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1. Decimals, Fractions and Percentages

Decimals

1. Evaluate $8.1 - 19.4 \div 4$ 2 KU

2. Evaluate $43 - 5.6 \times 4$ 2 KU

3. Evaluate $5.7 + 3.9 \times 4$ 2 KU

4. Evaluate $31\cdot4 - 27.09 \div 3$. 2 KU

Fractions

5. Evaluate $4\frac{5}{6} + 2\frac{3}{5}$ 2 KU

6. Evaluate $4\frac{2}{5} - 1\frac{2}{3}$ 2 KU

7. Evaluate $2\frac{3}{4} \times 1\frac{1}{3}$ 2 KU

8. Evaluate $5\frac{1}{2} \div 1\frac{3}{8}$ 2 KU

9. Evaluate: $\frac{3}{8}$ of $\left(1\frac{2}{3} - \frac{4}{7}\right)$. 2 KU

10. Evaluate $\frac{3}{7}\left(1\frac{5}{6} + \frac{3}{4}\right)$ 2 KU

Various

11. Evaluate $23 + (-6)^2 \times \frac{3}{4}$ 2 KU

12. Evaluate 32% of £850 2 KU

13. Find $\frac{3}{8}$ of 544 2 KU

Using Percentages

1. Bacteria in a test tube increase at the rate of 0.9% per hour.
   At 12 noon there are 4500 bacteria.
   At 3 pm, how many bacteria will be present?
   Give your answer **to 3 significant figures**. 4 KU

2. In January 2001, it was estimated that the number of flamingos in a colony was 7000.
   The number of flamingos is decreasing at the rate of 14% per year.
   How many flamingos are expected to be in this colony in January 2005 ?
   Give your answer **to the nearest 10.** 4 KU

3. In 1999, a house was valued at £70,000 and the contents were valued at £45,000.
   The value of the house **appreciates** by 7% each year.
   The value of the contents **depreciates** by 9% each year.
   What will be the **total** value of the house **and** contents in 2002 ? 3 KU

4. A factory was put on the market in January 2001.

   The site was in an excellent location so the value of the building has appreciated since then by 5.3% per year.

   Unfortunately the plant & machinery were poorly maintained and have depreciated by 8·5% per year.

   The value of the building was £435 000 and the value of the plant & machinery was £156 000 in January 2001.

   What would be the expected value of the complete factory in January 2003 ? 4 KU

5. How much would the Strachans pay for a new iron, priced £16.50 at Watsons ? 3 KU

WATSON'S SALE
$66\frac{2}{3}$ % off everything

6. In 1995, the price of 1 litre of a certain kind of petrol was 54.9 pence

   By 1996, the price of 1 litre of the same kind of petrol had risen to 56.3 pence.

   The percentage increase for each of the next four years is expected to be the same as the percentage increase between 1995 and 1996.

   What is the price of 1 litre of petrol expected to be in the year 2000? 4 RE

Reversing the change

7. A computer is sold for £695. This price includes VAT at 17.5%
   Calculate the price of the computer **without** VAT. 3 KU

8. During the Christmas Sales a shopkeeper sold 60% of his "Santa Claus Dolls"
   He then found he was left with 50 dolls.
   How many dolls had he in stock to begin with ? 3 KU

9. Kerry bought a new car in 1996. When she sold it four years later, she found that it had reduced in value by 60% and she received only £4640.
   How much had Kerry paid for the car in 1996 ? 3 KU

10. James bought a car last year. It has lost 12½ % of its value since then.
    It is now valued at £14 875.
    How much did James pay for his car. 2 KU

Standard Form

1. Each of these large oil containers holds 4.80×10^8 litres of the fuel. How many litres are there altogether in the full tanks shown ? Give your answer in scientific notation. 2 KU

2. A newspaper report stated "Concorde has now flown 7.1×10^7 miles This is equivalent to 300 journeys from the earth to the moon." Calculate the distance from the earth to the moon. Give your answer in **scientific notation correct to 2 significant figures.** 3 KU

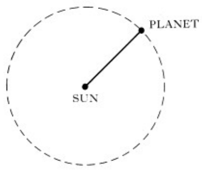
3. The planet Mars is at a distance of 2.3×10^8 kilometres from the Sun. The speed of light is 3.0×10^5 km per second. How long does it take light from the Sun to reach Mars ? **Give your answer to the nearest minute.** 3 KU

4. A planet takes 88 days to travel round the Sun.

   The approximate path of the planet round the Sun is a circle with diameter 1.2×10^7 kilometres.

   Find the speed of the planet as it travels round the Sun.

   Give your answer in kilometres per hour, correct to 2 significant figures. 4 KU



5. The mass of a proton is approximately 1.8×10^3 times greater than the mass of an electron. If the mass of an electron is 9.11×10^{-31} kg, calculate the mass of a proton. Give your answer in **scientific notation correct to 2 significant figures**. 2 KU

6. Large distances in space are measured in light years. A camera on a space telescope, photographs a galaxy, a distance of 50 million light years away. One light year is approximately 9.46×10^{12} kilometres. Calculate the distance of the galaxy from the space telescope in kilometres. **Give your answer in scientific notation** 2 KU

7. The annual profit (£) of a company was 3.2×10^9 for the year 1997. What profit did the company make per second. Give your answer to **three significant figures**. 2 KU

8. The total number of visitors to an exhibition was 2.925×10^7. The exhibition was open each day from 5 June to 20 September **inclusive**. Calculate the average number of visitors per day to the exhibition. 3 KU

9. The mass of the sun is 2.2×10^{30} kilograms. The mass of the earth is 5.97×10^{24} kilograms. Express the mass of the earth as a percentage of the mass of the sun. Give your answer in **scientific notation.** 3 KU

2. Algebra 1 – Basic Algebraic operations, Indices and Surds

Evaluation

1. Evaluate $30 - 3p^2q$ where $p = -1$ and $q = -6$ — 2 KU

Simplification

2. Simplify $4(3x-2)-5(4x+1)$ — 3 KU

3 Remove the brackets and collect like terms $(3a-b)(2a-5b)$ — 2 KU

4. Remove the brackets and simplify your answer $(2x-1)(x+3)+(x-4)^2$ — 4 KU

5. Remove the brackets and simplify $(3y-4)^2$ — 2 KU

6. Multiply out the brackets and simplify. $(2x-3)(3x^2+4x-1)$ — 3 KU

Factorisation

7. Factorise $6x^2-9x$ — 2 KU

8. Factorise $4a^2-9b^2$ — 2 KU

9. a) Factorise the expression $9x^2-y^2$ — 1 KU

   b) Hence simplify $\dfrac{6x+2y}{9x^2-y^2}$ — 2 KU

10. a) Factorise a^2-9b^2 — 1 KU

    b) Hence simplify $\dfrac{a^2-9b^2}{2a+6b}$ — 2 KU

11. a) Factorise x^2-9 — 1 KU

    b) Express $\dfrac{4(5x+3)}{25x^2-9}$ in its simplest form — 2 KU

12. Express $\dfrac{15x-20}{9x^2-16}$ in its simplest form — 3 KU

13. i) Factorise **completely** $2x^2-6x$ — 1 KU

    ii) Express $\dfrac{2x^2-6x}{x^2-9}$ in its simplest form. — 2 KU

14. Factorise $3x^2-13x-10$ — 2 KU

Solve Linear Equations

15. Solve the equation $5-2(1+3x)=27$ 3 KU

16. Solve the equation $5+3a=a-15$ 3 KU

Simultaneous Equations

17. Solve **algebraically**, the system of equations $\begin{aligned}2a+4b&=-7\\3a-5b&=17\end{aligned}$ 3 KU

18. Solve the system of equations $\begin{aligned}5a+3b&=9\\7a-2b&=25\end{aligned}$ 3 KU

Functions

1. $f(x)=x^2-2x$, evaluate $f(\text{-}2)$ 2 KU

2. $h(t)=15t-3t^2$ Find $h(\text{-}2)$ 2 KU

3. Given that $f(x)=\dfrac{x^3+x^2+2}{5x-1}$ evaluate $f(-3)$ 3 KU

4. $f(x)=9-6x$
   (a) Evaluate $f(\text{-}3)$ 1 KU
   (b) Given that $f(t)=11$, find t 2 KU

5. The function $f(x)$ is given by the formula $f(x)=3x^2-7$, where x is a real number.
   (a) Find the value of $f(-2)$. 2 KU
   (b) Find the **values** of a for which $f(a)=20$. 3 KU

6. $f(x)=\dfrac{4}{x^2}$ find $f\left(\frac{1}{2}\right)$ 2 KU

7. $f(x)=3^x$
   a) Find $f(4)$ 1 KU
   b) Given that $f(x)=\sqrt{27}$, find x. 3 KU

8. $f(x)=\dfrac{3}{\sqrt{x}}$ Find the **exact** value of $f(2)$
   Give your answer **as a fraction** with a rational denominator. 2 KU

9. $f(x)=3\sqrt{x}$ Find the exact value of $f(12)$,
   giving your answer as a **surd, in its simplest form**. 2 KU

Quadratic Equations

1. Solve **algebraically**, the equation $x^2 = 7x$ — 3 KU

2. Solve **algebraically**, the equation $6y - y^2 = 0$ — 2 KU

3. Solve **algebraically**, the equation $2x^2 - 9x - 5 = 0$ — 3 KU

4. Solve for x: $2x^2 + 7x - 15 = 0$ — 3 KU

5. Solve the equation $2x^2 + 5x - 12 = 0$ — 3 KU

6. Solve the equation $2p^2 - p - 10 = 0$ where p is a real number. — 3 KU

7. Two functions are given below:

   $f(x) = x^2 + 2x - 1$
   $g(x) = 5x + 3$

   Find the values of x for which $f(x) = g(x)$ — 3 KU

8. Find the two roots of the equation $2x^2 - 3x - 4 = 0$
   (Answer correct to 1 decimal place). — 4 KU

9. Solve the equation $x^2 + 2x - 6 = 0$
   Give your answers correct to 2 significant figures. — 5 KU

Inequalities

1. Solve the inequality $8 - x > 3(2x + 5)$ — 3 KU

2. Solve **algebraically** the inequality $3y < 4 - (y + 2)$ — 3 KU

3. Solve the inequality $3 - (x - 6) < 2x$ — 3 KU

4. Solve algebraically the inequality $6x - 2 < 5(1 - 3x)$ — 3 KU

5. Solve algebraically, the inequality $2 + 5x \geq 8x - 16$ — 3 KU

6. Solve the inequality $2 - 5(3x - 2) \geq 4(1 - 3x)$ where x is a **positive integer**. — 5 KU

7. An inequality, like $4x + 10 \leq 6x + 2 \leq 3x + 26$, can be solved by

   i) solving $4x + 10 \leq 6x + 2$ and
      solving $6x + 2 \leq 3x + 26$

   then ii) looking carefully at the two sets of answers to decide
      on the correct solution to the original inequality.

   a) Solve $3x + 1 \leq 5x + 3 \leq x + 23$ — 4 KU

   b) Write down the set of **all** possible solutions where x is an INTEGER. — 1 KU

Changing the subject of the formula

1. $Y = \dfrac{3(2v - w)}{5}$ Change the subject of the formula to v. 3 KU

2. $P = \frac{1}{3}(m - s)$ Change the subject of the formula to m 2 KU

3. $L = 8 + \dfrac{6}{Y}$ Change the subject of the formula to Y. 3 KU

4. Change the subject of the formula to k. $d = \dfrac{k - m}{t}$ 2 KU

5. $Q = p^2 + 3T$ Change the subject of the formula to T. 2 KU

6. $M = R^2 t - 3$ Change the subject of the formula to R. 3 KU

7. Change the subject of the formula to b. $A = \sqrt{4b^2 - c}$ 3 KU

8. a) Change the subject of the formula $Q = 2\sqrt{s} + t$, to s 3 KU

   b) Find the value of s when $Q = 3.5$ and $t = 2.2$ 2 KU

9. The frequency, F hertz of the sound you hear as you drive past a factory siren at a speed of v metres per second is given by the formula

$$F = f\left(1 - \frac{v}{s}\right)$$

where f is the true frequency of the sound emitted by the siren and s is the speed of sound. Change the subject of the above formula to v. 3 KU

Algebraic Fractions

1. Express as a single fraction in its simplest form $\frac{1}{2x} - \frac{1}{3x}, \quad x \neq 0$ 2 KU

2. Express as a single fraction in its simplest form

$$\frac{3}{x} + \frac{2-x}{x^2}, \quad x \neq 0$$ 3 KU

3. Express as a single fraction in its simplest form

$$\frac{5}{x} - \frac{3}{(x-2)}, \; x \neq 0 \quad \text{or} \quad x \neq 2$$ 3 KU

Fraction Equations

1. Solve the equation $\frac{2x+1}{3} - \frac{x}{4} = 2$ 3 KU

2. Solve the equation $\frac{x+4}{2} - \frac{2x+1}{3} = 1$, where x is a real number. 3 KU

3. Solve **algebraically** the equation $3x - \frac{(5x+2)}{4} = 3$ 3 KU

4. Solve the equation $\frac{x-3}{2} + \frac{2x-1}{3} = 4$ 4 KU

5. Solve this equation for x: $\frac{x-2}{3} - \frac{x}{2} = \frac{1}{4}$ 4 KU

6. Solve **algebraically**, the equation $\frac{x}{2} - \frac{(x+1)}{3} = 4$ 3 KU

7. Solve **algebraically**, the equation $\frac{m}{3} = \frac{(1-m)}{5}$ 3 KU

Indices

1. Evaluate $27^{\frac{2}{3}}$ 2 KU

2. Express in its simplest form $y^{10} \times \left(y^{4}\right)^{-2}$ 2 KU

3. Simplify $a^{3}(a^{-7}+5)$ 2 KU

4. Express $\dfrac{3y^{5} \times 4y^{-1}}{6y}$ in its simplest form. 3 KU

5. Express $\dfrac{y^{4} \times y}{y^{-2}}$ in its simplest form. 2 KU

6. Express $\dfrac{b^{\frac{1}{2}} \times b^{\frac{3}{2}}}{b}$ in its simplest form. 2 KU

7. Remove the brackets and simplify $b^{\frac{1}{2}}\left(b^{\frac{1}{2}}+b^{-\frac{1}{2}}\right)$ 3 KU

8. Remove the brackets and simplify $a^{\frac{1}{2}}\left(a+\dfrac{1}{a}\right)$ 2 KU

Surds

1. Express $\sqrt{50}$ as a surd in its simplest form. 1 KU

2. Simplify $\frac{\sqrt{72}}{\sqrt{3}}$ 2 KU

3. Simplify $\sqrt{48}-3\sqrt{3}$ 2 KU

4. Express $\sqrt{32}-\sqrt{2}$ as a surd in its simplest form. 2 KU

5. Express $\sqrt{72}-\sqrt{2}+\sqrt{50}$ as a surd in its simplest form 3 KU

6. Express $\sqrt{32}+\sqrt{8}$ as a surd in its simplest form. 3 KU

7. Multiply out the brackets $\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right)$

   Express your answer as a **surd** in its simplest form. 2 KU

8. $f(x)=3\sqrt{x}$

   Find the exact value of $f(12)$, giving your answer as a **surd, in its simplest form**. 2 KU

9. Express $\frac{3}{\sqrt{5}}$ as a fraction with a rational denominator. 2 KU

10. Simplify $\frac{\sqrt{3}}{\sqrt{24}}$ Express your answer as a fraction with a rational denominator 3 KU

11. $f(x)=\frac{3}{\sqrt{x}}$ Find the **exact** value of $f(2)$

    Give your answer **as a fraction** with a rational denominator. 2 KU

12. A function f is given by $f(x)=4^x$

    Find the value of $f\left(\frac{3}{2}\right)$ 2 KU

3. Data Handling

Simple Probability - **Note**: You should always give your answer in its **simplest form**

(Questions 3 to 8 in this section are not from Past Papers – but you should know how to do them.)

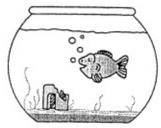
1. A bag contains **red**, **green**, **blue**, **yellow** and **white** balls.
There are 10 of **each** colour, numbered from 1 to 10.
The balls are placed in a drum and one is drawn out.

a) What is the probability that it is a **7** ? 1 KU

b) What is the probability it is a **blue 7** ? 1 KU

2. Roy and Zara go to the fairground.

A stall has a card game where a goldfish can be won if anyone can turn over a face card from a pack of 52 cards which are placed face down.



Calculate the probability, in its simplest form, of Zara winning the goldfish. 3 KU

3. A box contains **5 red, 6 green, 7 blue and 2 yellow** coloured pencils.
Jenny picks one out of the box

a) What is the probability that it is a **green pencil** 1

b) She does **NOT** replace the pencil, but draws another one
What is the probability that this is a blue pencil 2

4. A bag contains **10 red, 25 green, 9 blue and 6 yellow** marbles.

Sam picks one out of the bag, **replaces** it and then picks another one.

What is the probability that he picked a **Green** marble followed by a **Red** one 3

5. Michelle estimates that the probability that her hockey team will win their next game is 0.2, and the probability they will draw is 0.5

a) Calculate P(Win or Draw) 1

b) Calculate P(Lose) 1

6. Robin is the member of an archery club. On average 80% of his shots hit the target.

What is the probability that:

a) He misses the target 1

b) He hits the target 3 times in a row 1

c) He hits the target with the first shot, and misses with the next two shots. 1

7. When microprocessors are made, it is known that in any batch, 15% are defective.

a) What is the probability of picking a microprocessor that is **NOT** defective 1

b) A batch of 5000 microprocessors are produced. How many would be expected to have **NO** defects. 2

8. Three new students are about to join a class. Assuming that P(male) = ½

a) What is the probability that all three will be boys ? 1

b) If you are told that one is a boy, what is the probability now, that all three will be boys. 2

Probability from relative Frequency

1. A garage carried out a survey on 600 cars. The results are shown in the table below:

Engine size (cc)

		0 – 1000	1001 – 1500	1501 – 2000	2001 +
Age	Less than 3 years	50	80	160	20
	3 years or more	60	100	120	10

a) What is the probability that a car chosen at random, is less than 3 years old? 1 KU

b) In a sample of 4200 cars, how many would be expected to have an engine size greater than 2000cc **and** be 3 or more years old. 2 KU

2. The National Tourist Association carried out a survey amongst 500 adults from the UK to find out what would influence them most when choosing a holiday.

The results of the survey are shown in the table below.

Age	Price	Weather	Facilities	Scenery
35 and under	190	65	23	7
Over 35	95	35	12	73

a) What is the probability that any adult chosen at random would have scenery as their main priority when choosing a holiday ? 1 KU

b) A 25 year old adult is chosen at random. What is the probability that the facilities is his/her main concern when choosing a holiday ? 2 KU

c) What is the probability that any adult chosen at random **will not** have cost as their main concern when choosing a holiday ? 2 KU

3. A group of people who admitted to drinking bottled water were asked if they preferred FIZZY water or STILL water.

The results are shown in this table.

	FIZZY	STILL
aged 10 to 20	65	5
aged over 20	10	30

What is the probability that:

a) a person chosen at random **from this sample** will prefer STILL water. 1 KU

b) the person chosen will be over 20 years old **and** prefer FIZZY water. 2 KU

Note: to gain full credit in this question, both answers must be in their simplest form.

4. Smiley's Garage was asked to supply information on last month's sales. They were asked to identify the number of used and new cars purchased. The results are shown in the table.

	new car	used car
aged 18 to 40	17	30
aged over 40	23	50

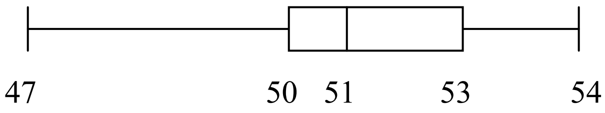
What is the probability that a person chosen at random **from this sample** will

a) have bought a new car ? 1 KU

b) be between 18 and 40 years old **and** have bought a used car ? 1 KU

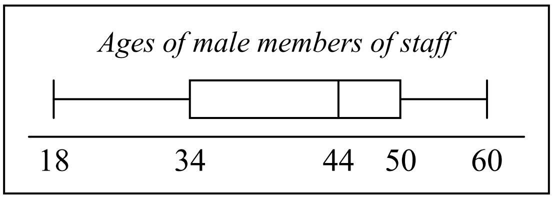
Statistical Diagrams

1. A random check is carried out on the contents of a number of matchboxes.
A summary of the results is shown in the boxplot below.



What percentage of matchboxes contains fewer than 50 matches. 1 RE

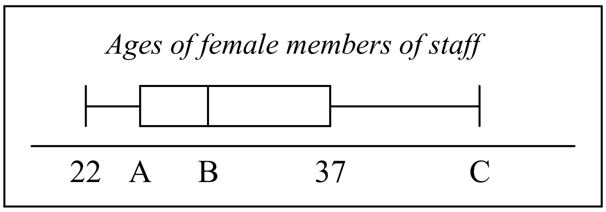
2. The ages of the male members of staff in a school were recorded and a box plot was drawn to show the results.



When the same study was carried out for the female members of staff in the same school, another box plot was drawn.

It was found that:

- the **range** of the ladies' ages was **half** that of the range of the mens'.
- the ladies' **median** age was **15 years less** than the men's median age.
- the **semi-interquartile range** of the ladies' was **three quarters** that of the men's.



Make a copy of the above females' box plot and complete it to show which ages are represented by the letters **A**, **B** and **C**. 3 RE

3. Fifteen medical centres each handed out a questionnaire to fifty patients.
The numbers who replied to each centre are shown below.

11	19	22	25	25
29	31	34	36	38
40	46	49	50	50

Also, they each posted the questionnaires to another fifty patients.
The numbers who replied to each centre are shown below.

15	15	21	22	23
25	26	31	33	34
37	39	41	46	46

Draw an appropriate statistical diagram to compare these two sets of data. 3 RE

4. A furniture maker investigates the delivery times, in days, of two local wood companies and obtains the following data.

Company	*Minimum*	*Maximum*	*Lower Quartile*	*Median*	*Upper Quartile*
Timberplan	16	56	34	38	45
Allwoods	18	53	22	36	49

a) Draw an appropriate statistical diagram to illustrate these two sets of data. 3 RE

b) Given that consistency of delivery is the most important factor, which company should the furniture maker use? Give a reason for your answer. 1 RE

5. Jamie conducted a survey.

He asked his classmates how they had travelled to school that day.

Here are their replies.

Walk	13
Bus	9
Car	6
Cycle	2

Draw an appropriate statistical diagram to illustrate this information 4 RE

6. The stem and leaf diagram shows a sample of 50 scores in a boy's golf tournament.

Golf Scores

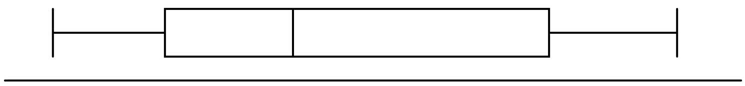
6	3 4
6	5 5 5 6 6 6 6 7 7 8 9 9 9
7	0 0 0 0 0 1 1 2 3 3
7	5 6 6 6 7 8
8	0 0 1 1 2 2 3
8	5 6 6 6 7 8 9
9	2 3 4
9	7 8

8 | 0 represents a score of 80

a) Write down the **median** golf score. 1 KU

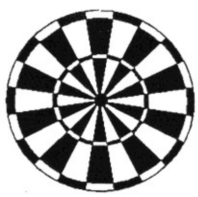
b) Calculate the **semi-interquartile range** for these scores. 3 KU

c) Sketch this boxplot and fill in the correct values to illustrate the golf scores in this sample. 2 KU



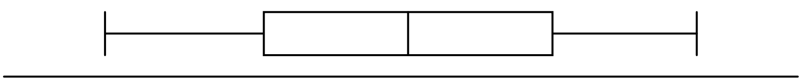
7. In a tournament, 13 men throw one dart each at a dart board and their scores are noted.

Alex	16	Nick	20	Steve	28
Norrie	6	George	9	Brian	18
Ted	24	James	22	Graeme	18
Tom	12	John	13	Tony	7
George	9				



a) Find the **median** and the upper and lower quartiles. 3 KU

b) Make a neat sketch of the following box plot and fill in all the missing values. 2 KU



Standard Deviation

1. Fiona checks out the price of a litre of milk in several shops.

   The prices in pence are:

   49 44 41 52 47 43

   a) Find the mean price of a litre of milk. 1 KU

   b) Find the standard deviation of the prices. 2 KU

   c) Fiona also checks out the price of a kilogram of sugar in the same shops and finds that the standard deviation of the prices is 2.6.
   Make one valid comparison between the two sets of prices. 1 RE

2. A group of fifth year students from Alloa High School were asked how many hours studying they did in the week prior to their exams.

   The results are shown below.

   14 7 9 12 19 10 16 15

   (a) Use an appropriate formula to calculate the mean and standard deviation of these times. 3 KU

   (b) A similar group of students from Alloa Academy were asked the same question
   The mean number of hours studied was 16 and the standard deviation was 2·2.

   How did the number of hours studied by students from Alloa High School compare with the number of hours studied by students from Alloa Academy ? 2 RE

3. The Mobile Phone Shop is advertising their five latest mobile phones on their website.

   Their prices are:

   £120 £135 £75 £235 £185

   Use an appropriate formula to calculate the mean and standard deviation of these prices.

   (Show all working) 4 KU

4. The price, in pence per litre, of petrol at 10 city garages is shown below:

84.2	84.4	85.1	83.9	81.0
84.2	85.6	85.2	84.9	84.8

   a) Calculate the mean and standard deviation of these prices. 3 KU

   b) In 10 rural garages, the petrol prices had a mean of 88.8 and a standard deviation of 2.4

   How do the rural prices compare with the city prices ? 2 RE

5. Jim typed six pages on his computer using his word processor.

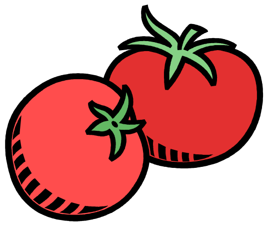
He did a "spell check" and discovered that he had made the following numbers of errors on the 6 pages.

page one	-	4 errors
page two	-	1 errors
page three	-	7 errors
page four	-	13 errors
page five	-	6 errors
page six	-	5 errors

a) Calculate the mean number of errors — 1 KU

b) Calculate the standard deviation. — 4 KU

6. After trying a new fertilizer on one of his tomato plants, a grower counted the number of tomatoes on each of its six bunches.



