Practice Unit tests

Use this booklet to help you prepare for all unit tests in Higher Maths.

Your formal test will be of a similar standard.

- Read the description of each assessment standard carefully to make sure you know what you could be tested on.
- Work through each practice test to check your understanding. There are slight differences and it is important that you do your best to prepare for these.
- Reasoning problems are identifiable in the booklet with a #2.1 or #2.1.

If you come across something you don't understand or can't remember then do something about it:

- Look in your course notes for a similar example
- Look in your textbook for notes and worked examples
- Look on SCHOLAR for further explanation and more practice questions
- Look on the Internet notes and worked examples (HSN is a useful site http://www.hsn.uk.net/)
- Ask your teacher or anyone else that is able to help you with Maths

Contents:

Unit title: <u>Applications</u>

Assessment Standards	**Making assessment judgements**
1.1 Applying algebraic skills to rectilinear shapes	The sub-skills in the Assessment Standard are: ♦ finding the equation of a line parallel to, and a line perpendicular to, a given line ♦ using $m = \tan\theta$ to calculate a gradient or angle
1.2 Applying algebraic skills to circles	The sub-skills in the Assessment Standard are: ♦ determining and using the equation of a circle ♦ using properties of tangency in the solution of a problem
1.3 Applying algebraic skills to sequences	The sub-skills in the Assessment Standard are: ♦ determining a recurrence relation from given information and using it to calculate a required term ♦ finding and interpreting the limit of a sequence, where it exists
1.4 Applying calculus skills to optimisation and area	The sub-skills in the Assessment Standard are: ♦ determining the optimal solution for a given problem ♦ finding the area between a curve and the x-axis ♦ finding the area between two curves or a straight line and a curve
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	Assessment Standard 2.1 and 2.1 are transferable across Units and can be attached to any of the sub-skills listed above. #2.1 is mostly about choosing an appropriate strategy. This skill is usually required to make a good start on a problem. #2.2 is about how well you answer a question, often with a summary statement at the end of a question.
2.2 Explaining a solution and, where appropriate, relating it to context	

My assessment record: Mathematics: Applications (Higher)

Keep this up to date as you go along so that you know if you have any areas that you need to do more work on

Assessment standard	First attempt		Second attempt (if required)	
	Mark(s)	Pass/Fail	Mark(s)	Pass/Fail
1.1 Applying algebraic skills to rectilinear shapes • finding the equation of a line parallel to, and a line perpendicular to, a given line • using $m = \tan\theta$ to calculate a gradient or angle				
1.2 Applying algebraic skills to circles • determining and using the equation of a circle • using properties of tangency in the solution of a problem				
1.3 Applying algebraic skills to sequences • determining a recurrence relation from given information and using it to calculate a required term • finding and interpreting the limit of a sequence, where it exists				
1.4 Applying calculus skills to optimisation and area • determining the optimal solution for a given problem • finding the area between a curve and the *x*-axis • finding the area between two curves or a straight line and a curve				
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy				
2.2 Explaining a solution and, where appropriate, relating it to context				

To pass each of the assessment standards 1.1 to 1.4 you need to get at least half of the marks overall or half of the marks for each sub-skill.

Assessment standards 2.1 and 2.2 can be gained across the whole course (you have to show that you can apply each skill on 2 separate occasions, i.e. twice for #2.1 and twice for #2.2) so if you don’t manage to get it in this unit you need to make sure that you do in either of the other units.

Unit title: Applications

Assessment standard: 1.1 Applying algebraic skills to rectilinear shapes

Sub-skills	♦ finding the equation of a line parallel to, and a line perpendicular to, a given line ♦ using $m = \tan\theta$ to calculate a gradient or angle

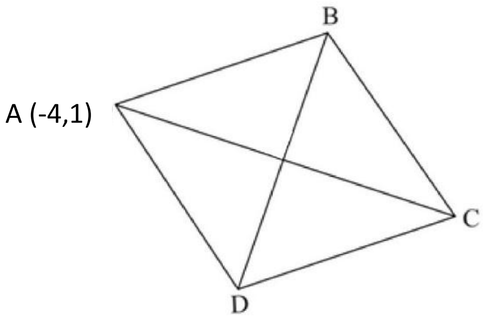
Practice test 1

1 Find the equation of the line passing thorough (5, -10), parallel to the line with equation $4x + y - 8 = 0$.

(2)

2. ABCD is a rhombus.
Diagonal BD has equation $y = 3x - 2$.
A has coordinates (–4, 1).

Find the equation of the diagonal AC

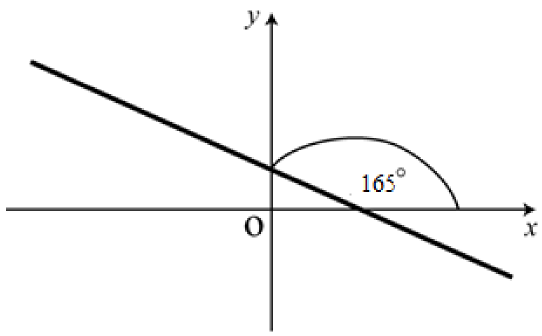


(#2.1 and 1)

3 A ski slope is categorised by its gradient as shown in the table.

Dry slope category	**Steepness (*s*) of slope**
Teaching and general skiing	$0 \leq s \leq 0{\cdot}35$
Extreme skiing	$s > 0{\cdot}35$

(a) What is the gradient of the line shown in the diagram? **(1)**



(b) To which category does the ski slope represented by the line in part (a) belong?

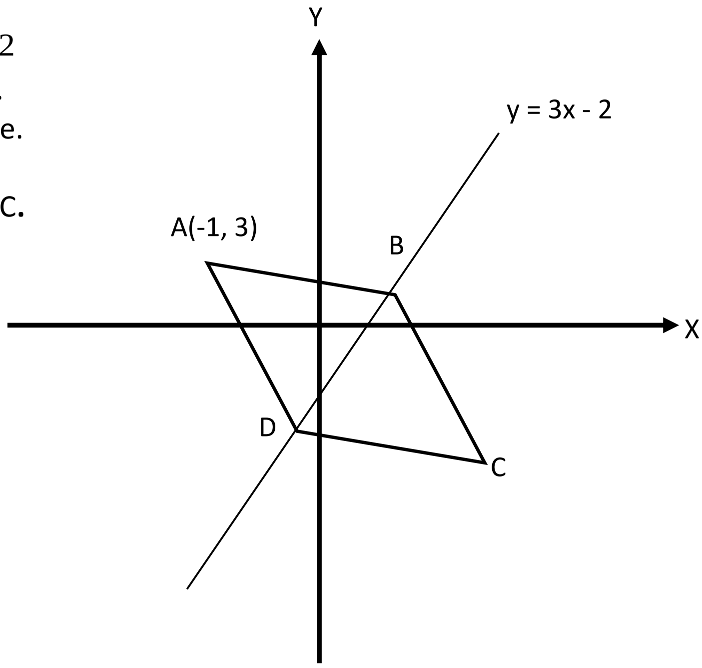
Give a reason for your answer. **(#2.2)**

Practice test 2

1 A straight line has the equation $-6x + y - 2 = 0$.
Write down the equation of the line parallel to the given line, which passes through the point (3, –5).
(2)

2. ABCD is a rhombus.
Diagonal BD has equation $y = 3x - 2$
and point **A** has coordinates (–1, 3).
Note that the diagram is not to scale.

Find the equation of the diagonal AC.

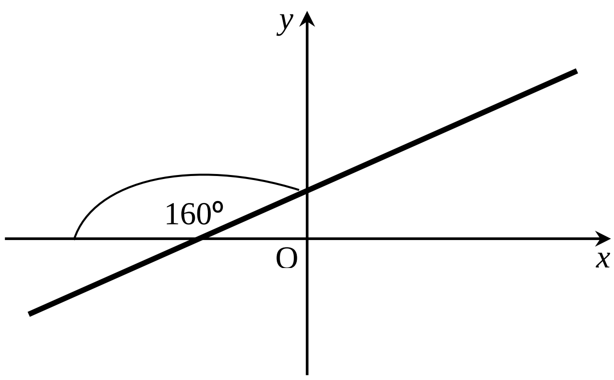


(#2.1 and 1)

3 A ramp is categorised by its gradient as shown in the table.

Category	**Steepness (*s*) of ramp**
Safe	$0 < s \leq 0.3$
Dangerous	$s > 0.3$

Which category does the ramp in the diagram below belong to?
Explain your answer fully.



(#2.2)

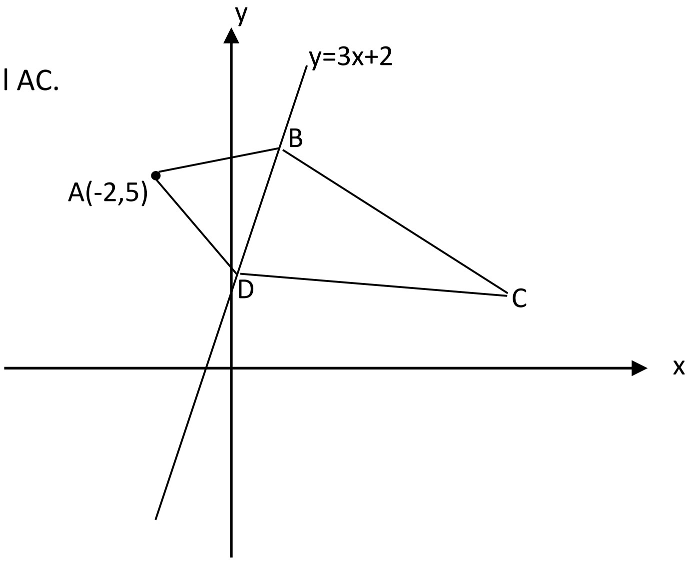
Practice test 3

1 A straight line has the equation $4x + y + 3 = 0$.
Write down the equation of the line perpendicular to the given line, which passes through the point (-3,1).

