2008 Mathematics

Higher – Paper 1 and Paper 2

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or **X**√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. - The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   - Only the mark should be written, **not** a fraction of the possible marks.
   - These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the “wrong” part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

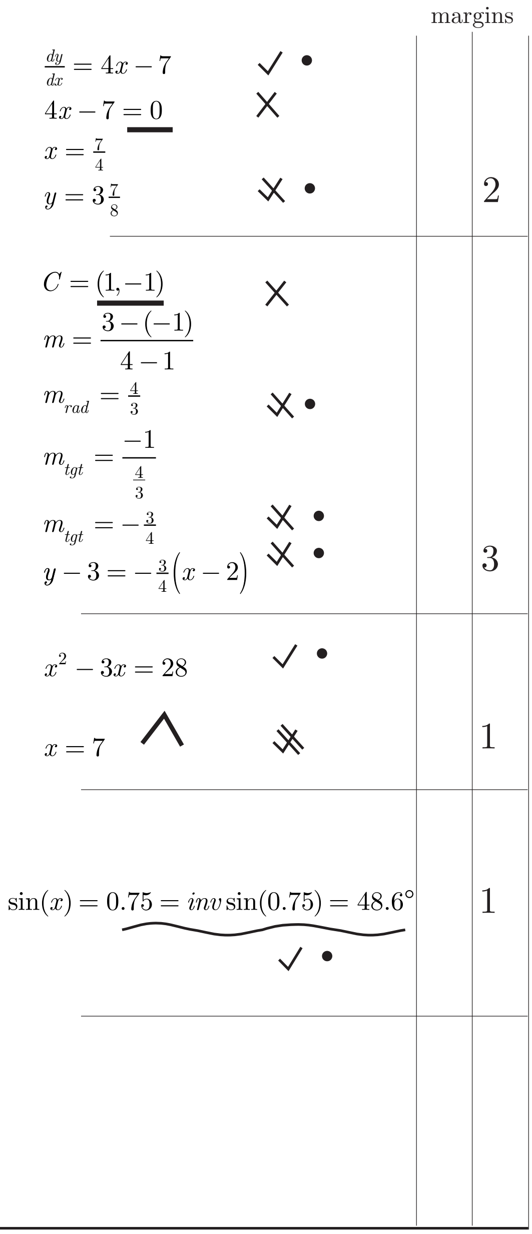
✓✗ The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

〜 The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

✓✗✗ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts



Remember - No comments on the scripts. No abreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

Year

1	2		UNIT 1	1	2		UNIT 2	1	2		UNIT 3
		A1	determine range/domain			A15	use the general equation of a parabola			A28	use the laws of logs to simplify/find equiv. expression
		A2	recognise general features of graphs:poly,exp,log			A16	solve a quadratic inequality			A29	sketch associated graphs
		A3	sketch and annotate related functions			A17	find nature of roots of a quadratic			A30	solve equs of the form $A = Be^{kt}$ for A,B,k or t
		A4	obtain a formula for composite function			A18	given nature of roots, find a condition on coeffs			A31	solve equs of the form $log_b(a) = c$ for a,b or c
		A5	complete the square			A19	form an equation with given roots			A32	solve equations involving logarithms
		A6	interpret equations and expressions			A20	apply A15-A19 to solve problems			A33	use relationships of the form $y = ax^n$ or $y = ab^x$
		A7	determine function(poly,exp,log) from graph & vv							A34	apply A28-A33 to problems
		A8	sketch/annotate graph given critical features								
		A9	interpret loci such as st.lines,para,poly,circle								
		A10	use the notation u_n for the nth term			A21	use Rem Th. For values, factors, roots			G16	calculate the length of a vector
		A11	evaluate successive terms of a RR			A22	solve cubic and quartic equations			G17	calculate the 3rd given two from A,B and vector AB
		A12	decide when RR has limit/interpret limit			A23	find intersection of line and polynomial			G18	use unit vectors
		A13	evaluate limit			A24	find if line is tangent to polynomial			G19	use: if $\mathbf{u}$, $\mathbf{v}$ are parallel then $\mathbf{v} = k\mathbf{u}$
		A14	apply A10-A14 to problems			A25	find intersection of two polynomials			G20	add, subtract, find scalar mult. of vectors
						A26	confiirm and improve on approx roots			G21	simplify vector pathways
						A27	apply A21-A26 to problems			G22	interpret 2D sketches of 3D situations
										G23	find if 3 points in space are collinear
										G24	find ratio which one point divides two others
		G1	use the distance formula			G9	find C/R of a circle from its equation/other data			G25	given a ratio, find/interpret 3rd point/vector
		G2	find gradient from 2 pts,/angle/equ. of line			G10	find the equation of a circle			G26	calculate the scalar product
		G3	find equation of a line			G11	find equation of a tangent to a circle			G27	use: if $\mathbf{u}$, $\mathbf{v}$ are perpendicular then $\mathbf{v.u=0}$
		G4	interpret all equations of a line			G12	find intersection of line & circle			G28	calculate the angle between two vectors
		G5	use property of perpendicular lines			G13	find if/when line is tangent to circle			G29	use the distributive law
		G6	calculate mid-point			G14	find if two circles touch			G30	apply G16-G29 to problems eg geometry probs.
		G7	find equation of median, altitude,perp. bisector			G15	apply G9-G14 to problems				
		G8	apply G1-G7 to problems eg intersect.,concur.,collin.								
		C1	differentiate sums, differences			C12	find integrals of px^n and sums/diffs			C20	differentiate $psin(ax+b)$, $pcos(ax+b)$
		C2	differentiate negative & fractional powers			C13	integrate with negative & fractional powers			C21	differentiate using the chain rule
		C3	express in differentiable form and differentiate			C14	express in integrable form and integrate			C22	integrate $(ax + b)^n$
		C4	find gradient at point on curve & vv			C15	evaluate definite integrals			C23	integrate psin(ax+b), pcos(ax+b)
		C5	find equation of tangent to a polynomial/trig curve			C16	find area between curve and x-axis			C24	apply C20-C23 to problems
		C6	find rate of change			C17	find area between two curves				
		C7	find when curve strictly increasing etc			C18	solve differential equations(variables separable)				
		C8	find stationary points/values			C19	apply C12-C18 to problems				
		C9	determinenature of stationary points								
		C10	sketch curvegiven the equation								
		C11	apply C1-C10 to problems eg optimise, greatest/least								
		T1	use gen. features of graphs of $f(x)=ksin(ax+b)$,			T7	solve linear & quadratic equations in radians			T12	solve sim.equs of form $kcos(a)=p$, $ksin(a)=q$
			$f(x)=kcos(ax+b)$; identify period/amplitude			T8	apply compound and double angle (c & da) formulae			T13	express $pcos(x)+qsin(x)$ in form $kcos(x \pm a)$ etc
		T2	use radians inc conversion from degrees & vv				in numerical & literal cases			T14	find max/min/zeros of $pcos(x)+qsin(x)$
		T3	know and use exact values			T9	apply c & da formulae in geometrical cases			T15	sketch graph of $y=pcos(x)+qsin(x)$
		T4	recognise form of trig. function from graph			T10	use c & da formulaewhen solving equations			T16	solve equ of the form $y=pcos(rx)+qsin(rx)$
		T5	interpret trig. equations and expressions			T11	apply T7-T10 to problems			T17	apply T12-T16 to problems
		T6	apply T1-T5 to problems								