The number of tomatoes was:

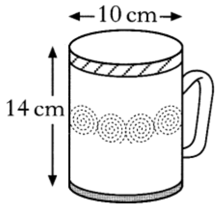
8, 14, 9, 16, 13, 18

a) Calculate the mean number of tomatoes. — 1 KU

b) Construct a table and use it to calculate the standard deviation. — 4 KU

4. Area & Volume

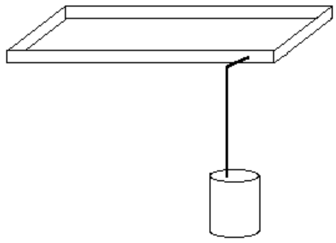
1. A mug is in the shape of a cylinder with diameter 10 centimetres and height 14 centimetres.



a) Calculate the volume of the mug. 2 KU

b) 600 millilitres of coffee are poured in.
Calculate the depth of the coffee in the cup. 3 RE

2. Rainwater is collected in a rectangular based tank on top of a flat roof and is drained periodically to a cylindrical tank on the ground where it is used for watering plants in dry weather.



The tank on the roof measures 3 metres by 9 metres and has a depth of 0·25 metres.

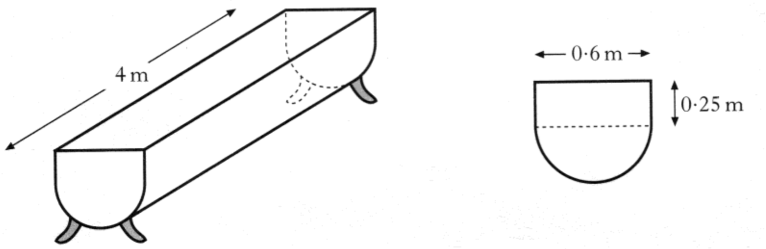
The tank on the ground is 1·85 metres high and has base radius of 0·55 metres.

Both tanks were empty, but after a heavy shower all the rainwater from the roof tank was drained to the ground tank and completely filled it.

Calculate the depth of rainwater, to the nearest millimetre, in the roof tank immediately before it was drained to the ground tank. 5 RE

3. A feeding trough, 4 metres long, is prism shaped.

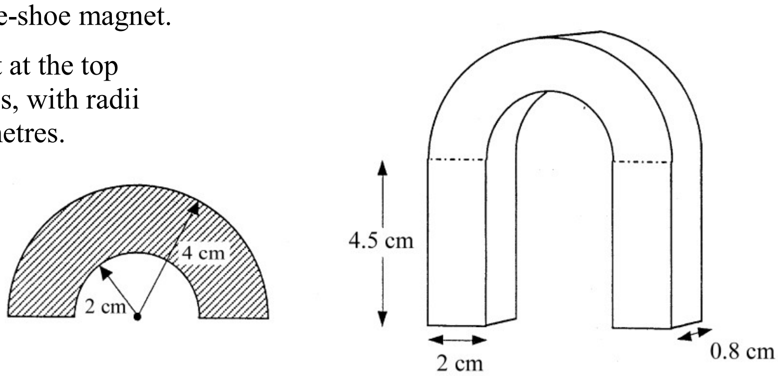
The uniform cross-section is made up of a rectangle and semi-circle as shown below.



Find the volume of the trough, **correct to 2 significant figures**. 5 KU

4. The diagram shows a horse-shoe magnet.

The face of the arched part at the top consists of two semi-circles, with radii 2 centimetres and 4 centimetres.



Calculate the shaded area and use this to calculate the volume of metal required to make the magnet.

Give your answer correct to 1 decimal place. 5 RE

5. A cylindrical soft drinks can is 15 centimetres in height and 6.5 centimetres in diameter.

A new cylindrical can holds the same volume but has a reduced height of 12 centimetres.

What is the height of the new can ?

Give your answer **to 1 decimal place.** 4 RE

6. A metal doorstop is prism shaped, as shown in Figure 1

The uniform cross-section as shown in Figure 2:

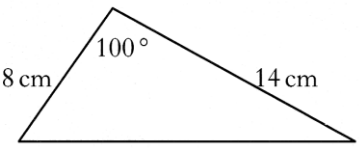


Figure 2

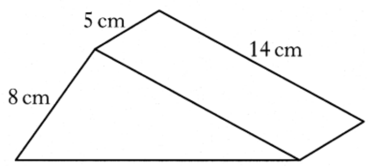
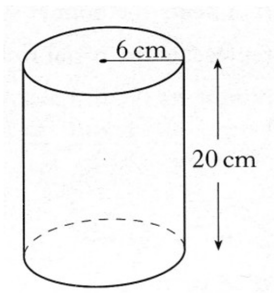


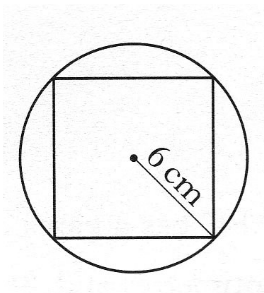
Figure 1

Find the volume of metal required to make the doorstop. 4 KU

7. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.



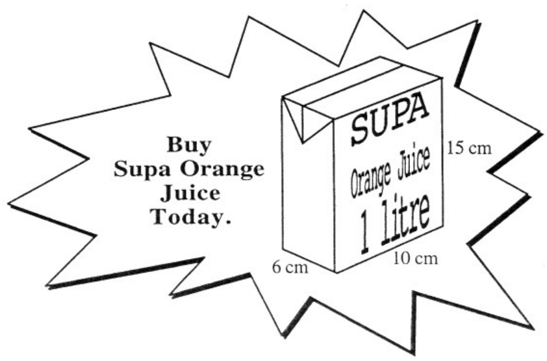
It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.



The corners of the vase touch the Inside of the cylinder as shown.

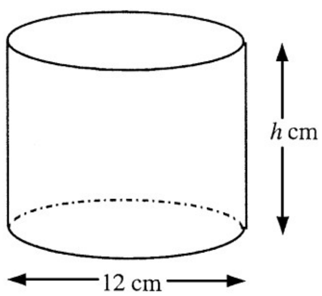
Show that the volume of the space between the vase and the cylinder is $720(\pi - 2)$ cubic centimetres. 5 RE

8. a) Explain what is wrong with this advert for a 1 litre carton of Orange Juice. 2 KU



b) The measurements 10 cm, 6 cm and 15 cm are correct.

All of the juice is poured into this cylindrical container with base diameter 12 cm and it is found to exactly half fill it.



Calculate the height of the container. 4 RE

9. A wooden toy box is prism-shaped as shown in figure 1.

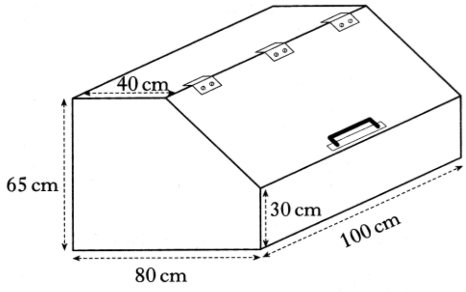


figure 1

The uniform cross-section of the box is as shown in figure 2.

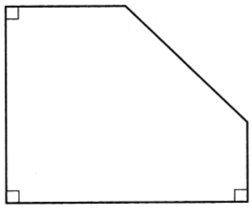
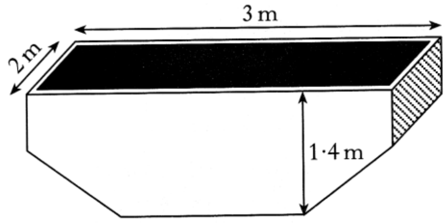


figure 2

Calculate the volume of the box in **cubic metres**. 4 KU

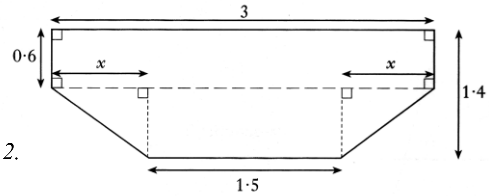
10. A skip is prism shaped as shown in figure 1.

Figure 1.



The cross section of the skip, with measurements in metres, is shown in figure 2.

Figure 2.

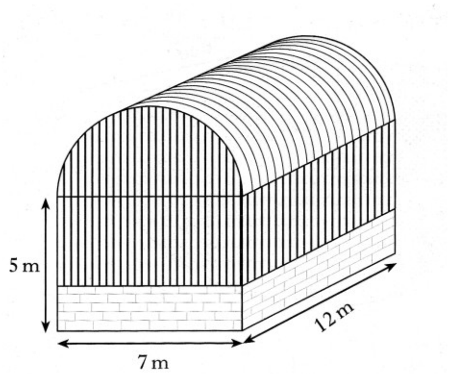


a) Find the value of x. 1 KU

b) Find the volume of the skip in cubic metres. 3 KU

11. A storage barn is prism shaped, as shown.

The cross-section of the storage barn consists of a rectangle measuring 7 metres by 5 metres and a semi-circle of radius 3.5 metres.

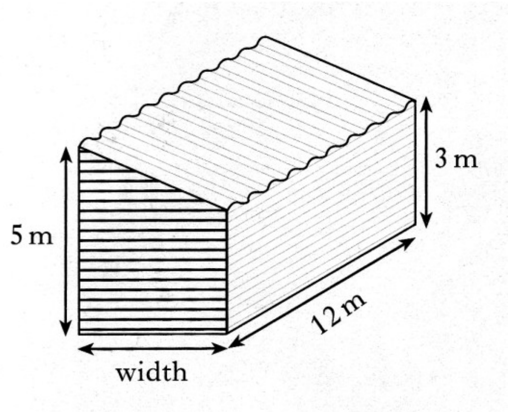


a) Find the volume of the storage barn.

Give your answers in cubic metres, **correct to2 significant figures**. 4 KU

b) An extension to the barn is planned to increase the volume by 200 cubic metres.

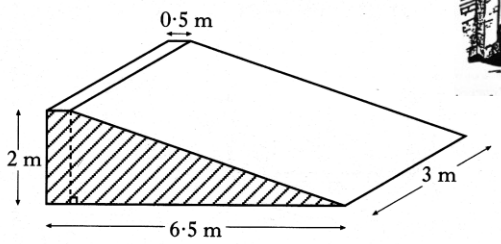
The uniform cross-section of the extension consists of a rectangle and a right angled triangle.

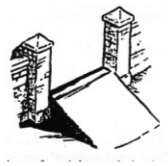


Find the width of the extension. 3 RE

12. A ramp is being made from concrete.

The uniform cross section of the ramp consists of a right angled triangle and a rectangle as shaded in the diagram.

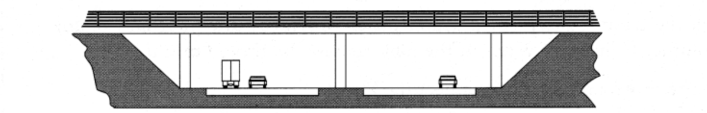




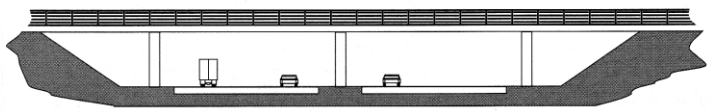
Find the volume of concrete required To make the ramp. 2 KU

13. Ground has to be blasted and removed so that a motorway can be widened.

The existing motorway and the motorway after widening are shown below.



Existing motorway



Motorway after widening

The uniform cross-section of the existing motorway consists of a rectangle and two congruent right angled triangles as shown in figure 1.

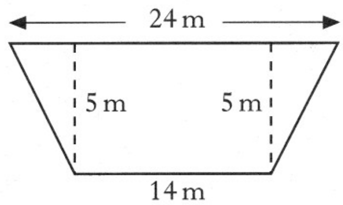


figure 1

The uniform cross-section of the motorway after widening consists of a rectangle and two congruent right angled triangles as shown in figure 2.

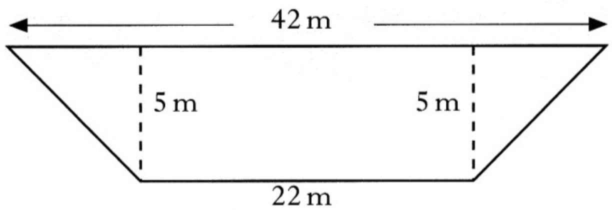
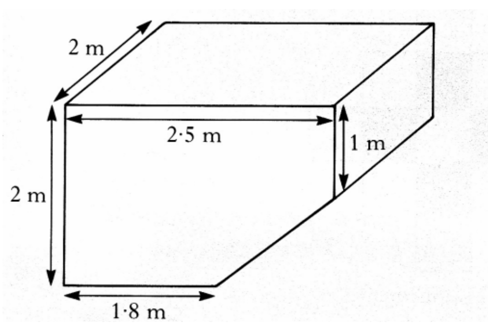


figure 2

The cost of blasting and removing each cubic metre of ground is £4.
10 kilometres of existing motorway is to be widened.
Find the total cost of blasting and removing the ground. 4 RE

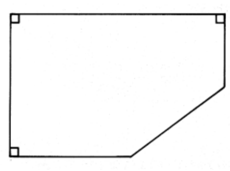
14. A bottle bank is prism shaped, as shown in figure 1.

Figure 1



The uniform cross-section is shown in figure 2.

Figure 2



Find the volume of the bottle bank. 4 KU

5. Similar Shapes and Similar Triangles

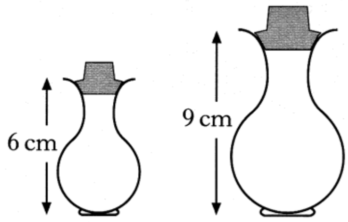
Similar Shapes – Area and Volume Scale Factors

1. Two perfume bottles are mathematically similar in shape.

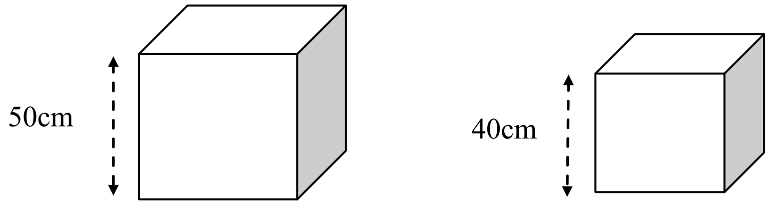
   The smaller one is 6 centimetres high and holds 30 millilitres of perfume.

   The larger one is 9 centimetres high.

   What volume of perfume will the larger one hold. 3 KU



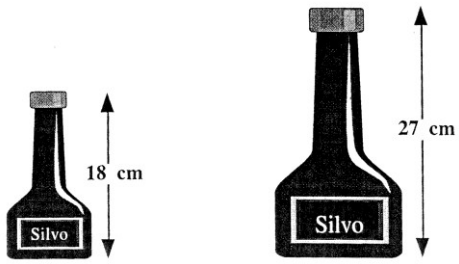
2. The two boxes below are mathematically similar and both have to be wrapped with decorative paper.



   If it requires 3.27 m^2 of paper to cover the large box, calculate the amount of paper needed to cover the smaller box. 3 KU

3. The diagram shows two bottles of Silvo Shampoo.

   The two bottles are **mathematically similar**, and the cost of the shampoo depends only on the volume of liquid in the bottle.

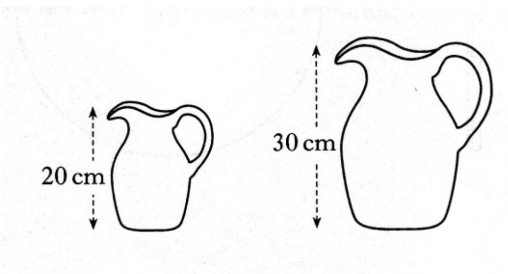


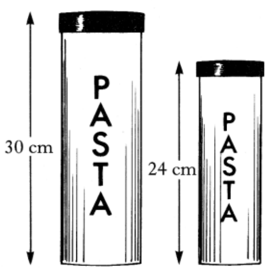
   If the small one costs 80p, what should the large one cost ? 3 KU

4. The diagram shows two jugs which are mathematically similar.

   The volume of the smaller jug is 0.8 litres.

   Find the volume of the larger jug. 3 KU



5. 

   The diagram shows two storage jars which are mathematically similar.

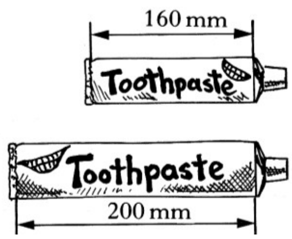
   The volume of the large jar is 1.2 litres.

   Find the volume of the smaller jar.

   Give your answer in litres correct to 2 significant figures. 4 KU

6. The diagram shows two tubes of toothpaste.

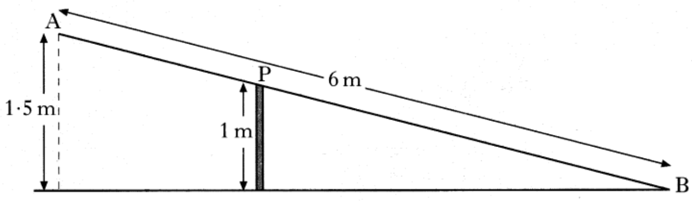
   Assuming that the tubes are mathematically similar, and that the price of toothpaste depends only on the volume of toothpaste in the tube, what would be the cost of the large tube when the small one costs £1.12? 3 KU



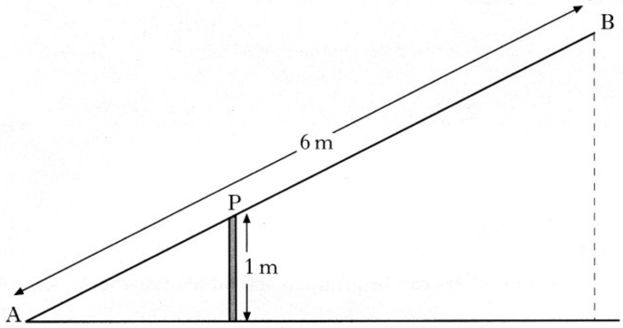
Similar Triangles

1. A metal beam, AB, is 6 metres long.
It is hinged at the top, P,
of a vertical post 1 metre high.
When B touches the ground,
A is 1.5 metres above the ground,
as shown in Figure 1.

Figure 1

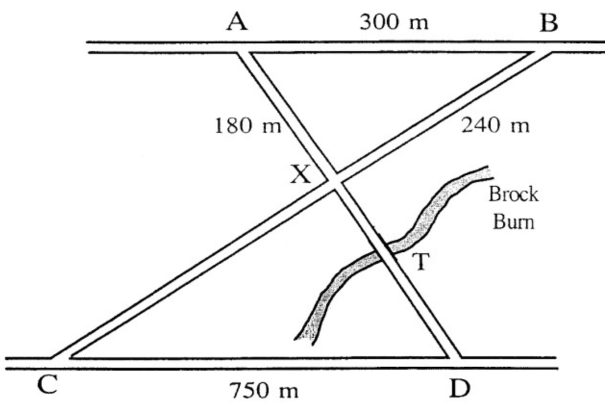


When A comes down to the ground,
B rises, as shown in Figure 2.



By calculating the length of AP,
or otherwise, find the height of B
above the ground.
Do not use a scale drawing. 5 RE

2. The road joining A to B is parallel to the road joining C to D in the diagram.



AB = 300 metres,
AX = 180 metres,
BX = 240 metres
and CD = 750 metres.

a) Prove that the two roads AX and BX are at right angles to one another 3 RE

b) The Brock Burn burst its banks at T and the road became impassable.
An alternative route had to be found in order to travel from A to D.

Calculate the length of the shortest route. 3 RE

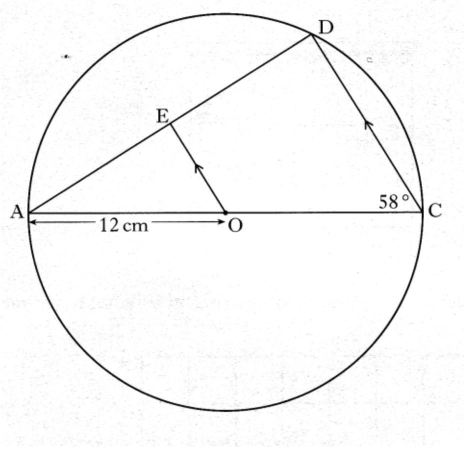
3. AC is the diameter of the circle.
with centre O, and radius 12 centimetres

AD is a chord of the circle.

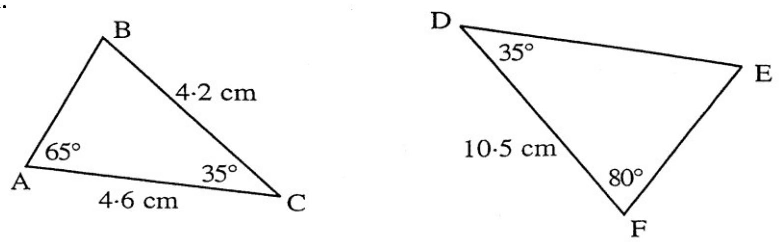
OE is parallel to CD

Angle ACD is 58°

Calculate the length of ED. 4 KU



4. Study the two triangles shown.



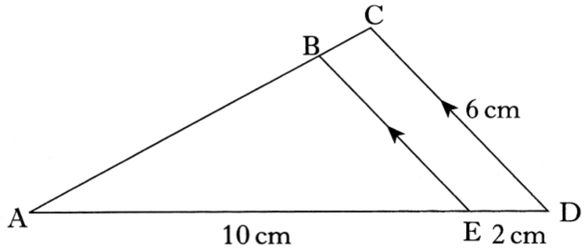
a) Explain clearly why the two triangles must be similar. 1 KU

b) Use the fact that the two triangles are similar to calculate the length of the line DE. 2 KU

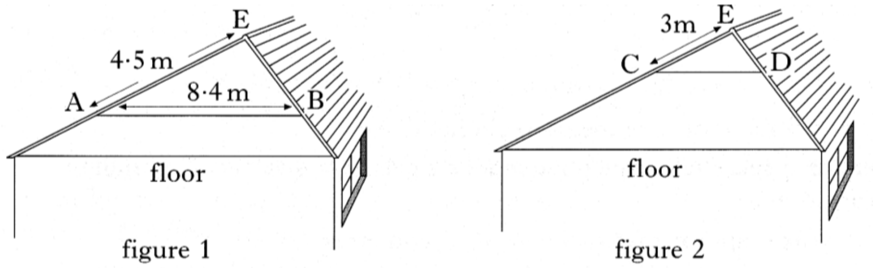
5. Triangles ABE and ACD with some of their measurements are shown opposite.

Triangle ABE is similar to triangle ACD.
Calculate the length of BE.

Do not use a scale drawing. 3 KU



6. The brown family want to convert the roof space in their bungalow into an extra room.



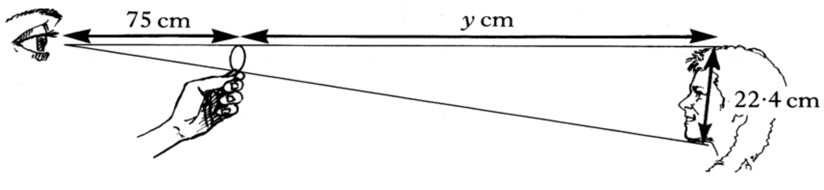
The position, AB, of the wooden beam must be changed to position CD, as shown in figure 2.

The wooden beam must always be parallel to the floor.

By considering the similar triangles EAB and ECD, calculate the length of the wooden beam in position CD.

Do not use a scale drawing. 3 KU

7. By holding a 10 pence coin at arms' length, it is possible to cover exactly the face of a person standing a distance away.



The diameter of the 10 pence coin is 2.8 cm and the length from the top to the bottom of the person's face is 22.4 cm.

If the distance from the observer's eye to the top of the coin is 75 cm, find the distance from the top of the 10 pence coin to the top of the person's head. 4 KU

6. Pythagoras

NB There is some overlap between these questions and those on the Circle, Similar Triangles and Trigonometry.

1. A sheep shelter is part of a cylinder as shown in Figure 1.

   It is 6 metres wide and 2 metres high.

   The cross-section of the shelter is a segment of a circle with centre O, as shown in Figure 2.

   OB is the radius of the circle.

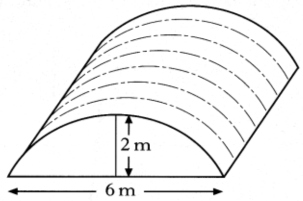


Figure 1

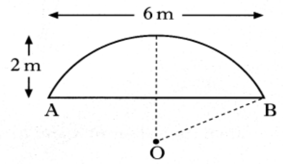


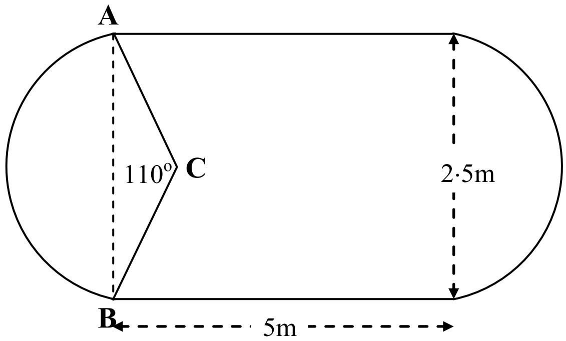
Figure 2

   Calculate the length of OB. 4 RE

2. A large shop display table is in the shape of a rectangle with a circle segment at both ends, as shown in the diagram below.

   The rectangle at the centre measures 5 metres by 2·5 metres.

   AC and BC are radii of the circle and angle ACB is 110°.



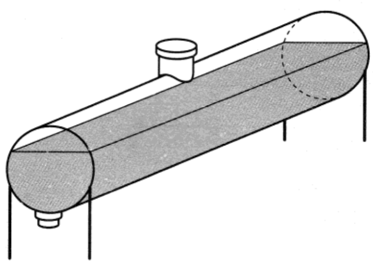
   (a) Show that AC, the radius of the segment, is 1.53 m correct to 3 significant figures. 3 RE

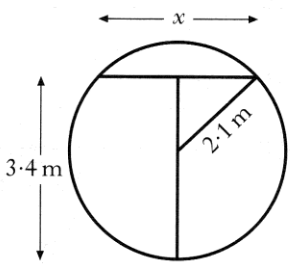
   (b) To stand comfortably around this table it is estimated that an average person requires 75 cm of table edge.