(3)

2 ABCD is a kite.
Diagonal BD has equation $y = 3x + 2$ and point A has coordinates (–2,5). Note that the diagram is not to scale.

Find the equation of the diagonal AC.



(2)

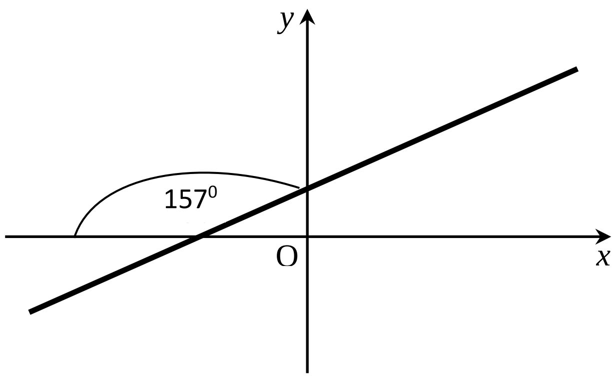
3 Calculate the size of the obtuse angle between the line $y = 4x - 3$ and the *x*-axis.

(2 + #2.2)

4 The ramp on a livestock trailer is categorised by its gradient as shown in the table.

Livestock category	**Steepness (*s*) of ramp**
Pigs/horses	0 < s < 0.36
Sheep/cattle	0 < s < 0.5

Which animals would be able to use the ramp in the diagram below?
Explain your answer fully.



(1 + #2.2)

Unit title: Applications

Assessment standard: 1.1 Applying algebraic skills to rectilinear shapes

Answers to Practice Tests

Practice test 1

1 $y = -4x + 10$

2 $x + 3y + 1 = 0$

3 gradient, m = -0·268
Therefore the steepness of the slope is 0·268 which means the slope is for teaching and general skiing because 0 ≤ 0·268 ≤ 0·35

Practice test 2

1 $y = 6x - 23$

2 $x + 3y - 8 = 0$

3 angle = 20°,
gradient, m = 0·364
Therefore the steepness of the ramp is 0·364 which dangerous because 0·364 > 0·3

Practice test 3

1 $x - 4y + 7 = 0$

2 $x + 3y - 13 = 0$

3 acute angle = 76·0°

Obtuse angle = 180 – 776·0 = 104·0°

4 angle = 23°,
gradient, m = 0·424
Therefore the steepness of the ramp is 0·424 which means it is only for sheep/cattle because 0·424 < 0·5 but it is not less than 0·35

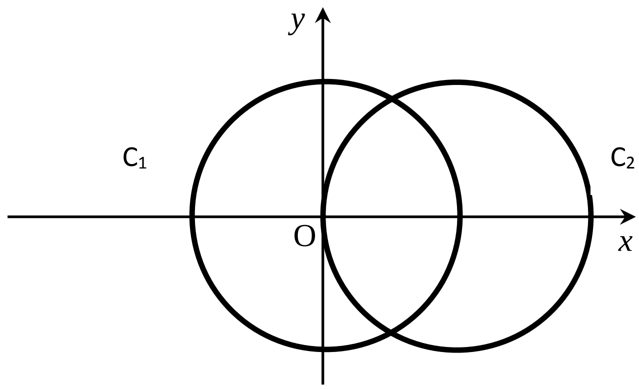
Unit title: Applications

Assessment standard: 1.2 Applying algebraic skills to circles

Sub-skills	♦ determining and using the equation of a circle ♦ using properties of tangency in the solution of a problem

Practice test 1

1 The diagram shows two identical circles, C_1 and C_2. C_1 has centre the origin and radius 8 units.



The circle, C_2, passes through the origin.
The x-axis passes through the centre of C_2.
Find the equation of circle C_2.

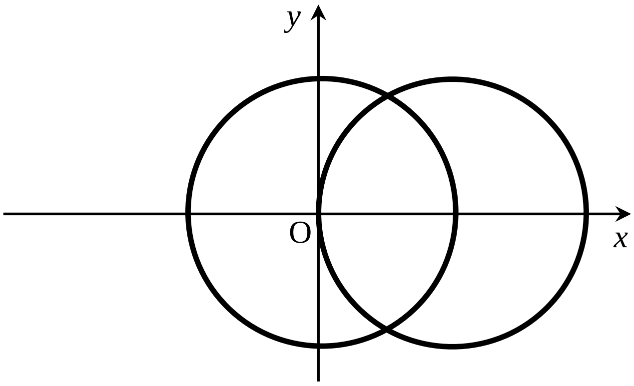
(#$^{2.1}$ and 1)

2 Determine if the line $y = 3x + 10$ is a tangent to the circle
$x^2 + y^2 - 8x - 4y - 20 = 0$

(3 and #$^{2.2}$)

Practice test 2

1 The diagram shows two congruent circles. One circle has centre the origin and diameter 18 units.



Find the equation of the other circle which passes through the origin and whose centre lies on the *x*-axis.

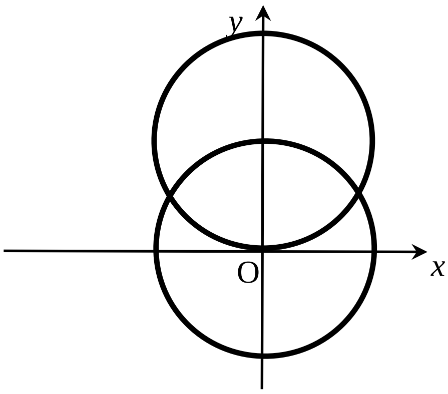
(#2.1 and 1)

2 Determine algebraically if the line $y = -3x - 10$ is a tangent to the circle $(x-4)^2 + (y+2)^2 = 40$

(3 + #2.2)

Practice test 3

1 The diagram shows two congruent circles.
One circle has centre the origin and diameter 16 units.



Find the equation of the other circle which passes through the origin and whose centre lies on the *y*-axis.

(2)

2 Determine algebraically if the line $y = x + 9$ is a tangent to the circle $(x+3)^2 + (y-2)^2 = 8$

(3 + #2.2)

Unit title: Applications

Assessment standard: 1.2 Applying algebraic skills to circles

Answers to Practice Tests

Practice test 1

1 $(x-8)^2 + y^2 = 64$

2 Solving simultaneously gives: $10x^2 + 40x + 40 = 0$
Testing for discriminant gives: $b^2 - 4ac = 0$
Since $b^2 - 4ac = 0$ there are 2 roots are real and equal (or repeated) so therefore the line a tangent to the circle.

Practice test 2

1 $(x-9)^2 + y^2 = 81$

2 Solving simultaneously gives: $5x^2 - 26x + 20 = 0$
Testing for discriminant gives: $b^2 - 4ac = 356$
Since $b^2 - 4ac > 0$ there are 2 distinct points of intersection so therefore the line is not a tangent to the circle.

Practice test 3

1 $x^2 + (y-8)^2 = 64$

2 Solving simultaneously gives: $x^2 + 10x + 25 = 0$
Testing for discriminant gives: $(x+5)^2 = 0, x = -5 \; or \; x = -5$
Since the roots are real and equal there is only one point of intersection so therefore the line is a tangent to the circle.

Unit title: Applications

Assessment standard: 1.3 Applying algebraic skills to sequences

Sub-skills	♦ determining a recurrence relation from given information and using it to calculate a required term ♦ finding and interpreting the limit of a sequence, where it exists

Practice test 1

1 A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$ where *m* and *c* are constants.
It is known that $u_1 = 3,\ u_2 = 7,$ and $u_3 = 23$
Find the recurrence relation described by the sequence and use it to find the value of u_5.

(4)

2 On a particular day at 09:00, a doctor injects a first dose of 400 mg of medicine into a patient's bloodstream. The doctor then continues to administer the medicine in this way at 09:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 11% of what it was at the start.

(a) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

(1)

The patient will overdose if the amount of medicine in their bloodstream exceeds 500 mg.

(b) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(2 + #2.2)

Practice Test 2

1 A Regular Saver Account offers 2% interest per year. Interest on the account is paid at the **end** of each year.
You open this account with your first deposit of £180 at the start of a particular year and deposit £700 into the account at the start of each subsequent year.

u_n represents the amount of money in the account *n* years after the account is opened, then $u_{n+1} = au_n + b$.
State the values of *a* and *b.*

(2)

Given that u_0 is the initial deposit, calculate the value of u_3.

2 Two sequences are generated by the recurrence relations
$u_{n+1} = au_n + 2$ and $v_{n+1} = 4av_n + 3$ with $u_0 = 1$ and $v_0 = -1$.
The two sequences approach the same limit as $n \to \infty$.
Determine the value of *a* and hence evaluate the limit.

(#2.1 and 2)

Practice Test 3

1 A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$ where *m* and *c* are constants.
It is known that $u_1 = 3, u_2 = 2, u_3 = -1$.

Find the recurrence relation described by the sequence and use it to find the value of u_7

(4)

2 On a particular day at 07:00, a vet injects a first dose of 65 mg of medicine into a dog's bloodstream. The vet then continues to administer the medicine in this way at 07:00 each day.

The vet knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 18% of what it was at the start.