2008 Higher Mathematics
Paper 1 Section A

1	C
2	D
3	C
4	B
5	A
6	B
7	C
8	D
9	B
10	A
11	B
12	C
13	A
14	B
15	C
16	A
17	C
18	C
19	B
20	D

1.21

QU	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.21	a	6	C8,C9	NC		1	3	2	6				6		
	b	5	A21,A22			1	3	1	5					5	
	c	4	C10					4	2	2			4		

A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

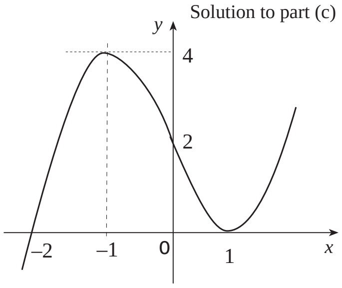
(a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

(b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

(c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

Solution to part (c)



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	set derivative to zero
$\bullet^2$	pd	differentiate
$\bullet^3$	pd	solve
$\bullet^4$	pd	evaluate y-coordinates
$\bullet^5$	ic	justification
$\bullet^6$	ic	state conclusions
$\bullet^7$	ss	know to use $x = 1$
$\bullet^8$	pd	complete eval. & conclusion
$\bullet^9$	ic	start to find quadratic factor
$\bullet^{10}$	pd	complete quadratic factor
$\bullet^{11}$	pd	factorise completely
$\bullet^{12}$	ic	interpret y-intercept
$\bullet^{13}$	ic	interpret x-intercepts
$\bullet^{14}$	ic	sketch : showing turning points
$\bullet^{15}$	ic	sketch : showing intercepts

Primary Method : Give 1 mark for each •

$\bullet^1$ $f'(x) = 0$

$\bullet^2$ $3x^2 - 3$

$$\begin{array}{cc|c|c|} & & \bullet^3 & \bullet^4 \\ \bullet^3 & x & -1 & 1 \\ \bullet^4 & y & 4 & 0 \end{array}$$

$$\begin{array}{cc|ccc|ccc|} & & & \bullet^5 & & & \bullet^6 & \\ & & \dots & -1 & \dots & \dots & 1 & \dots \\ \bullet^5 & f' & + & 0 & - & - & 0 & + \\ \bullet^6 & & \text{max} & at\,x = & -1 & \text{min} & at\,x = & 1 \end{array}$$

$\bullet^7$ know to use $x = 1$

$\bullet^8$ $1 - 3 + 2 = 0 \Rightarrow x - 1$ is a factor

$\bullet^9$ $(x - 1)(x^2 ...)$

$\bullet^{10}$ $(x - 1)(x^2 + x - 2)$

$\bullet^{11}$ $(x - 1)(x - 1)(x + 2)$ ***stated explicitly***

$\bullet^{12}$ $(0,2)$

$\bullet^{13}$ $(-2,0),\ (1,0)$

$\bullet^{14}$ Sketch with turning pts marked

$\bullet^{15}$ Sketch with $(0,2)$ or $(-2,0)$

Notes

1 The "=0" shown at $\bullet^1$ must appear at least once before the $\bullet^3$ stage.

2 An unsimplified $\sqrt{1}$ should be penalised at the first occurrence.

3 $\bullet^3$ is only available as a consequence of solving $f'(x) = 0$.