   How many people can stand comfortably at the table described above? 4 RE

3. An oil tank has a circular cross section of radius 2.1 metres.

   It is filled to a depth of 3.4 metres.

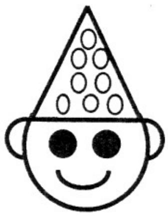


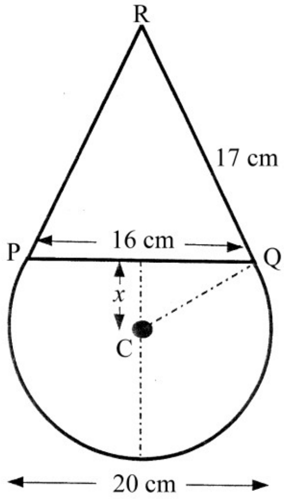


   a) Calculate x, the width in metres of the oil surface. 3 KU

   b) What other depth of oil would give the same surface width. 1 RE

4. A clown's face consists of an isosceles triangle PQR on top of a sector of a circle.





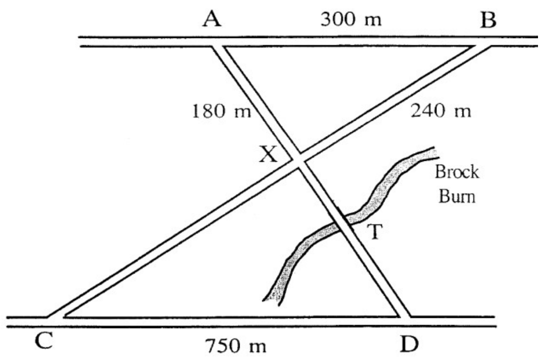
The diameter of the circle is 20 centimetres.

The base of the triangle is 16 centimetres and its sloping sides are 17 centimetres long.

a) Calculate x, the distance in centimeters from the centre of the circle to the base of the triangle. 3 KU

b) Calculate the total height of the figure. 3 RE

5. The road joining A to B is parallel to the road joining C to D in the diagram.



AB = 300 metres,

AX = 180 metres,

BX = 240 metres

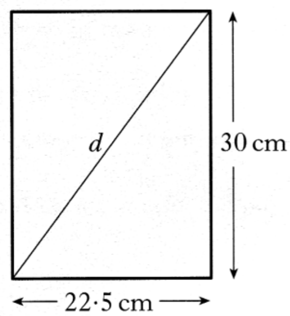
and CD = 750 metres.

a) Prove that the two roads AX and BX are at right angles to one another 3 RE

b) The Brock Burn burst its banks at T and the road became impassable. An alternative route had to be found in order to travel from A to D.

Calculate the length of the shortest route. 3 RE

6. A rectangular picture frame is to be made.



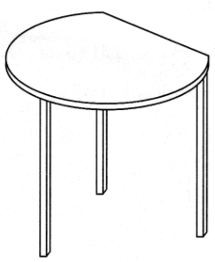
It is 30 centimetres high and 22.5 centimetres wide, as shown.

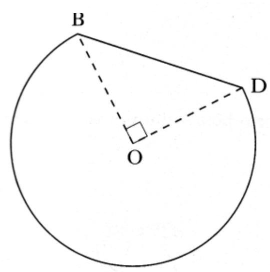
To check that the frame is rectangular, the diagonal, d, is measured.

It is 37.3 centimetres long.

Is the frame rectangular ? 4 RE

7. The diagram shows a table whose top is in the shape of part of a circle with centre, O, and radius 60 centimetres.





BD is a straight line.

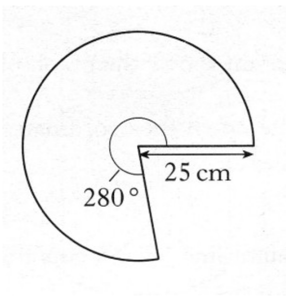
Angle BOD is 90°.

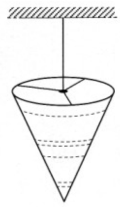
Calculate the perimeter of the table top. 3 RE

8. A lampshade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

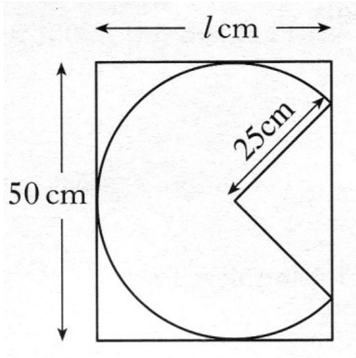
The circle has radius 25 centimetres and the angle of the sector is 280°





a) Find the area of the sector of the circle. 3 KU

Each sector is cut from a rectangular piece of material, 50 centimetres wide.

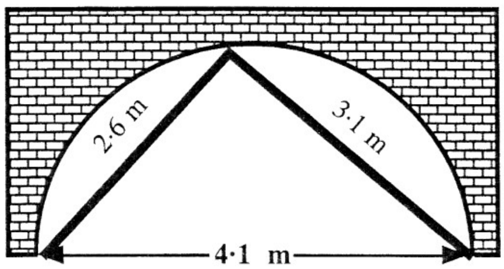


b) Find to the nearest centimetre the minimum length l, required for the piece of material. 4 RE

9. The central semi-circular archway under a bridge is to be strengthened.

While the work is being carried out, 2 metal beams are to be set in place to support the archway.

For safety reasons, the beams have to just meet on the circumference of the arch.

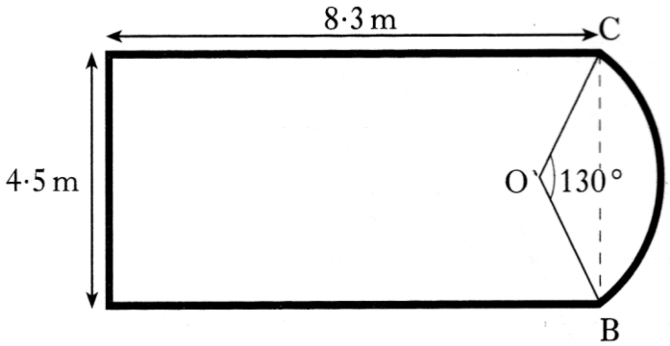


Will the beams fit this archway which is 4.1 metres wide ? 4 RE

10. The diagram shows a ceiling in the shape of a rectangle and a segment of a circle.

The rectangle measures 8.3 metres by 4.5 metres.

OB and OC are radii of the circle and angle BOC is 130°.



a) Find the length of OB. 2 RE

A border has to be fitted around the perimeter of the ceiling.

b) Find the length of border required. 4 RE

11. Figure 1 shows the circular cross section of a tunnel with a horizontal floor.

Figure 1.

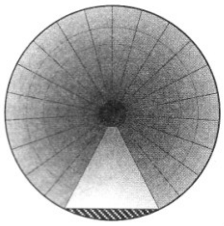
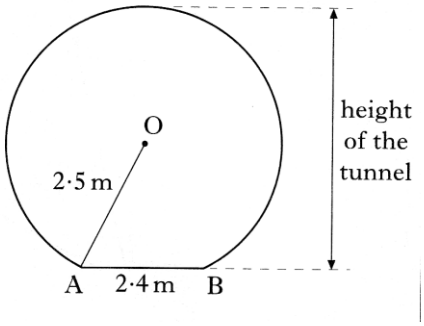


Figure 2



In figure 2, AB represents the floor.
AB is 2.4 metres.

The radius, OA, of the cross-section is 2.5 metres.

Find the height of the tunnel. 4 KU

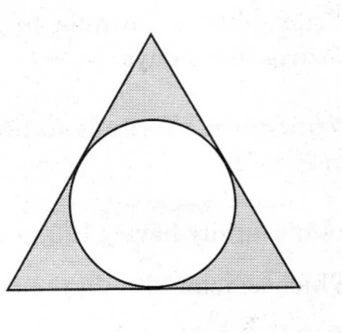
6. The diagram shows the design of an earring.

The earring consists of a circle
inside an equilateral triangle.

The sides of the triangle are tangents to the circle.

The radius of the circle is 8 mm

The distance from the centre of the circle
to **each** vertex of the triangle is 17mm.

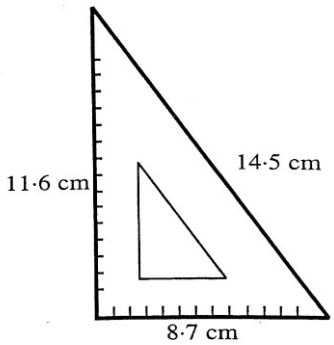


Calculate the perimeter of the triangle. 4 RE

10. MATRIX is a company which makes
mathematical instruments.

They intend to make a new size of set
square which **must have a perfect
right angle** at one of its corners..

If the set square has sides of length
8.7 cm, 11.6 cm and 14.5 cm, will it
be acceptable.



(Give reasons for your answer). 4 RE

11. Figure 1 shows a road bridge.

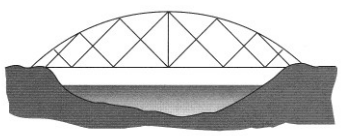


figure 1

The curved part of the bridge
is formed from the arc of a circle,
centre O, as shown in figure 2.

OA and OB are radii
of length 170 metres.

The height of the middle of the bridge
above its ends is 28 metres
as shown in figure 2.

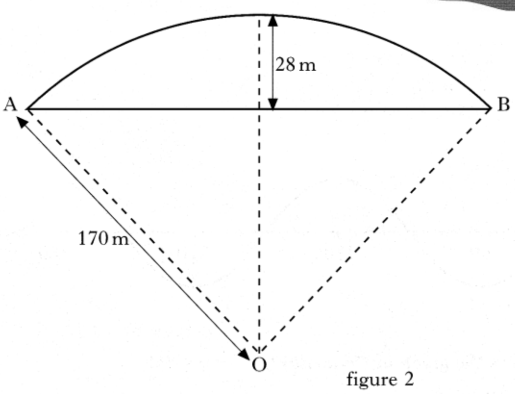


figure 2

Calculate the horizontal distance, AB.

Do not use a scale drawing. 4 KU

12. A loop of rope is used to mark out a triangular plot, ABC.

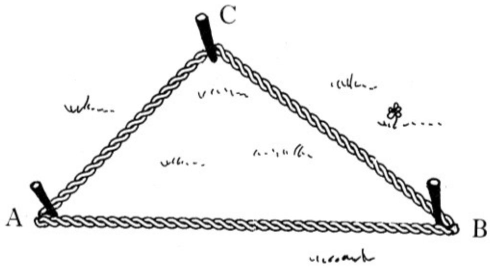
The loop of rope measures 6 metres.

Pegs are positioned at A and B such that AB is 2.5 metres.

The third peg is positioned at C such that BC is 2 metres.

Prove that angle ACB = 90°.

Do not use a scale drawing. 4 KU

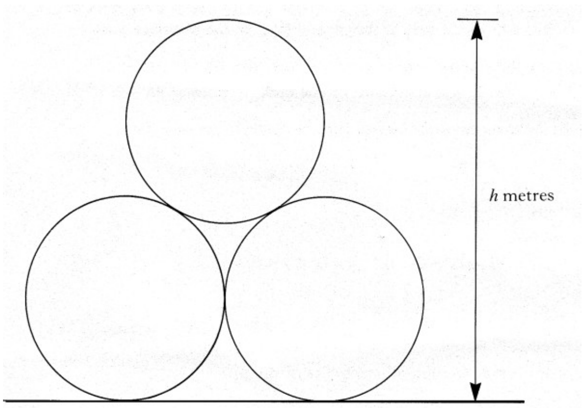


13. Three pipes are stored on horizontal ground as shown in the diagram.

Each pipe has a circular cross-section with radius 1 metre.

Calculate the height, *h* metres, of the stacked pipes. (Ignore the thickness of the pipes.)

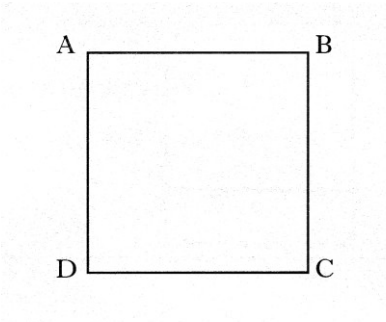
Give your answer in metres correct, to two decimal places. 4 RE



12. a) ABCD is a square of side 2 cms

Write down the ratio of the length AB to the length of AC. 2 KU

b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$ 3 RE



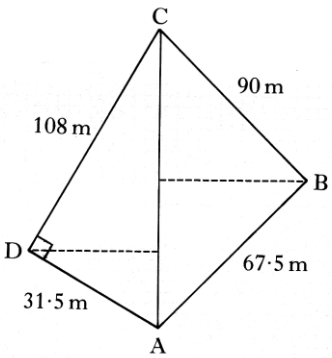
13. A school's playing fields have recently been surveyed and the following plan produced.

The plan is not drawn to scale.

AB = 67.5 metres
BC = 90 metres
AD = 31.5 metres
DC = 108 metres
Angle ADC = 90°

Without doing any further measurements, the surveyor realises that angle ABC is a right angle.

Prove that angle ABC =90° 4 KU



This next question is quite an involved one from 1990. It is unlikely to be set today, however, if you can do this then you have demonstrated an excellent understanding of mathematics.

14. The diagram of a rivet is shown opposite.
The body of the rivet is in the shape of a cylinder.

The head of the rivet is a cap of a sphere of radius R, which is obtained as shown in figure 2.

Figure 1

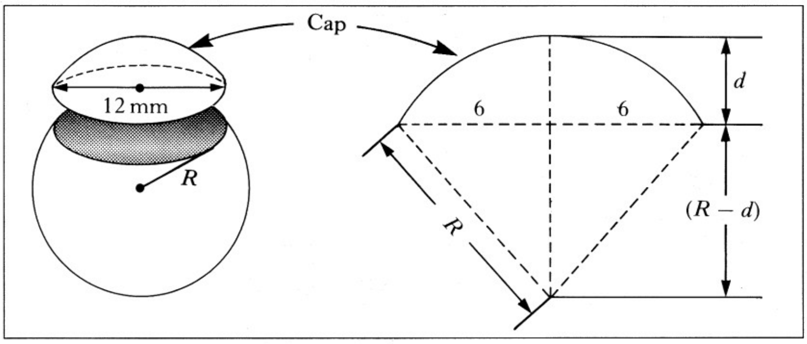


Figure 2

a) Find the value of R for this cap of width 12mm when its height in mm is given by d = 2. 4 RE

b) The length of the cylindrical body of this rivet is 8mm and the diameter of the base is 5mm.

Assuming that the volume of the cap of the sphere is given by

$$V = \frac{1}{3}\pi d^2 (3R - d)$$

show that the total volume of the rivet

is $\dfrac{262\pi}{3}$ mm^3 4 RE

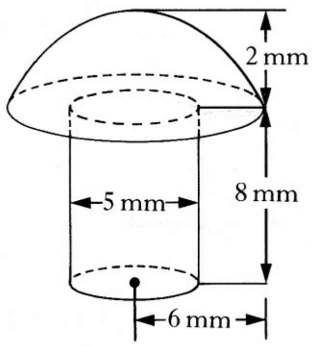


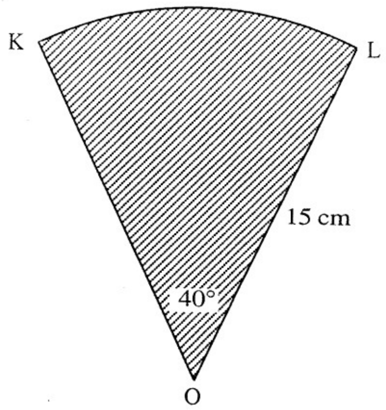
Figure 3

7. The Circle

NB There is considerable overlap between these questions and those on Pythagoras and Trigonometry.

1. Sector KOL of a circle centre O and radius 15 centimetres is shown opposite.

   Calculate the area of this sector. 2 KU

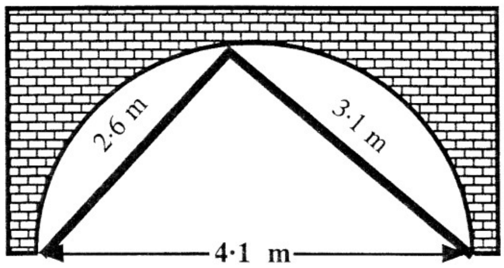


2. The central semi-circular archway under a bridge is to be strengthened.

   While the work is being carried out, 2 metal beams are to be set in place to support the archway.

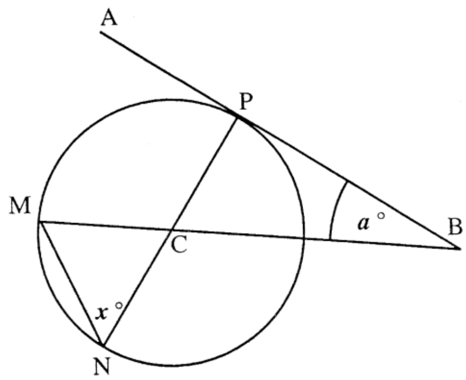
   For safety reasons, the beams have to just meet on the circumference of the arch.

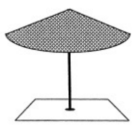
   Will the beams fit this archway which is 4.1 metres wide ? 4 RE



3. AB is a tangent to the circle with **centre C**. It meets the circle at the point P.

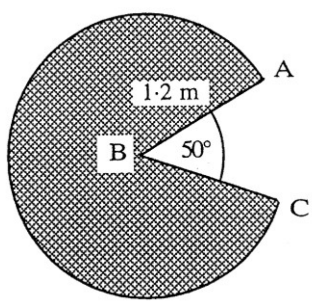
   Use the information in the diagram to find an expression for x in terms of **a**. 3 RE



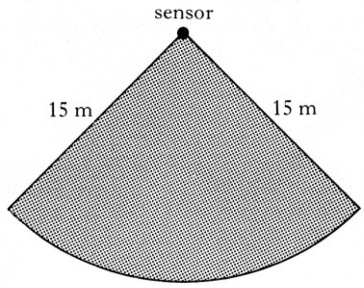
4. 

   June is replacing the fabric on her garden parasol. She uses a sector of a circle, with radius 1.2 metres.

   Calculate the area of fabric needed to replace the old material. 4 KU



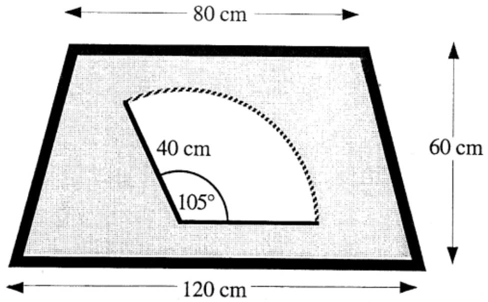
5. A sensor in a security system covers a horizontal area in the shape of a sector of a circle of radius 15 m.



The area of the sector is 200 square metres.

Find the length of the arc of the sector. 4 RE

6. The diagram shows the rear wiper on a car's back window.

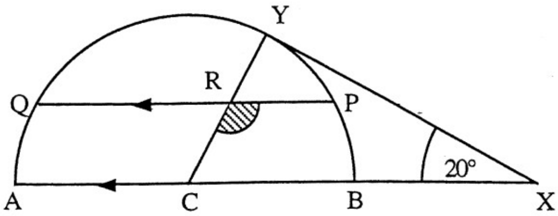


The rear glass is in the shape of a **trapezium** with sizes given.

The wiper blade is 40 centimetres long and it sweeps through an angle of 105°.

Calculate the area of glass **NOT** cleaned by the wiper blade. 4 RE

7. In this diagram, AB is the diameter of the circle, centre C.



X is a point of the line AB extended.

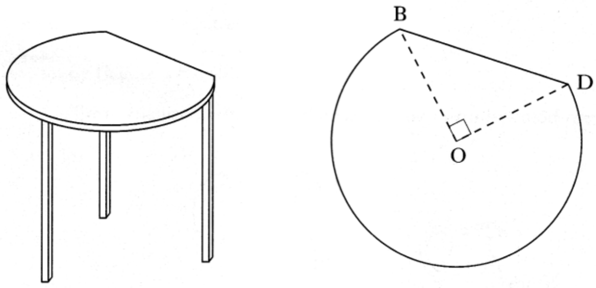
XY is a tangent from X.

QP is parallel to AB.

If ∠YXC = 20°, calculate the size of the shaded angle (∠PRC) 3 KU

(explain how you produced your answer)

8. The diagram shows a table whose top is in the shape of part of a circle with centre, O, and radius 60 centimetres.

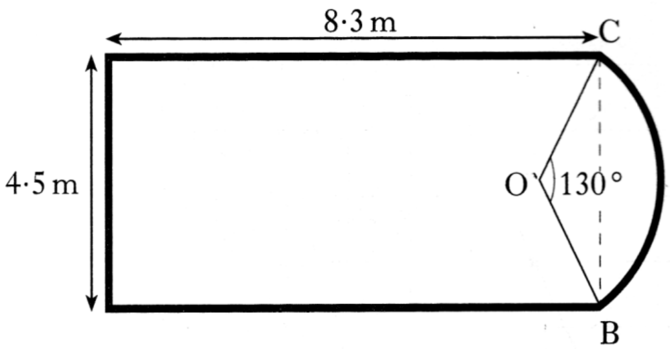


BD is a straight line.

Angle BOD is 90°.

Calculate the perimeter of the table top. 3 RE

9. The diagram shows a ceiling in the shape of a rectangle and a segment of a circle.



The rectangle measures 8.3 metres by 4.5 metres.

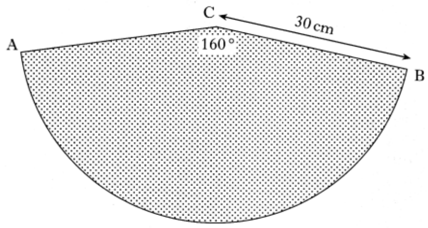
OB and OC are radii of the circle and angle BOC is 130°.

a) Find the length of OB. 2 RE

A border has to be fitted around the perimeter of the ceiling.

b) Find the length of border required. 4 RE

10. The diagram shows a sector of a circle, centre, C.

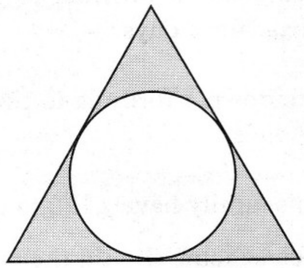


Angle ACB is 160°,
and the radius of the circle is 30 cm.

Calculate the length of the arc AB. 3 KU

11. The diagram shows the design of an earring.

The earring consists of a circle
inside an equilateral triangle.



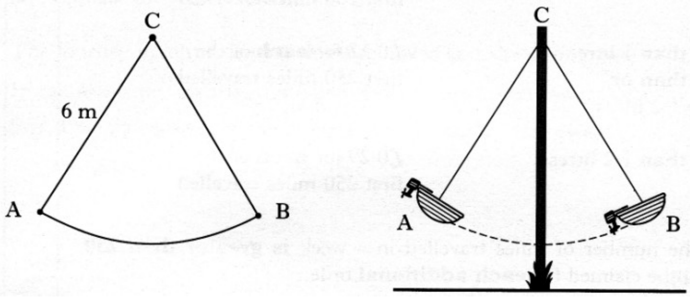
The sides of the triangle are tangents to the circle.

The radius of the circle is 8 mm

The distance from the centre of the circle
to **each** vertex of the triangle is 17mm.

Calculate the perimeter of the triangle. 4 RE

12. The boat on a carnival ride travels along an arc of a circle, centre C.



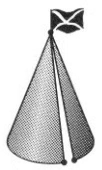
The boat is attached to C
by a rod 6 metres long.

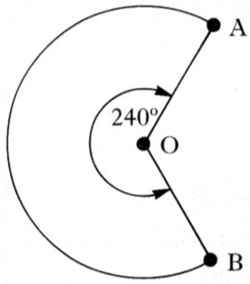
The rod swings from position CA
to position CB.

The length of the arc AB is 7 metres.

Find the angle through which the
rod swings from position A to position B. 4 RE

13. The diagram shows a tent.





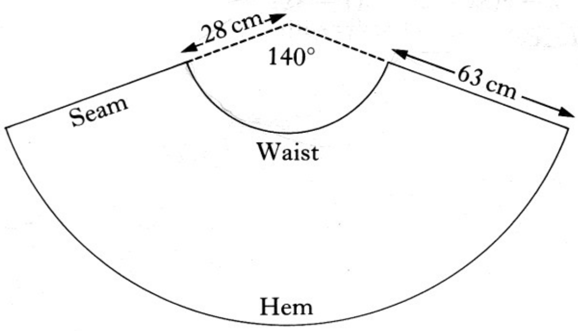
The shape of the material used
to make the tent is a sector of a
circle as shown in the diagram.

O is the centre of the circle.

OA and OB are radii of length 3 metres.
Angle AOB is 240°

Calculate the area of this piece of material. 3 KU

14. The pattern for a skirt consists
of part of the sector of a circle.

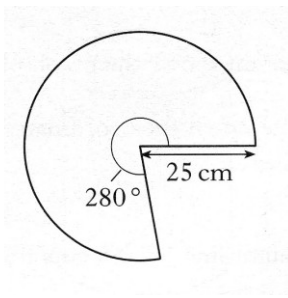


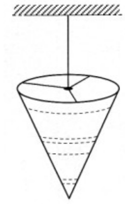
Calculate the length of the waist
shown on the pattern. 3 KU

15. A lampshade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

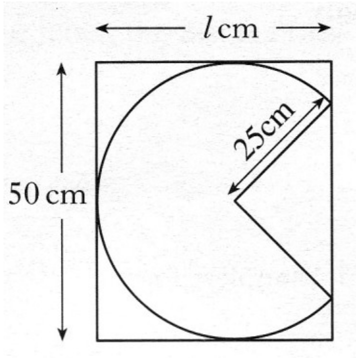
The circle has radius 25 centimetres and the angle of the sector is 280°





a) Find the area of the sector of the circle. 3 KU

Each sector is cut from a rectangular piece of material, 50 centimetres wide.

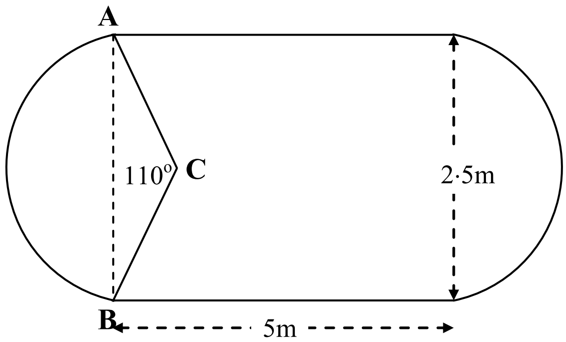


b) Find to the nearest centimetre the minimum length l, required for the piece of material. 4 RE

16. A large shop display table is in the shape of a rectangle with a circle segment at both ends, as shown in the diagram below.

