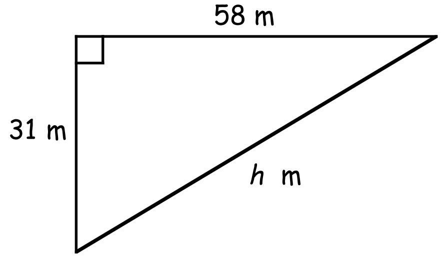
(f)



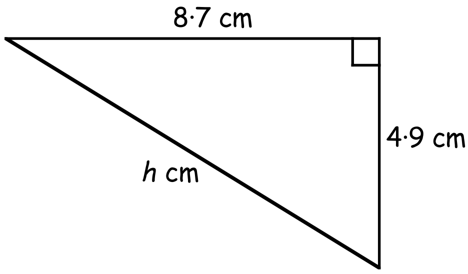
(g)



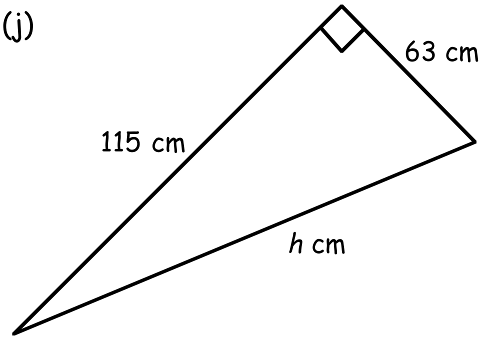
(h)



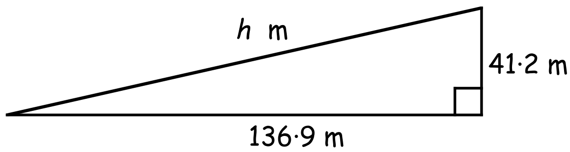
(i)



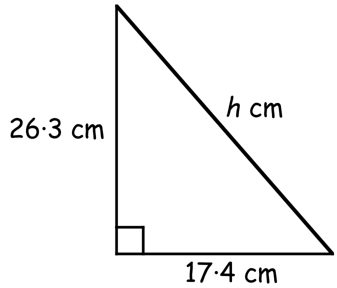
(j)



(k)

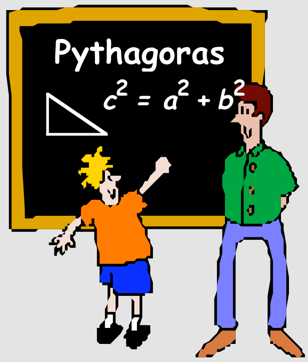


(l)

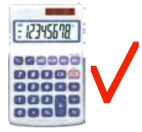


Problems involving Pythagoras Theorem

Whenever you come across a problem involving finding a missing side in a right angled triangle, you should consider using Pythagoras' Rule to calculate its length.



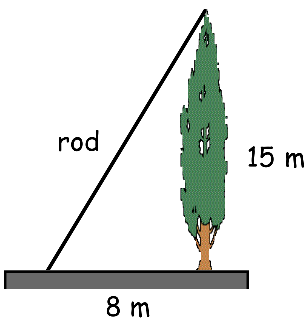
Exercise 5

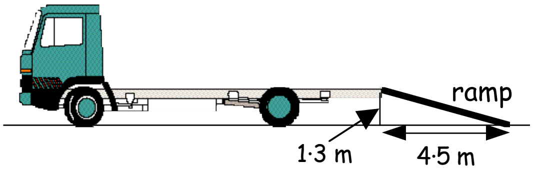


(The triangles in questions 1 to 7 are right-angled)

1. A steel rod is used to support a tree in danger of falling down.

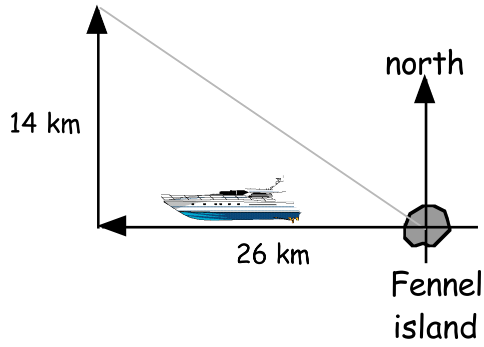
   Calculate the length of the rod.

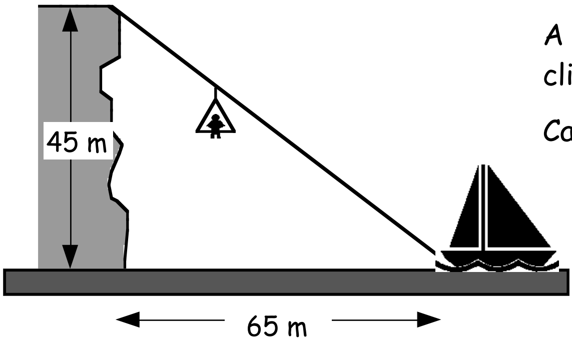


2. 

A ramp is used to help run wheelbarrows onto the back of a lorry. Calculate the length of the ramp.

3. A ship left Fennel Island. The captain sailed 26 kilometres West. He then sailed 14 kilometres due North. Calculate how far away the ship then was from Fennel Island.

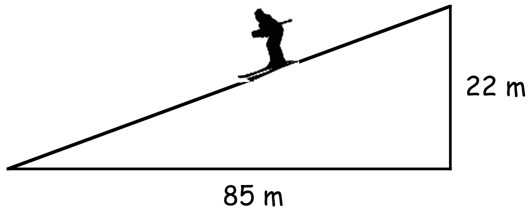


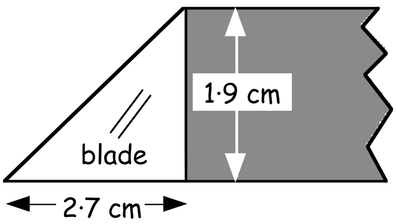
4. 

A breeches boy is used to transport an injured climber down from a cliff to a boat below.

Calculate the length of the cable used.

5. Shown is an artificial ski slope. Calculate the length of the slope.



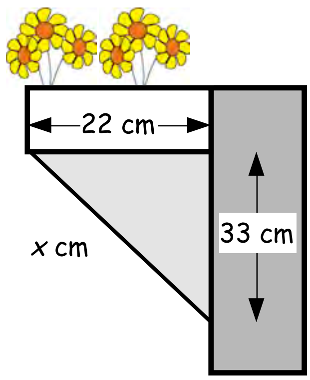
6. 

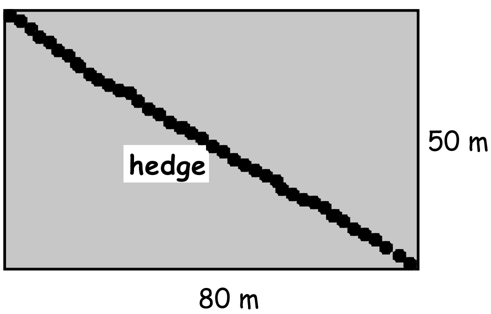
The blade of a Stanley Knife is shown opposite.

Calculate the length of the sloping edge of the blade.

7. A triangular metal bracket is used to support a window box, 22 centimetres wide.

Calculate the length of the sloping edge of the support bracket.



8. 

Farmer Black's field is in the shape of a rectangle 80 metres long by 50 metres wide.

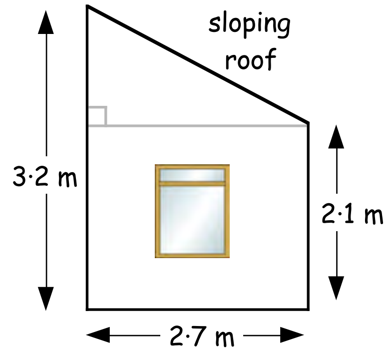
He plants a long hedge diagonally from one corner to the other to separate his sheep from his cattle.

Calculate the length of the hedge.

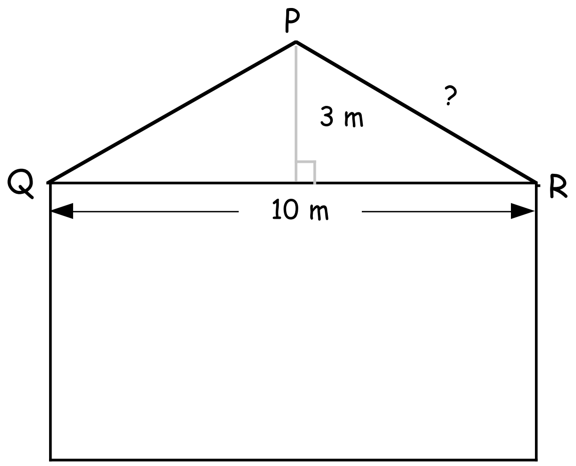
9. The picture shows the side view of a hut.

Calculate the length of the sloping roof.

(Hint :- just think about the right angled triangle at the top)



10.



Shown is the roof of a barn in the shape of an isosceles triangle PQR.

Calculate the length of the sloping roof PR.

(note :- it is **not** $10^2 + 3^2$ etc.)

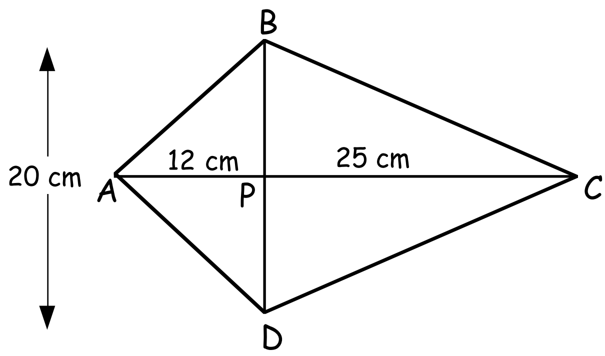
11. Kite ABCD has its 2 diagonals, AC and BD, crossing at right angles at P.

(a) Write down the length of the 2 lines, PB and PD.

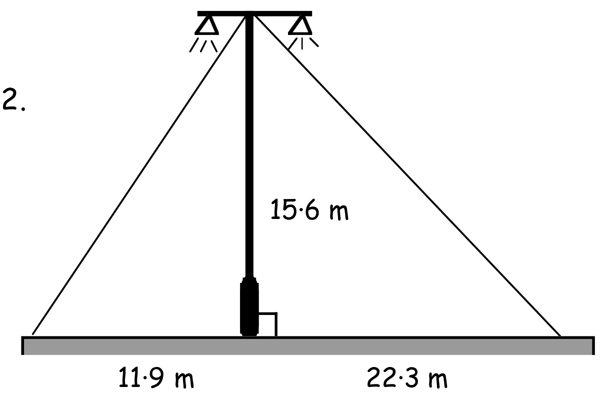
(b) Calculate the length of the line BC.

(c) Calculate the length of the line AB.

(d) Calculate the **PERIMETER** of the kite.



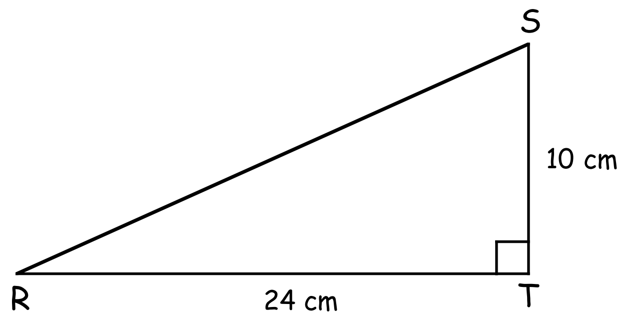
12.



Two wires are used to support a concrete motorway pole until the concrete hardens.

Calculate the **total** length of the support wires.

13. Calculate the **PERIMETER** of triangle RST.



Calculating the Length of the Smaller Side

You can use Pythagoras' Theorem to calculate one of the smaller sides of a right angled triangle.

This time, you are asked to find the length of the smaller side (a) :-

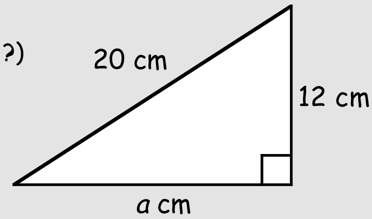
=> $a^2 = c^2 - b^2$

=> $a^2 = 20^2 - 12^2$

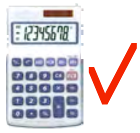
=> $a^2 = 400 - 144 = 256$

=> $a = \sqrt{256} = 16$ cm

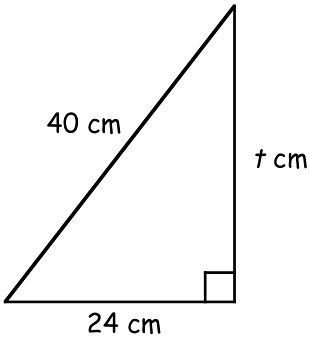
(can you see why the "–" sign ?)



Exercise 6



1. Calculate the length of the side of this right angled triangle marked with an t.



$a^2 = c^2 - b^2$

=> $t^2 = 40^2 - 24^2$

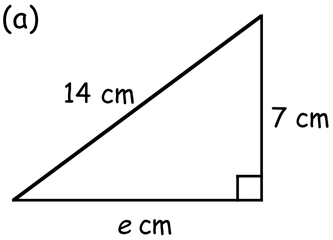
=> $t^2 = 1600 - 576$

=> $t^2 = 1024$

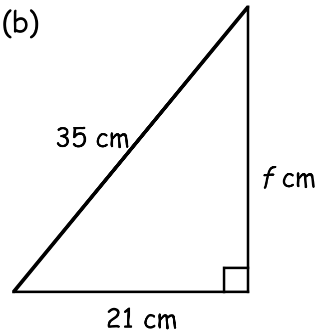
=> $t =$ .... cm

2. Calculate the size of each of the smaller sides in the following right angled triangles. (to 2 decimal places)

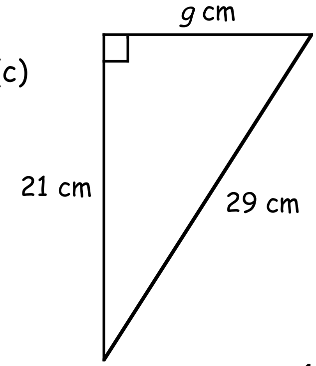
(a)



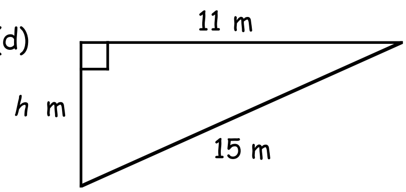
(b)



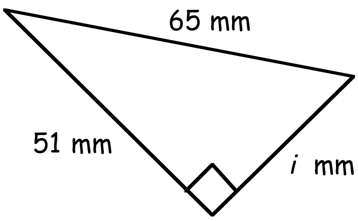
(c)



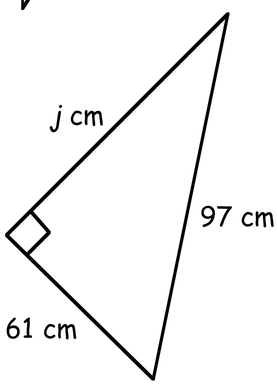
(d)



(e)

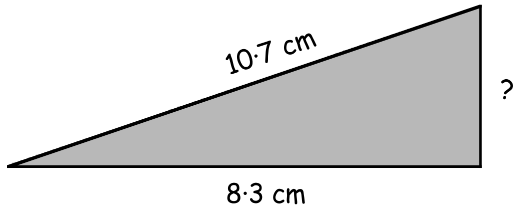


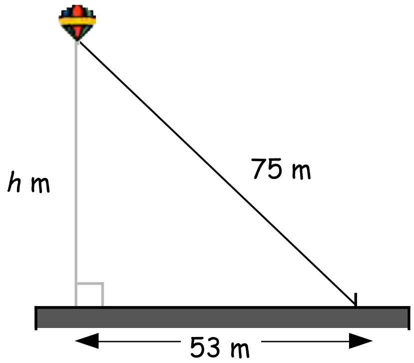
(f)



3. A door wedge has a sloping side of 10·7 cm and a horizontal side of 8·3 cm.

   Calculate the height of the wedge.