4 The nature table must reflect previous working from $\bullet^3$.

5 Candidates who introduce an extra solution at the $\bullet^3$ stage cannot earn $\bullet^3$.

6 The use of the 2nd derivative is an acceptable strategy for $\bullet^5$.

7 As shown in the Primary Method, ($\bullet^3$ and $\bullet^4$) and ($\bullet^5$ and $\bullet^6$) can be marked in series or in parallel.

8 The working for (b) may appear in (a) or vice versa. Full marks are available wherever the working occurs.

Notes

9 In Primary method $\bullet^8$ and alternative $\bullet^9$, candidates must show some acknowledgement of the resulting "0". Although a statement wrt the zero is preferable, accept something as simple as "underlining the zero".

Alternative Method : $\bullet^7$ to $\bullet^{10}$

$$\bullet^7 \quad \begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & & & \\ \hline & & & & \end{array}$$

$$\bullet^8 \quad \begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$\bullet^9$ $f(1) = 0$ *so* $(x - 1)$ *is a factor*

$\bullet^{10}$ $x^2 + x - 2$

Notes

10 Evidence for $\bullet^{12}$ and $\bullet^{13}$ may not appear until the sketch.

11 $\bullet^{14}$ and $\bullet^{15}$ are only available for the graph of a cubic.

Nota Bene

For candidates who omit the x^2 coeff. leading to

$\bullet^7$ X

$$\bullet^8 \ \surd \quad \begin{array}{c|ccc} 1 & 1 & -3 & 2 \\ & & 1 & -2 \\ \hline & 1 & -2 & 0 \end{array}$$

$\bullet^9$ $\surd$ $f(1) = 0$ *so* $(x - 1)$.........

$\bullet^{10}$ X $x^2 - 2x$

$\bullet^{11}$ $\surd$ $x(x - 1)(x - 2)$

but

$\bullet^{10}$ X $x - 2$

$\bullet^{11}$ X $(x - 1)(x - 2)$

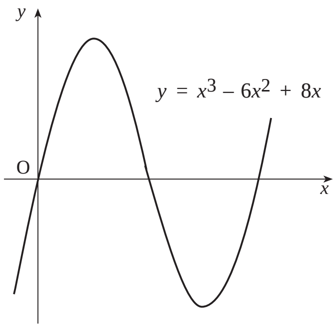
1.22

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.22	a	5	C4	NC		2	3		5				5		
	b	2	C11			1		1	2					2	

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.

(a) Find the coordinates of the points on the curve where the gradient of the tangent is -1. 5

(b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know to differentiate
$\bullet^2$	pd	differentiate
$\bullet^3$	ss	set derivative to -1
$\bullet^4$	pd	factorise and solve
$\bullet^5$	pd	solve for y
$\bullet^6$	ss	use gradient
$\bullet^7$	ic	interpret result

Primary Method : Give 1 mark for each •

$\bullet^1$ $\frac{dy}{dx} = ..(1\ \textit{term correct})\quad s\ /\ i\ by\ \bullet^2$

$\bullet^2$ $3x^2 - 12x + 8 \quad s\ /\ i\ by\ \bullet^3$

$\bullet^3$ $3x^2 - 12x + 8 = -1$

		$\bullet^4$	$\bullet^5$
$\bullet^4$	x	1	3
$\bullet^5$	y	3	-3

$\bullet^6$ $y = 4 - x$ has gradient $= -1$

$\bullet^7$ *check* $(3,-3)$ and reject
check $(1,3)$ and accept

Notes

1 *in (a)*

$\bullet^1$ √ $\frac{dy}{dx} = ..(1\ \textit{term correct})$

$\bullet^2$ √ $3x^2 - 12x + 8$

For candidates who now guess $x = 1$ and check that $\frac{dy}{dx} = -1$, only one further mark ($\bullet^3$) can be awarded. Guessing and checking further answers gains no more credit.

2 An "=0" must appear at least once in the two lines shown in the alternative for $\bullet^6$ and $\bullet^7$.

Common Error

$\bullet^1$ √ $\frac{dy}{dx} = ..(1\ \textit{term correct})$

$\bullet^2$ √ $3x^2 - 12x + 8$

$\bullet^3$ X $3x^2 - 12x + 8 = 0$

$\bullet^4$ X irrespective of what is written.