The rectangle at the centre measures 5 metres by 2·5 metres.

AC and BC are radii of the circle and angle ACB is 110°.



(a) Show that AC, the radius of the segment, is 1.53 m correct to 3 significant figures. 3 RE

(b) To stand comfortably around this table it is estimated that an average person requires 75 cm of table edge.

How many people can stand comfortably at the table described above? 4 RE

8. Trigonometry 1 – SOH-CAH-TOA

NB There is some overlap between these questions and those on Pythagoras and the Circle.

1. In the diagram

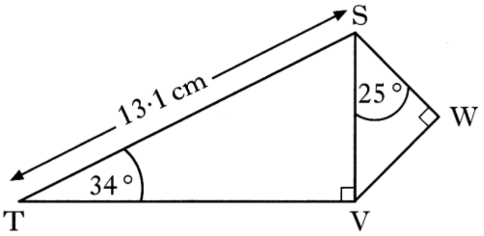
Angle STV = 34°

Angle VSW = 25°

Angle SVT = Angle SWV = 90°

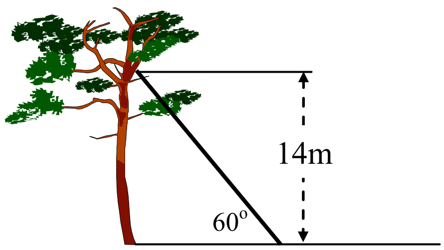
ST = 13.1 centimetres

Calculate the length of SW



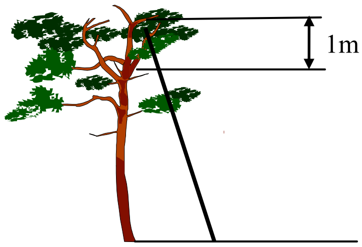
4 KU

2. A cat is trapped in a tree and a ladder is placed against the tree in an attempt to rescue it.



The ladder rests against the tree making an angle of 60° with the horizontal and reaching 14 metres up the tree, allowing the rescuer to reach the cat.

Just as the cat is about to be rescued, it jumps to a branch 1 metre above its original resting place.



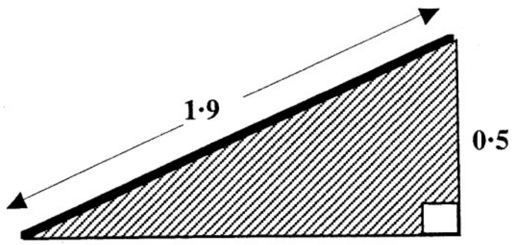
Calculate the size of the angle, to the nearest degree, that the ladder now has to make with the horizontal to allow the rescuer to reach the cat.

5 RE

3. The owners of Stately Hall Manor erected an entrance ramp

for disabled people at the main front entrance.

Local building regulations state that ramps must be built at an angle of **not more than** 15° to the horizontal ground.



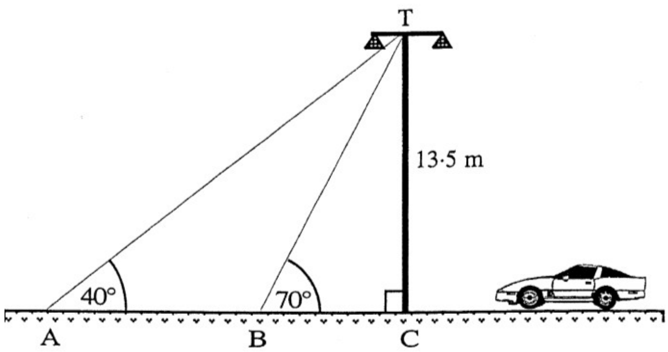
A side view of the ramp which was actually erected is shown above.

Does this ramp satisfy the local building regulations ?

You must explain your answer with mathematical reasoning.

4 RE

4. Two support cables, from the top (T) of a motorway light, are attached to a pair of points, A and B, on the ground, as shown in the diagram.



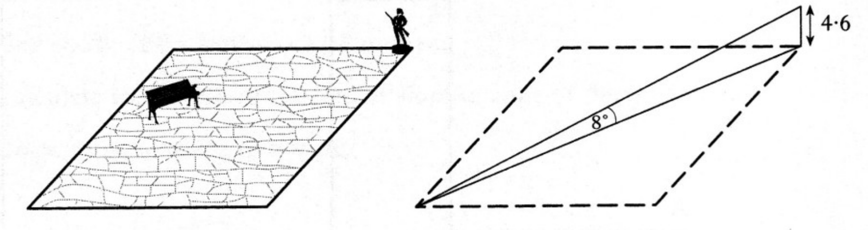
a) Calculate the distance from B to C.

2 KU

b) Calculate the distance from A to B.

3 RE

5. A statue stands at the corner of a square courtyard.



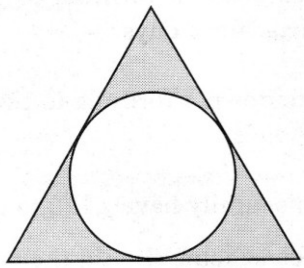
The statue is 4.6 metres high.

The angle of elevation from the opposite corner of the courtyard to the top of the statue is 8°.

a) Find the distance from the base of the statue to the opposite corner of the courtyard. 2 RE

b) Show that the length of the side of the courtyard is approximately 23 metres. 2 RE

6. The diagram shows the design of an earring.



The earring consists of a circle inside an equilateral triangle.

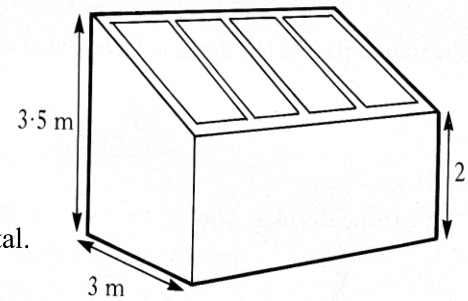
The sides of the triangle are tangents to the circle.

The radius of the circle is 8 mm

The distance from the centre of the circle to **each** vertex of the triangle is 17mm.

Calculate the perimeter of the triangle. 4 RE

7. The Scott family want to build a conservatory as shown in the diagram.



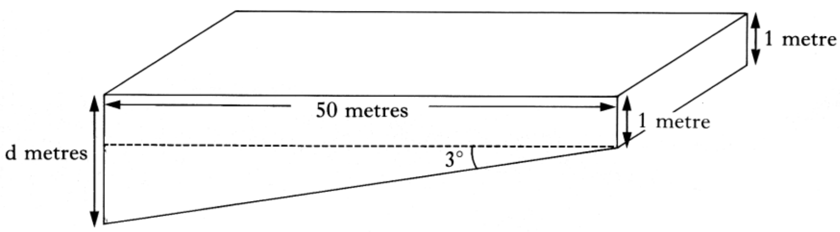
The conservatory is to be 3 metres wide.
The height of the conservatory at the lower end is to be 2 metres and at the higher end 3.5 metres.

To obtain planning permission, the roof must slope at an angle of (25 ± 2) degrees to the horizontal.

Should planning permission be granted.

Justify your answer. 4 RE

8. The diagram shows the design of a swimming pool 50 metres in length.



The pool is 1 metre deep at one end and its base slopes downwards at an angle of 3° to the horizontal.

Calculate the depth, *d* metres, of the other end of the pool, giving your answer to 2 significant figures.

Do not use a scale drawing. 5 KU

9. Trigonometry 2 – Sine, Cosine Rule, Area of Triangle

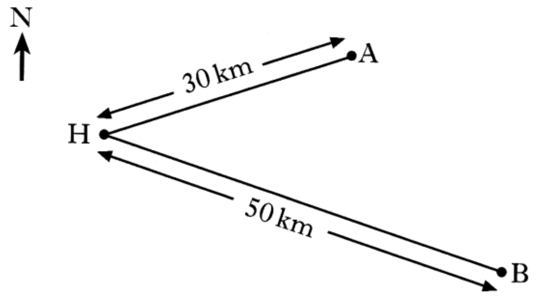
1. Two yachts leave from harbour H.

   Yacht A sails on a bearing of 072° for 30 kilometres and stops.

   Yacht B sails on a bearing of 140° for 50 kilometres and stops.

   How far apart are the two yachts when they have both stopped?

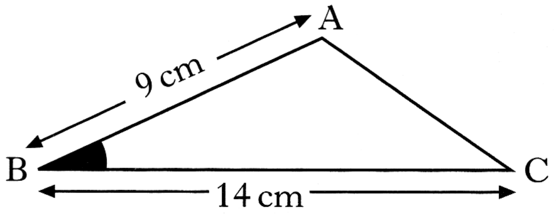
   Do not use a scale drawing.



4 RE

2. The area of triangle is 38 square centimeters.

   AB is 9 centimetres and BC is 14 centimetres.



   Calculate the size of the acute angle ABC

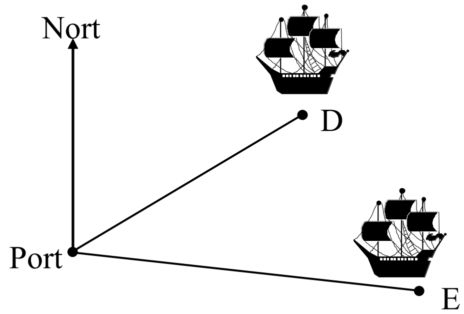
3 RE

3. Two boats leave port together.

   Boat D sails on a course of 057° at 13 miles per hour.

   Boat E sails on a bearing of 104° at 15 miles per hour.

   After 45 minutes Boat D receives a distress call from Boat E requesting their help as soon as possible.

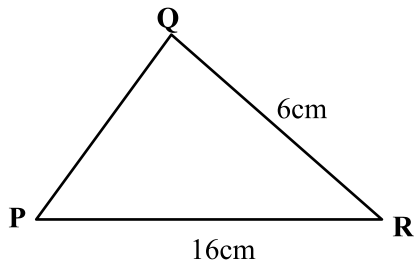


   How far, to the nearest mile, would Boat D have to travel to reach Boat E?

4 RE

4. The area of the triangle shown is 36 cm^2.

   Show that $\sin R = \frac{3}{4}$.

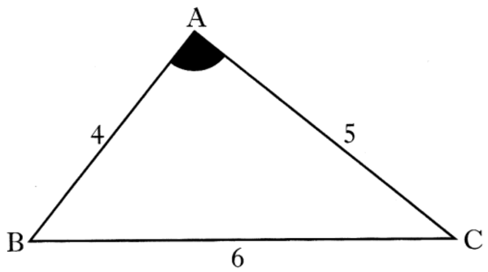


4 RE

5. In triangle ABC

   AB = 4 units
   AC = 5 units
   BC = 6 units

   Show that $\cos A = \frac{1}{8}$

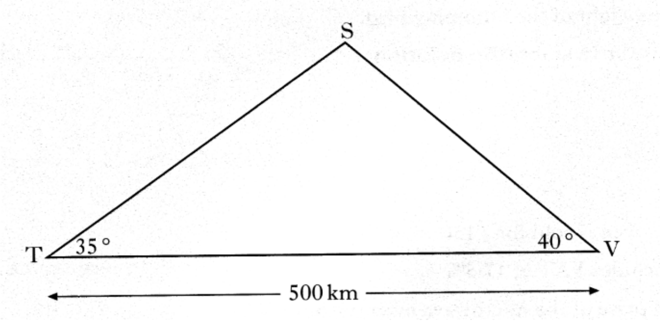


3 KU

6. A TV signal is sent from a transmitter T, via a satellite S, to a village V, as shown in the diagram.

The village is 500 kilometres from the transmitter.

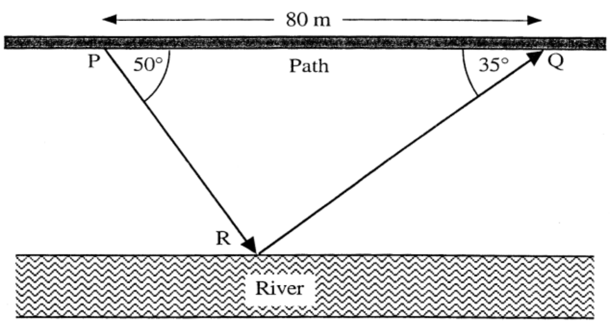
The signal is sent out at an angle of 35° and is received in the village at an angle of 40°.



Calculate the height of the satellite above the ground. 5 RE

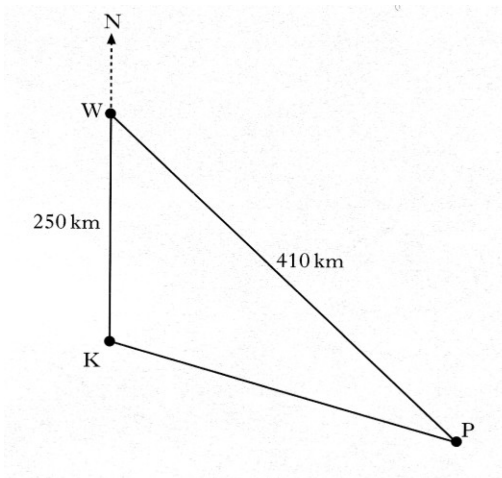
7. The path in the diagram opposite runs parallel to the river.

Jennifer leaves the path at P, walks to the river to bathe her feet at R and rejoins the path further on at Q.



Calculate the distance between the river and the path. 5 RE

8. The radio masts, Kangaroo (K), Wallaby (W) and Possum (P) are situated in the Australian outback.



Kangaroo is 250 kilometres due south of Wallaby.

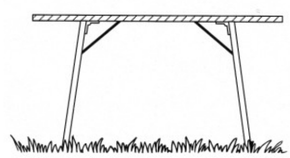
Wallaby is 410 kilometres from Possum

Possum is on a bearing of 130° from Kangaroo.

Calculate the bearing of Possum from Wallaby.

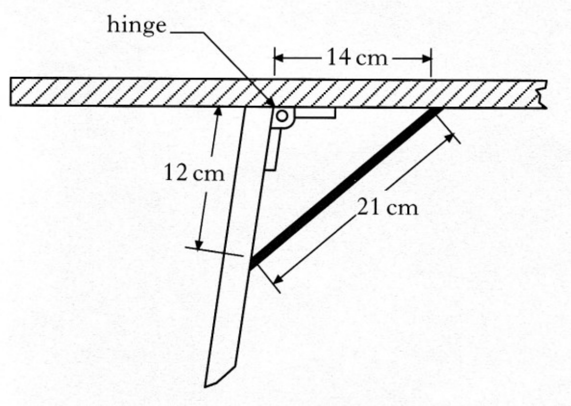
Do not use a scale drawing. 4 RE

9. Each leg of a folding table is prevented from opening too far by a metal bar.



The metal bar is 21 centimetres long.

It is fixed to the table **top** 14 centimetres from the hinge and to the table **leg** 12 centimetres from the hinge.



a) Calculate the size of the obtuse angle which the table top makes with the leg. 3 KU

b) Given that the table leg is 70 centimetres long, calculate the height of the table. 3 RE

10. A newspaper group advertises a new magazine on a helium balloon.

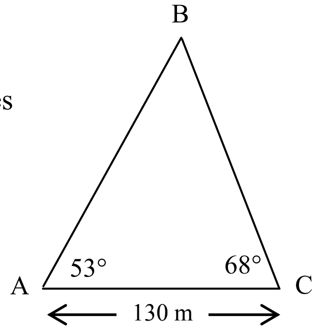
From the base of the balloon, B, two holding wires are attached to the ground at A and C.

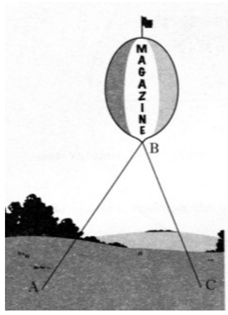
The distance from A to C is 130 metres.

From A, the angle of elevation of B is 53°.

From C, the angle of elevation of B is 68°

Calculate the height of point B above the ground.





Do not use a scale drawing 5 RE

11. The bonnet of a car is held open, at an angle of 57°, by a metal rod.

In the diagram,

PQ represents the bonnet

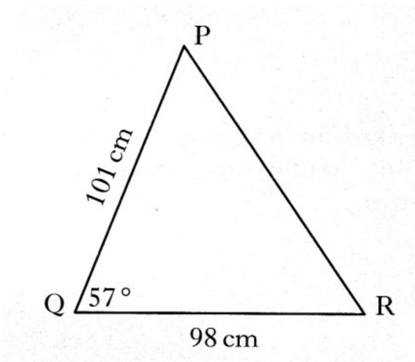
PR represents the metal rod.

QR represents the distance from the base of the bonnet to the front of the car.

PQ is 101 centimetres

QR is 98 centimetres

Calculate the length of the metal rod, PR.



Do not use a scale drawing. 4 KU

12. Triangle ABC has an area of 14 square centimetres.

AB is 6 centimetres and AC is 7 centimetres.

Calculate the possible **sizes** of angle BAC 4 RE

13. An orienteering course has 3 checkpoints – A, B and C.

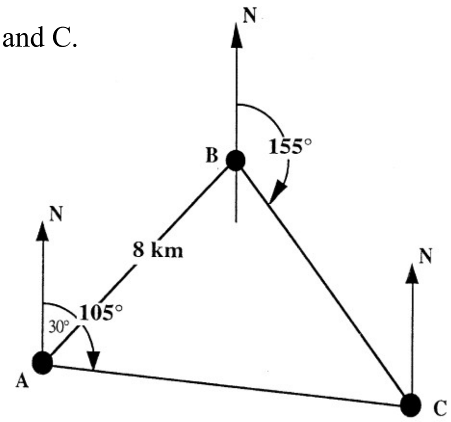
B is on a bearing of 030° and a distance of 8 km from A.

C is on a bearing of 155° from B and a bearing of 105° from A.

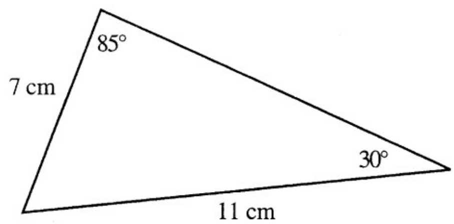
a) Explain clearly why ∠ABC = 55° 2 KU

b) Calculate the distance between points B and C.

Do not use a scale drawing. 4 RE

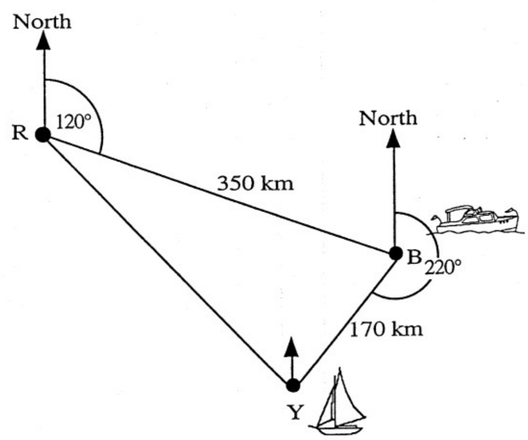


14. Calculate the area of the triangle. 3 RE



15. A rescue boat, at R, picks up a distress call from a boat B, 350 km away, on a bearing of 120°.

At the same time another distress call comes from a yacht Y, which is 170 km away from B and on a bearing of 220° from B.



a) Prove that $\angle RBY = 80°$ 2 KU

b) The rescue boat is obliged to respond to the nearest distress call first.

Will the people on the boat or those on the yacht be rescued first ?

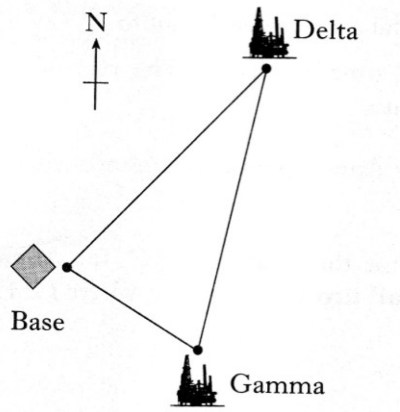
(You must support your answer by showing working). 4 RE

16. The diagram shows the position of a helicopter base and two oil rigs, Delta and Gamma.

From the helicopter base, the oil rig Delta is 35 kilometres away on a bearing of 050°.

From the same base, the oil rig Gamma is 20 kilometres away on a bearing of 125°.

Calculate the distance between Delta and Gamma.



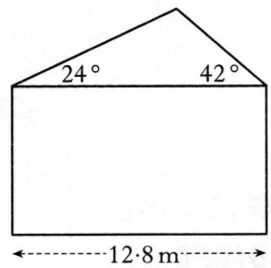
Do not use a scale drawing. 5 RE

17. The end wall of a bungalow is in the shape of a rectangle and a triangle as shown in the diagram.

The roof has one edge inclined at an angle of 24° to the horizontal and the other edge inclined at 42° to the horizontal.

The width of the house is 12.8 metres.

Calculate the length of the longer sloping edge of the roof.

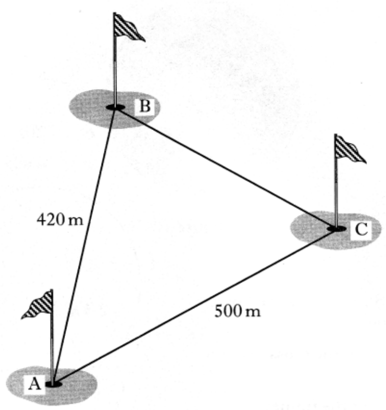


Do not use a scale drawing. 4 KU

18. The diagram shows part of a golf course.

The distance AB is 420 metres, the distance AC is 500 metres and angle BAC = 52°.

Calculate the distance BC.

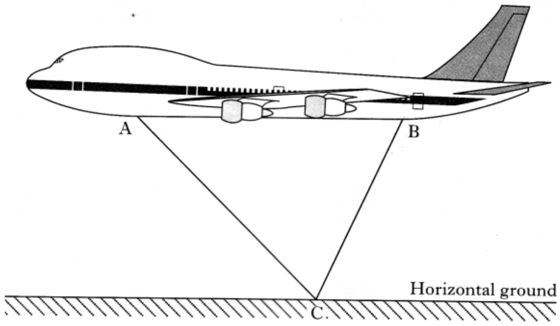


Do not use a scale drawing. 3 KU

19. An aeroplane is flying parallel to the ground.

Lights have been fitted at A and B as shown in the diagram.

When the aeroplane is flying at a certain height, the beams from these lights meet exactly on the ground at C.



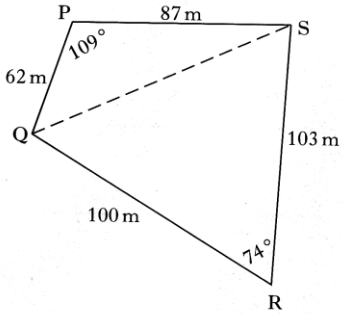
The angle of depression of the beam of light from A to C is 50°.

The angle of depression of the beam of light from B to C is 70°.

The distance AB is 20 metres.

Find the height of the aeroplane above C. 6 RE

20. The sketch shows a plot of ground, PQRS, split into two triangles.



Calculate the area of the plot of ground. 4 KU

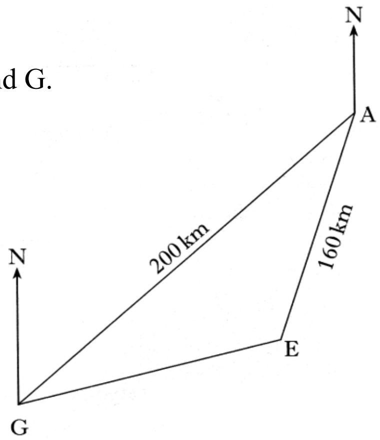
21. The diagram shows the position of three airports, A, E and G.

G is 200 kilometres from A

E is 160 kilometres from A

From G the bearing of A is 052°

From A the bearing of E is 216°



How far apart are airports G and E ? 6 RE

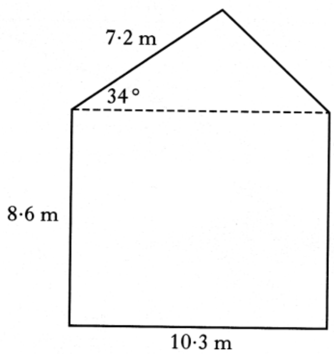
22. The side wall of a house, with measurements as shown in the diagram, requires painting.

The wall is in the shape of a rectangle and a triangle.

On average, a litre of paint will cover 8 square metres.

A painter estimates that he will require 12 litres of paint.

Will this be enough paint?



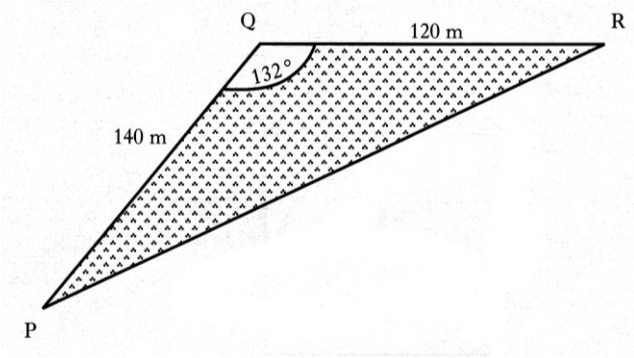
Justify your answer. 4 RE

23. A triangular field, PQR is shown in the diagram.

PQ = 140 metres,
QR = 120 metres
and angle PQR =132°

Calculate the length of PR.

Do not use a scale drawing. 4 KU



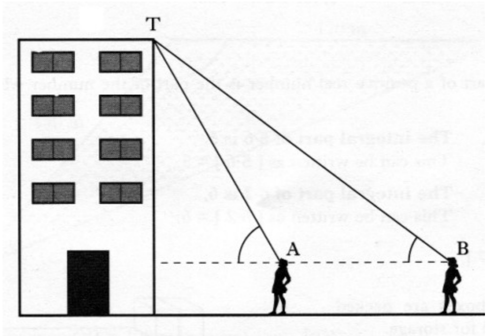
24. The diagram shows two positions of a student as she views the top of a tower.