(a) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The dog will overdose if the amount of medicine in its bloodstream exceeds 85 mg.

(b) In the long term, if the dog continues with this treatment, is there a danger it will overdose? **Explain your answer.**

(3 +#2.2)

Unit title: <u>Applications</u>

Assessment standard: 1.3 Applying algebraic skills to sequences

Answers to Practice Tests

Practice test 1

1 $u_2 = mu_1 + c \Rightarrow 7 = 3m + c$
$u_3 = mu_2 + c \Rightarrow 23 = 7m + c$
$m = 4, c = -5$
$u_{n+1} = 4u_n - 5$
$u_5 = 4 \times 87 - 5 = 343$

2 $U_{n+1} = 0.11U_n + 400$
$L = 0.11L + 400$ or $L = \frac{400}{1 - 0.11}$
$L = 449.44$
449.44 < 500 so looks like the patient would not be in danger of overdosing

Practice test 2

1 a = 1·02, b = 700
$u_1 = 883.6$, $u_2 = 1601.27$, $u_3 = 2333.30...$

2 know to equate, e.g. $\frac{2}{1-a} = \frac{3}{1-4a}$
$a = -\frac{1}{5}$ (or -0·2)
$L = \frac{1}{1-\left(-\frac{1}{5}\right)} = \frac{5}{3}$

Practice test 3

1 $u_2 = mu_1 + c$, $2 = 3m+c$
$u_3 = mu_2 + c$, $-1 = 2m + c$
$m = 3$, $c = -7$
$u_{n+1} = 3u_n - 7$
$u_7 = -361$

2 $u_{n+1} = 0.18u_n + 65$
L = 0.18 x L + 65 or $L = \frac{65}{1 - 0.18}$
L = 79.268 or $79\frac{11}{41}$
79.268 < 85 so looks like the dog would not be in danger of overdosing

Unit title: Applications

Assessment standard: 1.4 Applying calculus skills to optimisation and area

Sub-skills	♦ determining the optimal solution for a given problem ♦ finding the area between a curve and the *x*-axis ♦ finding the area between two curves or a straight line and a curve

Practice test 1

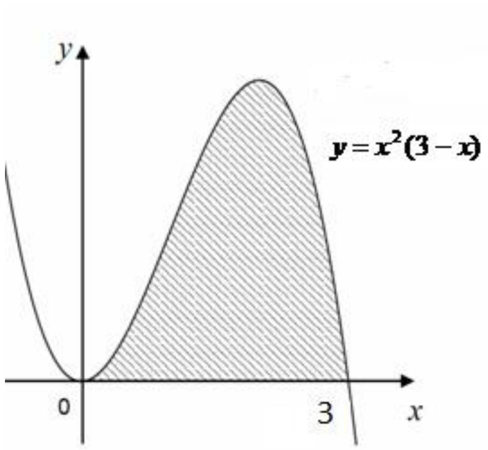
1 A box with a square base and open top has a surface area of 768 cm^2. The volume of the box can be represented by the formula:

$$V(x) = 75x - \frac{1}{9}x^3$$

Find the value of *x* which maximises the volume of the box.

(5)

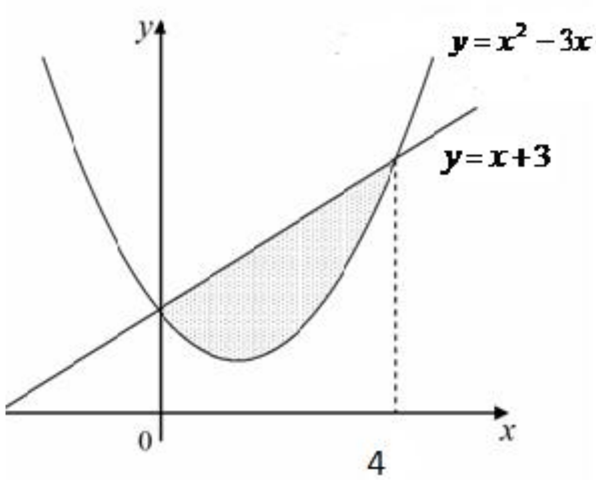
2 The curve with equation $y = x^2(3-x)$ is shown in the diagram.



Calculate the shaded area.

(#2.1 + 4)

3 The line with equation $y = x + 3$ meets the curve with equation $y = x^2 - 3x + 3$ when $x = 0$ and $x = 4$ as shown in the diagram.



Calculate the shaded area.

(5)

Practice test 2

1 The area of a rectangle can be represented by the formula

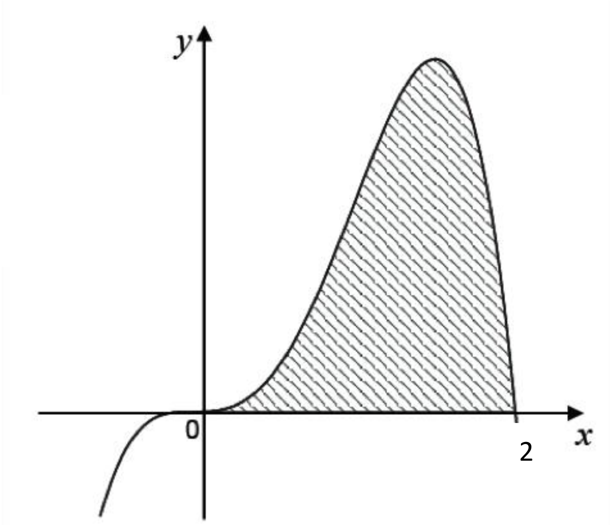
$$A(x) = 27x - 3x^3, \text{ where } x > 0.$$

Find the value of x which maximises the area of the rectangle.
Justify your answer.

(5)

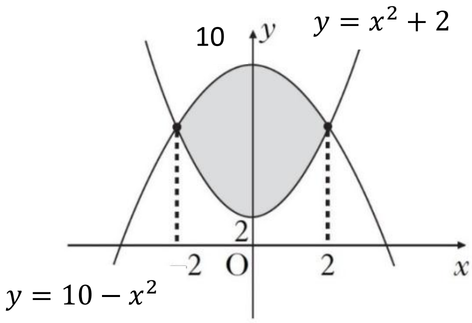
2 The curve with equation $y = x^3(2 - x)$ is shown in the diagram.

Calculate the shaded area.



(4)

3 The diagram shows graphs with equations $y = 10 - x^2$ and $y = x^2 + 2$



(a) Which of the following integrals represents the shaded area?

A $\int_2^{10}(2x^2 - 8)\,dx$ B $\int_{-2}^{2}(8 - 2x^2)\,dx$ C $\int_{-2}^{2}(2x^2 - 8)\,dx$ D $\int_2^{10}(8 - 2x^2)\,dx$

(1)

(b) Calculate the shaded area.

(3)

Practice test 3

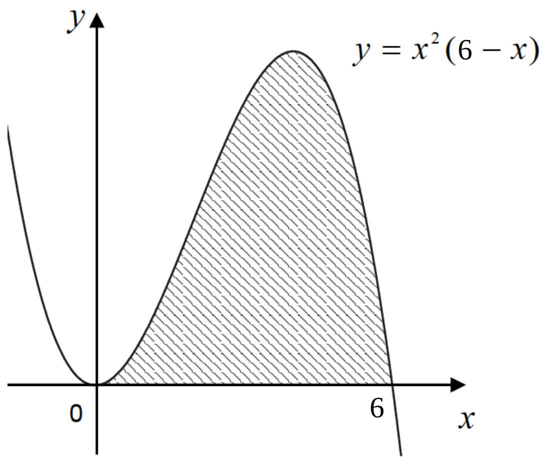
1 A box with a square base and open top has a surface area of 108 cm^2. The volume of the box can be represented by the formula:

$$V(x) = 27x - \frac{1}{4}x^3.$$

Find the value of *x* which maximises the volume of the box.

(4 + #2.2)

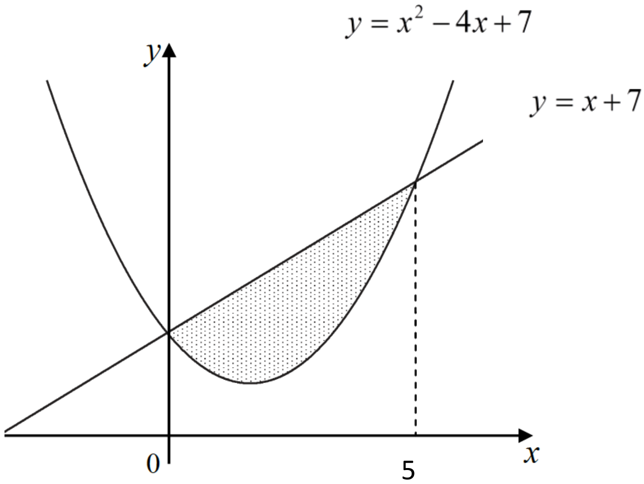
2 The curve with equation $y = x^2(6 - x)$ is shown In the diagram.



Calculate the shaded area.

(4)

3 The line with equation $y = x + 7$ and the curve with equation $y = x^2 - 4x + 7$ are shown in the diagram.



The line and curve meet at the points where *x* = 0 and *x* = 5.

Calculate the shaded area.