$\bullet^5$ X

Alternative for $\bullet^6$ and $\bullet^7$

$$\bullet^6\quad \begin{cases} x^3 - 6x^2 + 8x = 4 - x \\ x^3 - 6x^2 + 9x - 4 = 0 \\ (x-1)(x^2 - 5x + 4) \\ \qquad (x-4)(x-1) \end{cases}$$

$$\bullet^7\quad \begin{cases} \text{repeated root implies} \\ \text{tangent at } (1,3). \end{cases}$$

1.23

qu	part	mk	A3	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.23	a	3	A4	NC				3	3				3		
	b	5	A31			2	2	1		1	4				5

Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(a) Find expressions for $h\big(f(x)\big)$ and $h\big(g(x)\big)$. 3

(b) Hence solve $h\big(f(x)\big) - h\big(g(x)\big) = 3$ 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpretation composition
$\bullet^2$	ic	interpretation composition
$\bullet^3$	ic	interpretation composition
$\bullet^4$	ss	use log laws
$\bullet^5$	ss	convert to exponential form
$\bullet^6$	pd	process conversion
$\bullet^7$	pd	express in standard form
$\bullet^8$	ic	find valid solutions

Primary Method : Give 1 mark for each •

$\bullet^1$	$h\big(f(x)\big) = h\big(x^2 - x + 10\big)$ $s\,/\,i\ by\ \bullet^2$
$\bullet^2$	$\log_2\big(x^2 - x + 10\big)$
$\bullet^3$	$\log_2(5 - x)$
$\bullet^4$	$\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$
$\bullet^5$	$\dfrac{x^2 - x + 10}{5 - x} = 2^3$
$\bullet^6$	$x^2 - x + 10 = 8(5 - x)$
$\bullet^7$	$x^2 + 7x - 30 = 0$
$\bullet^8$	$x = 3,\ -10$

Notes

1 In (a) 2 marks are available for finding one of $h(f(x))$ or $h(g(x))$ and the third mark is for the other.

2 Treat $\log_2 x^2 - x + 10$ *and* $\log_2 5 - x$ as bad form.

3 The omission of the base should not be penalised in $\bullet^2$ to $\bullet^4$.

4 $\bullet^7$ is only available for a quadratic equation and $\bullet^8$ must be the follow-through solutions.

Common Error 1

$\bullet^4$	X	$\log_2(x^2 + 5) = 3$
$\bullet^5$	$\surd$	$x^2 + 5 = 2^3$
$\bullet^6$	X	$x^2 = 3$
$\bullet^7$	X	$x = \pm\surd 3$
$\bullet^8$	X	not available

Common Error 2

$\bullet^4$	$\surd$	$\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$ $\log_2\left(\dfrac{x^2 \cancel{-} \cancel{x} + \cancel{1}\cancel{0}}{\cancel{5} \cancel{-} \cancel{x}}\right)$ $\log_2\big(x^2 + 2\big) = 3$
$\bullet^5$	$X\surd$	$x^2 + 2 = 2^3$
$\bullet^6$	X	$x = \pm\surd 6$
$\bullet^7$	X	*not available*
$\bullet^8$	X	*not available*

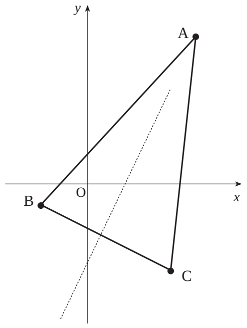
Common Error 3

$\bullet^4$	X	not available
$\bullet^5$	$\surd$	$\log_2(x^2 - x + 10) - \log_2(5 - x) = \log_2 8$
$\bullet^6$	X	$x^2 - x + 10 - (5 - x) = 8$
$\bullet^7$	X	not available
$\bullet^8$	X	not available

2.01

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.01	a	4	G7	CN		2		2	4				4		
	b	3	G7	CN		1	1	1	3				3		
	c	3	C8	CN		1	2		3				3		

The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.
The broken line represents the perpendicular bisector of BC.



(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. 4

(*b*) Find the equation of the median from C. 3

(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. 3

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know and find gradient
$\bullet^2$	ic	interpret perpendicular gradient
$\bullet^3$	ss	know and find midpoint
$\bullet^4$	ic	complete proof
$\bullet^5$	ss	know and find midpoint
$\bullet^6$	pd	calculate gradient
$\bullet^7$	ic	state equation
$\bullet^8$	ss	start to solve sim. equations
$\bullet^9$	pd	find one variable
$\bullet^{10}$	pd	find other variable

Primary Method : Give 1 mark for each •

$\bullet^1$	$m_{BC} = -\frac{1}{2}$	***stated explicitly***
$\bullet^2$	$m_{\perp} = 2$	*stated / implied by* $\bullet^4$
$\bullet^3$	midpoint of BC = (1, –3)	
$\bullet^4$	$y + 3 = 2(x - 1)$ and complete	
$\bullet^5$	midpoint of AB = (2,4)	
$\bullet^6$	$m_{\text{median}} = -3$	
$\bullet^7$	$y + 5 = -3(x - 5)$ *or* $y - 4 = -3(x - 2)$	
$\bullet^8$	use $y = 2x - 5$ $y = -3x + 10$	
$\bullet^9$	$x = 3$	
$\bullet^{10}$	$y = 1$	

Notes

In (a)

1 $\bullet^4$ is only available as a consequence of attempting to find and use both a perpendicular gradient and a midpoint.

2 To gain $\bullet^4$ some evidence of completion needs to be shown.
The minimum requirements for this evidence is as shown:

$$y + 3 = 2(x - 1)$$
$$y + 3 = 2x - 2$$
$$y = 2x - 5$$

3 $\bullet^4$ is only available for completion to $y = 2x - 5$ and nothing else.