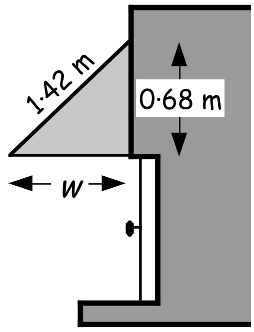
4. 

A hot air balloon is tethered by a rope to the ground as shown opposite.

Calculate the height of the balloon.

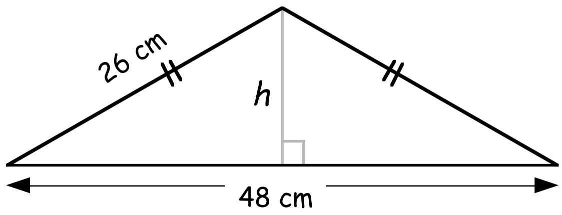
5. A triangular canopy is built to protect a front door from rain.

   Calculate the width (*w*) of the canopy.



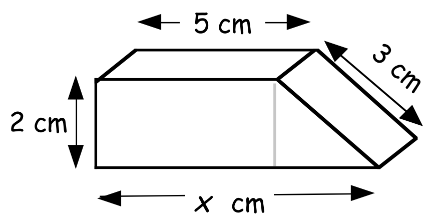
6. This isosceles triangle has a base of 48 cm and a sloping edge of 26 cm.

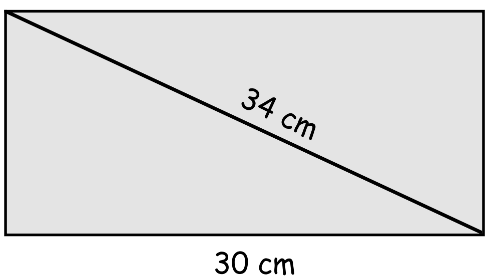
   Calculate the height of the triangle.



7. An eraser has its sloping edge 3 cm. Its height is 2 cm.

   Calculate the overall length (*x*) of the eraser.

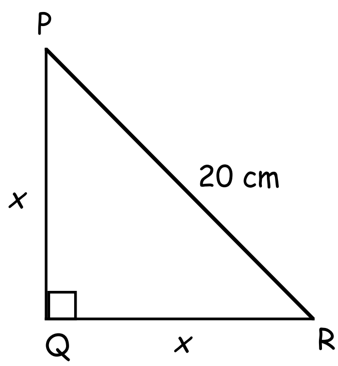


8. 

Calculate the **perimeter** of this rectangle of length 30 cm and with a diagonal of 34 cm.

9. Shown is a right angled isosceles triangle PQR.

   Calculate the value of x (tricky but not impossible !)

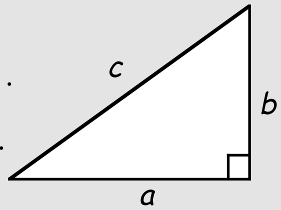


Mixed Examples

In the following exercise, if you are asked to find

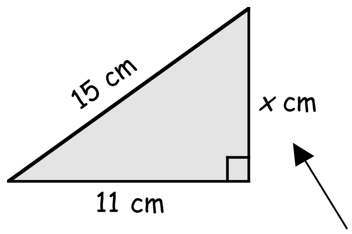
the hypotenuse → use $c^2 = a^2 + b^2$.

a shorter side → use $a^2 = c^2 - b^2$.



You must decide which formula you have to use.

Example 1



(here, you are looking for a short side)

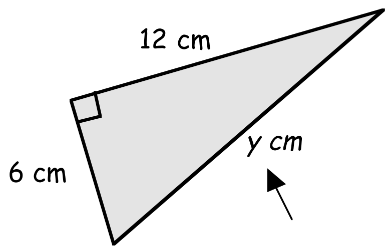
$$x^2 = 15^2 - 11^2$$ note

$$x^2 = 225 - 121$$

$$x^2 = 104$$

$$x = \sqrt{104} = 10{\cdot}20 \text{ cm}$$

Example 2



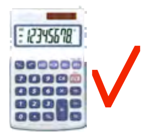
(here, you are looking for the hypotenuse)

$$y^2 = 12^2 + 6^2$$

$$y^2 = 144 + 36$$

$$y^2 = 180$$

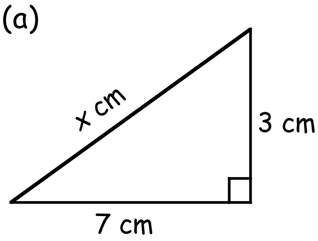
$$y = \sqrt{180} = 13{\cdot}42 \text{ cm}$$



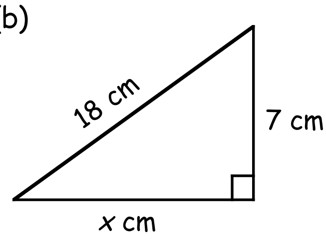
Exercise 7

1. Use the appropriate formula to find the value of x each time :-

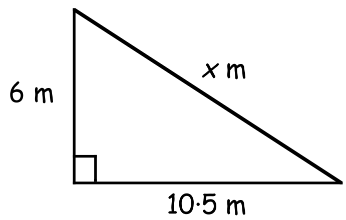
(a)



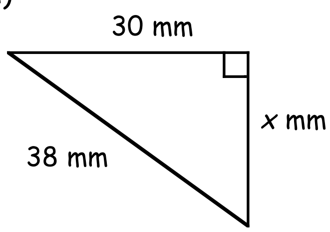
(b)



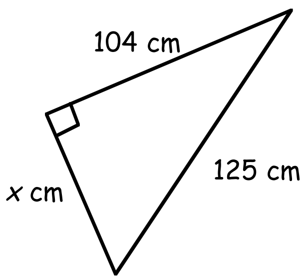
(c)



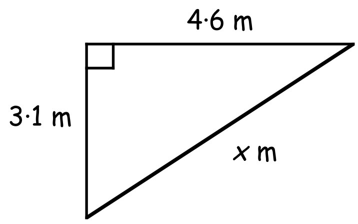
(d)



(e)

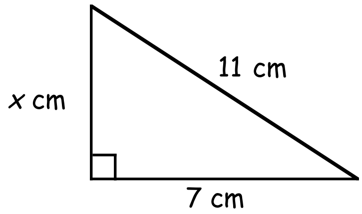


(f)



2. Tina, in her maths exam, tried to calculate the value of the missing side in this triangle. She wrote the following in her exam paper :–

$$x^2 = 11^2 + 7^2$$
$$x^2 = 121 + 49$$
$$x^2 = 170$$
$$x = \sqrt{170} = 13{\cdot}0 \text{ cm}$$

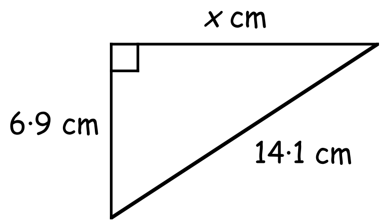


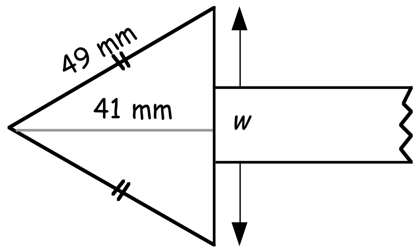
Why should Tina have known that her answer **<u>had</u>** to be **<u>wrong</u>**, by just comparing it to the other 2 sides of the triangle.

3. **ONE** of the following two answers is known to be the **correct** value for x in this question.

$x = 12{\cdot}3$ cm or $x = 15{\cdot}7$ cm

Without actually doing the calculation, say which one it must be and why the other is obviously wrong.



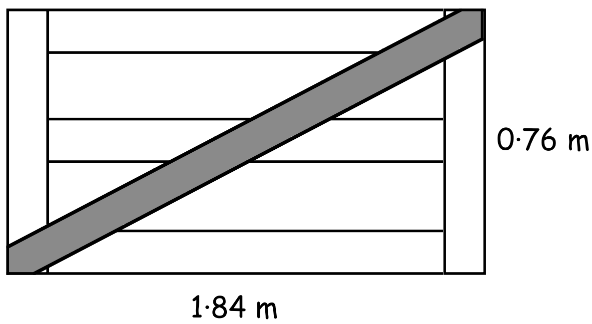
4. 

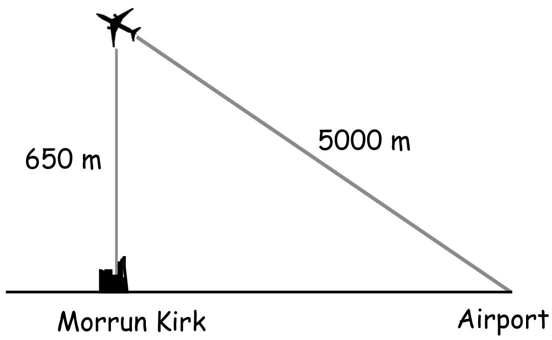
An arrowhead is in the shape of an isosceles triangle.

Calculate the width of the arrowhead. (*Remember to use the right angled triangle and not the isosceles one*).

5. A rectangular fence is strengthened by nailing on a diagonal plank.

What length must the diagonal plank be?

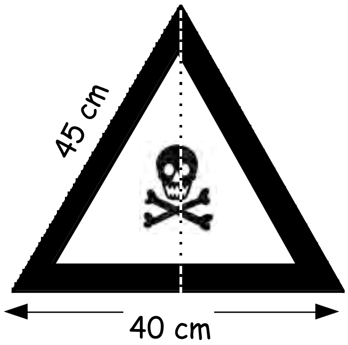


6. 

As a plane took off from Edinburgh Airport, it climbed steadily for 5000 metres until it was directly over Morrun Kirk.

Calculate the distance from Morrun Kirk to Edinburgh Airport.

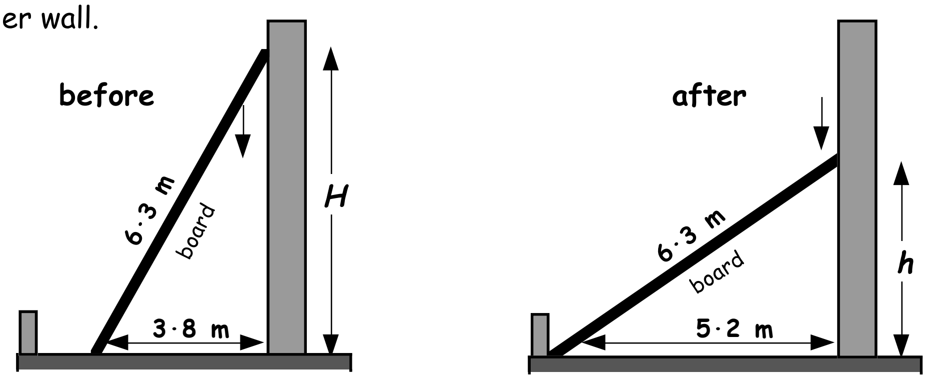
7.



This warning sign is in the shape of an isosceles triangle.

Calculate the height of the triangle.

8. A board, resting against a wall, slid partially downwards until it was stopped by a smaller wall.



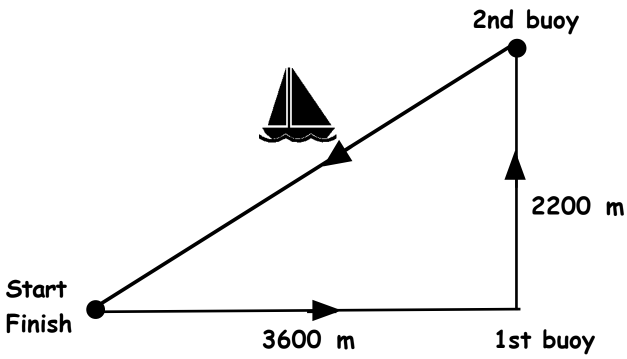
(a) Calculate the original height (H) of the top of the board.

(b) Calculate the new height (h) of the top of the board.

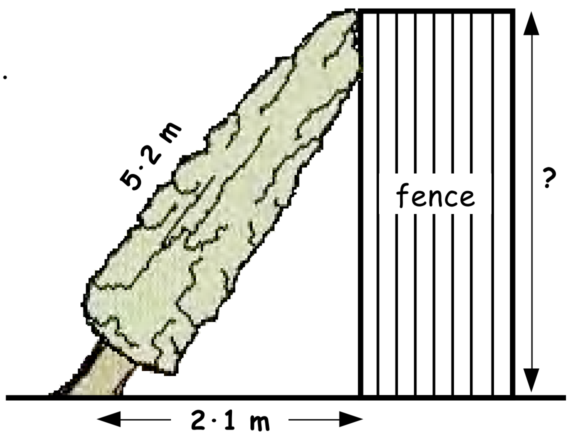
(c) By how many metres had the top of the board slipped ?

9. A yacht race is staged over a triangular course. From the start, the competitors sail East to the 1st buoy, North to the 2nd buoy then race back to the finishing line, where the race began.

Calculate the overall distance of the race.



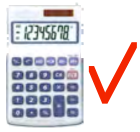
10.



A tree was blown over during a gale and came to rest with its top resting against the top of a wooden fence.

Calculate the height of the fence.

Pythagoras



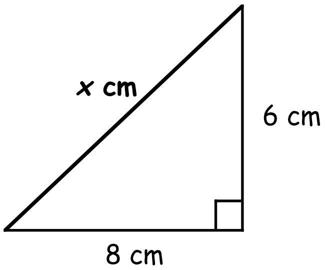
Topic in
a Nutshell

1. Find :- (a) 7^2 (b) 14^2

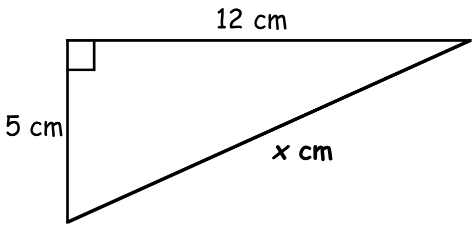
   (c) $\sqrt{16}$ (d) $\sqrt{169}$

2. Use Pythagoras' theorem to calculate the length of the hypotenuse in each triangle :-

   (a)

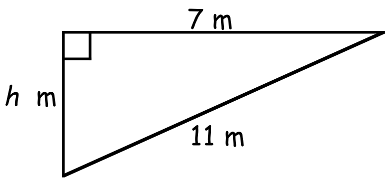


   (b)

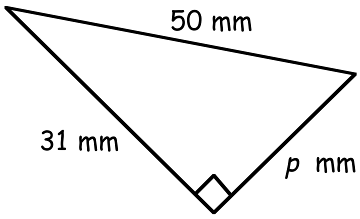


3. Calculate the size of each of the smaller sides in these triangles :-
   (*Give each answer to 2 decimal places*).

   (a)

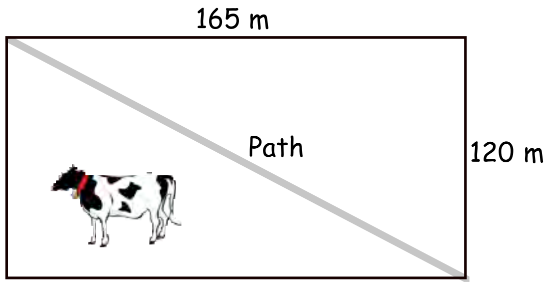


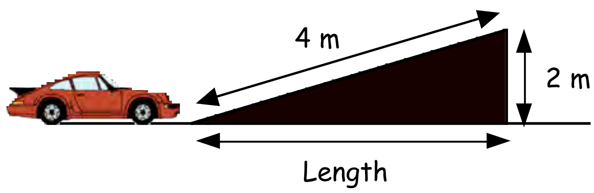
   (b)



4. (a) A farmer has a rectangular field, measuring 165 metres by 120 metres. A path runs diagonally across the field.