(5)

Unit title: <u>Applications</u>

Assessment standard: 1.4 Applying calculus skills to optimisation and area

Answers to Practice Tests

Practice Test 1

1 $V' = 75 - \frac{1}{3}x^2 = 0$ stated explicitly
$x = \pm 15$
use 2nd derivative or nature table
maximum at $x = 15$

2 $\frac{27}{4}$ or $6\frac{3}{4}$ square units

3 $\frac{32}{3}$ or $10\frac{2}{3}$ square units

Practice Test 2

1 $27 - 9x^2 = 0$ stated explicitly
$x = \pm\sqrt{3}$
Uses nature table or 2nd derivative
Max area when $x = \sqrt{3}$

2 $\frac{8}{5}$ square units or equivalent

3 Area = $21\frac{1}{3}$ square units, or equivalent

Practice Test 3

1 $V'(x) = 27 - \frac{3}{4}x^2$ and $V'(x) = 0$
$x = 6$
nature table or 2nd derivative
Maximum when $x = 6$

2 108 square units

3 $\frac{125}{6}$ or $20\frac{5}{6}$ square units

Unit title: Expressions and Functions

Assessment Standards	Making assessment judgements
1.1 Applying algebraic skills to logarithms and exponentials	The sub-skills in the Assessment Standard are: ♦ simplifying an expression, using the laws of logarithms and exponents ♦ solving logarithmic and exponential equations, using the laws of logarithms and exponents
1.2 Applying trigonometric skills to manipulating expressions	The sub-skills in the Assessment Standard are: ♦ applying the addition or double angle formulae ♦ applying trigonometric identities ♦ converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, α in 1st quadrant $k > 0$
1.3 Applying algebraic and trigonometric skills to functions	The sub-skills in the Assessment Standard are: ♦ identifying and sketching related algebraic functions ♦ identifying and sketching related trigonometric functions ♦ determining composite and inverse functions — including basic knowledge of domain and range
1.4 Applying geometric skills to vectors	The sub-skills in the Assessment Standards are: ♦ determining the resultant of vector pathways in three dimensions ♦ working with collinearity ♦ determining the coordinates of an internal division point of a line ♦ evaluating a scalar product given suitable information and determining the angle between two vectors
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	Assessment Standard 2.1 and 2.1 are transferable across Units and can be attached to any of the sub-skills listed above. #2.1 is mostly about choosing an appropriate strategy. This skill is usually required to make a good start on a problem. #2.2 is about how well you answer a question, often with a summary statement at the end of a question.
2.2 Explaining a solution and, where appropriate, relating it to context	

My assessment record: Mathematics: Expressions and Functions (Higher)

Keep this up to date as you go along so that you know if you have any areas that you need to do more work on

Assessment standard	First attempt		Second attempt (if required)	
	Mark(s)	Pass/Fail	Mark(s)	Pass/Fail
1.1 Applying algebraic skills to logarithms and exponentials • simplifying an expression, using the laws of logarithms and exponents • solving logarithmic and exponential equations, using the laws of logarithms and exponents				
1.2 Applying trigonometric skills to manipulating expressions • applying the addition or double angle formulae • applying trigonometric identities • converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, α in 1st quadrant $k > 0$				
1.3 Applying algebraic and trigonometric skills to functions • identifying and sketching related algebraic functions • identifying and sketching related trigonometric functions • determining composite and inverse functions — including basic knowledge of domain and range				
1.4 Applying geometric skills to vectors • determining the resultant of vector pathways in three dimensions • working with collinearity • determining the coordinates of an internal division point of a line • evaluating a scalar product given suitable information and determining the angle between two vectors				
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy				
2.2 Explaining a solution and, where appropriate, relating it to context				

Unit title: <u>Expressions and Functions</u>

Assessment standard: 1.1 Applying algebraic skills to logarithms and exponentials

Sub-skills	♦ simplifying an expression, using the laws of logarithms and exponents ♦ solving logarithmic and exponential equations, using the laws of logarithms and exponents

Practice test 1

1 (a) Simplify $\log_5 6a + \log_5 7b$.

(b) Express $\log_b x^7 - \log_b x^4$ in the form $k\log_b x$

[3]

2 Solve. $\log_4(x-1) = 3$

[2]

Practice test 2

1 (a) Simplify $\log_4 3p - \log_4 2q$.

(b) Express $\log_a x^2 + \log_a x^3$ in the form $k\log_a x$

[3]

2 Explain why *x* = 0·399 is a solution of the following equation to 3 significant figures:

$e^{5x+1} = 20$

[2 + #2.2]

Practice test 3

1 (a) Simplify $\log_5 4x + \log_5 6y$.

[1]

(b) Express $\log_a x^7 - \log_a x^4$ in the form $k\log_a x$.

[2]

2 Solve $\log_2(3x-5) - \log_2(x+2) = 1$

Unit title: Expressions and Functions

Assessment standard: 1.1 Applying algebraic skills to logarithms and exponentials

Answers to Practice Tests

Practice test 1

1 (a) $\log_5(6a \times 7b) = \log_5(42ab)$

(b) $\log_b\left(\frac{x^7}{x^4}\right) = \log_b\left(x^3\right)$

$= 3\log_b x$

2 $x - 1 = 4^3$ stated explicitly

$x = 65$

Practice test 2

1 (a) $\log_4 3p - \log_4 2q$

$= \log_4\left(\frac{3p}{2q}\right)$

(b) $\log_a x^2 + \log_a x^3$

$\log_a\left(x^5\right)$

$5\log_a x$

2

$e^{5x+1} = 20$

$\ln\left(e^{5x+1}\right) = \ln(20)$

$5x + 1 = \ln(20)$

$x = \frac{\ln(20) - 1}{5} = 0 \cdot 399$

OR

$e^{5x+1} = 20$

$\log_e(20) = 5x + 1$

$x = \frac{\ln(20) - 1}{5} = 0 \cdot 399$

- Apply logs to both sides OR convert from exponential to logarithmic form
- Rearrange equation for *x*

Practice test 3

1 (a) $\log_4(3p \times 5q) = \log_4 15pq$

(b) $5\log_a x - 3\log_a x$ OR $\log_a x^2$

$2\log_a x$

2 $\log_2(3x - 5) - \log_2(x + 2) = 1$

$3x - 5 > 0$ $x + 2 > 0$

$x > \frac{5}{3}$

$\log_2\left(\frac{3x - 5}{x + 2}\right) = 1$

$2^1 = \frac{3x - 5}{x + 2}$

$2(x + 2) = 3x - 5$

$x = 9$

Unit title: **Expressions and Functions**

Assessment standard: 1.2 Applying trigonometric skills to manipulating expressions

Sub-skills	♦ applying the addition or double angle formulae ♦ applying trigonometric identities ♦ converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, α in 1st quadrant $k > 0$

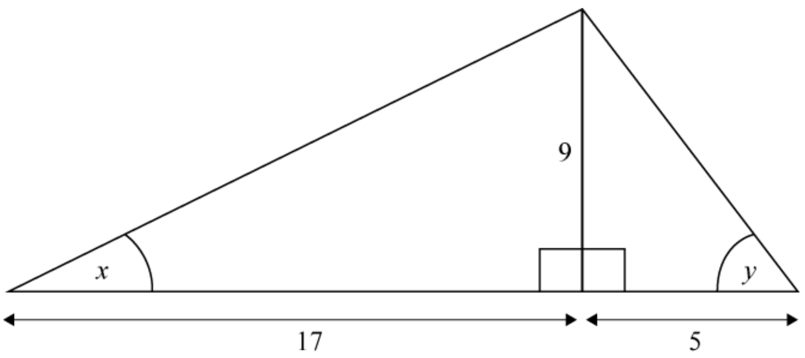
Practice test 1

1 Express 2 sin x + 3cosx in the form $k\sin(x+a)^{\circ}$ where $k>0$ and $0 \le a < 360$.
Calculate the values of *k* and *a*.

[4]

2 The diagram below shows two right-angled triangles with measurements as shown.
Find the exact value of sin (x-y).

[3]



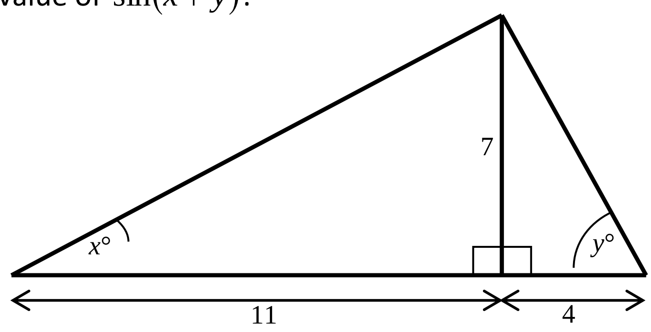
3 Show that (3 + 2 cos x) (3 – 2 cos x) = 4 $\sin^2$ x + 5.