4 Alternative for $\bullet^4$:
$\bullet^4$ may be obtained by using $y = mx + c$

Notes

In (b)

5 $\bullet^7$ is only available as a consequence of finding the gradient via a midpoint.

6 For candidates who find the equation of the perpendicular bisector of AB, only $\bullet^5$ is available.

In (c)

7 $\bullet^8$ is a strategy mark for juxtaposing the two correctly rearranged equations.

Follow - throughs

Note that from an incorrect equation in (b), full marks are still available in (c). Please follow-through carefully.

Cave

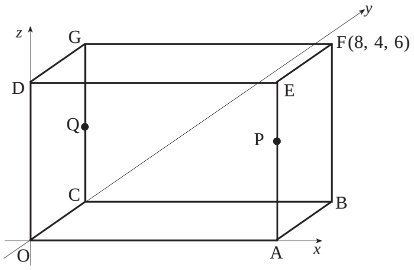
Candidates who find the median, angle bisector or altitude need to show the triangle is isosceles to gain full marks in (a).
For those candidates who do not justify the isosceles triangle, marks may be allocated as shown below:

	Altitude	Median
$\bullet^1$	√	√
$\bullet^2$	√	*X*
$\bullet^3$	*X*	√
$\bullet^4$	*X*	*X*

2.02

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.02	a	2	G25	CN	8202			2	2						2
	b	2	G25	CN			1	1	2						2
	c	5	G28	CR		1	4		5						5

The diagram shows a cuboid OABC,DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2:1.
Q is the midpoint of CG.



(*a*) State the coordinates of P and Q. 2

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. 2

(*c*) Find the size of angle QPA. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpret ratio
$\bullet^2$	ic	interpret ratio
$\bullet^3$	pd	process vectors
$\bullet^4$	ic	interpret diagram
$\bullet^5$	ss	know to use scalar product
$\bullet^6$	pd	find scalar product
$\bullet^7$	pd	find magnitude of vector
$\bullet^8$	pd	find magnitude of vector
$\bullet^9$	pd	evaluate angle

Primary Method : Give 1 mark for each •

$\bullet^1$	$\mathrm{P} = (8, 0, 4)$
$\bullet^2$	$\mathrm{Q} = (0, 4, 3)$
$\bullet^3$	$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$
$\bullet^4$	$\overrightarrow{\mathrm{PA}} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$
$\bullet^5$	$\cos \mathrm{QPA} = \dfrac{\overrightarrow{\mathrm{PQ}}.\overrightarrow{\mathrm{PA}}}{\lvert \overrightarrow{\mathrm{PQ}} \rvert \lvert \overrightarrow{\mathrm{PA}} \rvert}$ *stated / implied by* $\bullet^9$
$\bullet^6$	$\overrightarrow{\mathrm{PQ}}.\overrightarrow{\mathrm{PA}} = 4$
$\bullet^7$	$\lvert \overrightarrow{\mathrm{PQ}} \rvert = \sqrt{81}$
$\bullet^8$	$\lvert \overrightarrow{\mathrm{PA}} \rvert = \sqrt{16}$
$\bullet^9$	83·6°, 1.459 *radians*, 92.9 *gradians*

Notes

1 Treat coordinates written as column vectors as bad form.

2 Treat column vectors written as coordinates as bad form.

3 For candidates who do not attempt $\bullet^9$, the formula quoted at $\bullet^5$ must relate to the labelling in order for $\bullet^5$ to be awarded.

4 Candidates who evaluate $\mathrm{P\hat{O}Q}$ correctly gain 4/5 marks in (c) (74° or 75°)

Exemplar 1

$\bullet^3,\bullet^4$ X, X $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$ $\overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$

$\bullet^5$ X $\cos \mathrm{AOQ} = \dfrac{\overrightarrow{\mathrm{OA}}.\overrightarrow{\mathrm{OQ}}}{\lvert \overrightarrow{\mathrm{OA}} \rvert \lvert \overrightarrow{\mathrm{OQ}} \rvert}$

$\bullet^6$ $\surd$ $\overrightarrow{\mathrm{OA}}.\overrightarrow{\mathrm{OQ}} = 0$

$\bullet^7$ $\surd$ $\lvert \overrightarrow{\mathrm{OA}} \rvert = \sqrt{64}$

$\bullet^8$ $\surd$ $\lvert \overrightarrow{\mathrm{OQ}} \rvert = \sqrt{25}$

$\bullet^9$ $\surd$ 90°

Exemplar 2

$\bullet^3,\bullet^4$ X, X $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$ $\overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$

$\bullet^6$ $\surd$ $\overrightarrow{\mathrm{OA}}.\overrightarrow{\mathrm{OQ}} = 0$

$\bullet^9$ $\surd$ 90°

Alternative for $\bullet^5$ to $\bullet^8$

$\bullet^5$ $\cos \mathrm{QPA} = \dfrac{\mathrm{PA}^2 + \mathrm{PQ}^2 - \mathrm{QA}^2}{2\mathrm{PA} \times \mathrm{PQ}}$

$\bullet^6$ $\lvert \overrightarrow{\mathrm{PA}} \rvert = \sqrt{16}$

$\bullet^7$ $\lvert \overrightarrow{\mathrm{PQ}} \rvert = \sqrt{81}$

$\bullet^8$ $\lvert \overrightarrow{\mathrm{QA}} \rvert = \sqrt{89}$

2.03

qu	part	2	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.03	a	2	T4	CN	8203			2	2				2		
	b	4	T13	CR		1	2	1	4						4
	c	2	C20	CN			1	1	1	1					2

(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.
Write down the value of p.