From position B, the angle of elevation to T at the top of the tower is 64°.

From position A, the angle of elevation to T at the top of the tower is 69°.

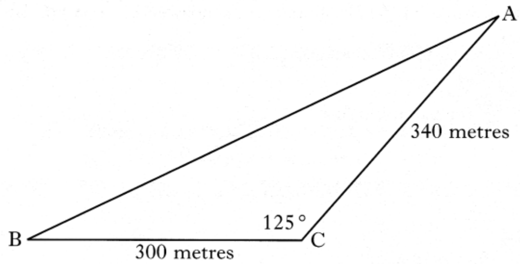
The distance AB is 4.8 metres and the height of the student to eye level is 1.5 metres.

Find the height of the tower. 6 RE



25. A field, ABC, is shown in the diagram.

Find the area of the field. 2 KU

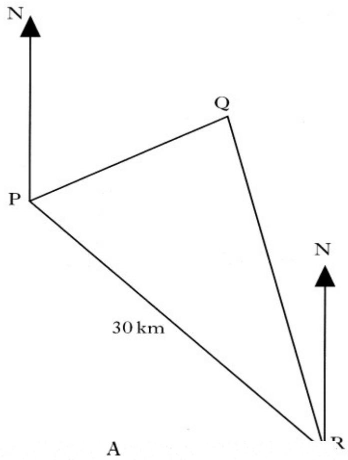


26. A ship, at position P, observes a lighthouse at position Q on a bearing of 040°.

The ship travels 30 kilometres on a bearing of 125° to position R.

From position R, the ship observes the lighthouse on a bearing of 340°.

When the ship is at position R, how far is it from the lighthouse? 6 RE

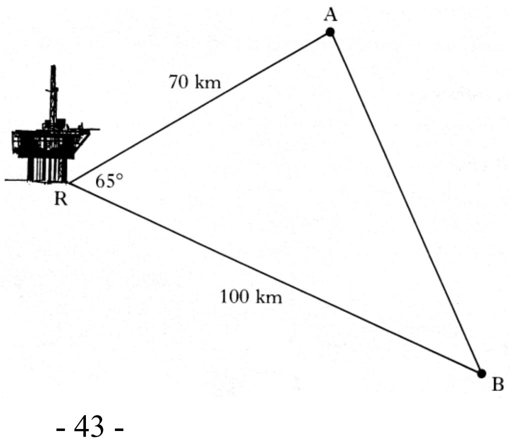


27. The diagram shows the positions of an oilrig and two ships.

The oilrig at R is 70 kilometres from a ship at A and 100 kilometres from a ship at B. Angle ARB = 65°.

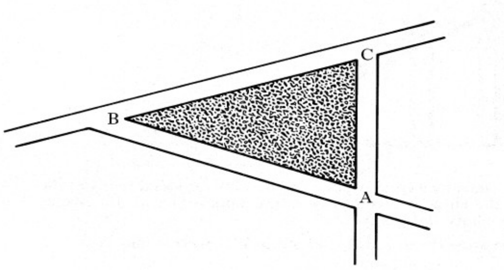
Calculate the distance AB.

Do not use a scale drawing. 4 KU



28. A traffic island, ABC, is shown in the diagram.

Find the area of the traffic island if AB = 12.6 metres, AC = 10 metres and angle BAC = 72° — 2 KU



29. The diagram shows the goalposts on a rugby field.

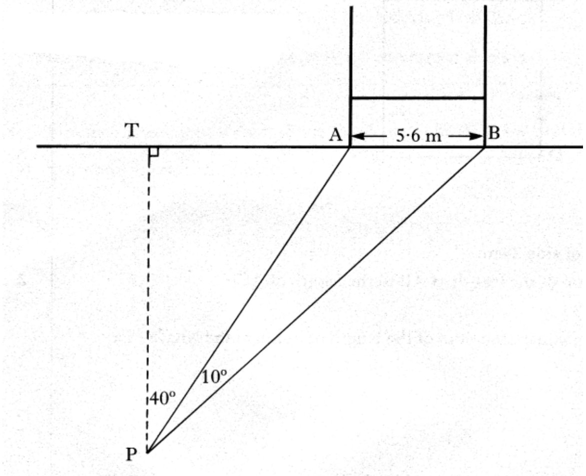
To take a kick at goal, a player moves from T to position P.

TP is perpendicular to TB.

Angle TPA =40° and angle APB = 10°

The distance AB between the goal posts is 5.6 metres.

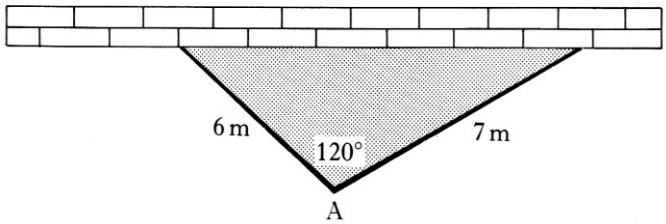
Find the distance from T to P. — 6 RE



30. A family wants to fence

off a triangular part of their garden for their pet rabbits.

They have a long straight wall available and two straight pieces of fencing 6 metres and 7 metres in length.



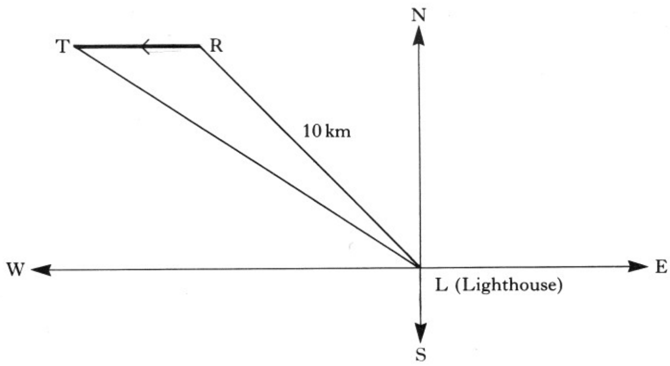
They first erect the fencing as shown in the diagram.

a) Find the area of garden enclosed by the wall and the two pieces of fencing. — 2 KU

b) What size should they make the angle at A so that the greatest area of garden is enclosed?

Give a reason for your answer. — 2 KU

31. A ship is first spotted at position R, which is on a bearing of 315° from a lighthouse, L. The distance between R and L is 10 kilometres. After the ship has travelled due West to position T, its bearing from the lighthouse is 300°.

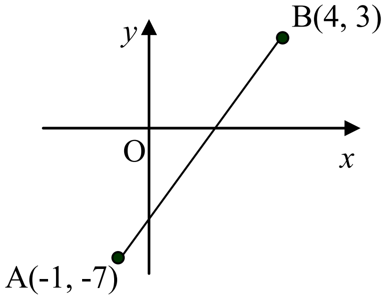


How far has the ship travelled from R to T ? — 5 RE

10. Gradients & The Straight Line

Finding Equations

1. In the diagram, A is the point (-1, 7) and B is the point (4, 3).



   a) Find the gradient of the line AB. 1 KU

   b) AB cuts the y-axis at the point (0, -5). Write down the equation of the line AB 1 KU

   c) The point $(3k, k)$ lies on AB. Find the value of k. 2 RE

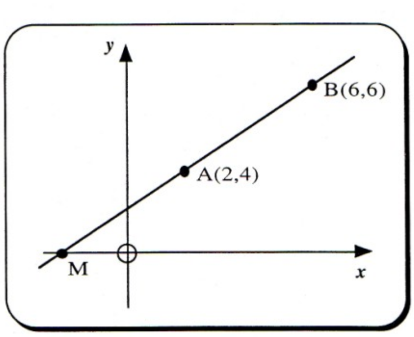
2. A is the point (a^2, a)

   T is the point (t^2, t) $a \neq t$

   Find the gradient of the line AT

   Give your answer in its simplest form. 3 KU

3. The straight line through the points A(2, 4) and B(6, 6) is shown in the diagram.

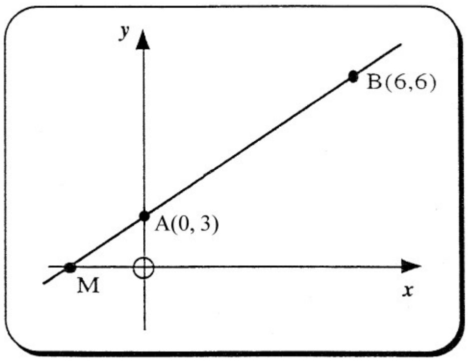


   The point M is where the line AB cuts the x-axis.

   a) Find the equation of the straight line AB. 4 KU

   b) Use this equation to find the coordinates of the point M. 2 RE

4. The straight line through the points A(0, 3) and B(6, 6) is shown in the diagram.

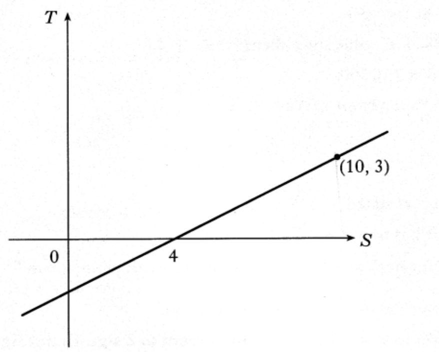


   The point M is where the line AB cuts the x-axis.

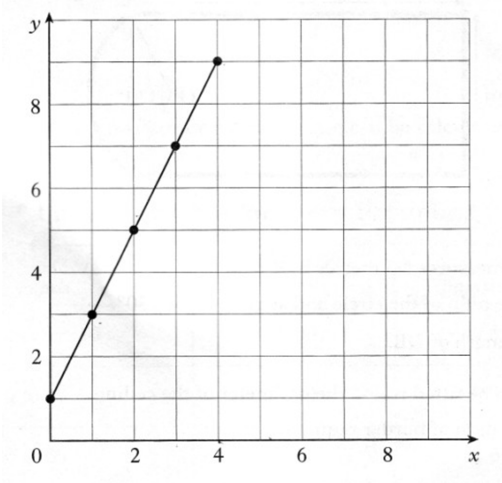
   a) Find the equation of the straight line AB. 4 KU

   b) Use this equation to find the coordinates of the point M. 2 RE

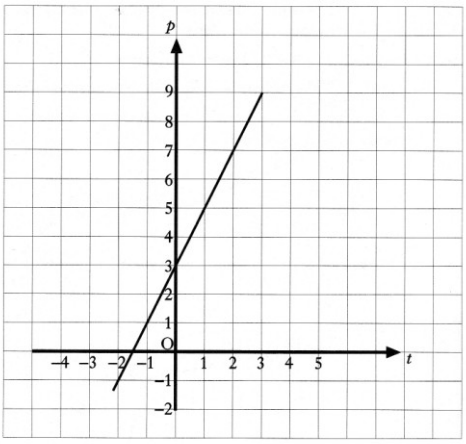
5. Find the equation of the given straight line in terms of S and T. 4 KU



6. Find the equation of the straight line. 3 KU



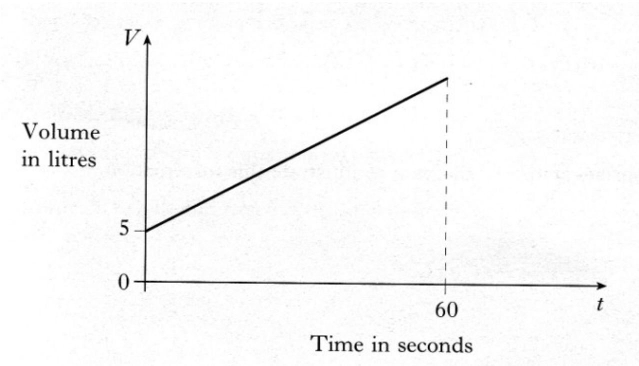
7.



Find the equation of the straight line in terms of p and t. 4 KU

8. The tank of a car contains 5 litres of petrol.

The graph below shows how the volume of petrol in this tank changes as a further 45 litres of petrol is pumped in at a steady rate for 60 seconds.

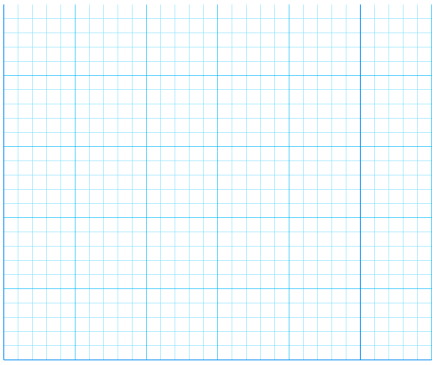


Find the equation of the straight line in terms of V and t. 4 KU

9. A tank contains 10 litres of water.

A further 30 litres of water is poured into the tank at a steady rate of 5 litres per minute.

a) On the 2mm square ruled graph paper provided, draw a graph of the volume, V litres, of water in the tank against the time, t minutes. 4 KU



b) Write down an equation connecting V and t. 2 KU

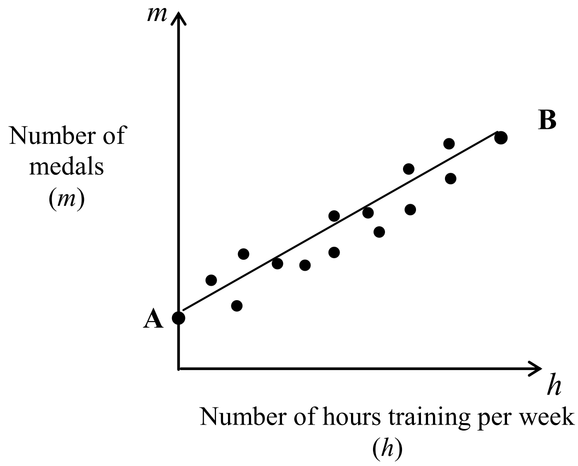
Applications of the Equation of a Straight Line

1. The graph shows the relationship between the number of hours (h) an athlete trains per week and the number of Championship medals (m) they have won.

A best fitting straight line AB has been drawn.

Athlete A does not train but has won 4 medals this year.

Athlete B who trains for 12 hours per week has won 40 medals this year.



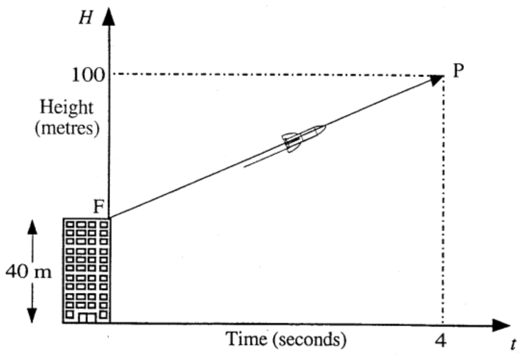
(a) Find the equation of the straight line AB in terms of m and h. 4 RE

(b) How many medals would you expect an athlete who trains 8 hours per week to have won ? 1 KU

2. A boy sets of a rocket from the top of a 40 metre high block of flats.

The diagram shows the path of the rocket over the first 4 seconds.

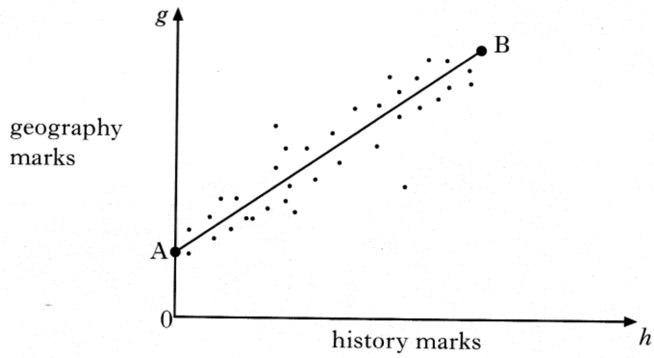
It is represented by the straight line in the graph.



After 4 seconds, the rocket has reached a point 100 metres above the ground.

Find the equation of the straight line FP in terms of H and t. 4 RE

3. The graph below shows the relationship between the history and geography marks of a class of students



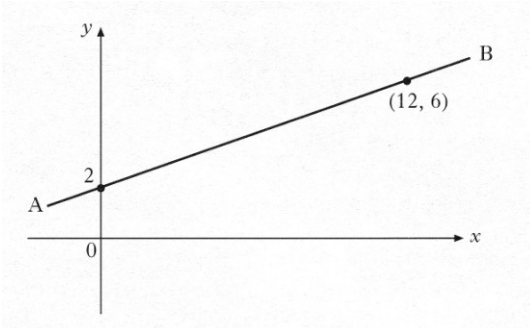
A best fitting straight line, AB has been drawn.

Point A represents 0 marks for history and 12 marks for geography.

Point B represents 90 marks for history and 82 marks for geography.

Find the equation of the straight line AB in terms of h and g. 4 RE

4. A water pipe runs between two buildings.
These are represented by the points A and B in the diagram below.

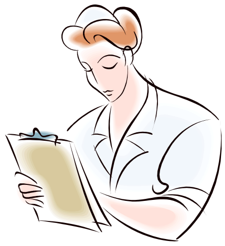


a) Using the information in the diagram, show that the equation of the line AB is $3y - x = 6$. 3 KU

b) An emergency outlet pipe has to be built across the main pipe.
The line representing this outlet pipe has equation $4y + 5x = 46$

Calculate the coordinates of the point on the diagram at which the outlet pipe will cut across the main water pipe. 4 RE

5. When a patient's blood pressure (B.P.), is taken, two measurements are made.



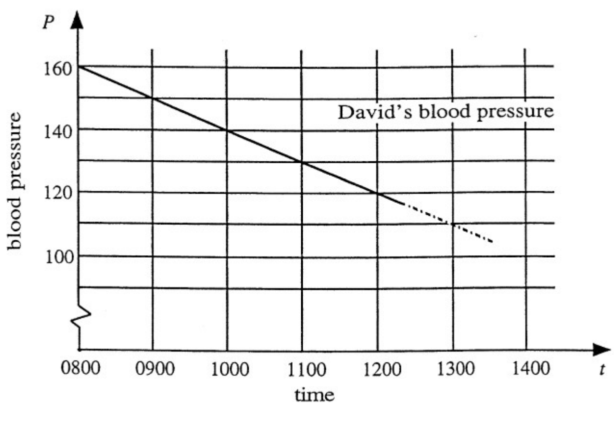
For example, in **"160 over 70"** (or $\frac{160}{70}$),

$\Rightarrow$ the 160 is the reading when the heart is pumping.

$\Rightarrow$ the 70 is the reading when the heart is at rest.

David has a heart problem, and has his blood pressure taken every hour.

The first number of these two measurements is monitored very carefully and the nurse plots a graph, showing the changes from 8 am.



a) Find the gradient of the line shown above. 2 KU

b) Write down the equation of the line in the form

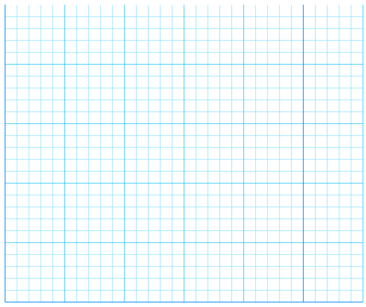
$P =$ ………….. 2 KU

c) It is known that if the blood pressure drops below 70, the patient will be in danger of losing consciousness.

If David's blood pressure continues to drop in the way indicated, when might he be expected to become unconscious. 3 RE

6. A tank contains 240 litres of water.

When the tap is opened, water flows from the tank at a steady rate of 20 litres per minute.

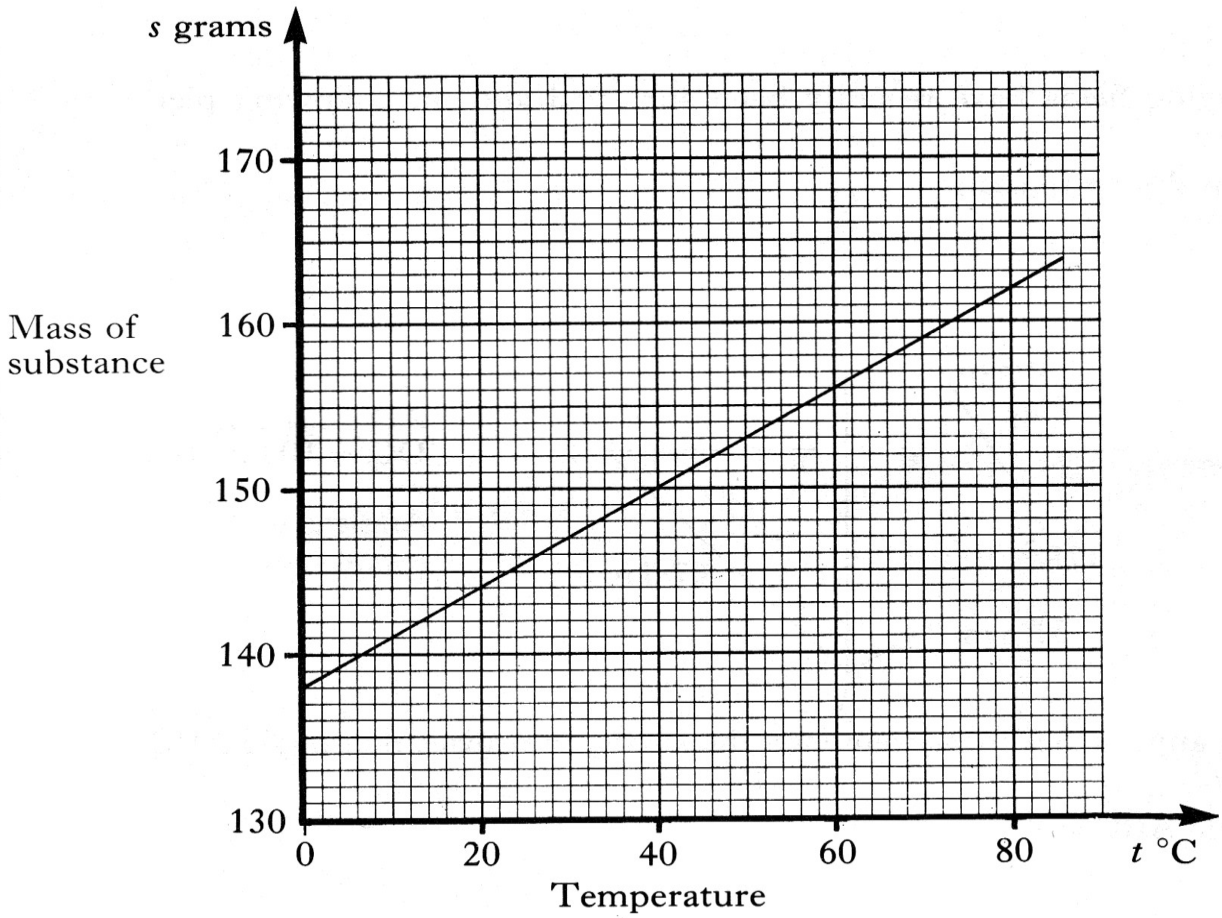


a) On the 2mm square-ruled paper provided, draw a graph of the volume V litres, of water in the tank against the time, t minutes. 3 KU

b) Write down an equation connecting V and t. 2 KU

7. The graph below shows the number of grams, s, of a substance that can be dissolved in a fixed quantity of water when the temperature of the water is t°C.

Find the equation of this straight line in terms of s and t. 4 KU



11. Simultaneous Equations

1. Andrew and Doreen each book in at the Sleepwell Lodge.

   a) Andrew stays for 3 nights and has breakfast on 2 mornings.
   His bill is £145

   Write down an algebraic equation to illustrate this. 1 KU

   b) Doreen stays for 5 nights and has breakfast on 3 mornings.
   Her bill is £240.

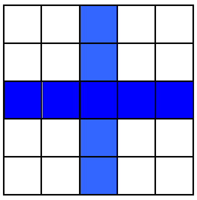
   Write down an equation to illustrate this. 1 KU

   c) Find the cost of one breakfast. 3 RE

2. The reception area in a council office block is to be tiled with a mixture of two types of ceramic tile – white and blue.

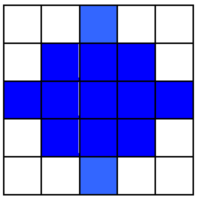
   The contractors left two samples, with their cost per square metre, as shown in the diagrams below.

Diagram 1



Cost: £25.20

Diagram 2



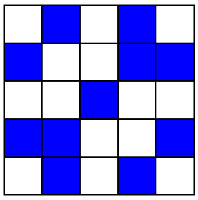
Cost: £26.40

   (a) Using Diagram 1 write down an equation in b and w, where b is the cost of a blue tile and w is the cost of a white tile. 1 KU

   (b) Using Diagram 2 write down a second equation in b and w. 1 KU

   Unfortunately the manager did not like any of the samples left and decided to use one of his own.

   His choice is shown in the diagram on the right.



   (c) How much per square metre would this design cost ? 4 RE

3. a) 4 peaches and 3 grapefruit cost £1.30
   Write down an algebraic equation to illustrate this. 1 KU

   b) 2 peaches and 4 grapefruit cost £1.20.
   Write down an algebraic equation to illustrate this. 1 KU

   c) Find the cost of 3 peaches and 2 grapefruit. 4 RE

4. The tickets for a Sports Club Disco cost £2 for members and £3 for non-members.

a) The total ticket money collected was £580.

x tickets were sold to members
and y tickets were sold to non-members.

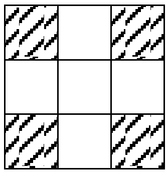
Use this information to write down an equation involving x and y. 2 RE

b) 250 people bought tickets for the disco.

Write down another equation involving x and y. 1 RE

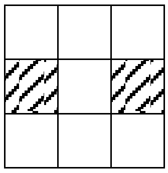
c) How many tickets were sold to members ? 3 RE

5. A small square patio required nine slabs to cover it.



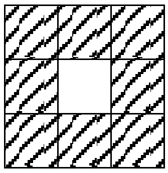
a) The cost of using 4 patterned slabs and 5 plain ones is £15.50

by letting £x be the cost of 1 patterned slab.
£y be the cost of 1 plain slab. 1
Write down an algebraic equation to illustrate this. 1 KU



b) If 2 patterned slabs and 7 plain ones are used instead, the cost becomes £14.50.