[#2.1, 2]

Practice test 2

1 Express $4\cos x - 5\sin x$ in the form $k\cos(x+a)°$ where $k>0$ and $0 \le a < 360$. **(4)**

2 The diagram below shows two right-angled triangles.
Find the exact value of $\sin(x+y)$.



(4)

3 Show that $(2+5\sin x)(2-5\sin x) = 25\cos^2 x - 21$. **(3 + 2.1)**

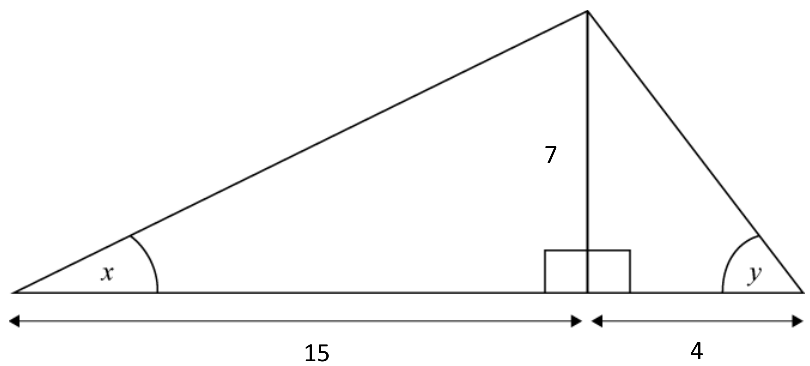
Practice test 3

1 Express $5sinx + 4cosx$ in the form $ksin(x+a)^0$ where $k>0$ and $0^o \le a^o \le 360^o$.

[4]

2 The diagram below shows two right-angled triangles.
Find the exact value of $\cos(x+y)$.

[4]



3 Show that $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$.

[#2.1 + 3]

Unit title: **Expressions and Functions**

Assessment standard: 1.2 Applying trigonometric skills to manipulating expressions

Answers to Practice tests

Practice test 1

1 $k=\sqrt{13}$ and a = 56.3º

2 $\frac{-108}{\sqrt{39220}}$

3 know to use identities

L.H.S. $= (3+2\cos x)(3-2\cos x)$

$= 9-4\cos^2 x$

$= 9-4(1-\sin^2 x)$

$= 9-4+4\sin^2 x$

$= 4\sin^2 x+5$

Practice test 2

1 $k=\sqrt{41}$ and a = 51.3°

2 $\frac{105}{\sqrt{11050}}$

3 know to use identities

L.H.S. $= 4-25\sin^2 x$

$= 4-25(1-\cos^2 x)$

$= 4-25+25\cos^2 x$

$= 25\cos^2 x-21$

$= RHS$

Practice test 3

1 $k=\sqrt{41}$ and a = 38.7°

2 $\frac{11}{\sqrt{17810}}$

3 know to use identities

L.H.S. $= \sin(x+y)\sin(x-y)$

$= (\sin x\cos y+\sin y\cos x)(\sin x\cos y-\sin y\cos x)$

$= \sin^2 x\cos^2 y-\sin^2 y\cos^2 x$

$= \sin^2 x(1-\sin^2 y)-\sin^2 y(1-\sin^2 x)$

$= \sin^2 x-\sin^2 x\sin^2 y-\sin^2 y+\sin^2 y\sin^2 x$

$= \sin^2 x-\sin^2 y$

= R.H.S.

Unit title: **Expressions and Functions**

Assessment standard: 1.3 Applying algebraic and trigonometric skills to functions

Sub-skills	♦ identifying and sketching related algebraic functions ♦ identifying and sketching related trigonometric functions ♦ determining composite and inverse functions — including basic knowledge of domain and range

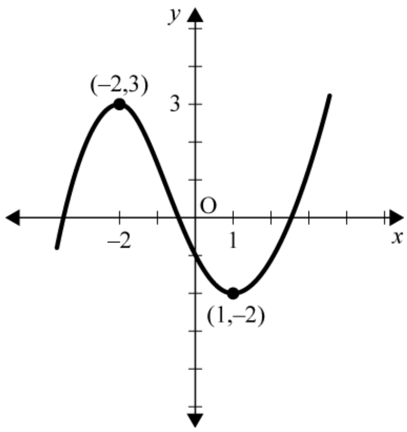
Practice test 1

1 Sketch the graph of $a\cos(x-\frac{\pi}{3})$ for $0 \le x \le 2\pi$ and $a > 0$.

Show clearly the intercepts on the *x*-axis and the coordinates of the turning points.

[4]

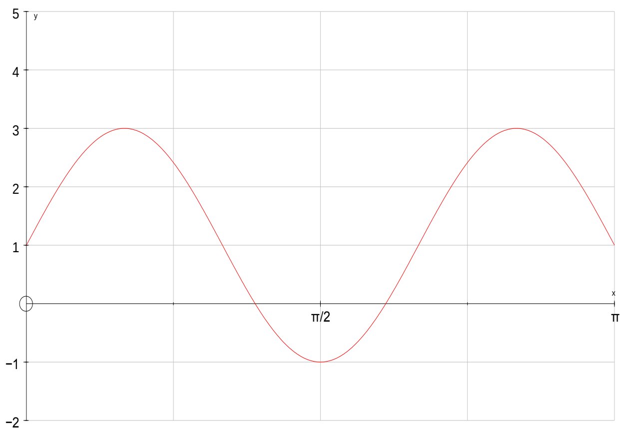
2 The diagram shows the graph of $y = f(x)$ with a maximum turning point at (–2, 3) and a minimum turning point at (1, –2).



Sketch the graph of $y = f(x+2)-1$

[3]

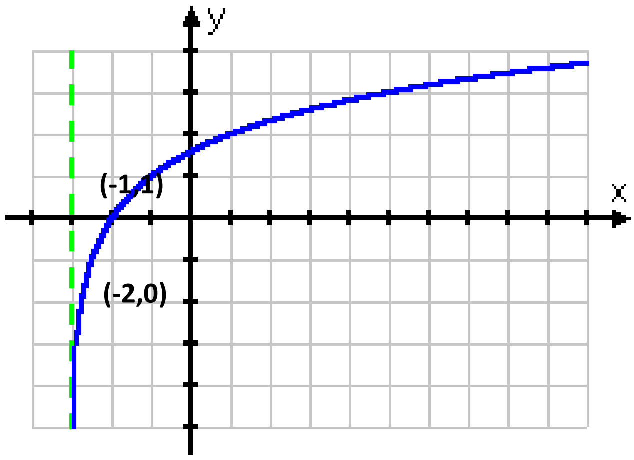
3 The diagram below shows the graph of $y = a\sin(bx)+c$.



Write down the values of *a*, *b* and *c*.

[3]

4 The diagram shows the graph of $y = \log_b(x-a)$.



Determine the values of *a* and *b*. [2]

5 The functions f and g, defined on suitable domains contained within the set of real numbers, $f(x) = 5x-2$, $g(x) = \sqrt{x-1}$. A third function *h*(*x*) is defined as $h(x) = g(f(x))$.

(a) Find an expression for $h(x)$.

[2]

(b) Explain why x = 0 is not in the domain of $h(x)$.

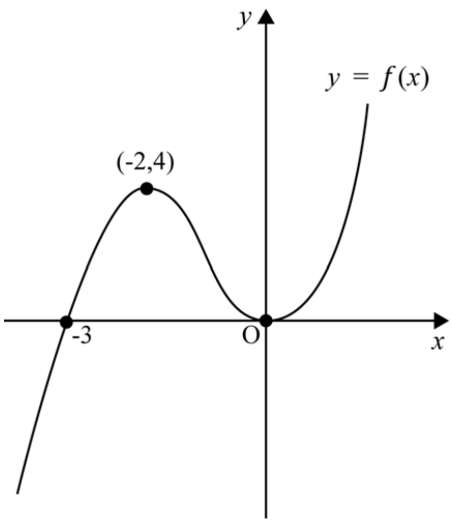
[#2.2]

6 A function is given by $f(x) = 4x+6$. Find the inverse function $f^{-1}(x)$.

[3]

Practice test 2

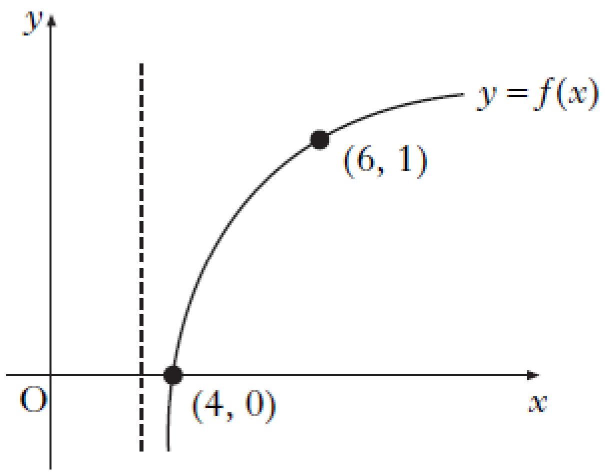
1 The diagram shows the graph of $y = f(x)$ with a maximum turning point at (–2, 4) and a minimum turning point at (0, 0).



Sketch the graph of $y = 1 - f(x-3)$.

[4]

2 The diagram shows the graph of $y = \log_b(x-a)$.



Determine the values of *a* and *b*.

[2]

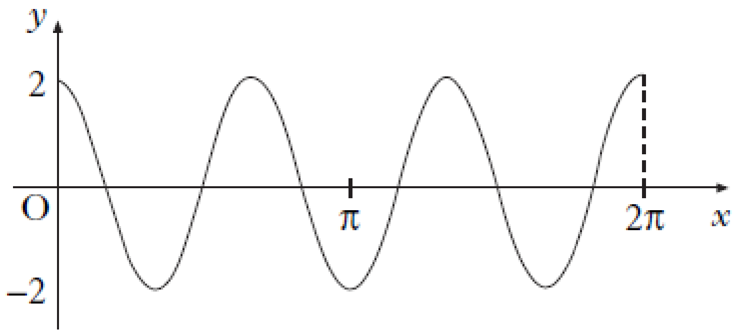
3 Sketch the graph of $y = 3\cos(x + \frac{\pi}{4})$ for $0 \le x \le 2\pi$.

Show clearly the intercepts on the *x*-axis and the coordinates of the turning points.

[4]

4 The diagram shows the graph of *y* = *a*cos(*bx*) for $0 \le x \le 2\pi$.

State the values of *a* and *b*.