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.
Write down the value of q. 2

(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2

Diagram 1

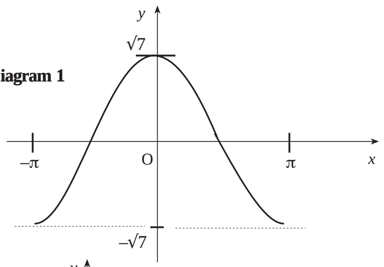
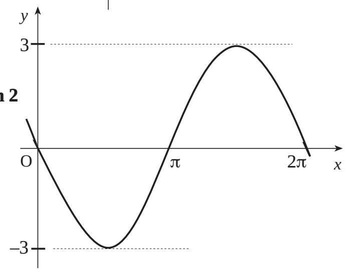


Diagram 2



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpret graph
$\bullet^2$	ic	interpret graph
$\bullet^3$	ss	expand
$\bullet^4$	ic	compare coefficients
$\bullet^5$	pd	process "k"
$\bullet^6$	pd	process "a"
$\bullet^7$	ss	state equation
$\bullet^8$	pd	differentiate

Primary Method : Give 1 mark for each •

$\bullet^1$	$p = \sqrt{7}$
$\bullet^2$	$q = -3$
$\bullet^3$	$k\cos x\cos a - k\sin x\sin a$ ***stated explicitly***
$\bullet^4$	$k\cos a = \sqrt{7}$ ***and*** $k\sin a = 3$ ***stated explicitly***
$\bullet^5$	$k = 4$
$\bullet^6$	$a \approx 0{\cdot}848$
$\bullet^7$	$4\cos(x + 0{\cdot}848)$
$\bullet^8$	$-4\sin(x + 0{\cdot}848)$

Notes

In (a)

1 For $\bullet^1$ accept $p = 2.6$ leading to $k = 4.0$, $a = 0.86$ in (b).

In (b)

2 $k(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$.

3 Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the $\bullet^4$ stage both contain k.

4 $4(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$ and $\bullet^5$.

5 $k = \sqrt{16}$ does not earn $\bullet^5$.

6 No justification is needed for $\bullet^5$.

7 Candidates may use any form of wave equation as long as their final answer is in the form $k\cos(x + a)$. If not, then $\bullet^6$ is not available.

Notes

8 Candidates who use degrees throughout this question lose $\bullet^6$, $\bullet^7$ and $\bullet^8$.

Common Error 1

(*sic*)

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 5.44 \ \ or\ -0.85$

$\bullet^2 X$, $\bullet^3\surd$, $\bullet^4\surd$, $\bullet^5\surd$, $\bullet^6\surd$

Common Error 2

(*sic*)

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 0.85$

$\bullet^2 X$, $\bullet^3\surd$, $\bullet^4\surd$, $\bullet^5\surd$, $\bullet^6 X$

Note that $\bullet^6$ is not awarded as it is not consistent with previous working.

Alternative Method (for $\bullet^7$ and $\bullet^8$**)**

If :

$f'(x) + g'(x) = -\sqrt{7}\sin x - 3\cos x$ ....

then $\bullet^7$ is only available once the candidate has reached e.g.

"choose $k\sin(x + a)$

$\Rightarrow k\sin a = -3,\ k\cos a = -7.$"

$\bullet^8$ is available for evaluating k and a.

2.04

qu	part	mk	code	calc	source	ss		ic	C	B	A		U1	U2	U3
2.04	a	2	G9	CN	8204			2	2					2	
	b	4	G14	CN		1	1	2	2	2				4	
	c	5	G12	CN		1	4			5				5	

(a) Write down the centre and calculate the radius of the circle with equation $x^2+y^2+8x+4y-38=0$. 2

(b) A second circle has equation $(x-4)^2+(y-6)^2=26$.
Find the distance between the centres of these two circles and hence show that the circles intersect. 4

(c) The line with equation $y=4-x$ is a common chord passing through the points of intersection of the two circles.
Find the coordinates of the points of intersection of the two circles. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	state centre of circle
$\bullet^2$	ic	find radius of circle
$\bullet^3$	ic	state centre and radius
$\bullet^4$	pd	find distance between centres
$\bullet^5$	ss	find sum of radii
$\bullet^6$	ic	interpret result
$\bullet^7$	ss	know to and substitute
$\bullet^8$	pd	start process
$\bullet^9$	pd	write in standard form
$\bullet^{10}$	pd	solve for x
$\bullet^{11}$	pd	solve for y

Primary Method : Give 1 mark for each •

$\bullet^1$ $(-4,-2)$

$\bullet^2$ $\sqrt{58}$ (≈ 7.6)

$\bullet^3$ $(4,6)$ ***and*** $\sqrt{26}$ (≈ 5.1) *s / i* $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $d_{centres}=\sqrt{128}$ *accept* 11.3

$\bullet^5$ $\sqrt{58}+\sqrt{26}$ *accept* 12.7

$\bullet^6$ compare 12.7 and 11.3

$\bullet^7$ $x^2+(4-x)^2+\ldots$

$\bullet^8$ $x^2+16-8x+x^2+\ldots$

$\bullet^9$ $2x^2-4x-6=0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	x	3	−1
$\bullet^{11}$	y	1	5

Notes

In (a)

1 If a linear equation is obtained at the $\bullet^9$ stage, then $\bullet^9$, $\bullet^{10}$ and $\bullet^{11}$ are not available.