   How long is the path ?





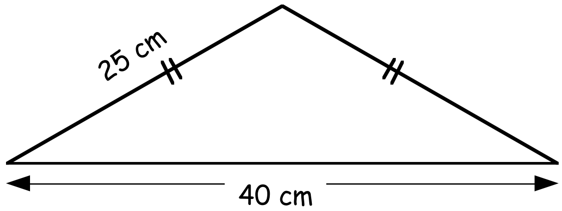
   (b) A car ramp has an inclined length of 4 metres and is 2 metres high.

   Find the horizontal length of the car ramp to the nearest centimetre.

5. An **equilateral** triangle has sides of length 8 centimetres.
   Calculate the height of the triangle.

6. This isosceles triangle has base 40 cm and sloping edges 25 cm.

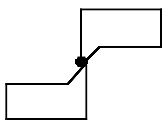
   Calculate the **area** of the triangle.



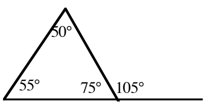
answers to LEVEL F

Answers to Chapter –1

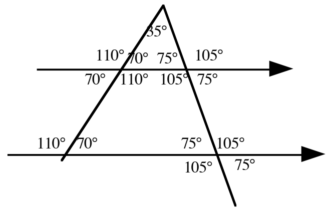
1. a. 5200 b. 6090 c. 620 d. 12 900
   e. 210 f. 32·7 g. 1·53 h. 21·6
   i. 0·043 j. 12·1 k. 5·4 l. 0·93
2. a. 57·55 b. 26·33 c. 117·04 d. 1·84
3. a. 0·3 b. 0·75 c. 0·4
4. a. tenths b. hundredths
   c. thousandths
5. £10·94
6. a. 19·7 b. 0·3 c. 0·7 d. 11·0
7. $9·90
8. 4·50€
9. 1·6 kg
10. a. $2/3$ b. $4/9$ c. $1/3$ d. $3/7$
11. a. 40 b. £90 c. 28 kg d. 360
12. 20
13. a. £60 b. £200 c. £6 d. 25p
    e. £12 f. £27 g. 26 h. 18 kg
14. a. 1 : 2 b. 3 : 4 c. 3 : 5 d. 12 : 7
15. 4 : 3
16. 30
17. a. 19, 23, 27 b. 46, 40, 34
    c. 64, 81, 100 d. 21, 28, 36
18. 59
19. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
20. a. 10, 13, 16
    b. bricks = 3 x (pattern number) + 1
    or b = 3p + 1
    c. 301
21. a. –25°C b. –35°C
22. 27°
23. –18°C
24. a. 4 b. 13
25. a. 17 b. 4 c. $4\frac{1}{2}$ d. $8\frac{1}{2}$
    e. 6 f. 5 g. 0 h. $2\frac{1}{2}$
26. a. {0, 1, 2, 3, 4} b. {3, 4, 5, 6, 7, 8, 9}
    c. { } d. {0, 1}
27. 12m^2
28. 63 mm
29. 3000 kg
30. 430 ml
31. 2·15 seconds
32. a. 60 cm^2 b. 49 cm^2 c. 30 cm^2
33. a. 47 cm b. 64 cm c. 36 cm
34. a. 56 cm^3 b. 27 cm^3
35. 2 cm
36. a. 5 cm b. 100 m c. 300 m
37. 520 m
38. a. parallelogram b. rhombus
    c. trapezium d. kite
39. 3
40. a. square and rhombus
    b. rectangle and rhombus
    c. kite and trapezium
    d. square and rectangle
41. Check diagrams
42. a. cube b. triangular prism
    c. square based pyramid
43. a. 060° b. 165° c. 260°
44. a. (–2, 3) b. check diagram c. 4
45. B, C and D
46.



47. Check diagram
48. a. straight b. acute
    c. obtuse d. right
    e. reflex
49. a. 85° b. 70° c. 110°
50.



51.



52. Check diagram
53. a. 25% b. Check diagram
54. £60
55. a. upward
    b. (i) 22°C (ii) 6°C
    c. 6 a.m. and 4 p.m.

Answers to Chapter 2

Ch 2 - Ex 1 (page 9)

1. a. 6°C b. –3°C c. –2°C d. –14°C
   e. –24°C f. –10°C g. –75°C h. –16°C
   i. –70°C j. –0·6°C
2. a. (i) £39·50 in the bank
   (ii) £22 overdrawn
   (iii) £119·75 in the bank
   (iv) Nothing in the bank.
   b. –£5 c. –£13 d. –£45
   e. –£40 f. £85 g. –£6
3. a. (i) 60 m (ii) 160 m
   (iii) 0 (iv) 100 m
   (v) –160 m (vi) –60 m
   (vii) –40 m (viii) –100 m
   (ix) 220 m
   b. 120 metres
4. a. (i) 1527 (ii) 655
   (iii) –35 (iv) –850
   b. 48 c. 50 d. 67
   e. 35 A.D. f. 73 B.C.

Ch 2 - Ex 2 (page 12)

2. a. 19°C b. 22°C c. 28°C d. 12°C
   e. 0°C f. 2°C g. –12°C h. 4°C
   i. –5°C j. –37°C k. –20°C l. –50°C
   m. –7°C n. –5°C
3. a. 5°C up b. 12°C down
   c. 80°C down d. 15°C up
   e. 13°C down f. 14°C up
   g. 20°C down h. 24°C down
   i. 100°C up j. 20°C down
4. –18°C
5. 76°C
6. 13°C
7. a. 12°C b. –4°C c. –20°C d. –60°C
8. a. –140°C b. –50°C c. 10°C d. 0°C
9. a. –5°C b. 20°C
   c. Feb - March d. –1°C
10. –100°C

Ch 2 - Ex 3 (page 14)

1. a. 15 b. 13 c. 23 d. 3
   e. 6 f. 0 g. –4 h. –10
   i. –20 j. 6 k. 0 l. 12
   m. –4 n. –7 o. –16 p. –13
   q. –18 r. –30 s. –8 t. –40
   u. –20 v. –90 w. –35 x. –6
2. a. 6 b. 0 c. 3 d. –2
   e. –5 f. –10 g. –15 h. –5
   i. –10 j. –16 k. –22 l. –35
   m. –20 n. –40 o. –200 p. –100
   q. –22 r. –20 s. –16 t. –60
   u. –2 v. –90 w. –26 x. –6
3. a. 10 b. –7 c. –10 d. 10
   e. 15 f. –8 g. –12 h. –100
   i. 15 j. –27 k. –27 l. –30
   m. 7 n. –7 o. 0 p. –30
   q. –13 r. –35 s. –2 t. –2

Ch 2 - Ex 4 (page 15)

1. a. 11 b. 20 c. 11 d. 50
   e. 19 f. 8
2. a. 15 b. 25 c. 11 d. 20
   e. 22 f. 50 g. 14 h. 1000
   i. 60 j. 10 k. 5·3 l. 1
3. a. 2 b. 5 c. 5 d. 4
   e. –10 f. 0
4. a. 4 b. 6 c. 3 d. –3
   e. 1 f. –6 g. 0 h. 70
   i. –20 j. 2 k. –0·5 l. 0
5. a. 7***x*** b. 18***x*** c. 5***x*** d. 13***a***
   e. 13***p*** f. 20***w*** g. 20***g*** h. 80***f***
   i. 4***m*** j. –5***k*** k. 0 l. ***b***
   m. –7***q*** n. 4***z*** o. 6***c*** p. 0
6. a. 5 b. –10 c. 11 d. 4
   e. 0 f. –14 g. –13 h. 10
   i. 20 j. 1 k. 14 l. 20
   m. 7***x*** n. 6***p*** o. 12***a*** p. –15***g***
   q. 2***a*** r. 5***p*** s. 200 t. –30***f***
   u. $-3a^2$ v. $8t^2$ w. 2000 x. –6

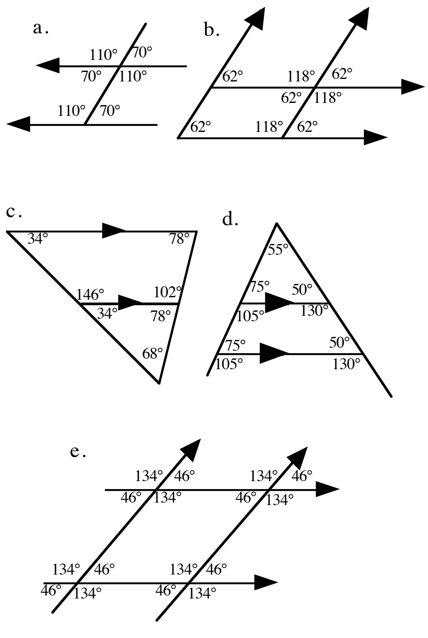
Ch 2 - Ex 5 (page 17)

1. a. –20 b. –42 c. –18 d. –25
   e. –24 f. –36 g. –22 h. –70
   i. –48 j. –24 k. –48 l. –49
   m. –9 n. –27 o. –20 p. –45
2. a. –5 b. –4 c. –8 d. –7
   e. –20 f. –9 g. –11 h. –8
   i. –1 j. –5 k. –9 l. –20
3. a. 6 b. –4 c. –24 d. –30
   e. –4 f. –9 g. –12 h. –20
4. a. 21 b. –18 c. –16
   d. –6 e. –20 f. –25
   g. –35 h. –2 i. –4
5. –5
6. a. –4 b. –4 c. –2 d. –5
   e. –9 f. –5 g. –7 h. –14
   i. –12 j. –20 k. –20 l. –7
   m. –1 n. $-3\frac{1}{2}$ o. $-7\frac{1}{2}$ p. $-\frac{1}{2}$
7. a. 12 b. 10 c. 63 d. 32
   e. 56 f. 64 g. 14 h. 90
   i. 25 j. 60 k. 200 l. 4000
8. a. 4 b. 6 c. 8 d. 11
   e. 4 f. 5 g. 9 h. 20
   i. 12 j. 20 k. 20 l. 56
9. a. –6 b. 4 c. 24
   d. 4 e. –6 f. 15
   g. 18 h. 42 i. 4
10. a. –24 b. –60 c. –120
    d. 9 e. 25 f. 100
    g. 1 h. –1 i. 1

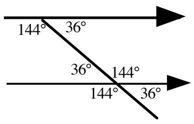
Answers to Chapter 3

Ch 3 - Ex 1 (page 21)

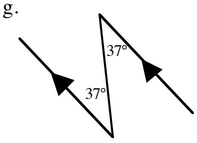
1. a. 63° b. 128° c. 105°
   d. 50° e. 48° f. 118°
   g. 34° h. 71° i. 61°
   j. 95° k. 23·5° l. 90°
   m. 129° n. 34° p. 50°
   q. 64° r. 56° s. 68°
   t. 113° u. 46° v. 120° 46
2



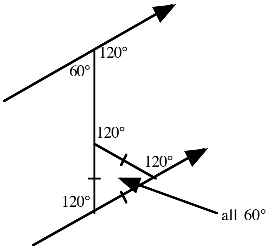
f.



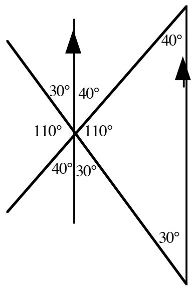
g.



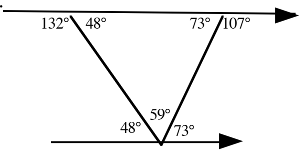
h.



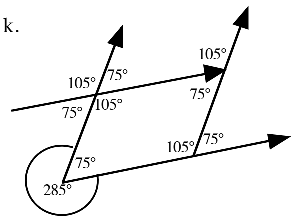
i.



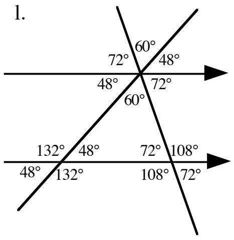
j.



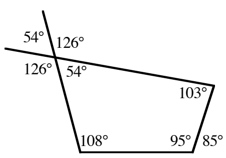
k.



l.



m.



Ch 3 - Ex 2 (page 23)

1. b. Octagon   c. Nonagon
   d. Decagon
2. a. 72°   b. 54°   c. 108°   d. 540°
3. a. 60°   b. 60°   c. 120°   d. 720°
4. a. 135°   b. 1080°
5. a. (i) 140°   (ii) 1260°
   b. (i) 144°   (ii) 1440°

6.

sides	4	5	6	7	8	9	10	11	12
sum	360	540	720	900	1080	1260	1440	1620	1800
interior	90	108	120	129	135	140	144	149	150

8. a. 60°   b. 45°   c. 36°
9. same

Ch 3 - Ex 3 (page 26)

1. a. 360°   b. 12   c. 30°
2. a. 60°   b. 90°   c. 120°   d. 150
   e. 180°   f. 90°
3. b. 105°
4. a. 135°   b. 15°   c. 75°   d. 15°
   e. 165°   f. 105°
5. a. 82·5°   b. 22·5°   c. 80°
6. 10 minutes and 50 seconds past 2

Answers to Chapter 5

Ch 5 - Ex 1 (page 28)

1. a. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
   b. 6, 12, 18, 24, 30, 36, 42, 48
   c. 10, 20, 30, 40, 50, 60
2. a. 12, 15, 18, 21, 24, 27, 30, 33, 36, 39
   b. 32, 36, 40, 44, 48
   c. 21, 28, 35, 42, 49
3. a. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
   b. Even numbers
   c. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19. No
   d. odd numbers
4. a. multiples of 5 from 40 to 70
   b. multiples of 2 from 22 to 36
   c. multiples of 6 from 84 to 108
   d. muliples of 10 from 150 to 190
   e. multiples of 14 from 28 to 84
   f. multiples of 50 from 200 to 400
5. a. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
   b. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
   c. 12, 24...
   d. 12
6. a. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72
   b. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80
   c. 12, 24, 36, 48, 60...
   d. 12
7. a. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
   b. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40
   c. 10, 20, 30, 40...
   d. 10
8. a. 15   b. 6   c. 28   d. 6
   e. 24   f. 30   g. 30   h. 63
   i. 70   j. 72   k. 12   l. 77
9. a. 12   b. 12   c. 30   d. 30
   e. 30   f. 24   g. 18
10. 180 seconds
11. 300 seconds
12. a $^{8}/_{15}$ b $^{3}/_{8}$ c $^{7}/_{12}$ d $^{7}/_{10}$
    e $^{3}/_{20}$ f $^{2}/_{3}$ g $^{4}/_{21}$ h $^{31}/_{30}$

Ch 5 - Ex 2 (page 30)