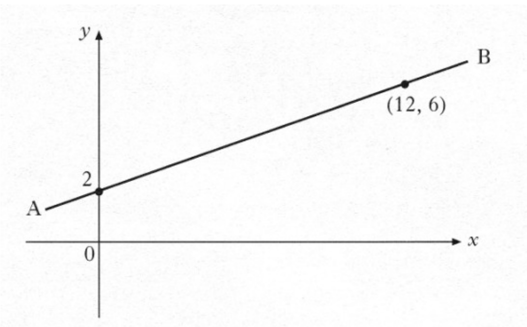
Write down an algebraic equation to illustrate this. 1 KU



c) Find the cost of this arrangement which is made up by using 8 patterned slabs and 1 plain one.

(show all your working clearly). 4 RE

6. A water pipe runs between two buildings.
These are represented by the points A and B in the diagram below.



a) Using the information in the diagram, show that the equation of the line AB is $3y - x = 6$. 3 KU

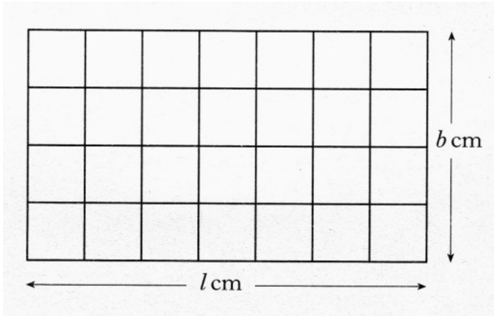
b) An emergency outlet pipe has to be built across the main pipe.
The line representing this outlet pipe has equation $4y + 5x = 46$

Calculate the coordinates of the point on the diagram at which the outlet pipe will cut across the main water pipe. 4 RE

7. A rectangular window has length, l centimetres and breadth b centimetres

A security grid is made to fit this window.

The grid has 5 horizontal wires and 8 vertical wires.



a) The perimeter of the window is 260 centimetres.

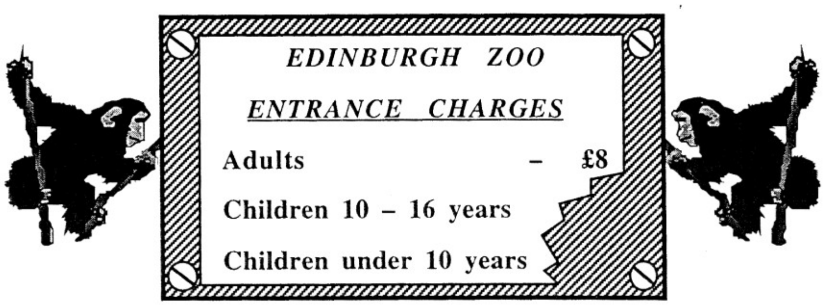
Use this information to write down an equation involving l and b. 1 RE

b) In total, 770 centimetres of wire are used.
Write down another equation involving l and b. 2 RE

c) Find the length and breadth of the window. 3 RE

8. Gillian and Laura took their children to the zoo.

The entrance cost for the zoo was as shown below, but the charges for children have been torn off.



a) **Gillian** paid for **herself** and:-
her **2** sons aged 13 and 15, and her **3** daughters all under 10 years of age.

Let the price for each 10 – 16 year old be £ x.
Let the price for each under 10 year old be £ y.

If Gillian paid £19 in total for herself and her own children, explain why the cost can be expressed in the form.

$$2x + 3y = 11$$ 1 RE

b) **Laura** paid for **herself** and:-
her **4** sons aged 10, 12, 13 and 16, and her **1** daughters aged 7.

Laura paid £15 in total.

Write down a second equation in x and y to indicate her total cost. 1 RE

c) Calculate the cost of:

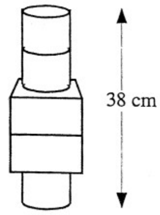
(i) a single ticket for a 14 year old child. 3 RE

(ii) a single ticket for a 7 year old child. 1 RE

9. A child has built a tower made of two types of brick.

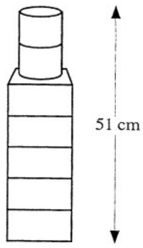
It has three cylinders and two cuboids.
The total height of his tower is 38 centimetres.

Let x cm be the height of one cylinder and
let y be the height of one cuboid.



a) Construct an equation connecting x and y 1 RE

He then built this second tower using two cylinders and five cuboids, and its height was 51 centimetres.



b) Form a second equation in x and y and calculate the height of both a cylinder and a cuboid. 4 RE

10. A number tower is built from bricks as shown in figure 1.

The number on the brick above is always equal to the sum of the two numbers below.

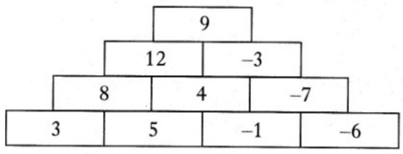


figure 1

a) Find the number on the shaded brick in figure 2. 1 KU

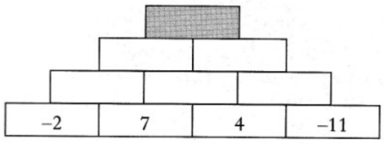


figure 2

b) In figure 3, two of the numbers on the base bricks are represented by p and q.

Show that $p+3q=10$ 2 RE

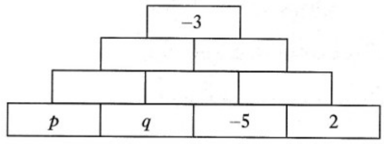


figure 3

c) Use figure 4 to write down a second equation in p and q. 2 RE

d) Find the values of p and q. 3 RE

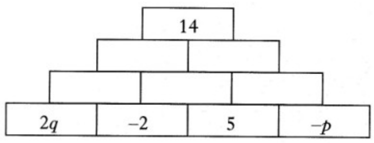


figure 4

11. Alloys are made by mixing metals. Two different alloys are made using iron and lead.

To make the first alloy, 3 cubic centimetres of iron and 4 cubic centimetres of lead are used.

This alloy weighs 65 grams.

a) Let x grams be the weight of 1 cubic centimetre of iron and y grams be the weight of 1 cubic centimetre of lead.

Write down an equation in x and y which satisfies the above condition. 2 RE

To make the second alloy, 5 cubic centimetres of iron and 7 cubic centimetres of lead are used.

This alloy weighs 112 grams.

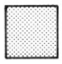
b) Write down a second equation in x and y which satisfies this condition. 2 RE

c) Find the weight of 1 cubic centimetre of iron and the weight of 1 cubic centimetre of lead. 3 RE

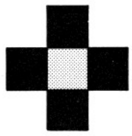
12. A large floor is to be covered with black and grey square tiles to make a chequered pattern.

The person laying the tiles must start at the centre of the floor and work outwards.

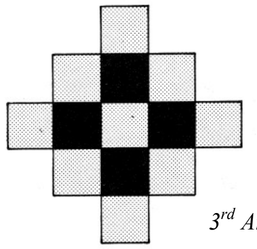
The instructions are as follows.



1st Arrangement



2nd Arrangement



3rd Arrangement

1. **Lay a grey tile in the centre of the floor**
2. **Place black tiles against the edges of the grey tiles**
3. **Place grey tiles against the edges of all the black tiles**
4. **Place black tiles against the edges of all the grey tiles.**
5. **And so on …..**

a) How many tiles are there in the 4th arrangement? 2 RE

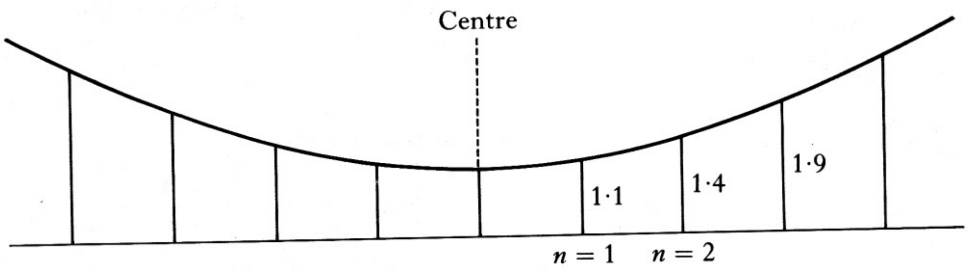
b) The number of tiles, T, needed to make the N th arrangement is given by the formula

$$T = 2N^2 + aN + b$$

Find the values of a and b. 4 RE

13. The heights in metres of the vertical rods of an early suspension bridge, as you move out from the centre, form the sequence

1.1, 1.4, 1.9, 2.6, ……



a) What are the likely heights of the 5th and 6th rods in this sequence 2 RE

b) The height, h metres, of the n^{th} rod in the sequence is given by the formula

$$h = A + bn^2$$

Find the values of A and b and write down the formula. 4 RE

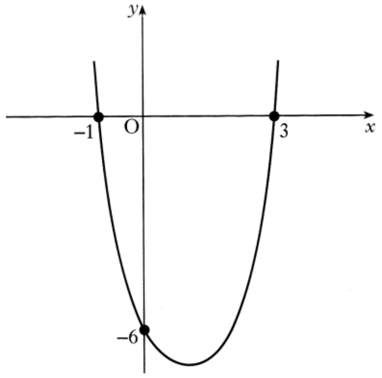
12. Functions

Properties of the parabola

1. The diagram shows part of the graph of a quadratic function, with equation of the form

$$y = k(x-a)(x-b)$$

The graph cuts the y-axis at (0, -6) and the x-axis at (-1, 0) and (3, 0)

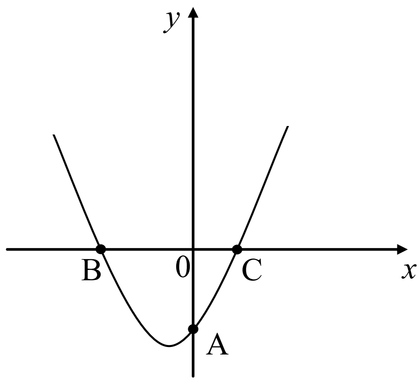


a) Write down the values of a and b. 2 KU

b) Calculate the value of k. 2 KU

c) Find the coordinates of the minimum turning point of the function 2 RE

2. The graph shown has equation $y = x^2 + x - 12$.



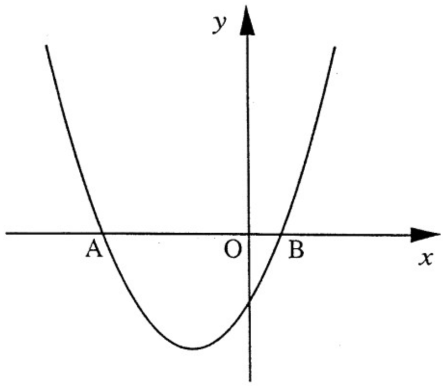
(a) Find the coordinates of A, the point where the curve cuts the y – axis. 1 RE

(b) Find the coordinates of B and C, the points where the curve cuts the x – axis. 3 RE

(c) Find the coordinates of the minimum turning point. 2 RE

3. The graph shows the parabola

$$y = 3x^2 + 7x - 2$$



By solving the quadratic equation

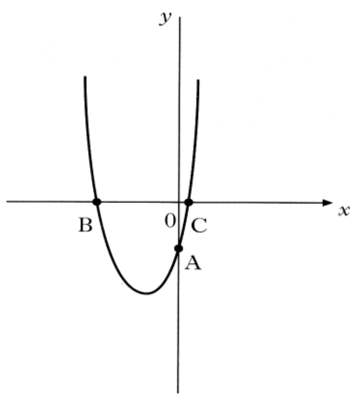
$$3x^2 + 7x - 2 = 0$$

find the coordinates of point A.

Give your answer **correct to 2 decimal places**. 4 KU

4. The diagram below shows part of the graph of $y = 4x^2 + 4x - 3$

The graph cuts the y-axis at A and the x-axis at B and C.



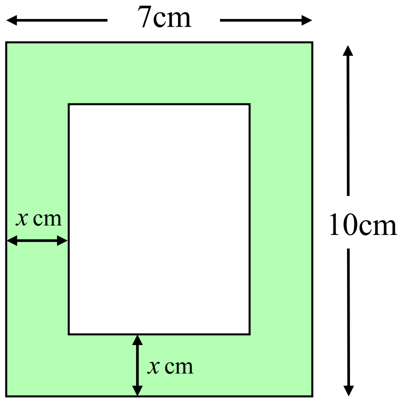
a) Write down the coordinates of A 1 KU

b) Find the co-ordinates of B and C. 3 KU

c) Calculate the minimum value of $4x^2 + 4x - 3$ 2 RE

Applications of the parabola

1. Jane found a small photo-frame and decided to put one of her favourite photographs in it. The diagram below shows the dimensions of the frame.



The width of the wooden surround is x cm.

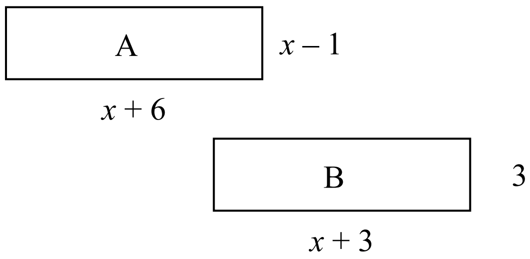
Unfortunately the glass in the centre of the frame was cracked and had to be replaced.

(a) Show that the area of glass needed for the centre of the frame can be given by the formula

$$A = \left(4x^2 - 34x + 70\right)\ \text{cm}^2$$ 4 RE

(b) If the area of glass needed was 28cm^2, find a possible value for x. 3 KU

2. Rectangle A, shown opposite, has length $x + 6$ units and breadth $x - 1$ units.



Rectangle B has length $x + 3$ units and breadth 3 units.

a) Write down expressions, in terms of x, for the area of Rectangle A and the area of Rectangle B. 2 KU

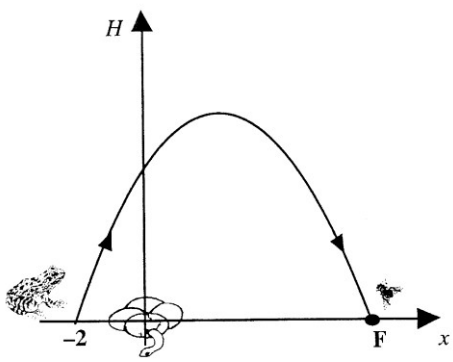
b) Given that both rectangles have the same area for a particular value of x, form an equation using your answers to part (a) and solve it to find this value of x. 3 RE

3. A frog is sitting 2 feet to the left of a snake.

The frog then notices a fly sitting on a rock on the other side of the snake. As the frog leaps over the snake to catch the fly, its path is described by the parabola with equation

$$H = 8 + 2x - x^2$$

where H is the height of the frog above the ground.



a) By considering the quadratic equation:

$$8 + 2x - x^2 = 0$$

find the co-ordinates of the point F, where the fly is sitting, and hence write down how far away the fly is from the frog. 4 KU

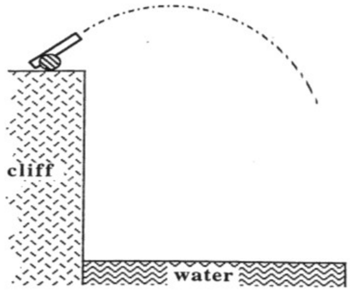
b) How high above the ground does the frog reach on its jump ? 3 RE

4. When a shell is fired from a cannon on top of a cliff, the height of the shell above the water surface is given by the formula:

$$H(t) = 9 + 6t - 3t^2$$

where t is the time in seconds and $H(t)$ is the height in metres after t seconds.

Calculate the height of the shell after 3 seconds.



Explain what your answer indicates. 2 KU

5. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

figure 1.

He plans his flower bed as shown in figure 2.

a) He uses a total of **6 meters of edging.**

AB = ED = x metres.
BC = DC

Show that the length in metres, of BC, can be expressed as BC = $3 - x$. 2 RE

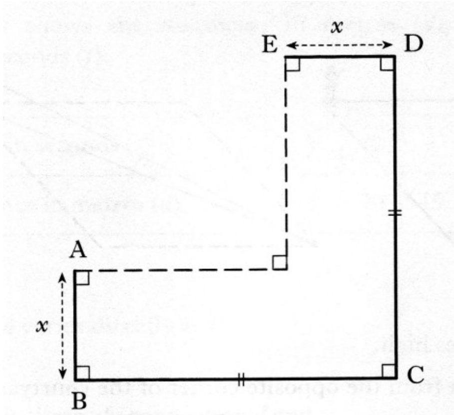


figure 2

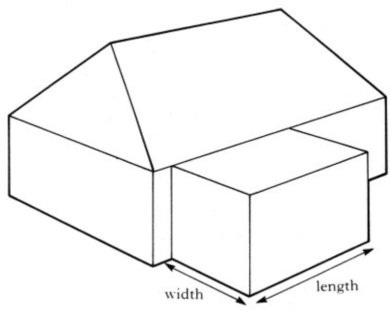
b) Hence show that the area, A, in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$ 3 RE

c) Calculate **algebraically** the maximum area of the flower bed. 3 RE

6. A family want to build an extension at the rear of their house.

An architect advises that the extension should have its length 2 metres more than its width.



a) If the width of the extension is w metres, write down an expression for its length. 1 KU

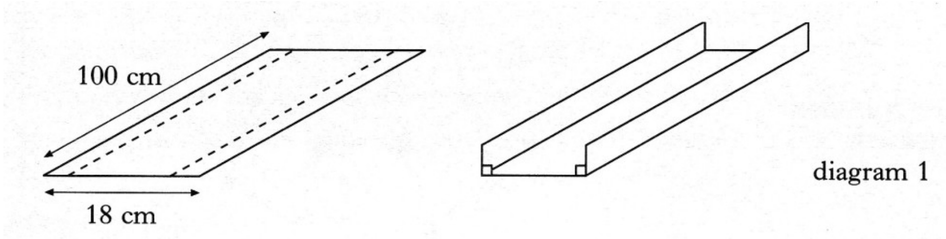
Planning regulations state that the area of the ground floor of the extension must not exceed 40% of the area of the ground floor of the original house.

b) The ground floor of the original house is 12 metres by 10 metres. Show that, if the largest extension is to be built, $w^2 + 2w - 48 = 0$. 3 RE

c) Find the dimensions of the largest extension which can be built. 2 RE

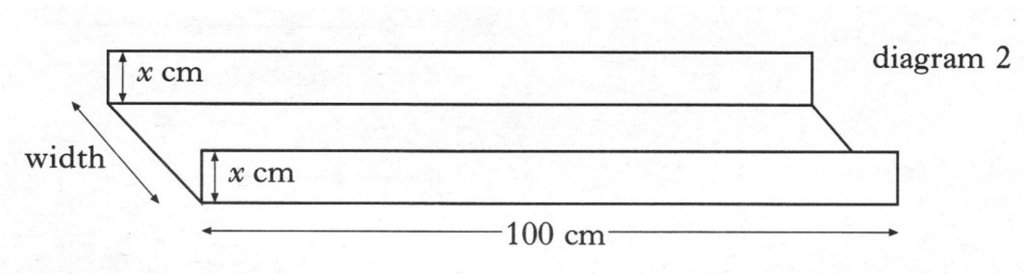
7. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.



a) The depth of the gutter is x centimetres as shown in diagram 2 below.

Write down an expression in x for the width of the gutter. 1 KU



b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$

2 RE

c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working. 4 RE

13. Making & Using Formulae

1. A rectangular clipboard has a triangular plastic pocket attached as shown in Figure 1.

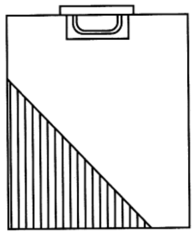


Figure 1

The pocket is attached along edges TD and DB as shown in Figure 2.
B is x centimeters from the corner C.

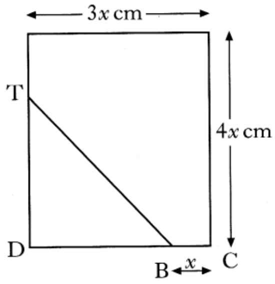


Figure 2

The length of the clipboard is $4x$ centimeters and the breadth is $3x$ centimeters.

The area of the pocket is a quarter of the area of the clipboard.

Find in terms of x, the length of TD. 4 RE

2. The number of diagonals, d, in a polygon with n sides is given by the formula:

$$d = \frac{n(n-3)}{2}$$

A polygon has 20 diagonals.

How many sides does it have? 4 RE

3. Esther has a new mobile phone and considers the following daily rates.

Easy Call	Green Call
25 pence per minute for the first 3 minutes	40 pence per minute for the first 2 minutes
5 pence per minute **after** the first three minutes.	2 pence per minute **after** the first two minutes.

a) For Easy Call, find the cost of ten minutes in a day. 1 KU

b) For Easy Call, find a formula for the cost of "m" minutes in a day, $m > 3$ 1 RE

c) For Green Call, find a formula for the cost of "m" minutes in a day, $m > 2$ 1 RE

d) Green Call claims that its system is cheaper.

Find **algebraically**, the least number of minutes (to the nearest minute) which must be used each day for this claim to be true. 3 RE

4. The intensity of light, I, emerging after passing through a liquid with concentration, c, is given by the equation

$$I = \frac{20}{2^c} \qquad c \geq 0$$

a) Find the intensity of light when the concentration is 3. 1 KU

b) Find the concentration of the liquid when the intensity is 10 2 KU

c) What is the maximum possible intensity ? 3 RE

5. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.

It is to be enlarged by increasing **both** the length and the width by x centimetres.

a) Write down the length of the new vent. 1 RE

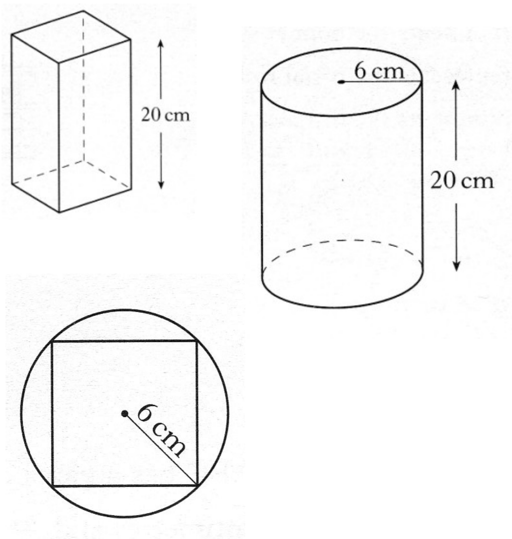
b) Show that the Area, A, square centimeters, of the new vent is given by

$$A = x^2 + 50x + 600$$ 2 RE

c) The area of the new vent **must** be **at least** 40% more than the original area.

Find the minimum dimensions to the nearest centimeters, of the new vent. 5 RE

6. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.



It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.

The corners of the vase touch the Inside of the cylinder as shown.

Show that the volume of the space between the vase and the cylinder is $720(\pi - 2)$ cubic centimetres. 5 RE

7. The cost of renting one of three apartments in Greece depends on the number of people sharing.

If there are **less** than the standard number of people sharing an apartment, (known as under-occupancy), an extra fee is charged.

If there are **more** than the standard number, then a reduction is given to **every** person in the room, *(based on **each** extra adult)*.

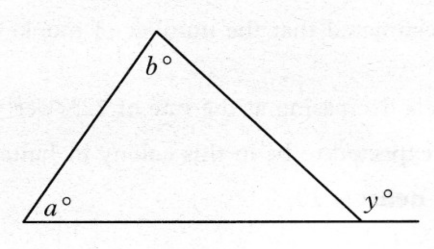
The table below shows how the cost is calculated.

Style of Appartment	*No. Rooms*	*Cost per person per week*	*Based on number sharing*	*Under-occupancy extra fee per person (£)*	*Reduction per extra adult (£)*
Mailia	1	425	2	40	30 (max. 2 extra)
Mavrikos	2	310	4	45	25 (max. 2 extra)
Tsilivi	3	450	6	55	40 (max. 4 extra)

a) Find the total cost of 4 adults staying at Malia Appartments for 1 week. 2 RE

b) Find a formula to calculate the total cost £C, of P people staying at Malia for 1 week, where P is greater than 2 but less than 5. 3 RE

8. Use the information in the diagram to find a relationship connecting a, b and y 2 RE



9. Anna hired a mobile phone at a fixed charge of £17.50 per month.

She is also charged for her total call time each month.

15 minutes of this total call time are **free**. The rest of her call time is charged at 35 pence per minute.

a) What is the total cost for Anna's phone in a month when her **total call time** is 42 minutes. 2 KU

b) Write down a formula for the total cost, £C, for Anna's phone in a month when her total call time is t minutes, where $t \geq 15$. 3 RE

10. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

figure 1.

He plans his flower bed as shown in figure 2.

a) He uses a total of **6 meters of edging.**

AB = ED = x metres.
BC = DC

Show that the length in metres, of BC, can be expressed as BC = 3 – x. 2 RE

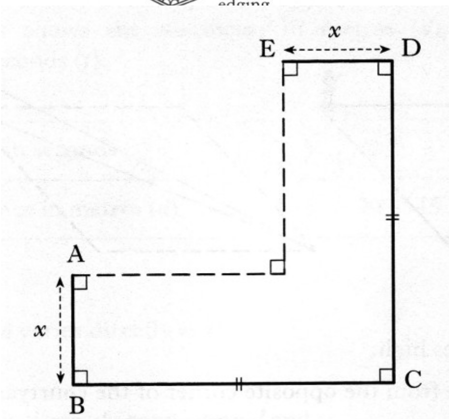


figure 2

b) Hence show that the area, A, in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$

3 RE

c) Calculate **algebraically** the maximum area of the flower bed. 3 RE

11. The cost of taking a school group to the theatre can be calculated from the information shown below.

*** 1 adult goes free for every 10 pupils ***

Number of pupils	*Cost per pupil*	*Cost per paying adult*
Less than 10	£5.00	£8.00
10 to 19	£4.50	£7.00
20 to 29	£4.00	£6.00
30 to 39	£3.00	£5.00

a) Find the cost for a group of 12 pupils and 3 adults. 2 RE

b) Write down a formula to find the cost, £C, of taking a group of p pupils and d adults where $20 \leq p \leq 29$. 4 RE

12. Traffic authorities are investigating the number of cars travelling along a busy stretch of road.

They assume that all cars are travelling at a speed of v metres per second.

The number of cars, N, which pass a particular point on the road in one minute is given by the formula

$$N = \frac{30v}{2+v}$$

In one minute, 26 cars pass a point on the road.

Find the speed of the cars in metres per second. 3 RE

13. While on holiday, John's family decide to hire a car.

There are two different schemes for hiring the same type of car, Eurocar and Apex.