2 Solving the circles simultaneously to obtain the equation of the common chord gains no marks.

3 The comment given at the $\bullet^6$ stage must be consistent with previous working.

alt. for $\bullet^7$ to $\bullet^{11}$:

$\bullet^7$ $(4-y)^2+\ldots$

$\bullet^8$ $y^2-8y+16+y^2+\ldots$

$\bullet^9$ $y^2-6y+5=0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	y	1	5
$\bullet^{11}$	x	3	−1

2.05

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.05		5	T10	CR		1	4			5				5	

Solve the equation $\cos 2x^\circ + 2\sin x^\circ = \sin^2 x^\circ$ in the interval $0 \leq x < 360$. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	use double angle formula
$\bullet^2$	pd	obtains standard form (i.e. " .... = 0")
$\bullet^3$	pd	factorise
$\bullet^4$	pd	process factors
$\bullet^5$	pd	completes solutions

Primary Method : Give 1 mark for each •

$\bullet^1$ $\cos 2x = 1 - 2\sin^2 x$

$\bullet^2$ $3\sin^2 x - 2\sin x - 1 = 0$

$\bullet^3$ $(3\sin x + 1)(\sin x - 1) = 0$

	$\bullet^4$	$\bullet^5$
$\bullet^4$	$\sin x = -\dfrac{1}{3}$	$\sin x = 1$
$\bullet^5$	199.5°, 340.5°	90°

Notes

1 $\bullet^1$ is not available for $1 - 2\sin^2 A$ with no further working.

2 $\bullet^2$ is only available for the three terms shown written in any correct order.

3 The "=0" has to appear at least once "en route" to $\bullet^3$.

4 $\bullet^4$ and $\bullet^5$ are only available for solving a quadratic equation.

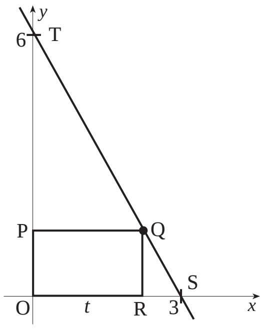
2.06

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.06		3	G3	CN	8206	1		2			3		3		
		6	C11	CN		2	2	2		6			6		

In the diagram Q lies on the line joining (0, 6) and (3, 0).
OPQR is a rectangle, where P and R lie on the axes and OR $= t$.

(a) Show that QR $= 6 - 2t$. 3

(b) Find the coordinates of Q for which the rectangle has a maximum area. 6



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know and use e.g. similar triangles, trigonometry or gradient
$\bullet^2$	ic	establish equation
$\bullet^3$	ic	find a length
$\bullet^4$	ss	know how and find area
$\bullet^5$	ss	set derivative of the area function to zero
$\bullet^6$	pd	differentiate
$\bullet^7$	pd	solve
$\bullet^8$	ic	justify stationary point
$\bullet^9$	ic	state coordinates

Primary Method : Give 1 mark for each •

$\bullet^1$	ΔOST, RSQ are similar *s / i* by $\bullet^2$
$\bullet^2$	$\frac{\text{QR}}{6} = \frac{3-t}{3}$ or equivalent
$\bullet^3$	$\text{QR} = 6 - 2t$
$\bullet^4$	$A(t) = t(6-2t)$
$\bullet^5$	$A'(t) = 0$
$\bullet^6$	$6 - 4t$
$\bullet^7$	$t = \frac{3}{2}$
$\bullet^8$	*e.g. nature table*
$\bullet^9$	$\text{Q} = \left(\frac{3}{2}, 3\right)$

Notes

1 "$y = 6 - 2x$" appearing *ex nihilo* can be awarded neither $\bullet^1$ nor $\bullet^2$.
$\bullet^3$ is still available with some justification *e.g.* OR $= t$ gives $y = 6 - 2t$.

2 The "=0" has to appear at least once before the $\bullet^7$ stage for $\bullet^5$ to be awarded.

3 Do not penalise the use of $\frac{\text{dy}}{\text{dx}}$ in lieu of $\text{A}'(t)$ for instance in the nature table.

4 The minimum requirements for the nature table are shown on the right.
Of course other methods may be used to justify the nature of the stationary point(s).