1. 1, 3, 5, 15
2. 1, 2, 4, 7, 14, 28
3. 1, 2, 3, 4, 6, 8, 12, 24
4. 1, 2, 3, 6, 9, 18
5. a. 1, 2, 4, 8   b. 1, 2, 3, 4, 6, 12
   c. 1, 2, 11, 22   d. 1, 2, 4, 7, 14, 28
   e. 1, 29   f. 1, 2, 3, 5, 6, 15, 30
   g. 1, 2, 4, 8, 16, 32
   h. 1, 2, 4, 5, 8, 10, 20, 40
   i. 1, 3, 5, 9, 15, 45   j. 1, 2, 5, 10, 25, 50
   k. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
   l. 1, 61
7. a. 1, 2, 4 (3 factors) b. 1, 5, 25 (3 factors)
   c. 1, 2, 3, 6, 12, 18, 36 (7 factors)
   d. 1, 3, 9 (3 factors) e. 1, 7, 49 (3 factors)
   f. 1, 2, 4, 8, 16 (5 factors)
   g. 1, 2, 4, 5, 10, 20, 25, 50, 100 (9 factors)
   h. 1, 2, 4, 8, 16, 32, 64 (7 factors)
8. b. square numbers
   c. repeated factor.
9. 1 row of 24, 2 rows of 12, 3 rows of 8
   4 rows of 6, 6 rows of 4, 8 rows of 3,
   12 rows of 2, 24 rows of 1
10. a. 1, 2, 3, 4, 6, 12
    b. 1, 2, 4, 8, 16
    c. 1, 2, 4
    d. 4
11. a. 1, 2, 4, 5, 10, 20
    b. 1, 2, 3, 5, 6, 10, 15, 30
    c. 1, 2, 5, 10
    d. 10
12. a. 2   b. 4   c. 5   d. 6
    e. 12   f. 20   g. 13   h. 6
13. a. 1   b. 1   c. 1   d. 1
14. a. 4   b. 5   c. 7   d. 8
15. 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360
16. 12

Ch 5 - Ex 3 (page 32)

1. 1, 2, 3, 4, 6, 12 (more than 2 factors)
2. 1, 11 (exactly 2 factors)
3. 4 factors (not prime)
4. Does not have two factors.
5. a. 1, 7 (prime)
   b. 1, 2, 5, 10 (not prime)
   c. 1, 3, 9 (not prime)
   d. 1, 13 (prime)
   e. 1, 3, 5, 15 (not prime)
   f. 1, 3, 7, 21 (not prime)
   g. 1, 23 (prime)
   h. 1, 29 (prime)
   i. 1, 2, 5, 10, 25, 50 (not prime)
   j. 1, 7, 49 (not prime)
   k. 1, 47 (prime)
   l. 1, 3, 5, 9, 15, 45 (not prime)
6. 30, 32, 33, 34, 35, 36, 38, 39, 40
7. h. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
8. f. 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199
9. a. divisible by 5   b. divisible by 2
   c. divisible by 10   d. divisible by 3

Ch 5 - Ex 4 (page 35)

1. 2 x 3 x 3 x 3
2. a. 2 x 3 x 5   b. 2 x 2 x 2 x 3
   c. 2 x 2 x 7
3. a. 2 x 2 x 2   b. 2 x 5
   c. 2 x 2 x 3   d. 2 x 2 x 5
   e. 5 x 5   f. 2 x 2 x 3 x 3
   g. 3 x 3 x 5   h. 2 x 5 x 5
   i. 2 x 2 x 2 x 2 x 2 x 2   j. 2 x 2 x 2 x 2 x 5
   k. 2 x 7 x 7   l. 2 x 2 x 5 x 5

Answers to Chapter 7

Ch 7 - Ex 1 (page 37)

1. $4{\cdot}6 \times 10^{4}$
2. a. $5{\cdot}9 \times 10^{3}$ b. $3{\cdot}4 \times 10^{2}$
   c. $8{\cdot}1 \times 10^{4}$
3. a. $5{\cdot}2 \times 10^{3}$ b. $4{\cdot}53 \times 10^{3}$
   c. $2{\cdot}7 \times 10^{4}$ d. $3{\cdot}59 \times 10^{4}$
   e. $8{\cdot}275 \times 10^{4}$ f. $7{\cdot}5 \times 10^{5}$
   g. $3{\cdot}78 \times 10^{5}$ h. $2{\cdot}047 \times 10^{5}$
   i. $8{\cdot}6 \times 10^{6}$

Ch 7 - Ex 2 (page 38)

1. $5{\cdot}7 \times 10^{4}$
2. a. $6{\cdot}9 \times 10^{4}$ b. $9{\cdot}3 \times 10^{3}$
   c. $2{\cdot}34 \times 10^{5}$ d. $5{\cdot}2 \times 10^{2}$
   e. $4{\cdot}287 \times 10^{3}$ f. $2{\cdot}6 \times 10^{5}$
   g. 8×10^{3} h. $4{\cdot}7 \times 10^{4}$
   i. 2×10^{4} j. $9{\cdot}32 \times 10^{6}$
   k. $4{\cdot}8 \times 10^{6}$ l. $2{\cdot}57 \times 10^{7}$
3. Austria : $8{\cdot}42 \times 10^{4}$ Australia : $7{\cdot}687 \times 10^{6}$
   Canada : $9{\cdot}976 \times 10^{6}$ France : $5{\cdot}472 \times 10^{5}$
   Iceland : $1{\cdot}033 \times 10^{5}$ USSR : $2{\cdot}2402 \times 10^{7}$
   Hong Kong : $9{\cdot}87 \times 10^{2}$
4. a. $6{\cdot}0 \times 10^{6}$ b. $8{\cdot}5 \times 10^{6}$
   c. $1{\cdot}98 \times 10^{6}$ d. $3{\cdot}5 \times 10^{6}$
   e. $1{\cdot}1 \times 10^{7}$ f. $2{\cdot}7 \times 10^{7}$
   g. $7{\cdot}4 \times 10^{6}$ h. $6{\cdot}23 \times 10^{6}$
   i. $1{\cdot}05 \times 10^{7}$ j. $1{\cdot}55 \times 10^{7}$
   k. $2{\cdot}745 \times 10^{6}$ l. $3{\cdot}25 \times 10^{6}$
   m. $5{\cdot}75 \times 10^{6}$
5. a. $3{\cdot}5 \times 10^{6}$ b. $4{\cdot}5 \times 10^{4}$
   c. $2{\cdot}225 \times 10^{7}$ d. $5{\cdot}0 \times 10^{8}$
   e. $8{\cdot}52 \times 10^{8}$ f. $2{\cdot}28 \times 10^{11}$

Ch 7 - Ex 3 (page 40)

1. 4560
2. a. 70 300 b. 2 730 000 c. 199 d. 34 750 000 e. 840 000 f. 3810 g. 62 300 h. 99 900 i. 145 000 000 j. 783 500 k. 70 000 l. 5 670 000 000 m. 1 000 000 000 000
3. 3070
4. a. 27 000 b. 340 c. 527 000 d. 2850 e. 45 230 f. 40 g. 6 000 000 h. 3 500 000 i. 90 170 j. 80 000 000 k. 43 700 000 l. 111 100 000
5. Portugal : 9 449 000 Malta : 304 200 Israel : 35 971 000 Greece : 3 548 000 Turkey : 40 160 000 China : 852 100 000 Greenland : 50 130
6. a. 4 900 000 b. 62 800 000 c. 1 450 000 000 000
7. a. 5 483 896 b. 48 mins 13 secs c. 63 days, 11 hrs, 20 mins and 16 secs d. 4387 pages

Ch 7 - Ex 4 (page 43)

1. a. 8×10^{-3} b. $6{\cdot}7 \times 10^{-5}$ c. $9{\cdot}31 \times 10^{-2}$ d. 7×10^{-6} e. $5{\cdot}58 \times 10^{-4}$ f. $1{\cdot}82 \times 10^{-1}$ g. $3{\cdot}15 \times 10^{-3}$ h. $9{\cdot}0 \times 10^{-5}$
2. a. $2{\cdot}5 \times 10^{-3}$ b. $2{\cdot}5 \times 10^{-7}$ c. $1{\cdot}7 \times 10^{-2}$
3. a. 0·034 b. 0·000 57 c. 0·00638 d. 0·000 040 5 e. 0·18236 f. 0·009 g. 0·000 04 h. 0·000 002 002
4. 1g less
5. a. 0·041 b. 0·603 c. 0·000 099 2 d. 0·001 01 e. 0·000 000 06 f. 0·000 002 003
6. a. (i) 0·0017 (ii) 2900 (iii) 0·000 08 (iv) 30000 (v) 0·000 001 67 (vi) 208 500
   b. (i) 9×10^{-3} (ii) $2{\cdot}64 \times 10^{2}$ (iii) $8{\cdot}1 \times 10^{-4}$ (iv) $5{\cdot}2 \times 10^{3}$ (v) $1{\cdot}39 \times 10^{-2}$ (vi) $3{\cdot}9 \times 10^{5}$ (vii) $7{\cdot}0 \times 10^{-6}$ (viii) $1{\cdot}25 \times 10^{8}$ (ix) $9{\cdot}0 \times 10^{-1}$
7. 7
8. a. 274 000 b. 0·000 003 35 c. 0·00103
9. a. $5{\cdot}95 \times 10^{9}$ b. $1{\cdot}65 \times 10^{8}$ c. $1{\cdot}494 \times 10^{5}$ d. $3{\cdot}3 \times 10^{-6}$ e. $6{\cdot}348781574 \times 10^{9}$

Answers to Chapter 11

Ch 11 - Ex 1 (page 46)

1. a. $2x$ b. $3w$ c. $5m$ d. c e. $2f$ f. x g. $7x$ h. $2e$ i. $4x + 2y$ j. $2b$ k. $5q$ l. $3x + 3w$ m. $8h - g$ n. $6w$ o. $2b^2$
2. a. $6t$ b. $4p$ c. a^2 d. w^2 e. $7f$ f. $15r$ g. $12m$ h. $8s$ i. $3x^2$ j. $20x$ k. $15d$ l. $24m$ m. $9a^2$ n. $7g^2$ o. pq p. $11mn$ q. $5pq$ r. $8k^2$ s. $15ab$ t. $27x^2$ u. $24d^2$ v. $5a^3$ w. $6w^3$ x. $16a^2$ y. $36x^2$ z. $4a^2b^2$
3. a. $4x^2$ b. $2x^2$ c. $3p^2$ d. $5v$ e. $12m^2$ f. $24n^2$ g. $11n$ h. $20xy$ i. $20xy$ j. $8a$ k. $6xy^2$ l. $4b$ m. $13t$ n. $x + y$ o. $8a + 1$ p. $5a^2$ q. y^2 r. $4a + b$ s. $7p + 1$ t. x^2 u. $d + 9$ v. $7 + 8h$ w. $8 + 5x$ x. $6a^2 - 4$ y. $11v^2$ z. $3a^2 + 4b^2$
4. a. 14 b. 12 c. 9 d. 14 e. 5 f. 19 g. 31 h. 9 i. 16 j. 23 k. 13 l. 25
5. a. $y^2 + 9y + 20$ b. $x^2 + 10x + 16$ c. $2w^2 + 6w + 4$ d. $4pq$ e. $9t + 3tv$ f. $36m^2$ g. $d^2 + 7d + 10$

Ch 11 - Ex 2 (page 48)

1. a. $2d + 8$ b. $3c + 3$ c. $4e + 20$ d. $5g + 40$ e. $6h + 42$ f. $2n - 6$ g. $4a - 8$ h. $5t - 5$ i. $2b - 6$ j. $7k - 42$ k. $5n - 45$ l. $9 + 9x$ m. $12 + 4y$ n. $8 + 8m$ o. $2 - 2p$ p. $16 - 4q$ q. $6a + 6b$ r. $2f + 2g$ s. $7g - 7k$ t. $10d - 1000$ u. $36 + 12b$ v. $15r + 30$ w. $20a - 100$ x. $50g - 200$
2. a. $4g + 6$ b. $12a + 3$ c. $5 + 10d$ d. $6 - 8k$ e. $36h - 6$ f. $30 - 70n$ g. $8a + 12y$ h. $15t + 5x$ i. $8b - 6c$ j. $80k - 24p$ k. $77n - 63x$ l. $18ab - 6d$ m. $xy + 5x$ n. $ap + 8a$ o. $wt - w$ p. $g^2 - 2g$ q. $an + 9a$ r. $wm - aw$ s. $ef - 10e$ t. $2x + x^2$ u. $2an + ag$ v. $4xy + 3ux$ w. $12a - 24a^2$ x. $30u^2 - 3uw$
3. a. $6a + 8b + 2$ b. $15x + 6y + 9$ c. $35c + 10d + 30$ d. $8k + 12j + 16f$ e. $12v + 24w + 30z$ f. $10p + 10q - 20r$ g. $10a - 4b - 8c$ h. $18p - 30q - 42$ i. $10x - 15y - 25t$ j. $8a - 16b - 40c$ k. $45 - 36f - 27g$ l. $a^2 - ab - 4ac$
4. a. $3x + 15$ b. $4x + 36$ c. $100 - 5x$ d. $16 - 2x$
5. a. $-2a - 2$ b. $-3x + 6$ c. $-15 - 5d$ d. $-20 + 4c$ e. $-p - q$ f. $-p + q$ g. $-6d - 6e$ h. $-5d + 5e$ i. $-p^2 - 4p$ j. $-h^2 + h$ k. $-x - x^2$ l. $-2m^2 - 6m$ m. $-4a^2 + a$ n. $-5h^2 - 4hk$ o. $-5xy + 4x^2$ p. $-2x^2 + 6xk$

Ch 11 - Ex 3 (page 50)

1. a. $3x + 14$ b. $2a + 6$ c. $4b + 25$ d. $2c + 1$ e. $6w + 1$ f. $4r$ g. $5y + 6$ h. $15x + 8$ i. $m + 12$ j. $5n + 24$ k. $3h + 40$ l. $6x + 3$ m. $9a - 10$ n. $13p + 2$ o. $21q - 6$ p. $4v - 1$ q. $5a + 6b$ r. $11x + 8y$ s. $16x - 28y$ t. $190p + 10q$ u. $3h + 10$ v. $-12y$ w. $26w + 12v$ x. $2p$
2. a. $4x + 6$ b. $7a + 13$ c. $11d + 23$ d. $6m + 2$ e. $7c$ f. $9n + 3$ g. $v + 17$ h. $x + 7$ i. $19q$ j. $14d + 4$ k. $3h + 19$ l. $8v + 12w$
3. a. $2x$ b. $a + 2$ c. $b + 1$ d. $4c + 2$ e. $14p$ f . $2x + 18$ g. $2x + 11$ h. $14e + 4$ i. $2v + 8$ j. $x^2 + 3x - 2$ k. $n^2 + 2n - 4$ l. $3w^2 - 7w + 16$
4. a. $-2y + 1$ b. $-2p + 7$ c. $-3d + 6$ d. $3h + 7$ e. $-8c + 18$ f. $2u + 2$ g. $9b - 26$ h. $-2n + 5$ i. $4m - 12$ j. $2x - 3$ k. $6k - 18$ l. $9w - 4$
5. a. $2x + 12$ b. $y^2 - 4y + 16$

Ch 11 - Ex 4 (page 51)

1. a. 7 b. 1 c. 18 d. 1 e. 17 f. 2 g. 9 h. 27 i. 1 j. 18 k. 30 l. 0
2. a. 15 b. 30 c. 25 d. 100 e. 125 f. 50 g. 250 h. 5
3. a. 13 b. 28 c. 8 d. 13 e. 17 f. -2 g. -15 h. 20
4. a. 10 b. 25 c. 1 d. 18 e. 100 f. 27 g. 17 h. 0 i. 0
5. a. 9 b. 25 c. 1 d. 7 e. 49 f. 32 g. 162 h. 4 i. 2 j. 2 k. 2 l. 5
6. a. 4 b. 1 c. 4 d. 12 e. 26 f. 5 g. 11 h. 0 i. 0
7. a. 2 b. 2 c. 6 d. 5 e. 2 f. 4 g. 4 h. 2 i. 12 j. 4 k. 2 l. 2
8. a. 18 b. 13 c. 6 d. 29 e. -3 f. -8 g. -2 h. 8 i. 5