[2]

5 The functions f and g are defined on suitable domains contained within the set of real numbers,

$f(x) = \frac{1}{x^2 - 16}$ and $g(x) = x - 2$.

A third function, *h,* is defined as $h(x) = f(g(x))$.

(a) Find an expression for $h(x)$.

[2]

(b) Find a suitable domain for $h(x)$.

[#2.1 + 2]

6 A function is given by $f(x) = 2 - \sqrt[3]{x}$. Find the inverse function $f^{-1}(x)$.

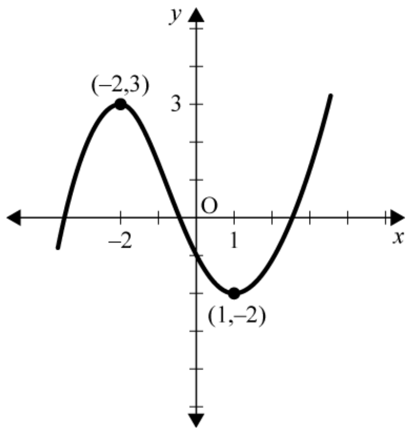
[3]

Practice test 3

1 Sketch the graph of $y = a\sin(x - \frac{\pi}{6})$ for $0 \le x \le 2\pi$ and $a > 0$, clearly showing the maximum and minimum values and where it cuts the *x*-axis.

[3]

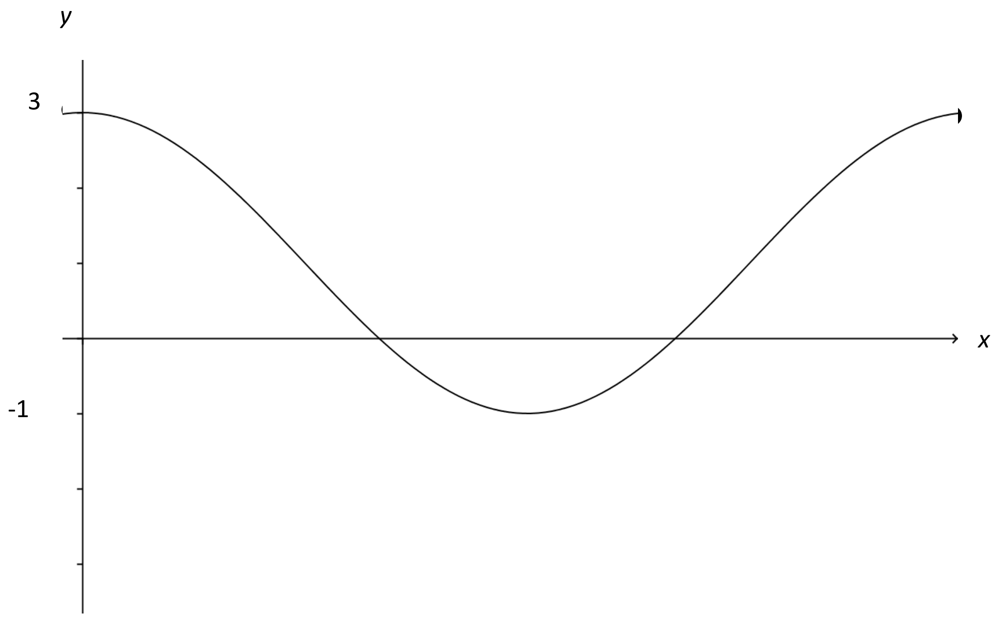
2 The diagram shows the graph of $y = f(x)$ with a maximum turning point at (–2, 3) and a minimum turning point at (1, –2).



Sketch the graph *of* $y = f(x-1) - 2$.

[3]

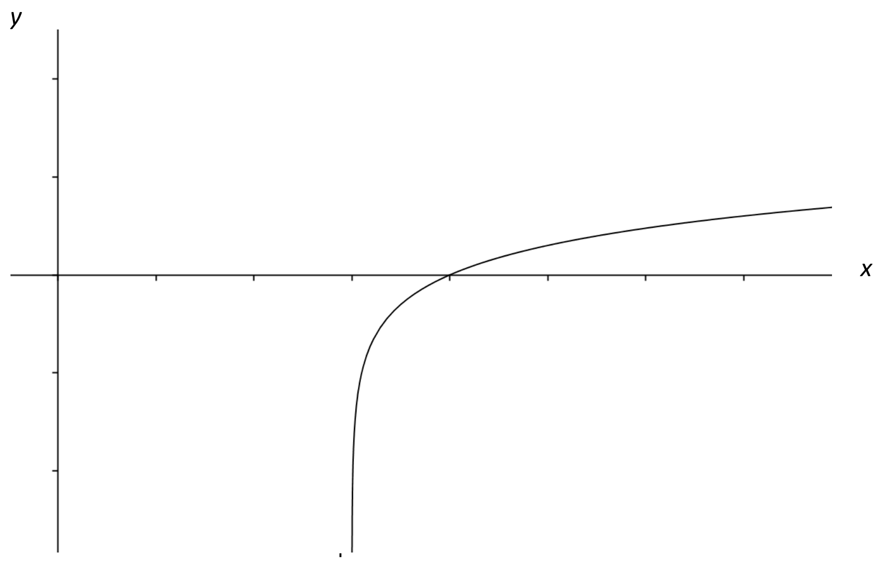
3 The diagram below shows the graph of $y = a\cos(bx) + c$.



Write down the values of *a*, *b* and *c*.

[3]

4 The diagram shows the graph of $y = \log_b(x-a)$.



Determine the values of *a* and *b*.

[2]

5 The functions f and g, defined on suitable domains, are given by $f(x) = 2x-3$, $g(x) = \sqrt{x}$.
A third function *h*(*x*) is defined as $h(x) = g(f(x))$.

(a) Find an expression for $h(x)$.

[3]

(b) Explain why the largest domain for $h(x)$ is given *by* $x \geq \frac{3}{2}$.

[#2.2]

6 A function is given by $f(x) = 3x+4$. Find the inverse function $f^{-1}(x)$.

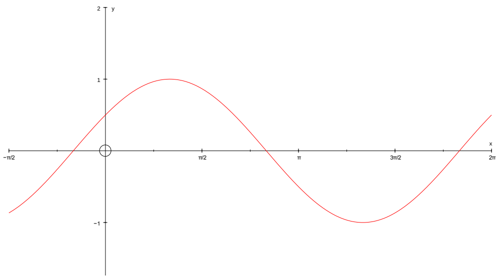
[2]

Unit title: **Expressions and Functions**

Assessment standard: 1.3 Applying algebraic and trigonometric skills to functions

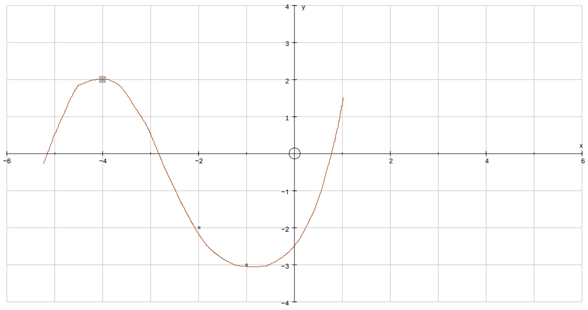
Answers to Practice Tests

Practice test 1

1 

x-intercepts: $\left(\frac{5\pi}{6}, 0\right)$ and $\left(\frac{11\pi}{6}, 0\right)$

Max = a $\left(\frac{\pi}{3}, a\right)$ Min = -a $\left(\frac{4\pi}{3}, -a\right)$

2 

(-4,2), (-1,-3) and (-2,-2) clearly annotated

3 *a* = 2, *b* = 3, *c* = 1

4 *a* = -3, *b* = 2

5 (a) $h(x) = \sqrt{5x-3}$

(b) $5x - 3 \geq 0$

$5x \geq 3$

$x \geq \frac{3}{5}$

Because on the set of real numbers, the square root of a negative number cannot be found.

6 $f^{-1}(x) = \frac{1}{4}(x-6)$

Practice test 2

1
- Horizontal translation (3 units to the right)
- Reflection in the *x*-axis
- Vertical translation (1 unit up)
- Each image annotated clearly: (-3, 0) to (0, 1)
  (-2, 4) to (1, -3)
  (0, 0) to (3, 1)

2 *a* = 3, *b* = 3

3
- Amplitude correct: Max value = 3, Min value = -3
- Correct turning points: Min. T.P. $\left(\frac{3\pi}{4},\ -3\right)$ Max T.P. $\left(\frac{7\pi}{4},\ 3\right)$
- Correct x-intercepts $\left(\frac{\pi}{4},\ 0\right)\left(\frac{5\pi}{4},\ 0\right)$
- Correct shape, i.e. cosine curve

  Note: y-intercept = $\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 2\cdot121...$

4 $a = 2$, $b = 3$

5 (a) $h(x) = \frac{1}{(x-2)^2 - 16}$

(b)
$$(x-2)^2 - 16 = 0$$
$$(x-2)^2 = 16$$
$$x - 2 = \pm\ 4$$
$x = -2,\ 6$ domain = $\{x \in R : x \neq -2, x \neq 6\}$

6 $f^{-1}(x) = (2-x)^3$

Practice test 3

1 max a at $(\frac{2\pi}{3}, a)$ and min $-a$ at $(\frac{5\pi}{3}, -a)$

x-intercepts $(\frac{\pi}{6}, 0)$ and $(\frac{7\pi}{6}, 0)$

correct shape, i.e. sine wave

2 Correct horizontal translation (1 units to the right)
Correct vertical translation (2 units down)
Key points annotated clearly: (-2, 3) to (-1, 1),
(1, -2) to (2, -4)

3 $a = 3$, $b = 2$, $c = 1$

4 $a = 3$, $b = 3$

5 (a) $g\big(f(x)\big) = \sqrt{2x - 3}$

(b) Because on the set of real numbers, the square root of a negative number cannot be found.