Variation 1:

$\bullet^1$ $\tan\text{'S'} = \frac{6}{3}$

$\bullet^2$ $\tan\text{'S'} = \frac{\text{QR}}{3-t}$ *and equate*

Variation 2:

$\bullet^1$ $\surd m_{\text{line}} = -2$ *s / i by* $\bullet^2$

$\bullet^2$ $\surd$equation of line $: y = -2x + 6$

Variation 3

$\bullet^1$ $\surd$ $m_{\text{line}} = -2$

$\bullet^2$ $\surd$equation of line $: y = 6 - 2x$

Variation 4

$\bullet^1$ X *(nothing stated)*

$\bullet^2$ Xequation of line $: y = 6 - 2x$

Alternative Method : (for $\bullet^5$ *to* $\bullet^8$**)**

$\bullet^5$ strategy to find roots $\Rightarrow$ t.p.s

$\bullet^6$ $t = 0,\ t = 3$

$\bullet^7$ max *t.p.* since coeff of "t^2" < 0

$\bullet^8$ turning pt at $t = \frac{3}{2}$

Nature Table

minimum requirements for •^{8}

			$\frac{3}{2}$	
	A'	$+$	0	$-$
$\bullet^8$		⋰	…	⋱

2.07

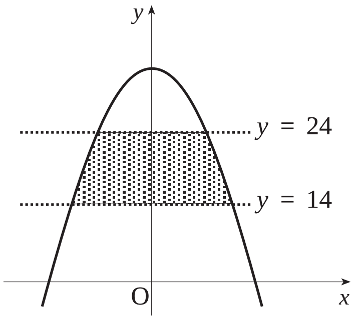
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
2.07		8	C19	CN		3	4	1			8		8	

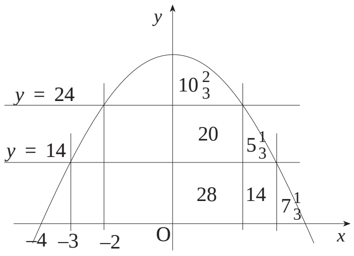
The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. 8





The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpret limits
$\bullet^2$	pd	find both x-values
$\bullet^3$	ss	know to integrate
$\bullet^4$	pd	integrate
$\bullet^5$	ic	state limits
$\bullet^6$	pd	evaluate limits
$\bullet^7$	ss	select "what to add to what"
$\bullet^8$	pd	completes a valid strategy

Primary Method : Give 1 mark for each •

$\bullet^1$	$32 - 2x^2 = 24$ ***or*** 14
$\bullet^2$	$x = 2$ ***and*** 3
$\bullet^3$	$\int (32 - 2x^2)\ dx$
$\bullet^4$	$32x - \frac{2}{3}x^3$
$\bullet^5$	$\left[\ldots\right]_2^3$
$\bullet^6$	$19\frac{1}{3}$
$\bullet^7$	*e.g.* $19\frac{1}{3} - 14 + 20$ and then double $s\,/\,i$ by $\bullet^8$
$\bullet^8$	$50\frac{2}{3}$

Notes

1 For $\int_{14}^{24} (32 - 2x^2)dx \;= \left[32x - \frac{2}{3}x^3\right]$

may be awarded $\bullet^3$ and $\bullet^4$ *ONLY*.

2 For integrating "along the *y* - axis"

$\bullet^1$ strategy: choose to integrate along y-axis

$\bullet^2$ $x = \sqrt{\left(16 - \frac{1}{2}y\right)}$

$\bullet^3$ $\int \left(16 - \frac{1}{2}y\right)^{\frac{1}{2}}\ dy$

$\bullet^4$ $-2.\frac{2}{3}\left(16 - \frac{1}{2}y\right)^{\frac{3}{2}}$

$\bullet^5$ $\left[\ldots\right]_{14}^{24}$

$\bullet^6$ $-\frac{4}{3}(4^{\frac{3}{2}} - 9^{\frac{3}{2}})$

$\bullet^7$ $2\times\ldots\ldots\ldots$

$\bullet^8$ $50\frac{2}{3}$

Exemplar 1 ($\bullet^3$ *to* $\bullet^8$)

$\bullet^3$ $\int (32 - 2x^2 - 14)\ \ dx$

$\bullet^4$ $18x - \frac{2}{3}x^3$

$\bullet^5$ $\left[\ldots\right]_{-3}^{3}$

$\bullet^6$ 72

$\bullet^7$ *e.g.* $72 - \int_{-2}^{2} (32 - 2x^2 - 24)\ \ dx$

$\bullet^8$ $50\frac{2}{3}$

or

$\bullet^5$ $\left[\ldots\right]_{0}^{3}$

$\bullet^6$ 36

$\bullet^7$ *e.g.* $2 \times \left[36 - \int_{0}^{2} (32 - 2x^2 - 24)\ \ dx\right]$

Variations ($\bullet^3$ *to* $\bullet^6$)

The following are examples of sound opening integrals which will lead to the area after one more integral at most.

$$\int_0^2 (32 - 2x^2)dx \quad = \ldots\ldots = 58\tfrac{2}{3}$$

$$\int_0^3 (32 - 2x^2)dx \quad = \ldots\ldots = 78$$

$$\int_2^3 (32 - 2x^2)dx \quad = \ldots\ldots = 19\tfrac{1}{3}$$

$$\int_0^2 (32 - 2x^2 - 24)dx \quad = \ldots\ldots = 10\tfrac{2}{3}$$

$$\int_0^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 36$$

$$\int_2^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 5\tfrac{1}{3}$$