Ch 11 - Ex 5 (page 54)

1. a. $2(p + q)$ b. $3(a + 2b)$ c. $2(3x + 2y)$ d. $a(x + y)$ e. $m(n + 1)$ f. $w(v + w)$ g. $p(r^2 + 1)$ h. $2h(x + y)$ i. $4(2v - 3g)$ j. $8(3m - 2n)$ k. $4k(g - 2)$ l. $3a(2 - 5a)$
2. a. $5(x + 2)$ b. $4(a + 3)$ c. $8(x - 5)$ d. $7(x + y)$ e. $8(a - b)$ f. $20(f - g)$ g. $7(n - 3)$ h. $12(v + 4)$ i. $2(2p + 3q)$ j. $14(h - 2k)$ k. $5(2u - 3w)$ l. $4(a - 6b)$ m. $3(3y + 5)$ n. $8(3n - 1)$ o. $16(2e - 5d)$ p. $8(3x + 4y)$ q. $3(2u + 3v)$ r. $2(a + 3b + 4c)$ s. $6(2x + 5y + 6z)$ t. $20(3r + 2s - 4t)$
3. a. $a(4 + c)$ b. $v(6 - g)$ c. $x(y + z)$ d. $p(p + 9)$ e. $g(3 - g)$ f. $n(n - 4)$ g. $7x(r + s)$ h. $3j(k - 2h)$ i. $12w(v - 1)$ j. $d(3d + 8)$ k. $3g(3g - 5e)$ l. $n(2n - 1)$ m. $2a(2 + 7a)$ n. $p(p - 2p)$ o. $3c(c - 4d)$ p. $8b(2a + 3b)$
4. a. $x(x + 3w - 5)$ b. $a(9b - 9c + 1)$ c. $w^2(w + 1)$ d. $3m(m^2 - 5)$ e. $xy(x + y)$ f. $6de(3e - 4)$ g. $2p(2p - 3q)$ h. $^1/_2c(b + d)$ i. $y(x + {}^1/_3my)$ j. $4jz(5j + 2z)$ k. $a(b - c + a)$ l. $3g(g - 4h + 1)$

Answers to Chapter 13

Ch 13 - Ex 1 (page 56)

1. a. tens b. hundredths c. tenths d. units e. thousandths
2. a. 6·1 b. 8·16 c. 4·754
3. a. 0·19 b. 0·07 c. 0·0175
4. a. 5·56 b. 7·03 c. 0·34 d. 7·28
5. a. 4·2 b. 7·4 c. 30·2 d. 0·8 e. 34·0
6. 28·1
7. 2·61 km
8. a. 44·65 b. 393·44 c. 12·84 d. 8·77
9. a. 120·23 kg b. 5·45 kg
10. a. 25·74 b. 907·6 c. 6920 d. 7000 e. 0·27 f. 0·012473 g. 0·3645 h. 0·5
11. a. 0·0234g b. £3562
12. a. 52·2 b. 135·1 c. 31·68 d. 113·85 e. 1·7 f. 14·5 g. 32·7 h. 74·8
13. 118·3 m^3
14. £6·87
15. £245·60
16. £429·80
17. MHV – 80p, CD WORLD – 0·79p (cheaper)

Ch 13 - Ex 2 (page 58)

1. a. 59·31 b. 36·69 c. 25·707 d. 29·005
2. a. 8·717 b. 6·656 c. 20·748 d. 8·317 e. 1·365 f. 1·505 g. 34·43 h. 30·122 i. 4·812
3. a. 2·384 m b. 1·157 m
4. 0·182 kg

5. a. 1896·525 ml
b. (i) 254·45 ml (ii) 220·475 ml
6. 28·2°C
7. a. 5·7°C b. 3·4°C
8. a. Totals = £21·35, £2·06, £7·22, -£3·13
b. £9·63
9. a. 6·817 + 1·273 = 8·090
b. 9·396 – 4·309 = 5·087
c. 4·173 + 3·695 + 2·619 = 10·487
10. a. 2·2 b. -2·9 c. -5·0 d. -0·42
e. 2·905 f. 0·955 g. 0·9 h. -7·85
i. 5·00 j. 10·00 k. -3·8 l. 1·03

Ch 13 - Ex 3 (page 60)

1. a. 1·5 b. 5·4 c. 3·2 d. 4·9
e. 4·5 f. 13·2 g. 14·4 h. 35·7
i. 18·6 j. 55 k. 200 l. 477·0
m. 0·84 n. 4·32 o. 1·38 p. 6·75
2. a. 18 b. 48 c. 72 d. 30
e. 120 f. 480 g. 200 h. 630
i. 1200 j. 5600 k. 4500 l. 5400
3. a. 1200 kg b. 36 kg
c. 480 kg d. 6000 kg
4. a. £0·40 b. £4·80 c. £24 d. £560
5. a. -2·0 b. -7·2 c. -2 d. -4·2
e. -4·2 f. 12 g. 32 h. 45

Ch 13 - Ex 4 (page 61)

1. a. 1·4 b. 0·9 c. 0·7 d. 0·9
e. 0·8 f. 0·9 g. 0·9 h. 0·4
i. 0·4 j. 0·9 k. 0·6 l. 0·4
m. 0·009 n. 0·007 o. 0·006 p. 0·002
2. a. 0·0036 b. 0·0018 c. 0·0012 d. 0·0009
e. 0·0015 f. 0·0008 g. 0·0009 h. 0·0007
3. £0·40
4. 0·35 g
5. 0·97 ml
6. 2·8 km
7. a. -4·8 b. -0·7 c. -0·8 d. -0·5
e. -0·9 f. -0·6 g. -0·7 h. 0·007

Ch 13 - Ex 5 (page 62)

1. a. 2·8 b. 28 c. 280 d. 2800
e. 0·28 f. 0·028 g. 0·0028 h. 0·00028
2. a. 0·42 b. 0·18 c. 0·16 d. 0·25
e. 0·021 f. 0·072 g. 0·048 h. 0·001
i. 0·0136 j. 0·0135 k. 0·0188 l. 0·049
m. 0·0102 n. 0·0378 o. 0·0768 p. 0·0477
3. a. 3600 b. 0·00012 c. 0·024 d. 0·21
e. 5·4 f. 2400 g. 3 h. 30
4. £27
5. 1·025 cm
6. a. 0·0006 b. 0·0028 c. 0·0045 d. 0·0032
e. -0·42 f. -0·0006 g. 0·0006 h. 0·000014

Ch 13 - Ex 6 (page 63)

1. a. 40 b. 40 c. 50 d. 80
e. 80 f. 90 g. 1000 h. 110
2. a. 2 b. 13 c. 7 d. 9
e. 5·1 f. 15·4 g. 31·8 h. 46·7
3. a. 400 b. 500 c. 140 d. 90
e. 0·3 f. 0·9 g. 0·5 h. 0·3
4. a. 90 b. 160 c. 80 d. 5
e. 13 f. 4 g. 5·1 h. 151
5. a. 0·7 b. 0·9 c. 0·03 d. 0·03
e. 0·06 f. 0·004 g. 0·03 h. 0·07
6. 154 mb
7. a. 20 b. 500 c. 1250 d. 9
8. a. 0·0007 kg b. 0·7 g
9. a. 50000 b. 100000 c. 11100 d. -0·6
e. -0·3 f. 0·6 g. 0·1 h. 0·1

Ch 13 - Ex 7 (page 64)

3. a. 4·2 b. 3·5 c. 6·5 d. 8·0
2. a. 7·58 b. 9·63 c. 3·99 d. 5·40
3. a. 2·792 b. 7·505 c. 8·299 d. 25·403
e. 31·457 f. 6·081 g. 34·999 h. 3·000
4. a. 11·16 b. 1017·90 c. 1·56 d. 0·47
5. a. 1·672 b. 2·48 c. 22·861
d. 0·0700 e. 0·9 f. 0·0713
6. a. 85 p b. £1·36

Ch 13 - Ex 8 (page 65)

1. a. 3 b. 4 c. 3 d. 3
e. 2 f. 6 g. 3
2. a. 3 b. 3 c. 4 d. 3
e. 3 f. 6 g. 4 h. 3
i. 2 j. 7 k. 4 l. 2
m. 7 n. 6 o. 1 p. 2
3. a. 40 b. 700 c. 7000 d. 90000
e. 1000 f. 2000 g. 6 h. 0·1
i. 0·7 j. 0·003 k. 0·0007 l. 40
4. a. 510 b. 8100 c. 41000 d. 480000
e. 27 f. 38 g. 6·4 h. 0·13
i. 0·45 j. 0·0066 k. 0·043 l. 30
5. a. 4870 b. 63100 c. 85000 d. 781000
e. 7·21 f. 12·8 g. 0·287 h. 0·288
i. 0·00569 j. 0·0107
k. 0·0469 l 0·0400
6. 12000 g
7. 28100 ml
8. £660
9. £43·24
10. £2330
11. £21000
12. 400 kg

Ch 13 - Ex 9 (page 67)

1. £120·75
2. a. £1783·25 b. £36·75
3. a. 12 b. £127·20 c. £19·20
4. a. £3847 b. £597
5. a. £2072 b. £472·01
6. a. £30 b. £17
7. a. £16530·50 b. £4940·50
8. £356·20
9. a. £391·20 b. £334·15 c. £57·05
10. Jan – £9·42, Dan – £10·15 – Dan better
11. £10151·88
12. £354·55
13. a. £14·60 b. £87·60
14. a. £7·35 b. £73·50
15. a. 462 euros b. 64·68 euros
c. $672 d. 605 dollars
e. 1309 euros f. £1736
16. a. £600 b. £75 c. £480
d. £5·20 e. £300 f. £1200
17. £350
18. £1792
19. a. £1300 b. £270
20. a. 2310 euros b. £222·22

Answers to Chapter 17

Ch 17 - Ex 1 (page 73)

1. a. 5 b. 10 c. 15
d. 0·5 e. 35 f. 12·7
2. a. 5 b. 15 c. 1·7 d. 153
3. a. 7·5 b. 8·5 c. 7 d 1·35
4. a. 7 b. 13 c. 2·1
d. 1 e. 124 f. $^3/_4$
5. a. (i) 8 (ii) 14 (iii) 8 (iv) 1·5
b. (i) 7 (ii) 44 (iii) 4·3
(iv) 1 (v) 189 (vi) $^{11}/_{20}$
6. a. 55
b. mean = 10, median = 5, mode = 1
c. median
d. mean is distorted by the large (56) value
mode (1) does not indicate "centre" value
7. a. mean = 5, med = 2, mode = 2, range = 15
b. mean = 4·2, med = 4·3, mode = 4·3,
range = 3·4
c. mean = 104, med = 106, mode = 106,
range = 21
d. mean = 34·25, med = 31, mode = 30,
range = 32
e. mean = 15·5, med = 15·5, mode = 17,
range = 4
f. mean = 15400, med = 15000,
mode = 12000, range = 9000
8. a. 23 kg
b. mode = 40 kg, median = 48·5 kg
c. median – indicates "centre" value better
9. a. mean = 7·8, med = 8, mode = 8
b. 4
10. a. 12·9 b. 38·5
11. a. Bob chose the mode, Bill chose the mean
Ben chose the median
b. Bob's (mode)
12. 33 kg
13. 15 years
14. 50 years
15. a. Mean is only 15 b. 26
16. 47
17. 132·5
18. No – she would need to get 102% in her last
test

Ch 17 - Ex 2 (page 76)

1. a. frequences = 6, 5, 3, 6, 5, 1, 6
b. 12
c. see bar graph (labelled)
2. a. 10 b. 8
c. frequences – 2, 7, 4, 5, 5, 4, 3, 3
d. 33
e. 13 f see bar graph (labelled)
3. frequences = 4, 4, 11, 7, 3, 1
4. a. range = 31
b. probably 0 – 4
c. 0 - 4 (10) 5 - 9(6) 10 - 14(8)
15 - 19(9) 20 - 24(3) 25 - 29(3)
30 - 34(1)
d. see bar graph
5. a. 26 b. 0 - 4, 5 - 9, etc
c. 0 - 4(6) 5 - 9(7) 10 - 14(10)
15 - 19(7) 20 - 24(7) 25 - 29(3)
6. a.

0-4	5
5-9	5
10-14	8
15-19	10
20-24	5
25-29	4
30-34	3

b.

10-19	6
20-29	7
30-39	6
40-49	9
50-59	5
60-69	5
70-79	2

c.

100-109	2
110-119	4
120-129	7
130-139	3
140-149	3
150-159	6
160-169	8
170-179	3

d.

0-0·9	1
1-1·9	5
2-2·9	8
3-3·9	2
4-4·9	7
5-5·9	9
6-6·9	4

Ch 17 - Ex 3 (page 78)

1. a. (f x x) = 0, 7, 24, 15, 20 (tot = 66)
b. 30 c. 66 d. 2·2
2. a. (f x x) = 0, 6, 20, 9, 8 (tot = 43)
b. 25 c. 43 d. 1·72
3. a. 2·67 b. 4·9 c. 7·2
4. 7
5. a. 25 b. 8 c. 14·72 d. 14
6. a. 13 - 8 b. (i) 14 (ii) 4 (iii) 15 (iv) 15

14 - 14
15 - 3
16 - 9
17 - 11

Ch 17 - Ex 4 (page 80)

1. a. Cum freq tots = 4, 13, 24, 48, 64, 71, 73
b. 73 c. 64 d. week 4 e. 4
2. a. (i) C.F. Tots 2, 13, 30, 38, 42, 44, 45 (ii) 2
b. (i) C.F. Tots 1, 4, 8, 18, 39, 46, 50 (ii) 4
c. (i) C.F. Tots 7, 14, 24, 44, 59, 79, 84 (ii)13

Ch 17 - Ex 5 (page 81)

1. a. Angles 40°, 180°, 20°, 120°
b. see pie chart
2. a. Angles 40°, 160°, 144°, 16°
b. see pie chart
3. a. Angles 120°, 144°, 84°, 12°
b. see pie chart
4. a. Angles 180°, 100°, 20°, 60° – pie chart
b. Angles 200°, 120°, 35°, 5° – pie chart
5. a

America	5
France	10
Italy	6
Spain	15
U.K.	4

b. angles :–
45°, 90°, 54°, 135°, 36°
See pie chart

Ch 17 - Ex 6 (page 83)

1. a. 24, 26, 28 – 30, 31, 33
   44, 44, 45, 46, 47, 49
   50, 53, 54, 59 – 61, 64, 65, 66
   b. (i) 24 (ii) 66 c. 20
2. a. 31, 37, 39 b. 42, 42, 43, 46, 48, 48
   c. (i) 21 (ii) 68 d. 40's (forties)
3. a. £1·10, £1·20, £1·50, £1·70, £1·80,
   £2·00, £2·10, £2·20, £2·60,
   £3·40, £3·40, £3·40, £3·50, £3·80, £3·80,
   £4·90, £5·00, £5·30
   b. £3 level
   c. £3·40 d. 18
4. a. 1 | 6 = 16 secs
   b. (i) 9 secs, (ii) 47 secs c. 12
   d. 3 e. 21 secs f. 21 secs
5. a.