Therefore domain: $2x - 3 \geq 0$

$$2x \geq 3$$

$$x \geq \frac{3}{2}$$

6 $f^{-1}(x) = \frac{x-4}{3}$

Unit title: **Expressions and Functions**

Assessment standard: 1.4 Applying geometric skills to vectors

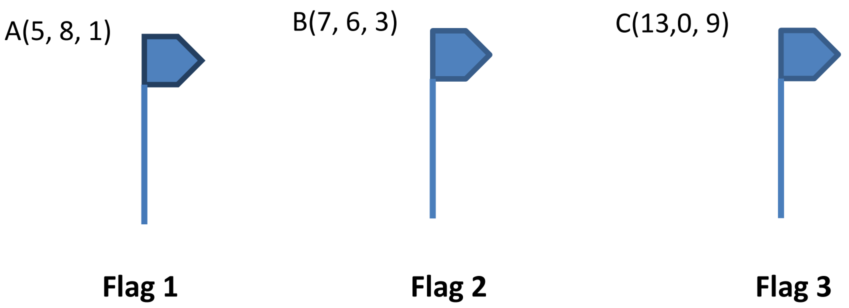
Sub-skills	♦ determining the resultant of vector pathways in three dimensions ♦ working with collinearity ♦ determining the coordinates of an internal division point of a line ♦ evaluating a scalar product given suitable information and determining the angle between two vectors

Practice test 1

1 An engineer positioning marker flags needs to ensure that the following two conditions are met:

- The poles are in a straight line.
- The distance between flag 2 and flag 3 is three times the distance between flag 1 and flag 2.

Relative to suitable axes, the top of each flag can be represented by the points A (2, 3, 1), B (5, 1, 3), and C (11,–3, 7) respectively. All three poles are vertical.

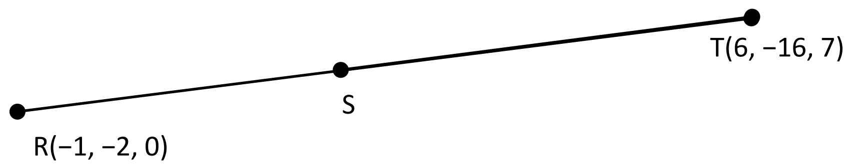


Has the engineer satisfied the two conditions?

You must justify your answer.

#2.1 #2.2 [4]

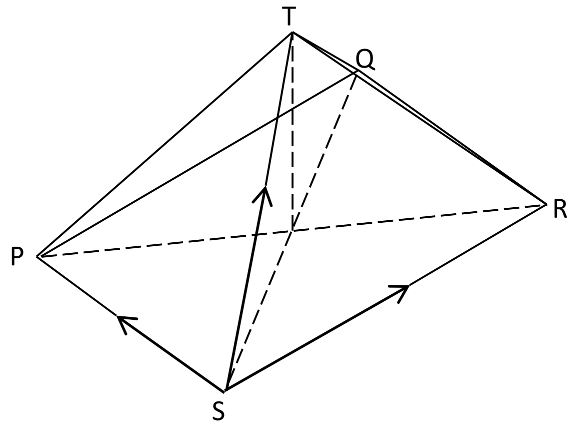
2 The points R, S and T lie in a straight line, as shown. S divides RT in the ratio 3:4.



Find the coordinates of S.

[3]

3 TPQRS is a pyramid with rectangular base PQRS.



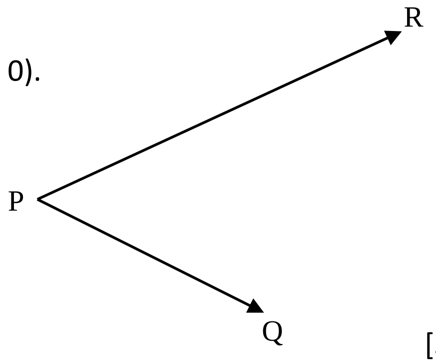
The vectors $\overrightarrow{SP}$ and $\overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = \begin{pmatrix} -2 \\ 10 \\ -6 \end{pmatrix}; \quad \overrightarrow{ST} = \begin{pmatrix} 4 \\ 16 \\ 12 \end{pmatrix}$$

Express $\overrightarrow{PT}$ in component form.

[2]

4 The diagram shows vectors $\overrightarrow{\mathrm{PR}}$ and $\overrightarrow{\mathrm{PQ}}$.
P, Q and R have coordinates P(4, –1, –2), Q(6, –2, 2) and R(8, –3, 0).



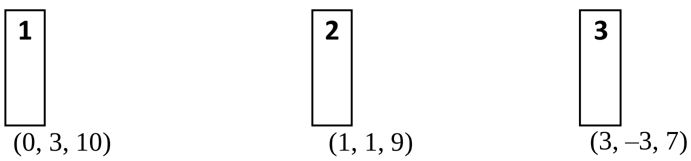
Find the size of the acute angle QPR.

[5]

Practice test 2

1 An engineer positioning concrete posts needs to ensure that the following two conditions are met:
- ♦ The poles are in a straight line.
- ♦ The distance between post 1 and post 2 is twice the distance between post 2 and post 3

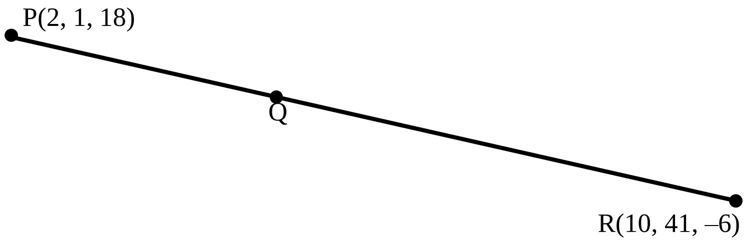
Relative to a suitable axes, the posts can be represented by the points A(0, 3, 10), B(1, 1, 9) and C(3, –3, 7) respectively.



Has the engineer satisfied the two conditions? You must justify your answer.

[4 +#2.1 + #2.2]

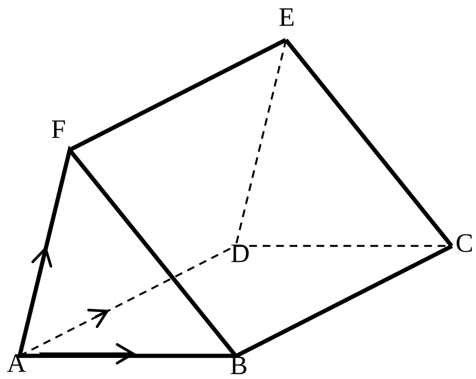
2 The points P, Q and R lie in a straight line, as shown. Q divides PR in the ratio 3:5.



Find the coordinates of Q.

[3]

3 ABCDEF is a triangular prism as shown.



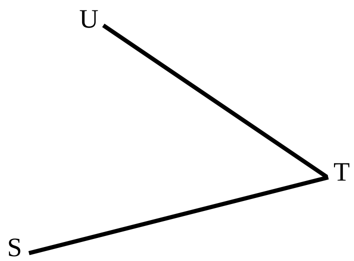
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AF}$ are given by:

$$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 8 \\ 4 \end{pmatrix}; \quad \overrightarrow{AD} = \begin{pmatrix} 10 \\ 4 \\ 2 \end{pmatrix}; \quad \overrightarrow{AF} = \begin{pmatrix} -1 \\ -4 \\ 13 \end{pmatrix}$$

Express $\overrightarrow{FB}$ in component form.

[3]

4 Points S, T and U have coordinates S(3, 0, 2), T(7, 1, -5) and U(4, 3, -2).



Find the size of the acute angle STU.

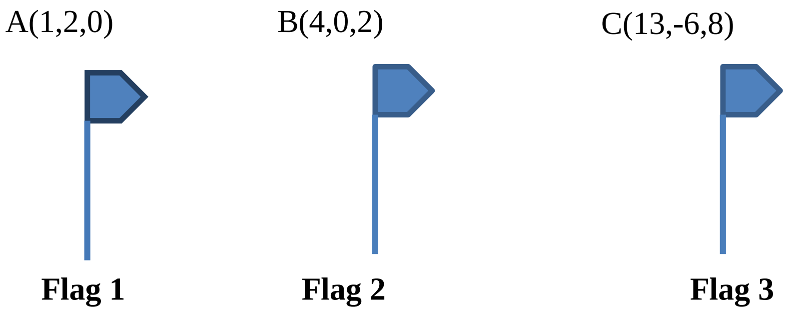
[5 + #2.1]

Practice test 3

1 An engineer laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is 3 times the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points A (1, 2, 0), B (4, 0, 2), and C (13,–6, 8) respectively. All three flags point vertically upwards.

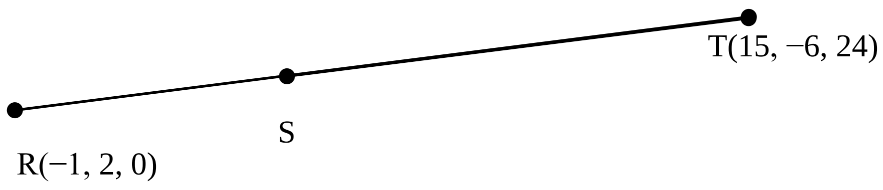


Has the engineer laid the flags correctly? You must justify your answer.