EUROCAR HIRE	APEX HIRE
No deposit required £15 per day	£50 deposit required **plus** £10 per day

a) Write down a formula to find the cost, £C, of hiring the car from Eurocar for d days. 1 KU

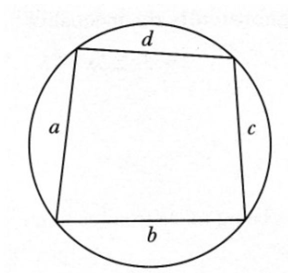
b) Write down a formula to find the cost, £C, of hiring the car from Apex for d days. 2 KU

c) John's family have £170 to spend on car hire.

Which scheme should they use to have the car as long as possible?

Show clearly all your working. 4 RE

14. The area, A, of a quadrilateral drawn inside a circle can be found using the formula

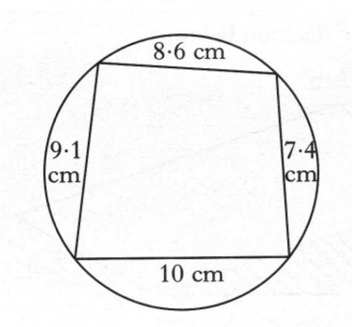


$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where

$$s = \frac{(a+b+c+d)}{2}$$

Use this formula to find the area of the quadrilateral shown in the diagram.



Give your answer correct to 2 significant figures. 3 KU

15. The travelling expenses claimed by a salesperson depend on the engine capacity of the car and the number of miles travelled per week as shown in the table below.

ENGINE CAPACITY	EXPENSES PER MILE
Less than or equal to 1 litre	£0.25 for **each** of the first 250 miles travelled
greater than 1 litre but **less than or equal to** 1.2 litres	£0.27 for **each** of the first 250 miles travelled
greater than 1.2 litres	£0.29 for **each** of the first 250 miles travelled
Where the number of miles traveled in a week **is greater than 250**, £0.15 can be claimed for **each additional** mile.	

a) Find the expenses claimed by a salesperson in a week when 550 miles are travelled and the engine capacity is 1.6 litres. 2 RE

b) Write down a formula to find the expenses £E, claimed for t miles travelled, where t is greater than 250, and the engine capacity is 1.6 litres 3 RE

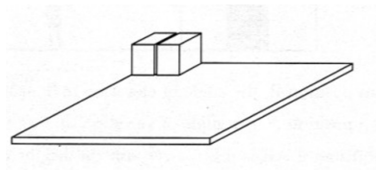
16. The integral part of a positive real number is the part of the number which is an integer.

EXAMPLES **The integral part of 5.6 is 5**
This can be written as **[5.6] = 5**

The integral part of 6.2 is 6
This can be written as **[6.2] = 6**

a) Find [16.7] 1 RE

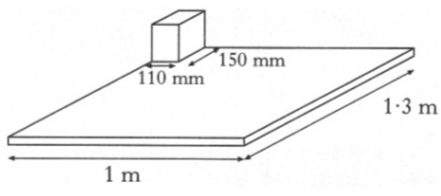
b) Identical boxes are packed on a board for storage. The boxes are all packed the same way round (two boxes are shown in the diagram).



i) The base of each box measures 150 millimetres by 110 millimetres.

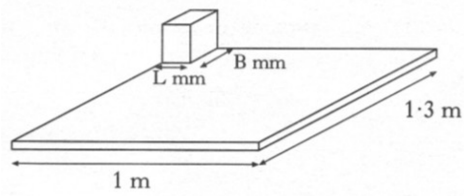
The board measures 1.3 metres by 1 metre.

The number of boxes that can fit along the 1.3 metre length is given by



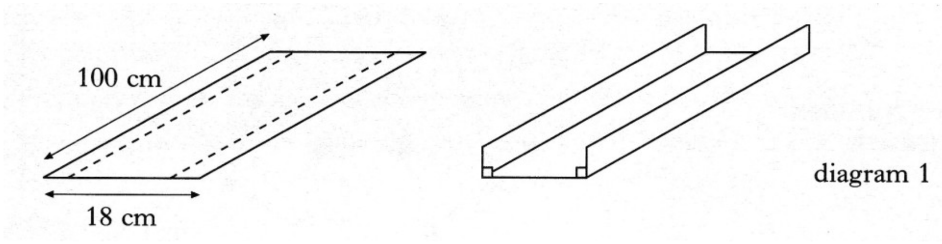
$\left[\dfrac{1300}{150}\right]$ Find $\left[\dfrac{1300}{150}\right]$ 1 RE

ii) Write down an expression for the number of boxes which can be packed on the board shown on the right. 2 RE



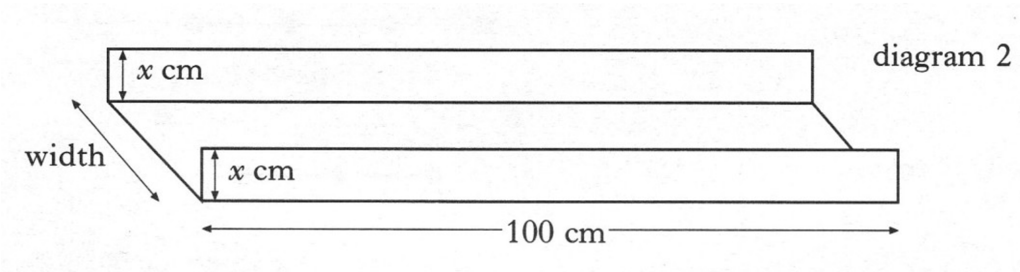
17. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.



a) The depth of the gutter is x centimetres as shown in diagram 2 below.

Write down an expression in x for the width of the gutter. 1 KU



b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$

2 RE

c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working. 4 RE

18. The cost of sending a parcel depends on the weight of the parcel and the time of delivery. The cost is calculated as shown below.

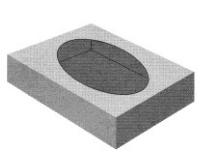
TIME OF DELIVERY	**COST**
by 10 am the next working day	**£18.20** for 10kg and £0.85 for **each extra kg.**
by noon the next working day	**£13.50** for 10kg and £0.75 for **each extra kg.**
by 5 pm the next working day	**£10.50** for 10kg and £0.50 for **each extra kg.**

a) Find the cost of sending a parcel, of weight 14 kg, for delivery **by noon** the next working day. 2 RE

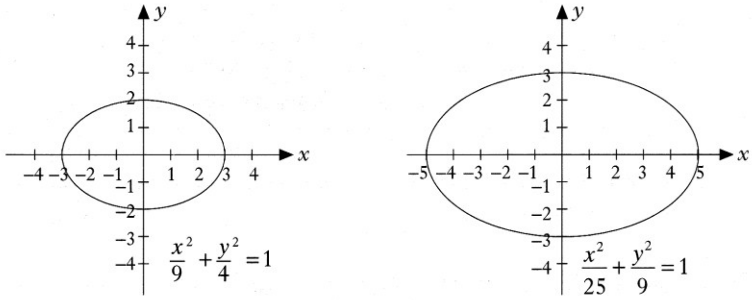
b) Write down a formula to find the cost, £C, of sending a parcel, of weight w kg, where w is greater than 10.

The parcel has to be delivered **by noon** the next working day. 3 RE

19. The opening on this box of tissues is in the shape of an ellipse.

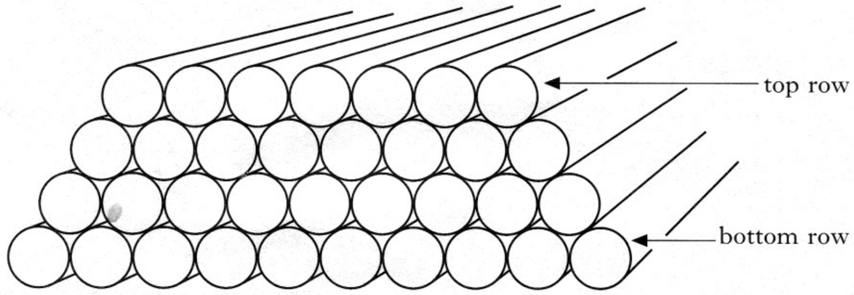


The graphs of two ellipses and their equations are shown below.



Sketch the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{16} = 1$ 3 RE

20. Pipes with equal diameters are arranged in a stack.



To find the number of pipes, P, in the stack, the following formula can be used.

$$P = \frac{(b+a)(b-a+1)}{2}$$

where *b* is the number of pipes on the bottom row and *a* is the number of pipes on the top row.

a) Use this formula to find the number of pipes in a stack where $b = 40$ and $a = 15$. 1 KU

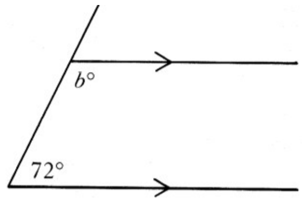
b) In a particular stack, the number of pipes on the bottom row is twice the number on the top row.

Show that in this stack $P = \frac{3a^2 + 3a}{2}$ where *a* is the number of pipes on the top row. 3 RE

c) Would it be possible to arrange exactly 975 pipes in the kind of stack described in part b)

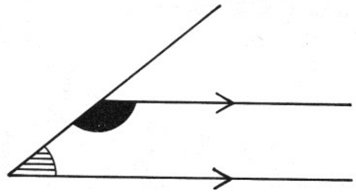
Justify your answer. 3 RE

21. The diagram opposite shows two parallel lines meeting a third at 72°.



a) Find the value of b. 1 KU

b) The diagram opposite shows the general case of two parallel lines meeting a third line.



Prove that in every case, the sum of the shaded angles is 180°. 3 RE

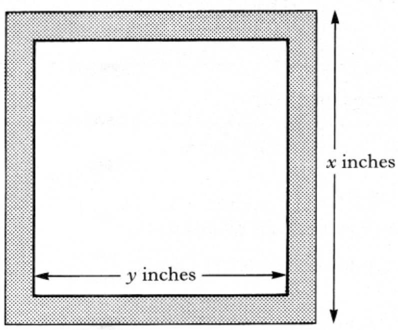
22. An extract from a camping holiday brochure is shown below.

Season	For 14 nights					Over 14 nights
	Two adults	Each extra adult	Each young adult aged 14 to17	Each child aged 10 to 13	Each child aged 0 to 9	Each additional night per family
Low	£399	£74	£40	Free	Free	£19
Mid	£555	£85	£50	Free	Free	£29
High	£699	£95	£60	£46	Free	£39

a) Find the cost of a holiday for 2 adults and a child, aged 8, for 17 nights during mid-season. 2 RE

b) Write down a formula to find the cost, £C, of a holiday in mid-season for 2 adults and a child aged 8 lasting *t* nights, where *t* is greater than 14. 3 RE

23. A square picture frame is shown.



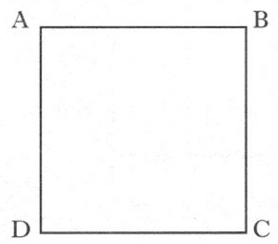
The border of the frame (shaded in the diagram) has uniform width and an area of 48 square inches.

a) Show that $(x-y)(x+y)=48$ 2 RE

b) Given that x and y are whole numbers each greater than 10, find suitable replacements for x and y. 3 RE

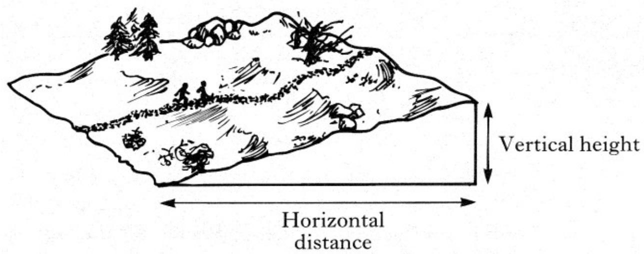
24. a) ABCD is a square of side 2 cms

Write down the ratio of the length AB to the length of AC. 2 KU



b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$ 3 RE

25. The total time a walk takes in hillwalking depends on the horizontal distance covered (h kilometres) and the vertical height climbed v metres.



For **each kilometre** of horizontal distance, 12 minutes should be allowed.

a) i) Write down the time which should be allowed for h kilometres of horizontal distance. 1 KU

ii) for **each 100 metres** of vertical height, 10 minutes should be allowed.

Write down the time which should be allowed for v metres of vertical height. 2 RE

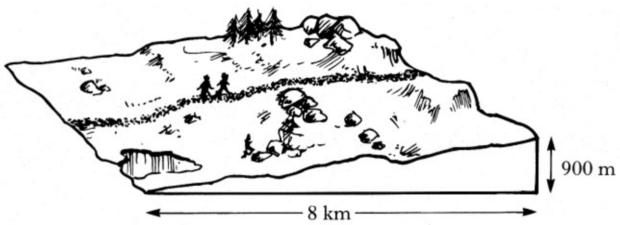
iii) Show that the **total** time T **hours** which should be allowed for the walk is given by the formula

$$T = \frac{120h + v}{600}$$

3 RE

b) For safety reasons, hillwalkers should be off the hills by 1900 hours.

Would it be safe to start the walk shown at 1300 hours? 3



27. Mr and Mrs Paton want to have their house valued before putting it up for sale.

The fee they have to pay for having this done depends on the value of their house.
The fee is calculated as follows

Value of house	Fee to be paid
First £2000 of value	£5.00
Each additional £500 up to £15000	£1.00 per £500
Each additional £1000 over £15000	£1.00 per £1000

a) The Paton's house is valued at £33 000
What fee will they have to pay? 4 RE

b) Write down a formula to find the total fee payable when a house is valued at
£*P* thousand, where *P* is a whole number greater than 15. 3 RE

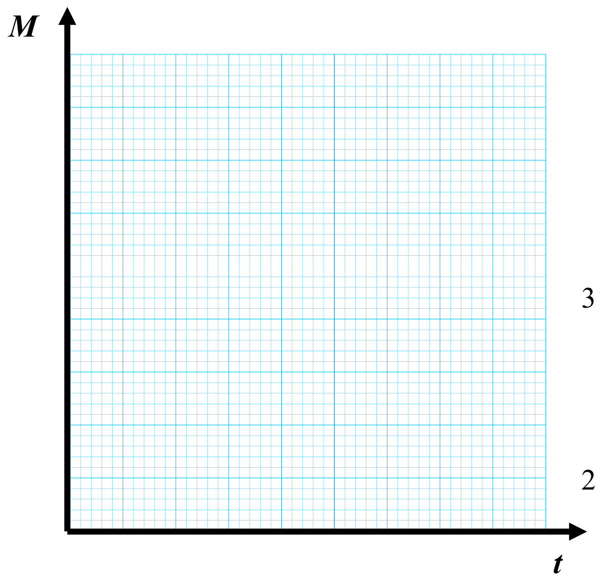
26. The mass, *M* grams, of a given radio-active isotope decreases with time according to the formula

$$M = 80(2)^{-t}$$

where *t* is the time in years.

a) The isotope weighs 80 grams at the start.

Show on the grid below,
how the mass of this isotope changes
over the following 4 years. 3 KU



b) Calculate how many years it takes for
an isotope weighing 80 grams to
decrease to a weight of $\frac{5}{8}$ of a gram. 2 KU

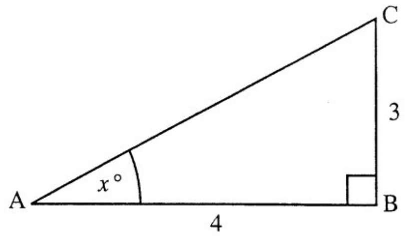
14. Trigonometry 3 - Graphs & Equations

Graphs, triangles, maxima and minima

1. ABC is a right angled triangle with AB = 4 units and BC = 3 units

   Prove that for the angle marked $x°$

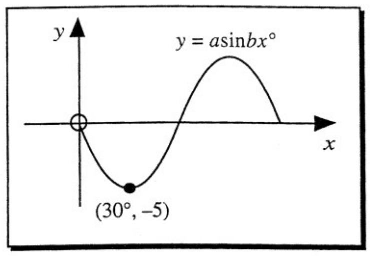
   $$\sin^2 x + \cos^2 x = 1$$



2 KU

2. Shown is the graph of $y = a \sin bx°$

   Write down the values of a and b.



2 KU

3. On a certain day the depth, D metres, of water at a fishing port, t hours after midnight, is given by the formula

   $$D = 12.5 + 9.5\sin(30t)°$$

   a) Find the depth of water at 1.30 pm — 3 RE

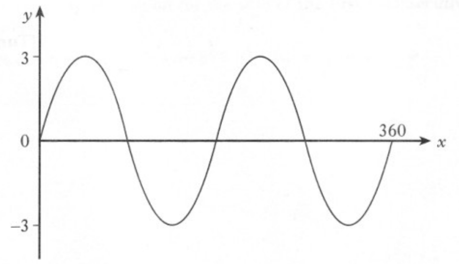
   b) The depth of water in the harbour is recorded each hour. What is the maximum difference in the depths of water in the harbour, over the 24 hour period ?

   Show clearly all your working. — 3 RE

4. The diagram shows the graph of

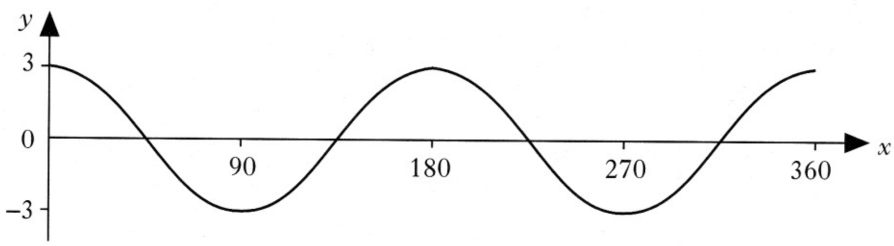
   $$y = k \sin ax°, \ 0 \le x \le 360$$

   Find the values of a and k.



2 RE

5.



The diagram shows the graph of $y = a \cos bx°, \quad 0 \le x \le 360$

Find the values of a and b. — 2 KU

Solving Equations

1. Solve the equation $3\tan x^o + 5 = 0$, for $0 \le x \le 360.$ 4 KU

2. Solve **algebraically** the equation $2 + 3\sin x° = 0$ for $0 \le x \le 360$ 3 KU

3. Solve **algebraically**, the equation $7\cos x° - 2 = 0$ for $0 \le x \le 360$ 3 KU

4. Solve **algebraically**, the equation $5\tan x - 9 = 0$, for $0 \le x \le 360$ 3 KU

5. Solve the equation $5\sin x° + 2 = 0$, *for* $0 \le x \le 360$ 3 KU

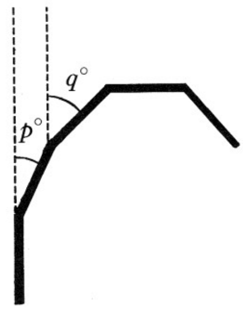
6. Solve algebraically the equation: $\tan 40° = 2\sin x° + 1$ $0 \le x \le 360$ 3 KU

7. The diagram opposite shows part of a natural crystal of topaz.

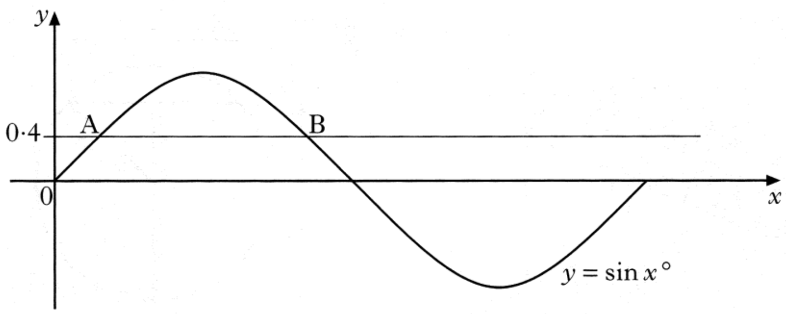
The relationship between the angles marked $p°$ and $q°$ is

$$2\tan p° = \tan q°$$

Find the value of q when p = 24. 3 KU



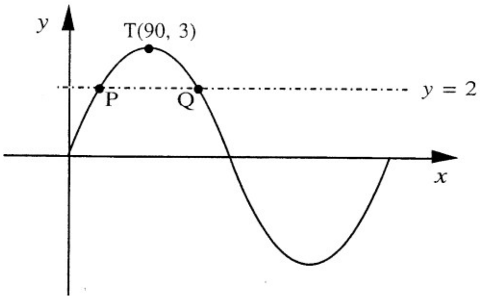
8. The diagram shows part of the graph of $y = \sin x$.



The line $y = 0.4$ is drawn and cuts the graph of $y = \sin x$ at A and B.

Find the x-coordinates of A and B. 3 RE

9. The graph shown has equation $y = a \sin bx°$.



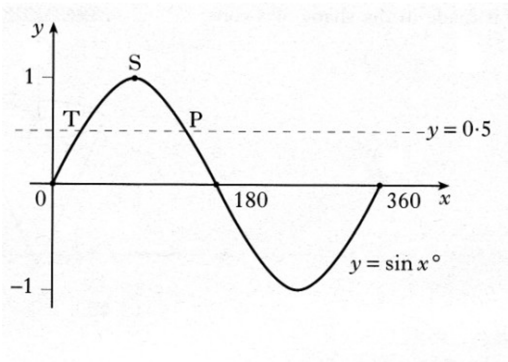
It has a maximum at the point T(90, 3).

a) Write down the values of a and b. 1 KU

Also shown in the figure is the line with equation y = 2, which meets the curve at the points P and Q.

b) Find the x-coordinate of the point Q. 3 RE

10. The diagram shows the graph of $y = \sin x°$, $0 \le x \le 360$

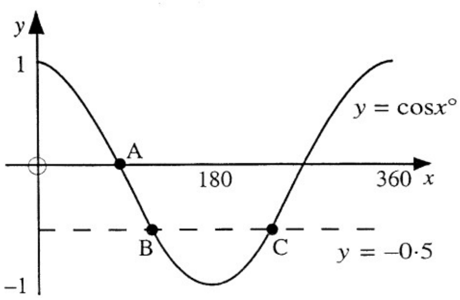


a) Write down the coordinates of point S. 1 KU

The straight line y = 0.5 cuts the graph at T and P.

b) Find the coordinates of T and P. 2 KU

11. The diagram shows the graph of $y = \cos x°$, $0 \le x \le 360$.



a) Write down the coordinates of point A. 1 KU

The straight line $y = -0.5$ cuts the graph at B and C.

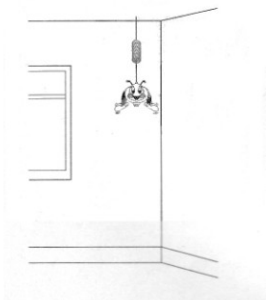
b) Find the coordinates of B and C. 3 KU

12. A toy is hanging by a spring from the ceiling.

Once the toy is set moving, the height, H metres, of the toy above the floor is given by the formula

$$h = 1.9 + 0.3\cos(30t)°$$

t seconds after starting to move.



a) State the maximum value of *H*. 1 KU

b) Calculate the height of the toy above the floor after 8 seconds. 3 RE

c) When is the height of the toy first 2.05 metres above the floor? 3 RE

13. The volume of water, V millions of gallons, stored in a reservoir during any month is to be predicted by using the formula

$$V = 1 + 0.5\cos(30t)°$$

where t is the number of the month. (For January $t = 1$, February $t = 2$ …)

a) Find the volume of water in the reservoir in October. 3 RE

b) The local council would need to consider water rationing during any month in which the volume of water stored is likely to be less than 0.55 million gallons.

Will the local council need to consider water rationing?

Justify your answer. 4 RE

15. Ratio & Proportion

1. School theatre visits are arranged for parents, teachers and pupils.

   The ratio of parents to teachers to pupils **must** be 1 : 3 : 15.

   a) 45 pupils want to go to the theatre.
   How many teachers must accompany them? 1 KU

   b) The theatre gives the school 100 tickets for a play.

   What is the maximum number of pupils who can go to the play? 3 RE

2. A coffee shop blends its own coffee and sells it in one-kilogram tins.

   One blend consists of two kinds of coffee, Brazilian and Columbian,
   in the ratio 2 : 3.

   The shop has 20 kilograms of Brazilian and 25 kilograms of Columbian in stock.

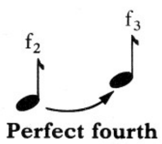
   What is the maximum number of one-kilogram tins of this blend
   which can be made. 3 RE

This is a question from 1990 and is unlikely to be asked today.
However, if you can do it, you are demonstrating a good understanding of ratio and proportion

3. Each of the examples below gives information about the relation between the
   frequencies of two musical notes.

When note 2, of frequency f_2, is a **perfect fifth** above note 1, of frequency f_1,

their frequency ratio $f_2 : f_1 = 3 : 2$ i.e. $\frac{f_1}{f_2} = \frac{3}{2}$

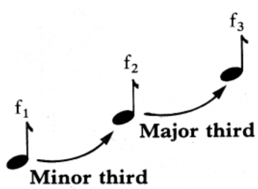


When note 3, is a **perfect fourth** above note 2, their frequency ratio $f_3 : f_4 = 4 : 3$

a) In a given piece of music, note 2 is a **perfect fifth** above note 1,
And note 3 is a **perfect fourth** above note 2.

Show that the frequency ratio of note 3 to note 1 is 2 : 1. 4 RE

b) It is also known that, when one note is a **minor third** above another note,
their frequency ratio is 6 : 5.



For the notes shown opposite, the second note is a **minor third** above the first and the third note is a **major third** above the second.

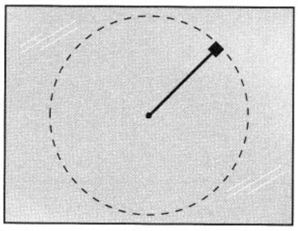
If note 3 is a **perfect fifth** above note 1,
find the frequency ratio of a **major third.**

Show all your working. 4 RE

16. Variation & Proportion

1. A weight on the end of a string is spun in a circle on a smooth table.

   The tension, T, in the string varies directly as the square of the speed, v, and inversely as the radius, r, of the circle.



   a) Write down a formula for T in terms of v and r. 1 KU

   b) The speed of the weight is multiplied by 3 and the radius of the string is halved.

   What happens to the tension in the string. 2 RE

2. The electrical resistance, R, of copper wire varies directly as its length, L metres, and inversely as the square of its diameter, d millimetres .

   Two lengths of copper wire, A and B, have the same resistance.

   Wire A has a diameter of 2 millimetres and a length of 3 metres.