0	9
1	
2	2 2 6 7 8
3	1 2 6 9 9 9
4	0 1 2 5

   b. 3 | 1 = 31 metres
   c. (i) 45 m (ii) 9 m
   d. No one threw between 10 and 19 metres
   e. (i) 39 m (ii) 34 m
6. a. (i)

0	1 2 3 4 4 7 8 9 9 9
1	1 1 2 3 3 4 4 4 5 5 5 5 5 6 6 6 8 8 9
2	0 0 2 3 3 5 5 5 7
3	0 2 4

   2 | 0 = 20

   (ii) mode = 15 median = 15

   b. (i)

1	0 1 9
2	1 2 2 4 5 7 9
3	0 1 3 4 4 4 7
4	1 2 5 5 7 7 9
5	2 2 2 2 3 9 9
6	0 0 1 1 2 2 3 5 8

   4 | 1 = 41

   (ii) mode = 52 median = 45

   c. (i)

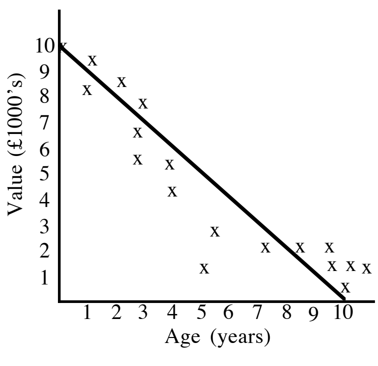
10	1 4 7
11	0 1 1 9
12	3 3 4 5
13	2 2 5 7 8 9
14	0 0 2 3
15	1 2 3 4 4 6 7 9
16	0 0 1 4 4 4 5

   12 | 3 = 123

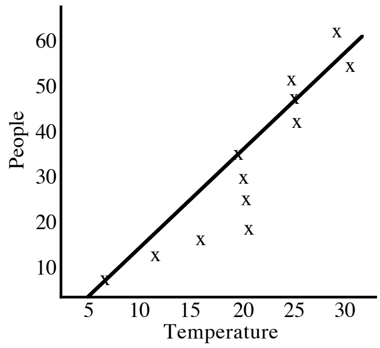
   (ii) mode = 164 median = 140

Ch 17 - Ex 7 (page 85)

1. Sam (20 kg, 150 cm) Jim (25 kg, 130 cm)
   Tim (35 kg, 165 cm) Gary (40 kg, 150 cm)
   Dave (50 kg, 140 cm) Joe (50 kg, 160 cm)
   Bob (60 kg, 170 cm)
2. a. (i) Bill (ii) Bill (iii) Paul (iv) Paul
   b. Bill (3, 10 kg) May (4, 15 kg)
   Ally (6, 12·5 kg) Mary (8, 20 kg)
   Tod (10, 20 kg) Tam (10, 27·5 kg)
   Paul (12, 30 kg)
3. a. yes b. yes c. yes
   d. yes e. yes f. yes
4. a. 80 soups b. 5°C
5. a/b. See graph with line
   c. strong positive d. line
   e. about £3·00
6. a.



b.



Ch 17 - Ex 8 (page 87)

1. Impossible 2. Evens
3. Unlikely 4. Likely
5. Unlikely 6. Certain
7. Impossible 8. Certain

Ch 17 - Ex 9 (page 88)

1. P(Black) = $\frac{1}{3}$
2. a. $\frac{1}{6}$ b. $\frac{1}{3}$ c. $\frac{1}{2}$ d. 0
3. a. $\frac{1}{6}$ b. (i) $\frac{1}{6}$ (ii) $\frac{1}{2}$ (iii) 0
4. a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{5}{12}$ d. $\frac{1}{2}$
5. a. $\frac{1}{5}$ b. $\frac{7}{10}$ c. $\frac{1}{10}$ d. $\frac{9}{10}$
6. a. $\frac{1}{3}$ b. $\frac{2}{9}$ c. $\frac{1}{6}$ d. $\frac{1}{9}$
   e. $\frac{1}{12}$ f. $\frac{1}{18}$ g. $\frac{1}{36}$ h. $\frac{7}{18}$
   i. $\frac{11}{18}$
7. a. $\frac{5}{11}$ b. $\frac{2}{11}$ c. $\frac{5}{11}$ d. $\frac{6}{11}$
8. $\frac{4}{7}$
9. a. (H,H,H) (H,H,T) (H,T,H) (T,H,H)
   (T,T,H) (T,H,T) (H,T,T) (T,T,T)
   b. (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$
10. a. $\frac{1}{20}$ b. $\frac{2}{5}$ c. $\frac{1}{2}$ d. $\frac{2}{5}$
11. 3

Ch 17 - Ex 10 (page 89)

1. a. shows bias
   b. shows bias
   c. shows bias
2. a. see questionnaire
   b. see questionnaire
   c. see questionnaire
3. a. discrete
   b. continuous
   c. continuous
   d. continuous

Answers to Chapter 19

Ch 19 - Ex 1 (page 92)

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$
2. a. £2·50 b. £42 c. 70p d. 32p
   e. £315 f. £12 g. £3·60 h. £1·20
   i. £7200 j. £250000
   k. £12·40 l. 7p m. £220 n. 9p
   o. £1·25 p. £16 q. £1·50 r. 5p
3. 98
4. a. 220 b. 99 c. 341
5. a. $\frac{28}{100}$ = 0·28 b. $\frac{35}{100}$ = 0·35
   c. $\frac{61}{100}$ = 0·61 d. $\frac{23}{100}$ = 0·23
   e. $\frac{58}{100}$ = 0·58 f. $\frac{4}{100}$ = 0·04
   g. $\frac{12}{100}$ = 0·12 h. $\frac{7}{100}$ = 0·07
   i. $\frac{12·5}{100}$ = 0·125 j. $\frac{2·5}{100}$ = 0·025
6. a. $\frac{3}{20}$ b. $\frac{2}{5}$ c. $\frac{9}{20}$ d. $\frac{7}{10}$
   e. $\frac{3}{20}$ f. $\frac{3}{4}$ g. $\frac{1}{50}$ h. $\frac{7}{20}$
   i. $\frac{1}{20}$ j. $\frac{12}{25}$ k. $\frac{3}{5}$ l. $\frac{19}{20}$
7. a. 18% b. 20% c. 16% d. 40%
   e. 35% f. 65% g. 96% h. 12·5%
   i. 28% j. 62·5% k. 75% l. 87·5%
   m. 95% n. 50%
8. a. 70% b. 46% c. 25% d. 30%
9. a. £4·80 b. £12·00 c. £16·80 d. £2240
   e. £3·06 f. £28·80 g. £1·90 h. £2·80
   i. 36p j. £43·75
10. 203 mm
11. 1·625 million (1625000)
12. £888·25

Ch 19 - Ex 2 (page 94)

1. 84p 2. £1·62
3. £8·20 4. 450 ml
5. 9 6. 42600
7. a. £61·20 b. £372 c. £315 d. £2160
   e. £951·75 f. £39488
8. 32200 euros 9. £17000
10. a. $143750 b. $172500
11. £2898·72
12. 5 years (£3017)
13. 6 years
14. a. £515 b. £530·45
15. £877·50

Ch 19 - Ex 3 (page 97)

1. a. 50% b. 25% c. 20% d. 10%
   e. 12·5% f. 5% g. 2% h. 1%
2. a. 50% b. 40% c. 25% d. 1%
   e. $66\frac{2}{3}$% f. 5%
3. a. 20% b. 80%
4. 24%
5. 8·3%
6. a. Chem – 70%, Physics – 72% (better)
   b. Only in Chemistry
7. 12·5%
8. 60%
9. 70%
10. 5900%

Ch 19 - Ex 4 (page 98)

1. £40
2. £75
3. £7·50
4. a. 50 kg b. 70 kg c. 90 kg
5. £500
6. £80
7. £132000
8. a. £150 b. £26·25 c. £65
9. a. £235 b. £456 c. £1220 d. £2350
10. £80

Answers to Chapter 23

Ch 23 - Ex 1 (page 101)

1. a. 18 cm^2 b. 36 cm^2 c. 32 cm^2
   d. 90 cm^2 e. 25 cm^2 f. 6 cm^2
2. a. 25 cm^2 b. 196 cm^2 c. 0·25 cm^2
3. a. 25 cm^2 b. 16 cm^2 c. 30 cm^2
   d. 6 cm^2
4. a. 64 cm^2 b. 2400 cm^2 c. 5 m^2
   d. 225 m^2 e. 51 cm^2 f. 26 m^2
   g. 2200 mm^2 h. 40·5 cm^2
5. a. 500 cm^2 b. 280 ft^2
   c. 410 cm^2 d. 298 cm^2

Ch 23 - Ex 2 (page 104)

1. a. 20 cm^2 b. 27 cm^2 c. 55 cm^2
   d. 68 cm^2 e. 60 cm^2 f. 175 cm^2
   g. 460 cm^2 h. 787·5 mm^2 i. 4182 mm^2
2. a. 575 m^2 and 592 m^2 (Triangle 2)
3. 675 cm^2
4. 2·85 m^2
5. a. 275 mm^2 b. 175 cm^2 c. 25 cm^2
6. 396 cm^2
7. obtuse – 1·08 m^2, acute – 1·08 m^2
8. a. 18 cm b. 8 cm

Ch 23 - Ex 3 (page 107)

1. a/b. See drawing
   c. 48 cm^2 d. 24 cm^2
2. a. 20 cm^2 b. 60 cm^2 c. 30 cm^2
   d. 85 cm^2 e. 84·5 cm^2 f. 187·5 cm^2
3. a. 48 cm^2 b. 70 cm^2 c. 40·5 cm^2
   d. 675 cm^2 e. 12·3 cm^2 f. 16·92 cm^2

4. a. 24 cm^2 b. 49·5 cm^2 c. 10·5 cm^2
d. 21 cm^2 e. 500 cm^2 f. 5400 cm^2
5. 270 cm^2
6. 6·375 m^2
7. 112·5 cm^2
8. 108 cm^2
9. 576 cm^2
10. 0·96 cm^2
11. 3·78 cm^2
12. 196 cm^2
13. a. 500 cm^2 b. 48 m^2 c. 7200 cm^2
14. 9 cm

Ch 23 - Ex 4 (page 110)

1. a. 75 cm^2
2. a. 63 cm^2 b. 48 cm^2 c. 70 cm^2
d. 78 cm^2 e. 240 cm^2
f. 33 cm^2 g. 102 cm^2
h. 35 cm^2 i. 34·125 cm^2
3. 4400 cm^2 4. 108 m^2 5. 9 m^2
6. 7700 cm^2 7. 180 cm^2 8. 5·98 m^2
9. a. AFDC and ABDE
b. 360 cm^2 and 192 cm^2
10. 14 cm

Ch 23 - Ex 5 (page 112)

1. a. 26 cm^2 b. 40 cm^2 c. 133 cm^2
d. 67·5 cm^2 e. 660 mm^2 f. 2112 cm^2
2. a. 840 cm^2 b. 600 cm^2 c. 252 cm^2
d. 11 m^2 e. 585 cm^2 f. 700 cm^2
3. 4·4 m^2
4. 12·5 cm^2
5. 15300 cm^2
6. 528 cm^2
7. 8 cm
8. 16·5 cm

Ch 23 - Ex 6 (page 114)

1. a. 576 cm^2 b. 162 cm^2 c. 336 cm^2
d. 3240 cm^2 e. 15300 cm^2 f 1250 cm^2
2. 120·5 cm^2
3. a. 115 cm^2 b. 48 cm^2 c. 4·25 cm^2
d. 82 cm^2 e. 171·5 cm^2 f. 4·5 m^2
4. 225 cm^2
5. 8·25 m^2
6. a. 262 cm^2 b. 1567·5 cm^2

Answers to Chapter 29

Ch 29 - Ex 1 (page 118)

1. 21·98 cm
2. a. 9·42 cm b. 31·4 cm c. 1·57 cm
d. 26·06 cm
3. a. 32·97 cm b. 188·4 cm c. 15·07 cm
4. 62·8 cm
5. a. 12·56 cm b. 34·54 cm c. 3·14 cm
d. 94·2 cm
6. a. 62·8 mm b. 100·48 cm c. 160·14 cm
7. 47·1 cm
8. 20·56 m
9. 6·28 m
10. A = 44 cm B = 43 cm C = 43·96 cm
11. a. 6·455 m b. 294·2 m c. 48·56 cm
d. 285·6 mm
12. a. 25·7 m b. 36·54 m
13. 12·078 m (1207·8 cm)

Ch 29 - Ex 2 (page 121)

1. 7 cm
2. a. 9 cm b. 18 cm c. 3295 mm
d. 1 mm e. 5 mm
3. a. 4 cm b. 60 mm c. 72 m
4. a. 2 cm b. 30 cm c. 36 m
5. 25 cm
6. 75 m
7. a. 19·1 cm b. 1·0 m c. 15·9 cm
d. 2·4 cm e. 0·16 mm
8. 95·5 cm
9. a. 11·46 mm b. 12·56 mm

Ch 29 - Ex 3 (page 123)

1. 28·26 cm^2
2. a. 153·86 cm^2 b. 254·34 cm^2
c. 176·6 mm^2
3. a. 346·19 mm^2 b. 38·47 cm^2
c. 2863·81 cm^2
4. a. 50·24 cm^2 b. 3·14 m^2
c. 314 mm^2
5. a. 3846·5 cm^2 b. 530·66 cm^2
c. 1·1304 m^2 d. 1·76625 m^2
6. a. 11304 cm^2 b. 28800 cm^2
c. 17496 cm^2
7. 100·48 m^2

Ch 29 - Ex 4 (page 125)

1. a. 226·08 cm^2 b. 76·93 mm^2 c. 47·49 m^2
2. a. 61·68 cm b. 35·98 mm c. 28·27 m
3. a. 19·625 cm^2 b. 50·24 cm^2 c. 0·196 m^2
4. a. 17·85 cm b. 28·56 cm c. 1·785 m
5. a. (i) area = 129·12 cm^2
(ii) perimeter = 46·56 cm
b. (i) area = 232·26 cm^2
(ii) perimeter = 89·42 cm
c. (i) area = 114·24 cm^2
(ii) perimeter = 44·56 cm
d. (i) area = 79·625 cm^2
(ii) perimeter = 41·85 cm
6. 19075·5 cm^2
7. 13·76 cm^2
8. a. 58·875 cm^2 b. 12·56 cm^2
9. 10000 cm^3
10. 14·13 cm^2
11. 31·4 cm
12. The side of the square is 20 cm
(diameter needed is 20·3 cm) – too big

Answers to Chapter 31

Ch 31 - Ex 1 (page 129)

1. a. $7\frac{1}{2}$ b. $3\frac{5}{6}$ c. $1\frac{4}{9}$ d. $3\frac{2}{5}$
2. a. $3\frac{2}{3}$ b. $5\frac{1}{4}$ c. $4\frac{5}{6}$ d. $4\frac{1}{2}$
e. $6\frac{2}{5}$ f. $8\frac{1}{8}$ g. $7\frac{3}{10}$ h. $1\frac{11}{20}$
3. a. $2\frac{3}{4}$ b. $4\frac{3}{5}$ kg c. $2\frac{5}{6}$ *l*
4. $3\frac{1}{3}$
5. a. $2\frac{1}{2}$ b. $4\frac{1}{2}$ c. $2\frac{1}{4}$ d. $3\frac{1}{5}$
e. $3\frac{1}{3}$ f. $4\frac{3}{4}$ g. $2\frac{1}{4}$ h. $1\frac{3}{4}$
6. a. 4 b. 8 c. 3 d. 11
e. $\frac{11}{4}$
7. a. 12 b. 2 c. 14 d. $\frac{14}{3}$
8. a. $\frac{17}{5}$ b. $\frac{13}{10}$ c. $\frac{23}{8}$ d. $\frac{97}{9}$
9. a. $\frac{13}{3}$ b. $\frac{18}{7}$ c. $\frac{44}{5}$ d. $\frac{29}{10}$
10. a. $\frac{3}{2}$ b. $\frac{23}{4}$ c. $\frac{32}{5}$ d. $\frac{57}{8}$
e. $\frac{32}{3}$ f. $\frac{39}{10}$ g. $\frac{23}{9}$ h. $\frac{48}{7}$
11. a. 4 b. 10 c. 7 d. 21
12. a. 9 b. 5 c. 7 d. 20
13. a. 8 b. 7 c. 17 d. 14
14. $7\frac{1}{5}$