   Wire B has a diameter of 3 millimetres

   What is the length of wire B. 4 RE

3. A frictional force is necessary for a car to round a bend.

   The frictional force, F kilonewtons, varies directly as the square of the car's speed, V metres per second, and inversely as the radius of the bend, R metres.

   a) Write down a relationship between F, V and R. 1 KU

   A frictional force of 20 kilonewtons is necessary for a car, travelling at a given speed to round a bend.

   b) Find the frictional force necessary for the same car, travelling at **twice** the given speed, to round the same bend. 2 RE

4. The table below shows the distances, in metres, (d), travelled by a snowboarder in seconds (t).

Time in seconds (t)	1	2	3	4
Distance in metres (d)	5	20	45	80

   a) Explain why d varies directly as t^2 1 RE

   b) Write down the formula connecting d and t. 1 KU

   c) How does the distance change when the time is multiplied by six ? 2 RE

5. The time, T minutes, taken for a stadium to empty varies directly as the number of spectators, S, and inversely as the number of open Exits, E.

   a) Write down a relationship connecting T, S and E. 1 KU

   It takes 12 minutes for a stadium to empty when there are 20 000 spectators and 20 open exits.

   b) How long does it take the stadium to empty when there are 36 000 spectators and 24 open exits ? 3 KU

6. The number of litres of petrol, L, used by a car on a journey varies directly as the distance, D kilometres, travelled, and as the square root of the average speed, S kilometres per hour.

a) Write down a relationship connecting L, D and S. 1 KU

The car uses 30 litres of petrol for a journey of 550 kilometres when it travels at an average speed of 81 kilometres per hour.

b) How many litres of petrol does the car use for a journey of 693 kilometres travelling at an average speed of 100 kilometres per hour. 3 KU

7. The surface area of a planet, A square kilometers, varies directly as the square of the diameter, D kilometres of the planet.

The surface area of the Moon is 3.8×10^7 square kilometres.

Calculate the surface area of a planet with diameter double the diameter of the Moon. **Give your answer in scientific notation**. 3 KU

8. A table of pairs of values of x and y is shown below.

x	1.5	2	2.5
y	6	4.5	3.6

a) Explain why y varies inversely as x. 2 KU
b) Write down the formula connecting x and y 1 KU

9. The number of letters, N, which can be typed on a sheet of paper varies inversely as the square of the size, s, of the letters used.

a) Write down a relationship connecting N and s. 1 KU

b) The size of the letters used is doubled.

What effect does this have on the number of letters which can be typed on the sheet of paper. 2 KU

10. The time, T seconds, taken by a child to slide down a chute varies directly as the length, L metres, of the chute and inversely square root of the height, H metres, of the chute above the ground.

It takes 10 seconds to slide down a chute which is 3.75 metres long and 2.25 metres high.

a) Find a formula connecting T, L and H. 3 KU

b) How long does it take to slide down a chute which is 5 metres long and 2.56 metres high? 2 KU

11. The power, P watts, produced by a windmill varies directly as the cube of the wind velocity, V metres per second.

At 4 pm on a given day, the wind velocity was 4 metres per second and the windmill was producing 75 watts of electrical power.

By 10 pm the wind velocity had doubled.

How many watts of electrical power were now being produced? 3 KU

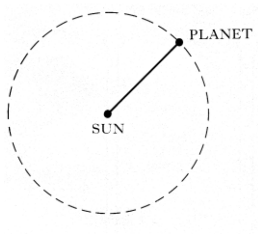
17. Distance, Speed & Time and Graphs

Calculations

1. a) A driver travels from A to B, a distance of x miles at a constant speed of 75 kilometres per hour.
Find the time taken for the journey in terms of x. 1 KU

b) The time for the journey from B to A is $\frac{x}{50}$ hours
Hence calculate the driver's average speed for the whole journey. 4 RE

2. A planet takes 88 days to travel round the Sun.
The approximate path of the planet round the Sun is a circle with diameter 1.2×10^7 kilometres.



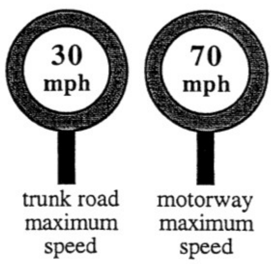
Find the speed of the planet as it travels round the Sun.

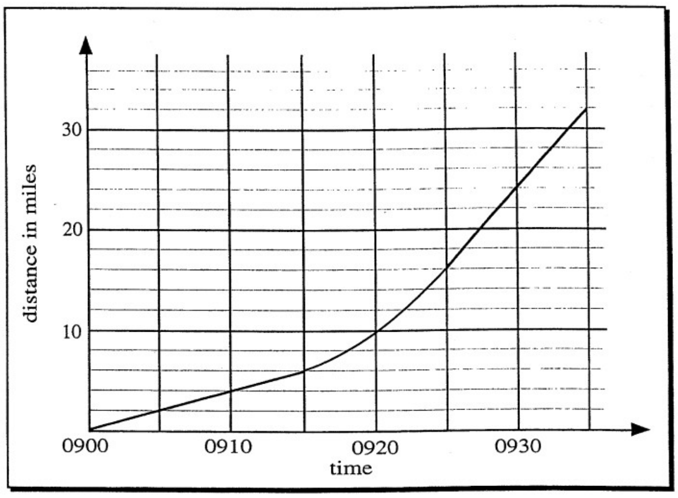
Give your answer in kilometres per hour, correct to 2 significant figures. 4 KU

3. The planet Pluto is at a distance of 5.9×10^9 kilometres from the Sun and the speed of light is 3.0×10^5 kilometres per second.
Calculate, to the nearest half hour, the time taken for light from the Sun to reach Pluto. 4 KU

4. The planet Mars is at a distance of 2.3×10^8 kilometres from the Sun.
The speed of light is 3.0×10^5 km per second.
How long does it take light from the Sun to reach Mars ?
Give your answer to the nearest minute. 3 KU

5. Jennifer is driving to work.
Part of her journey is on a trunk road.
At 0915 she joins the motorway.
The graph shows her journey.





a) Calculate Jennifer's average speed along the trunk road. 2 KU

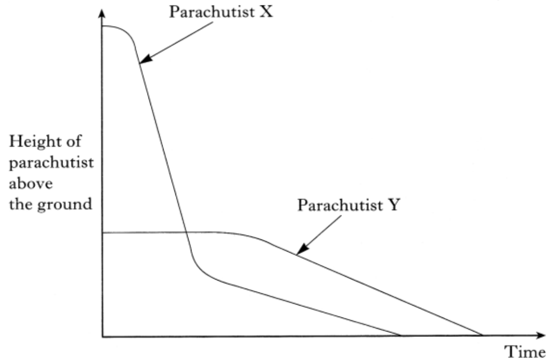
b) Explain what the graph indicates is happening between 0915 and 0925. 1 RE

c) Where on her way to work, did Jennifer appear to break the speed limit ?

(Give a reason for your answer) 3 RE

Graphs & Interpretation

1. Two parachutists, X and Y, jump from two separate aircrafts at different times.

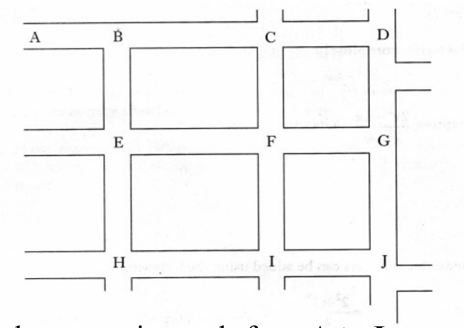


The graph shows how their height above the ground changes over a period of time.

a) Which parachutist jumped first ? 1 RE

b) Which parachutist did not open his parachute immediately after jumping ?
Explain your answer clearly. 2 RE

2. The diagram opposite shows part of the street plan of a town.



Vehicles can travel in both directions along each street.

As a vehicle travels on the straight parts of any street, it can reach the maximum speed.

The speed is always reduced on the bends.

The graph in figure 2 shows how the speed of a vehicle changes as it travels from **A to J**.

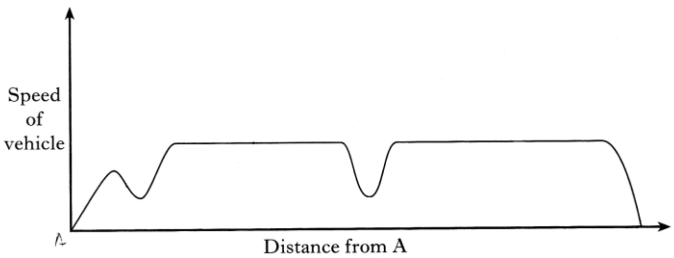
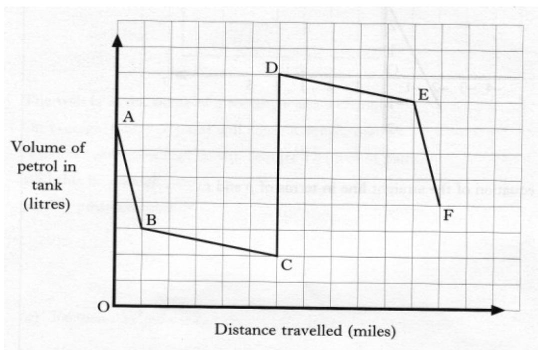


figure 2

a) What route did the vehicle travel? Use the letters from figure 1 to indicate this route. 2 RE

b) Another vehicle took the route A, B, C, F, G and J. Sketch a graph to show how the speed of this vehicle changes during the journey. 3 RE

3. The graph shows the volume of petrol in a car's tank during a journey.



a) Explain the significance of CD. 1 RE

The journey involves driving through towns and along motorways.

In the towns the car uses more petrol per mile than on the motorways.

b) Which two parts of the graph show driving on motorways?
Explain your answer clearly. 2 RE

This is a question from 1990 and would probably be considered to be too long for today's examination. However, it is good practice to attempt it.

4. The gate G of a country park lies on a 400 metre stretch of road which runs in a north-south direction.

See Figure 1.

A car leaves the park, travels northwards with increasing speed, and reaches the end of the stretch of road 24 seconds later.
A motor-cycle leaves the park at the same time as the car and also travels northwards.

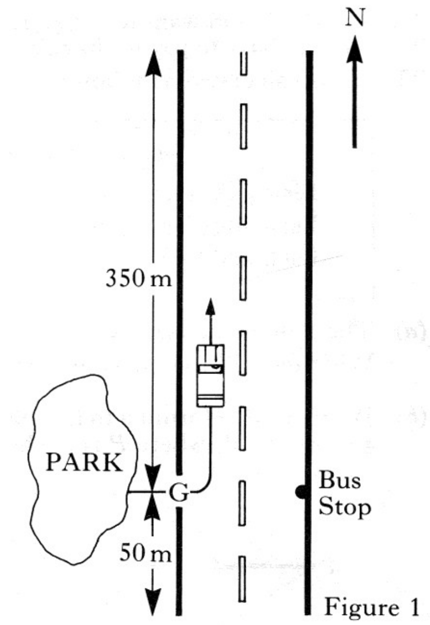


Figure 1

The progress of the two vehicles is shown on the graph below, Figure 2.

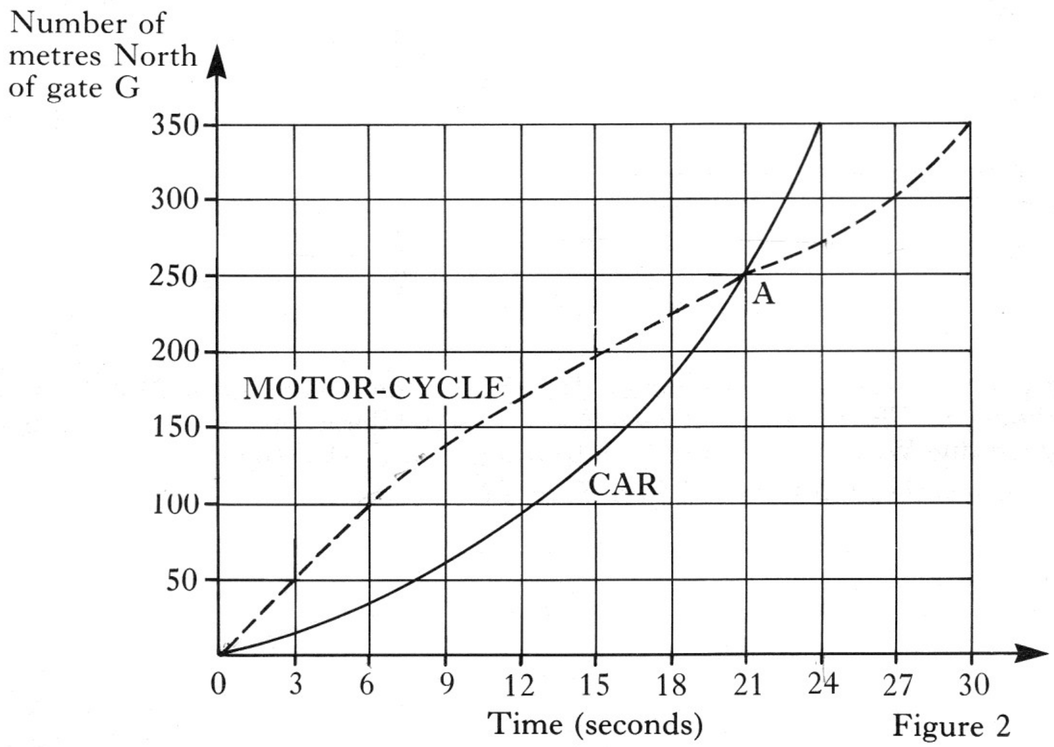


Figure 2

a) Describe the progress of the motor-cycle as it travels along the road, making particular reference to the significance of the point A. 2 RE

4 (continued)

b) The progress of a bus on the same road is also shown on the graph below, Figure 3.

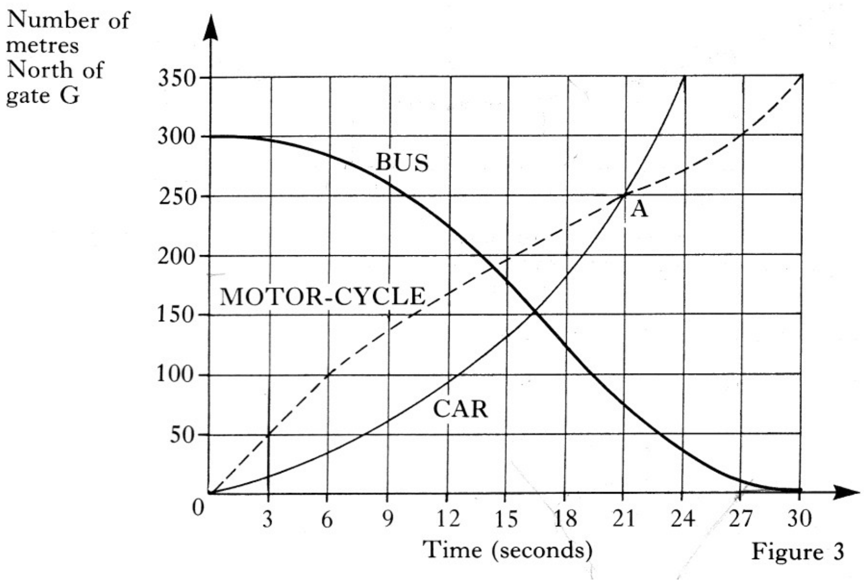


Figure 3

Describe the progress of the bus. 3 RE

c) Some time later, a taxi enters the same road at point T, in Figure 4, and travels southwards at a steady speed.

It reaches the roundabout R after 18 seconds, drives slowly round the roundabout and enters the gate G, 9 seconds later.

Draw a graph of the progress of the taxi. 3 RE

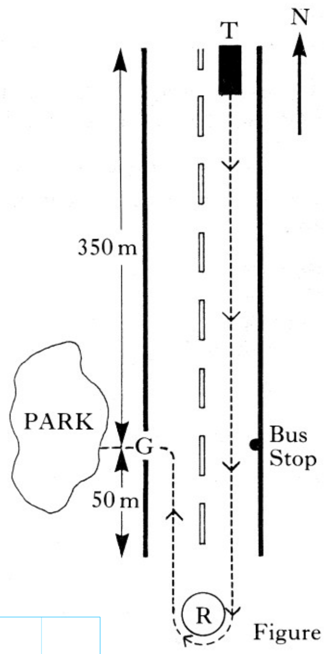
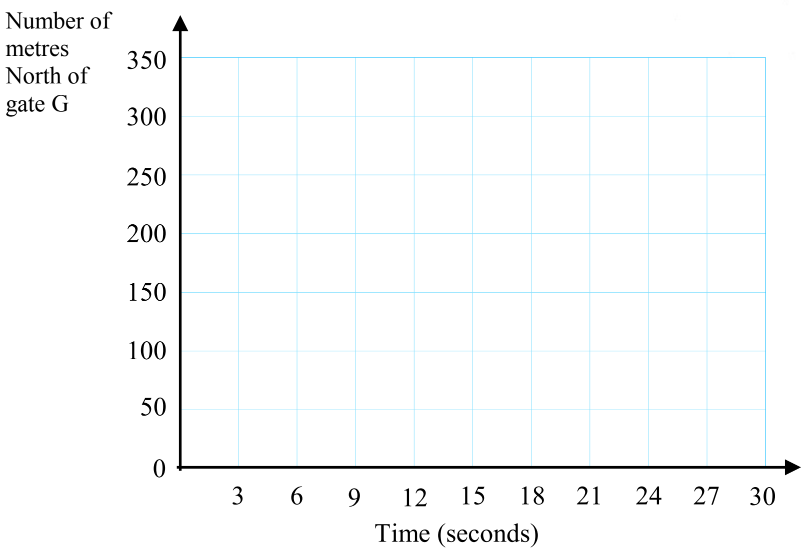


Figure 4



18. Sequences

1. Using the sequence

   1, 3, 5, 7, 9, …..

   a) Find S_3, the sum of the first 3 numbers. 1 RE

   b) Find S_n, the sum of the first n numbers. 2 RE

   c) Hence or otherwise, find the $(n + 1)^{th}$ term of the sequence 2 RE

2. The median of seven consecutive even integers is $2p+2$.

   (a) Write down, in terms of p , expressions for the seven integers. 2 RE

   (b) Show that the mean can be expressed as $2(p+1)$. 2 RE

3. a) Solve the equation

   $$2^n = 32$$ 1 KU

   b) A sequence of numbers can be grouped and added together as shown.

The sum of 2 numbers:	(1 + 2)	=	4 – 1
The sum of 3 numbers:	(1 + 2 + 4)	=	8 – 1
The sum of 4 numbers:	(1 + 2 + 4 + 8)	=	16 – 1

   Find a similar expression for the sum of 5 numbers. 1 RE

   c) Find a formula for the sum of the first n numbers of this sequence. 2 RE

4. Study the pattern of numbers given below:

1^{st} pattern	$2 \times (1) - 1$	=	1
2^{nd} pattern	$2 \times (1 + 2) - 2$	=	4
3^{rd} pattern	$2 \times (1 + 2 + 3) - 3$	=	9
4^{th} pattern	$2 \times (1 + 2 + 3 + 4) - 4$	=	16

   a) Write down a similar expression for the 5^{th} pattern. 1 KU

   b) Write down the general formula for the n^{th} pattern. 2 RE

   c) If $2 \times (1 + 2 + 3 + ….. + t) - t = 289$, find the value of t. 3 RE

5. A number pattern is shown below:

   $$1^3 + 1 = (1+1)(1^2 - 1+1)$$
   $$2^3 + 1 = (2+1)(2^2 - 2+1)$$
   $$3^3 + 1 = (3+1)(3^2 - 3+1)$$

   a) Write down a similar expression for $7^3 + 1$ 1 RE

   b) Hence write down an expression for $n^3 + 1$ 1 RE

   c) Hence find an expression for $8p^3 + 1$ 2 RE

6. 1, 3, 5, 7, ….

The **first** odd number can be expressed as $1 = 1^2 - 0^2$

The **second** odd number can be expressed as $3 = 2^2 - 1^2$

The **third** odd number can be expressed as $5 = 3^2 - 2^2$

a) Express the **fourth** odd number in this form. 1 RE

b) Express the number 19 in this form 1 RE

c) Write down a formula for the n^{th} odd number and simplify this expression. 2 RE

d) **Prove** that the product of two consecutive odd numbers is always odd 3 RE

7. A pattern of numbers is found as follows:

$3 + 2 - 1$	1^{st} term
$6 + 3 - 3$	2^{nd} term
$9 + 4 - 5$	3^{rd} term
………..	
………..	

a) Write down the next 2 terms in this pattern 2 RE

b) Write an expression for the n^{th} term in this pattern and express it in its simplest form. 3 RE

8. The difference between squares of any two consecutive whole numbers can be found using the following pattern.

$$2^2 - 1^2 = 3 = 2 + 1$$

$$3^2 - 2^2 = 5 = 3 + 2$$

$$4^2 - 3^2 = 7 = 4 + 3$$

a) Use this to find the difference between 24^2 and 23^2 2 RE

b) Write down an expression for the difference between the **squares of any two consecutive numbers**, and simplify it as much as possible. 4 RE

[Hint: let one of the consecutive numbers be n.]

9.

> A Fibonacci sequence is a sequence of numbers.
>
> After the first two terms, each term is the sum of the previous two terms.
>
> $13 = 5 + 8$ ↑
>
> e.g. **2, 3, 5, 8, 13, …..**
>
> ↓ $5 = 2 + 3$

a) Write down the next three terms of this Fibonacci sequence.

5, -1, 4, ____, ____, ____, 1 RE

b) For the Fibonacci sequence

4, -3, 1, -2, -1, -3, -4, …….

Show that the sum of the first six terms is equal to four times the fifth term. 2 RE

c) If p and q are the first two terms of a Fibonacci sequence, **prove** that the sum of the first six terms is equal to four times the fifth term. 3 RE

10. A sequence of terms, starting with 1, is

$$1, \; 5, \; 9, \; 13, \; 17, \; \ldots\ldots$$

Consecutive terms in this sequence are formed by adding 4 to the previous term.

The total of consecutive terms of this sequence can be found using the following pattern.

Total of the first 2 terms:	$1 + 5$	$=$	2×3
Total of the first 3 terms:	$1 + 5 + 9$	$=$	3×5
Total of the first 4 terms:	$1 + 5 + 9 + 13$	$=$	4×7
Total of the first 5 terms:	$1 + 5 + 9 + 13 + 17$	$=$	5×9

a) Express the total of the first 9 terms of this sequence in the same way. 2 RE

b) The first n terms of this sequence are added.
Write down an expression, in n, for the total. 3 RE

11. A 3×3 square has been identified on the calendar shown opposite.

The numbers in the diagonally opposite corners of the square are multiplied.

These products are then subtracted in the order shown below.

$$\mathbf{(23 \times 11) - (25 \times 9) = 28}$$

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

a) Repeat the process for a different 3×3 square.

Show clearly all your working. 1 RE

b) Prove that in every 3×3 square on the calendar above, the process gives the answer 28. 3 RE

12. Consecutive cubic numbers can be added using the following pattern.

$$1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

$$1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2 \times 5^2}{4}$$

a) Express $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$ 2 RE

b) Write down an expression for the sum of the first n consecutive cubic numbers. 3 RE

c) Write down an expression for $8^3 + 9^3 + 10^3 + \ldots\ldots + n^3$ 2 RE

13. The sequence of odd numbers starting with 3 is $3, \; 5, \; 7, \; 9, \; 11, \; \ldots\ldots$

Consecutive numbers from this sequence can be added using the following pattern.

$$3 + 5 + 7 + 9 = \mathbf{4 \times 6}$$

$$3 + 5 + 7 + 9 + 11 = \mathbf{5 \times 7}$$

$$3 + 5 + 7 + 9 + 11 + 13 = \mathbf{6 \times 8}$$

a) Express $3 + 5 + \ldots\ldots + 25$ in the same way. 2 RE

b) The first n numbers in this sequence are added. Find a formula for the total. 3 RE

14. A sequence of numbers is **1, 5, 12, 22, ……..**

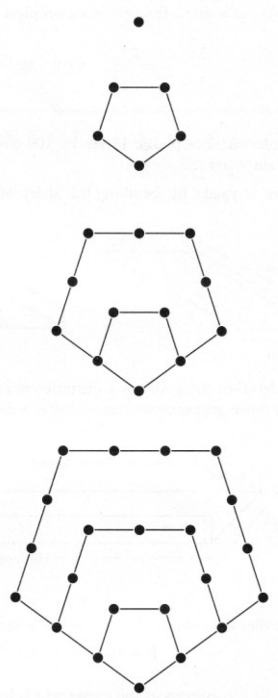
Numbers from this sequence can be illustrated in the following way using dots.

First Number
(N = 1)

Second Number
(N = 2)

Third Number
(N = 3)

Fourth Number
(N = 4)



a) What is the fifth number in this sequence?

Illustrate this in a sketch. 2 RE

b) The number of dots, D, needed to illustrate the Nth number in this sequence is given by the formula

$$D = aN^2 - bN$$

Find the values of a and b. 4 RE

15. Brackets can be multiplied out in the following way.

$$(y+1)(y+2)(y+3) = y^3 + (1+2+3)y^2 + (1\times2 + 1\times3 + 2\times3)y + 1\times2\times3$$
$$(y+2)(y+3)(y+4) = y^3 + (2+3+4)y^2 + (2\times3 + 2\times4 + 3\times4)y + 2\times3\times4$$
$$(y+3)(y+4)(y+5) = y^3 + (3+4+5)y^2 + (3\times4 + 3\times5 + 4\times5)y + 3\times4\times5$$

a) In the same way, multiply out $(y+4)(y+5)(y+6)$ 2 RE

b) In the same way, multiply out $(y+a)(y+b)(y+c)$ 2 RE

16. The following number pattern can be used to sum consecutive square whole numbers.

$$1^2 + 2^2 = \frac{2\times3\times5}{6}$$
$$1^2 + 2^2 + 3^2 = \frac{3\times4\times7}{6}$$
$$1^2 + 2^2 + 3^2 + 4^2 = \frac{4\times5\times9}{6}$$

a) Express $1^2 + 2^2 + 3^2 + \text{........} + 10^2$ in the same way. 1 RE

b) Express $1^2 + 2^2 + 3^2 + \text{........} + 10^2$ in the same way. 2 RE