Ch 31 - Ex 2 (page 131)

1. a. $\frac{4}{5}$ b. $\frac{2}{9}$ c. $\frac{2}{5}$ d. $\frac{3}{4}$
2. a. $\frac{6}{7}$ b. $\frac{2}{3}$ c. $\frac{1}{2}$ d. $1\frac{3}{5}$
e. $\frac{4}{11}$ f. $1\frac{1}{3}$ g. 1 h. $\frac{1}{2}$
3. a. 6 b. $4\frac{1}{2}$ c. $7\frac{2}{3}$ d. $4\frac{2}{9}$
e. $5\frac{1}{2}$ f. $7\frac{5}{7}$ g. $4\frac{5}{9}$ h. $14\frac{2}{5}$
4. $\frac{1}{2}$ km
5. $7\frac{1}{4}$
6. $8\frac{2}{5}$ kg
7. a. $3\frac{3}{5}$ m b. $4\frac{1}{2}$ *l*
c. $3\frac{2}{3}$ km d. $1\frac{4}{5}$
e. $38\frac{3}{7}$ kg
8. a. $3\frac{1}{5}$ ft b. $17\frac{2}{5}$ ft
9. a. $4\frac{1}{2}$ tonne b. $5\frac{1}{8}$ tonne
c. $5\frac{3}{4}$ tonne
10. $7\frac{1}{2}$ inches

Ch 31 - Ex 3 (page 133)

1. a. $1\frac{1}{12}$ b. $\frac{2}{15}$ c. $\frac{1}{8}$ d. $1\frac{11}{21}$
2. a. $\frac{13}{15}$ b. $\frac{1}{4}$ c. $1\frac{7}{24}$ d. $1\frac{3}{10}$
e. $\frac{1}{2}$ f. $\frac{1}{12}$ g. $1\frac{1}{10}$ h. $\frac{5}{18}$
3. a. $1\frac{1}{12}$ b. 0 c. $\frac{23}{30}$
4. a. $7\frac{5}{6}$ b. $2\frac{5}{12}$ c. $6\frac{5}{8}$ d. $8\frac{1}{10}$
e. $1\frac{1}{2}$ f. $5\frac{7}{24}$ g. $8\frac{9}{20}$ h. $4\frac{2}{5}$
i. $7\frac{1}{9}$ j. $6\frac{11}{20}$ k. $5\frac{1}{2}$ l. $\frac{1}{12}$
5. a. $3\frac{2}{3}$ b. $1\frac{3}{5}$ c. $2\frac{3}{8}$ d. $\frac{2}{5}$
6. a. $2\frac{4}{5}$ b. $2\frac{3}{7}$ c. $4\frac{1}{6}$ d. $1\frac{2}{5}$
e. $\frac{3}{10}$ f. $5\frac{5}{8}$ g. $5\frac{2}{7}$ h. $4\frac{2}{3}$
7. $2\frac{5}{8}$ m
8. $4\frac{2}{5}$ km
9. a. $3\frac{9}{10}$ b. $1\frac{31}{40}$ c. $5\frac{9}{20}$
10. a. $2\frac{7}{10}$ b. $4\frac{23}{30}$ c. $1\frac{3}{4}$ d. $1\frac{5}{8}$
e. $2\frac{5}{6}$ f. $4\frac{9}{14}$ g. $4\frac{19}{30}$ h. $2\frac{23}{30}$
11. $5\frac{3}{4}$ kg
12. $\frac{3}{8}$
13. $2\frac{3}{4}$ hours
14. $\frac{7}{12}$ kg
15. $4\frac{2}{5}$ m

Ch 31 - Ex 4 (page 136)

1. a. $\frac{8}{15}$ b. $\frac{5}{18}$ c. $\frac{5}{8}$
2. a. $\frac{4}{15}$ b. $\frac{1}{2}$ c. $\frac{4}{21}$ d. $\frac{1}{4}$
e. $\frac{3}{10}$ f. $\frac{1}{3}$ g. $\frac{11}{40}$ h. $\frac{1}{5}$
3. $\frac{5}{16}$ m^2
4. $\frac{3}{10}$
5. a. $3\frac{1}{2}$ b. $7\frac{1}{12}$ c. $4\frac{2}{3}$
6. a. $3\frac{8}{9}$ b. $10\frac{1}{2}$ c. 20 d. 6
e. 10 f. $15\frac{7}{12}$ g. $5\frac{19}{30}$ h. $11\frac{1}{10}$
i. $6\frac{5}{12}$ j. $26\frac{2}{5}$ k. 9 l. $5\frac{1}{5}$
7. $12\frac{3}{16}$ sq inches
8. $9\frac{3}{8}$ kg
9. $52\frac{1}{2}$ kg
10. $21\frac{7}{16}$ seconds

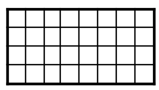
Ch 31 - Ex 5 (page 138)

1. a. $1\frac{1}{4}$ b. $1\frac{1}{4}$ c. $\frac{9}{10}$
2. a. $\frac{3}{5}$ b. $1\frac{3}{7}$ c. $\frac{1}{2}$ d. $\frac{3}{8}$
e. $\frac{9}{20}$ f. $\frac{2}{3}$ g. $1\frac{1}{10}$ h. $1\frac{1}{3}$
i. $1\frac{1}{6}$ j. $1\frac{11}{24}$ k. $1\frac{5}{27}$ l. $1\frac{2}{5}$
3. $\frac{3}{4}$
4. $5\frac{1}{3}$
5. a. $1\frac{7}{8}$ b. $3\frac{1}{3}$ c. $\frac{5}{6}$
6. a. $2\frac{2}{9}$ b. $1\frac{7}{15}$ c. $1\frac{19}{33}$ d. $\frac{27}{56}$
e. $1\frac{13}{72}$ f. $2\frac{8}{9}$ g. $\frac{12}{35}$ h. $5\frac{1}{4}$
i. $3\frac{1}{3}$ j. $\frac{6}{7}$ k. 4 l. 3
7. $4\frac{1}{2}$ inches
8. a. $2\frac{1}{4}$ kg b. $7\frac{5}{16}$ kg
9. $1\frac{11}{24}$ m
10. $5\frac{5}{9}$ minutes

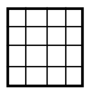
Answers to Chapter 37

Ch 37 Ex 1 (Page 141)

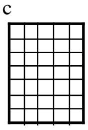
1. a



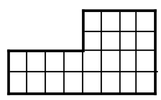
b



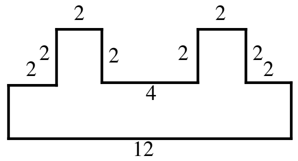
c



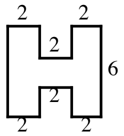
d



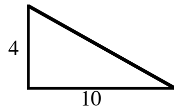
e



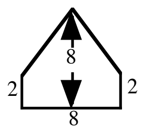
f



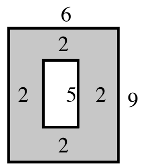
g



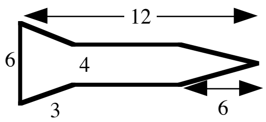
h



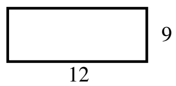
i



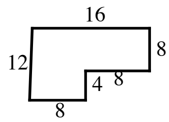
j



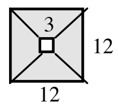
2. a



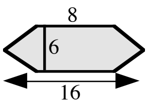
b



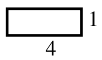
c



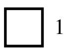
d



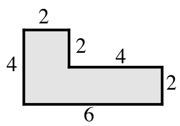
e



f



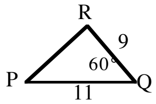
g

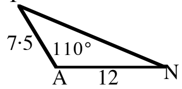


3. a a = 4·5 b c = 3
   b = 4 d = 5
c g = 2 d j = 27
   h = 4·5
e k = 12 f v = 22
   p = 10 w = 9
g x = 50 h q = 90
   y = 5 u = 1·3

Ch 37 Ex 2 (Page 146)

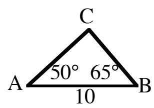
1. Drawing of triangle
2. Drawing of 3 triangles
3. a b

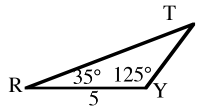




4. Drawing of triangle.
5. Drawing of 3 triangles.

6. a b

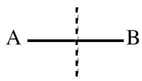




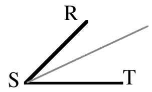
7. Drawing of triangle.
8. Drawing of 3 triangles.

Ch 37 Ex 3 (Page 149)

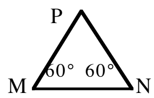
1.



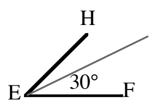
2.



3.



4. Drawing of Rectangle.
5. Drawing of Rectangle.
6.



7. Drawing of Rhombus.
8. a/b Drawing of Rhombus/Kite.

Ch 37 Ex 4 (Page 150)

1. a 4 cm / 40 cm b 9 cm / 450 cm
   c 6·5 cm / 65 m d 9·3 cm / 4·65 km
2. a 4·8 cm b 6 cm
   c 10·8 cm d 16·3 cm
3. a 2·4 km b 3 km
   c 5·4 km d 8·15 km
4. 4·575 km
5. 2·14 km
6. 42·5 m
7. a i 8 cm by 4 cm ii 3 cm by 1·4 cm
   iii 0·8 cm by 1·4 cm
   b i 16 m by 8 m ii 6 m by 2·8 m
   iii 1·6 m by 2·8 m
8. 6 cm b 1 : 200000 c 14 km

Ch 37 Ex 5 (Page 152)

1. a 190° b 265°
   c 330° d 010°
   e 067° f 114°
2. a F-E 2·9 km E-T 3·4 km F-T 4·25 km
   b i 065° ii 162° iii 120°
   c i 245° ii 342° iii 300°
3. a i 0·75 km ii 1·02 km
   b 0·69 km
   c 132° d 312°
4. 76°
5. angle ABC = 75°
   angle ACB = 62°
   angle BAC = 43°
6. Proof that triangle is isosceles.
7. a 248° b 340° c 295°

Answers to Chapter 41

Ch 41 Ex 1 (Page 157)

1. a 2 : 3 b 3 : 2
2. a 4 : 3 b 4 : 1 c 3 : 1
   d 1 : 2 e 1 : 2 f 2 : 1
3. a 122 : 59 b 59 : 169
   c 169 : 59 d 169 : 122
4. a 19 : 21 b 11 : 21 c 40 : 11 d 11 : 51
5. a 4 : 3 b 2 : 7 c 1 : 3 d 6 : 7
   e 4 : 5 : 12

Ch 41 Ex 2 (Page 158)

1. a 2 : 3 b 5 : 7 c 10 : 11
   d 1 : 9 e 100 : 121
2. a 1 : 12 b 1 : 4 c 1 : 6 d 1 : 8
   e 1 : 11 f 11 : 4 g 2 : 9 h 1 : 3
   i 1 : 5 j 1 : 3 k 3 : 22 l 1 : 43
   m 5 : 18 n 1 : 9 o 1 : 50 p 1 : 6
   q 50 : 1 r 5 : 1 s 1:2:5 t 2 : 12 : 3
3. a 2 : 3 b 2 : 1 c 1 : 4
   d 5 : 3 e 1 : 1
4. a 16 : 9 b 70 : 11
5. 1 : 6
6. a 9 : 5 b 3 : 10 c 9 : 6 : 5
7. a 1 : 6 b 1 : 15 c 1 : 12 d 1 : 4
   e 1 : 36 f 1 : 48 g 1 : 75 h 1 : 64
   i 1 : 91 j 1 : 300 k 1 : 2 l 4 : 1
8. a 1 : 6 b 2 : 15 c 1 : 20 d 1 : 25
   e 2 : 7 f 9 : 5 g 7 : 6 h 25 : 162
   i 3 : 250 j 3 : 44 k 1 : 200 l 4 : 5
9. a 2 : 1 b 1 : 2 c 1 : 5 d 5 : 2
10. a 1 : 2 b 5 : 3 c 10 : 1 d 1 : 4
   e 100 : 1 f 100 : 1 g 15 : 1
   h 10000 : 1 i 20000 : 1
   j 1 : 48 k 7 : 30 l 1 : 1
11. a 2 : 125 b 1 : 5

Ch 41 Ex 3 (Page 160)

1. a 36 b 20 c 33
2. a 20 b 18
3. a £350 b £150
4. a 350 b 744
5. a 27600 b 330000
6. a 10 m b 20 cm
7. a light b very dark
   c light d dark
   e purple f light

Ch 41 Ex 4 (Page 161)

1. £80 £120
2. £4000 £14000
3. a £16000 £20000 b £7000 £5000
   c £2·15 £6·45 d 176 euros 132 euros
   e $650000 $350000
4. a 2 : 1 b £160 & £80
5. £120000
6. £40
7. a £20 : £30 : £50 b 120$: 150$: 330$
   c 30 : 10 : 35
8 a 200 km b 400 km c 1400km
9. Henry 20 *l* Seb 10 *l* Tim 5 *l*

Ch 41 Ex 5 (Page 163)

1. 70p
2. a 14p b £5 c £8
   d 10p e 30p f £1·80
3. 20
4. 6 km/hr
5. 1·7 euros to the £.
6. 1·50 kg
7. 2·5
8. 16
9. £24 £22 Tim

Ch 41 Ex 6 (Page 164)

1. £84·70
2. £8·10
3. 81
4. 13500 cm^2
5. 375
6. a £2·40 b £9·10
7. a 50 b 350 c 3000 d 180000
8. a No b No c No d Yes
9. a 50 min b 300
10. a 52 min b 54
11. £6
12. a £22·50 b 3·5 hr
   c 8·4 kg d £3·00
   e £14·00 f 1950 tonnes
13. a £90·00 b £120

Ch 41 Ex 7 (Page 166)

1. a

1	2	3	4	5	6
30	60	90	120	150	180

   b Diagram with straight line passing through the origin
   c If you buy 0 pots => it costs you cost 0 p.

2. a

1	2	3	4	5	6
40	80	120	160	200	240

   b Diagram with straight line passing through the the origin
   c (ii) yes

3. a

1	2	3	4	5
10	20	30	40	50

   b Diagram with straight line passing through the origin
   c 80 km

4. Not a straight line => NOT direct Proportion
5. a

100	200	300	400	500
2	4	6	8	10

   b Yes - lies on straight line through the origin
   c £20

6. a No b No c Yes
   d No e No f Yes
7. For Q6c and Q6f - straight line graph thro' O

Ch 41 Ex 8 (Page 168)

1. 10 hr
2. 9·6 hours (or 9 hr 36 min)
3. 400 km/hr
4. 5 hr
5. 37·5 min
6. 4 days
7. 4 weeks
8. 10 more men

Ch 41 Ex 9 (Page 169)

1. 96p
2. 12·5 min
3. 6·75 km
4. 44 km/hr
5. £225
6. 7
7. 1 hour
8. 8 days
9. 38

Answers to Chapter 43

Ch 43 Ex 1 (Page 171)

1. a 6 b 13 c 3 d 0
   e 9 f 6 g 15 h 90
   i –2 j 5 k –11 l 50
   m –3 n 0 o –7 p –7
   q 0 r –30
2. a 9 b 6 c 4 d 6
   e 8 f 1 g 2·5 h 0
   i 0·5 j 50 k 18 l 4·5
   m $2\frac{3}{4}$ n $4\frac{4}{5}$ o $2\frac{6}{7}$ p $3\frac{1}{2}$
   q $\frac{3}{4}$ r $8\frac{1}{3}$
3. a 4 b 5 c 6 d 2
   e 5 f 10 g 7 h 2
   i 7 j 7 k 1 l 10
   m 3 n 20 o 8 p 29
   q –1 r 3·5 s –4 t 2·5
   u 5·25

Ch 43 Ex 2 (Page 173)

1. a. 5 b 7
2. a 3 b 5 c 6 d 5
   e 4 f 4 g 7 h 3·5
   i $2\frac{2}{3}$ j $1\frac{3}{4}$ k $4\frac{1}{5}$ l $1\frac{1}{3}$
3. a 6 b 5 c 10
   d 6 e 6·5 f 6
   g 5 h –3 i 7
4. a $3x = x + 24$ b 12
5. a $4x + 5 = 2x + 21$ b 8

Ch 43 Ex 3 (Page 174)

1. a 3 b 3 c 9 d 3
   e 4 f 0 g 6 h 5
   i 4·5 j 4 k 7 l –2
2. a 4 b 5 c 1 d 2
   e 5 f 3 g 5 h 5
   i 7 j 9 k 5 l –20
3. a 8 b 5 c 3
   d 6 e 6 f 3
   g 4 h 3 i 8
   j 6 k 3 l 2
   m 6 n –15

Ch 43 Ex 4 (Page 175)

1. a. 18 b 10
2. a 8 b 8 c 24 d 9
   e –5 f $5\frac{1}{3}$ g $3\frac{1}{3}$ h $1\frac{3}{5}$
   i $\frac{5}{9}$ j 6·5 k –2·5 l $7\frac{3}{11}$
   m 12 n 20 o 24 p $1\frac{1}{6}$
   q $\frac{2}{5}$ r –3·6

Ch 43 Ex 5 (Page 176)

1. a. 18 b $\frac{36}{31}$
2. a 13 b 17 c 11 d 10
   e 8 f –14 g 6 h 7
   i 14 j 6 k 8 l 6
   m 7 n 6 o 5 p 6
   q 14 r 3

Ch 43 Ex 6 (Page 177)

1. a $x > 2$ b $x < 7$ c $x \le 17$
   d $x \ge 13$ e $x \le 6$ f $x \ge 8$
2. a $x < 5$ b $x > 6$ c $x < 7$
   d $x \ge 6$ e $x \le 5$ f $x > 12$
3. a $x < 6$ b $x > 4$ c $x < 3$ d $x \ge 7$
   e $x \le 7$ f $x > 9$ g $x \le 0$ h $x < 5$
   i $x > 6$ j $x < 2$ k $x \ge 2$ l $x \le 4·5$
   m $x < 7$ n $x > 10$ o $x \ge 15$ p $x \le 6$
   q $x > 1$ r $x \le 0$ s $x < 4$ t $x > 4$
   u $x \ge 16$ v $x \le 3$ w $x < 6$ x $x \le 4$

Answers to Chapter 47

Ch 47 Ex 1 (page 179)

1. a 0430 b 0515 c 0800 d 1440
   e 1745 f 1900 g 0625 h 2155
   i 0135 j 0733 k 1200 l 0040
2. a 3.50 am b 10.45 am
   c 7.05 am d 3.30 pm
   e 5.35 pm f 11.45 pm
   g 1.10 am h 6.25 pm
   i 8.20 pm j 1.02 pm
   k midnight l 6.49 am
3. a 3 hrs b 3 hrs 10 min
   c 6 hrs 15 min d 2 hrs 55 min
   e 1 hr 40 min f 5 hrs 40 min
   g 2 hrs 5 min h 1hr 35 min
   i 1hr 15 min j 1hr 20 min

Ch 47 Ex 2 (Page 180)

1. a 18 km b 120 km c 15 km d 66 km
   e 840 m f 24 m g 54 m h 135 km
2. a 120 m b 12·5 m c 70 m
   d 35 km e 31500 miles
3. a 600 m b 9 m c 135 m
   d 120 km e 14 km f 28 km

Ch 47 Ex 3 (Page 181)

1. a 5 mph b 5 km/hr
   c 44 mph d 75 km/hr
   e 56 mph f 35 km/hr
2. a 25 km/hr b 70 mph
   c 30 m/s d 6·5 km/hr
   e 8000 km/hr f 50 m/s
   g 37·5 ft/s h 70 yd/m
3. a 230 mph b 60 km/hr
   c 80 mph d 8 mph
   e 65 cm/hr f 9·5 cm/hr
   g 42 mph h 61 mph

Ch 47 Ex 4 (Page 182)

1. a 1 hr b 6 hrs c 90 sec d 6 hrs
   e 1 min f 3 sec g 4 hrs h 5 hrs
2. a 2 hr 30 min b 5 hr 15 min
   c 3 hr 45 min d 6 hr 30 min
   e 8 hr 15 min f 3 hr 30 min
   g 1 hr 45 min h 15 min
3. a $2\frac{1}{2}$ hr 2·5 hr b $4\frac{1}{4}$ hr 4·25 hr
   c $3\frac{3}{4}$ hr 3·75 hr d $2\frac{1}{4}$ hr 2·25 hr
   e $5\frac{1}{2}$ hr 5·5 hr f $8\frac{3}{4}$ hr 8·75 hr
4. a 1 hr 30 min b 1 hr 15 min
   c 1 hr 45 min d 30 min
   e 2 hr 15 min f 1 hr 15 min
   g 15 min h 2 hr 45 min

Ch 47 Ex 5 (Page 183)

1. a 45 km/hr b 4 hr
   c 111 miles d 40 km/hr
   e 70 m f 2 hr 15 min
2. 36 km/ hr
3. 1 hr 15 min
4. 133 km
5. 70 mph
6. 1hr 45 min
7. 11200 km
8. a 3m/hr b twice as fast
9. 360 miles
10. 9 min
11. a 48 mph b 54 mph
    c 51 mph Andy fastest !
12. a 70 km/hr 80 km/hr 100 km/hr
    b 80 km/hr

Ch 47 Ex 6 (Page 185)

1. a 0·6 hr b 0·4 hr c 0·2 hr
   d 0·7 hr e 0·3 hr f 0·9 hr
   g 0·25 hr h 0·15 hr i 0·55 hr
2. a 0·83 h b 0·22 h c 0·33 h d 0·97 h
   e 0·67 h f 0·13 h g 1·17 h h 1·67 h
3. a 4·2 h b 2·6 h c 1·4 h
   d 3·55 h e 6·85 h f 3·3 h
   g 5·35 h h 4·33 h i 8·05 h
4. 144 km
5. a 14·4 m b 9 m c 24 m
   d 10·8 m e 5·4 m f 32·4 m
6. a 15 km b 24 m c 7 km
   d 55 m e 6 m
7. 48 km 44 km George, by 4 km
8. 67·2 km
9. a 1728 m b 60 m c 40 km d 377 km
10. 80 km/hr
11. a 250 mph b 80 km/hr
    c 640 mph d 120 mph
    e 25km/hr f 60 mph
    g 6000 mph h 48 mph

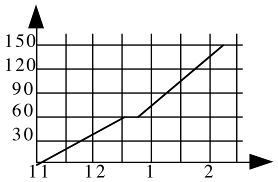
Ch 47 Ex 7 (Page 187)

1. a 39 b 24 c 18
   d 15 e 57 f 40
2. 4 hr 42 min
3. a 2 hr 18 min b 4 hr 30 min
   c 2 hr 39 min d 3 hr 36 min
   e 1 hr 51 min f 6 hr 54 min
   g 2 hr 40 min h 5 hr 50 min
   i 7·5 min
4. a 3 hr 45 min b 2 hr 24 min
   c 1 hr 40 min

5. a 3·2 hrs b 3 hr 12 min
6. a 2·4 hrs b 2 hr 24 min
7. a 3 hr 18 min b 20 min
   c 21 min d 2 hr 45 min
8. a 45 min b 40 min
   c 2 hr 15 min
9 a 8 m/s b 28·8 km/hr
10. a 36 b 72 c 1080 d 45
11. car, by 1 km/hr

Ch 47 Ex 8 (Page 189)

1. a 100 km b 1 hour c 2 pm (1400)
   d 50 km/hr 0 km/hr 20 km/hr
2. a 45 min b 240 mph
   c 200 mph d slowed
3. a 40 km/hr b 120 km/hr
   c 8.45 am d 30 km
4. a P slope is steeper b D-24 mph B-12 mph
   c 0730 d 1030
5. a noon > 1pm > 1.30pm
   > 2.30pm > 2.50pm > 3.20 pm
   b i 60 ii 50
   c i 60 km/hr ii 40 km/hr
   iii 100 km/hr iv 45 km/hr
6. a 1hr 30 min b 60 mph
   c See graph below



Answers to Chapter 53

Ch 53 Ex 1 (Page 193)

1. a

1	2	3	4	5	6
6	12	18	24	30	36

   b 6 c 6 times d L = 6 x B
2. a

1	2	3	4	5	6
2	4	6	8	10	12

   b 2 c 2 times d C = 2 x H d £24
3. a

1	2	3	4	5	6
5	10	15	20	25	30

   b 5 times c T = 5 x C d 75
4. a

1	2	3	4	5	6
12	24	36	48	60	72

   b 12 times c C = 12 x T d
£240
5. a 25 cm b L = 25 x B
   c 2500 cm (25 m)
6. a £1·25 b C = 1·25 x B c £25

7. a

1	2	3	4	5	6
4	8	12	16	20	24

L = 4C

   b

1	2	3	4	5	6
16	32	48	64	80	96

W = 16C

   c

3	4	5	6	7	8
18	24	30	36	42	48

C = 6P

   d

2	4	6	8	10	12
18	36	54	72	90	108

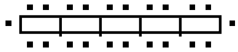
C = 9S

8. a $y = 2x$ b $y = 3x$ c $y = 5x$
   d $y = 0·5x$ e $y = 3·5x$ f $y = x$

9. a $y = 3x$ Plot (0,0) (1,3) (2,6) (3,9) & join
   $y = 5x$ Plot (1,5)(2,10)(3,15)(4,20) & join
   $y = 0·5x$ Plot (1,0·5) (2,1)(3,1·5)(4,2)join
   $y = 3·5x$ Plot (1,3·5) (2,7) (3,10·5)(4,14)
   $y = x$ Plot (0,0) (2,2) (4,4) (6,6) & join
   b all lines pass through O(0,0).
10. i a $y = 6x$ b $y = 8x$ c $y = 10x$
   d $y = x$ e $y = 2x$ f $y = 0·5x$
   ii drawings to show all lines pass through O.
11. The larger the number in front of x, the steeper the line eg $y = 8x$ steeper than $y = 2x$

Ch 53 Ex 2 (Page 197)

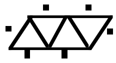
1. a



   b

1	2	3	4	5	6
6	10	14	18	22	26

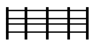
   c 4 d P = 4 x T + 2 e 42 f 86
2. a



   b

1	2	3	4	5	6
3	4	5	6	7	8

   c 1 d P = 1 x T + 2 e 11 f 42
3. a



   b

2	3	4	5	6	7
4	8	12	16	20	24

   c 4 d S = 4 x P – 4 e 56
4. a

1	2	3	4	5	6
12	17	22	27	32	37

C = 5D + 7

   b

1	2	3	4	5	6
18	21	24	27	30	33

D = 3T + 15

   c

3	4	5	6	7	8
1	5	9	13	17	21

T = 4V – 11

   d

10	11	12	13	14	15
53	59	65	71	77	83

A = 6L – 7

   e

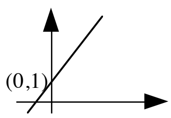
2	4	6	8	10	12
4	12	20	28	36	44

$y = 4x – 4$

5. a $y = 2x + 1$ b $y = 3x + 4$
   c $y = x + 5$ d $y = 3x + 7$
   e $y = 5x + 3$ f $y = 4x + 1$
6. a

0	1	2	3
1	5	9	13

   b/c $y = 4x + 1$



7. a $y = 3x + 1$ b $y = 5x + 2$
   c $y = x + 2$ d $y = 2x – 3$
   e $y = 4x – 2$ f $y = 3x + 1$
8 a Each of the form $y = ax + b$
   Cuts y - axis at $(0, b)$
   b The bigger the value of a, the steeper line

Answers to Chapter 59

Ch 59 Ex 1 (Page 202)

1. a 49 b 9 c 16 d 81
   e 36 f 4 g 64 h 1
   i 121 j 25 k 144 l 0·25
2. a 225 b 169 c 400 d 625
   e 1444 f 10000 g 361 h 40000
   i 484 j 1225 k 2025 l 3025
3. a $49\,cm^2$ b $225\,cm^2$ c $900\,cm^2$ d $196\,cm^2$

Ch 59 Ex 2 (Page 203)

1. a 4 b 3 c 7
   d 5 e 10 f 6
2. a 2 b 8 c 1 d 11
3. a 12 b 25 c 13 d 15
   e 18 f 40 g 14 h 17
   i 1·3 j 2·1
4. a 3·87 b 4·58 c 5·57 d 8·49
   e 9·85 f 10·63 g 13·89 h 22·36
   i 25·30 j 28·55
5. 10·49 cm
6. 16·43 cm

Ch 59 Ex 3 (Page 204)

1. a 9 16 25 b 25 c check
2. a 25 144 169 b 169 c check
3. a 81 144 225 b 225 c check

Ch 59 Ex 4 (Page 205)

1. 20 cm
2. 26 cm
3. a 25 cm b 8·5 cm c 50 cm
4. 10·30 cm
5. 14·76 cm
6. 17·80 cm
7. 26·40 cm
8. 15·39 cm
9. a 12·21 cm b 20·81 cm
   c 14·32 cm d 15·52 m
   e 37·20 mm f 35·36 cm
   g 30·89 mm h 65·76 m
   i 9·98 cm j 131·13 m
   k 142·97 m l 31·53 cm

Ch 59 Ex 5 (Page 208)

1. 17 m
2. 4·68 m
3. 29·53 km
4. 79·06 m
5. 87·80 m
6. 3·30 cm
7. 39·66 cm
8. 94·34 m
9. 2·92 m
10. 5·83 m
11. a PB = PD = 10 cm b 26·93 cm
   c 15·62 cm d 85·1 cm
12. 19·62 m + 27·21 m = 46·83 m
13. 60 cm

Ch 59 Ex 6 (Page 211)

1. 32 cm
2. a 12·12 cm b 28 cm
   c 20 cm d 10·20 m
   e 40·30 mm f 75·42 cm
3. 6·75 cm
4. 53·07 m
5. 1·25 m
6. 10 cm
7. 7·24 cm
8. 92 cm
9. 14·14 cm

Ch 59 Ex 7 (Page 213)

1. a 7·62 cm b 16·58 cm
   c 12·09 m d 23·32 mm
   e 69·35 cm f 5·55 m

2. Tina's answer (13 cm) is longer than the hypotenuse (11 cm) which is not possible !
3. 12·3 cm (it is shorter than the hypotenuse).
4. 53·67 mm
5. 1·99 m
6. 4957·57 m
7. 40·31 cm
8. a 5·02 m b 3·56 m c 1·46 m
9. 10019 m
10. 4·76 m