[#2.1, #2.2, 4]

2 The points R, S and T lie in a straight line, as shown. S divides RT in the ratio 3:5.

Find the coordinates of S.



[3]

3 TPQRS is a pyramid with rectangular base PQRS.

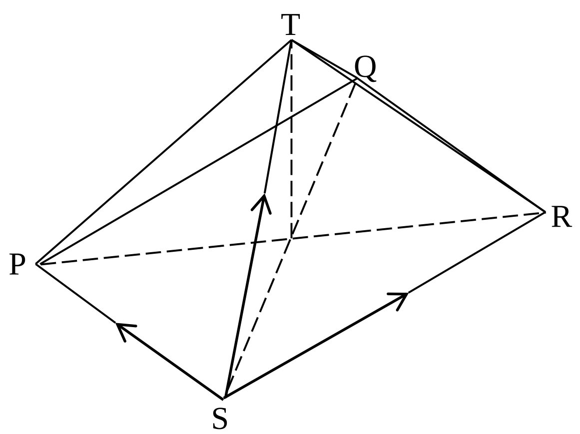
The vectors $\overrightarrow{SP}, \overrightarrow{SR}, \overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$$

$$\overrightarrow{SR} = 12\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$$

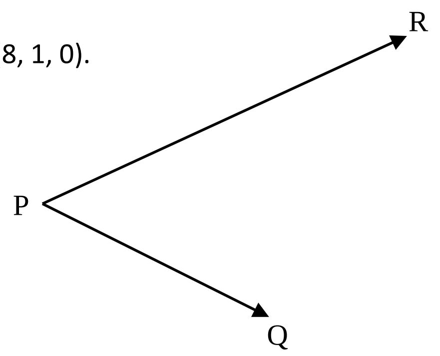
$$\overrightarrow{ST} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$$

Express $\overrightarrow{PT}$ in component form.



[3]

4 Points P, Q and R have coordinates P(5, –3, –1), Q(7, –4, 2) and R(8, 1, 0).



Find the size of the acute angle QPR.

[5]

Unit title: **Expressions and Functions**

Assessment standard: 1.4 Applying geometric skills to vectors

Answers to Practice Tests

Practice test 1

1 $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix} = 3\overrightarrow{AB}$ therefore $BC : AB = 1:3$

BC=3AB hence vectors are parallel and B is a common point so A, B and C are collinear.

Yes, the engineer has placed the flags correctly because A, B and C lie on the same straight line and $BC : AB = 3:1$ so the distance between flags 2 and 3 is 3 times bigger that the distance between flags 1 and 2.

2 $S = (2, -8, 3)$

3 $PT = 6i + 6j + 18k$ or $\begin{pmatrix} 6 \\ 6 \\ 18 \end{pmatrix}$

4 ∠QPR = 36·7° (or 0·64 radians)

Practice test 2

1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} = 2\overrightarrow{AB}$ therefore $BC : AB = 1:2$

$\overrightarrow{BC} = 3\overrightarrow{AB}$ hence the vectors are parallel. B is a common point so the A, B and C are collinear.

The posts have **not** been laid out correctly because although the posts lie in a straight line the distance between post 1 and post 2 is one half of the distance between posts 2 and 3.

2 Q = (5, 16, 9)

3 $-3\mathbf{i} + 12\mathbf{j} - 9\mathbf{k}$ OR $\begin{pmatrix} -3 \\ 12 \\ -9 \end{pmatrix}$

4 ∠STU = 35·6° or 0·62 (radians)

Practice test 3

1 $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 9 \\ -6 \\ 6 \end{pmatrix} = 3\overrightarrow{AB}$ therefore $\overrightarrow{BC} = 3\overrightarrow{AB}$ or $\overrightarrow{AB} = \frac{1}{3}\overrightarrow{BC}$

$\overrightarrow{BC} = 3\overrightarrow{AB}$ hence the vectors are parallel. B is a common point so the A, B and C are collinear.

Yes, the engineer has placed the flags correctly because A, B and C lie on the same straight line and $BC : AB = 3:1$ so the distance between flags 2 and 3 is 3 times bigger that the distance between flags 1 and 2.

2 S (5, -1, 9)

3 $4\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ OR $\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

4 $\angle$QPR = 74·8° or 1·306 (radians)

Unit title: Relationships and Calculus

Assessment Standards	Making assessment judgements
1.1 Applying algebraic skills to solve equations	The sub-skills in the Assessment Standard are: ♦ factorising a cubic polynomial expression with unitary x^3 coefficient ♦ solving cubic polynomial equations with unitary x^3 coefficient ♦ given the nature of the roots of an equation, use the discriminant to find an unknown
1.2 Applying trigonometric skills to solve equations	The sub-skills in the Assessment Standard are: ♦ solve trigonometric equations in degrees and radian measure, involving trigonometric formulae, in a given interval
1.3 Applying calculus skills of differentiation	The sub-skills in the Assessment Standard are: ♦ differentiating an algebraic function which is, or can be simplified to, an expression in powers of x ♦ differentiating $k\sin x,\ k\cos x$ ♦ determining the equation of a tangent to a curve at a given point by differentiation
1.4 Applying calculus skills of integration	The sub-skills in the Assessment Standard are: ♦ integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ integrating functions of the form $f(x)=(x+q)^n,\ n\neq -1$ ♦ integrating functions of the form $f(x)=p\cos x$ and $f(x)=p\sin x$ ♦ calculating definite integrals of polynomial functions with integer limits
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	Assessment Standard 2.1 and 2.1 are transferable across Units and can be attached to any of the sub-skills listed above. #2.1 is mostly about choosing an appropriate strategy. This skill is usually required to make a good start on a problem. #2.2 is about how well you answer a question, often with a summary statement at the end of a question.
2.2 Explaining a solution and, where appropriate, relating it to context	

My assessment record: Mathematics: Expressions and Functions (Higher)

Keep this up to date as you go along so that you know if you have any areas that you need to do more work on

Assessment standard	First attempt		Second attempt (if required)	
	Mark(s)	Pass/Fail	Mark(s)	Pass/Fail
1.1 Applying algebraic skills to solve equations • factorising a cubic polynomial expression with unitary x^3 coefficient • solving cubic polynomial equations with unitary x^3 coefficient • given the nature of the roots of an equation, use the discriminant to find an unknown				
1.2 Applying trigonometric skills to solve equations • solve trigonometric equations in degrees and radian measure, involving trigonometric formulae, in a given interval				
1.3 Applying calculus skills of differentiation • differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x* • differentiating $k\sin x,\ k\cos x$ • determining the equation of a tangent to a curve at a given point by differentiation				
1.4 Applying calculus skills of integration • integrating an algebraic function which is, or can be, simplified to an expression of powers of *x* • integrating functions of the form $f(x)=(x+q)^n,\ n\neq -1$ • integrating functions of the form $f(x)=p\cos x,\ f(x)=p\sin x$ • calculating definite integrals of polynomial functions with integer limits				
2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy				
2.2 Explaining a solution and, where appropriate, relating it to context				

Unit title: <u>Relationships and Calculus</u>

Assessment standard: 1.1 Applying algebraic skills to solve equations

Sub-skills	♦ factorising a cubic polynomial expression with unitary x^3 coefficient ♦ solving cubic polynomial equations with unitary x^3 coefficient ♦ given the nature of the roots of an equation, use the discriminant to find an unknown

Practice test 1

1 A function f is defined by $f(x) = x^3 + 2x^2 - 5x - 6$ where x is a real number.

(a) (i) Show that $(x+1)$ is a factor of $f(x)$.

(ii) Hence factorise $f(x)$ fully.

[4]

(b) Solve $f(x) = 0$.

[1]

2 Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $(x-3)$, the remainder is zero
- when the graph of $y = f(x)$ passes through the point $(-2, 0)$
- $(x-4)$ is a factor of $f(x)$

[#2·2]

3 The graph of the function $f(x) = kx^2 - 8x + 4$ does not touch or cross the x-axis.
What is the range of values for k ?

[#2·1, 1]

Practice test 2

1 Solve the cubic equation $f(x) = 0$ given the following:

- when $f(x)$ is divided by $x+2$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point $(-4, 0)$
- $(x-1)$ is a factor of $f(x)$.

[#2·2]

2 Solve the equation $x^3 - 4x^2 + x + 6 = 0$

[#2.1+ 5]

Unit title: **Relationships and Calculus**

Assessment standard: 1.1 Applying algebraic skills to solve equations

Answers to Practice Tests

Practice test 1

1 (a) (i) f(-1) = 0 so (x +1) is a factor
Either synthetic division or substitution may be used to show that
(ii) $(x-2)(x+3)(x+1)$
(b) $x=-3, x=-1, x=2$

2 $x=3, x=4, x=-2$

3 k>4

Practice test 2

1 x = -4, x = -2, x = 1

2 x = -1, x = 2, x = 3

Unit title: **Relationships and Calculus**

Assessment standard: 1.2 Applying trigonometric skills to solve equations

Sub-skills	♦ solve trigonometric equations in degrees and radian measure, involving trigonometric formulae, in a given interval

Practice test 1

1 Solve $\sqrt{2}\cos 2x^\circ = 1$, for 0° ≤ x° ≤ 180°.

[3]

2 Solve $4\sin 2t^\circ - \cos t^\circ = 0$, for 0° ≤ t° ≤ 180°

[4]

3 Given that $5\sin x^\circ + 3\cos x^\circ = \sqrt{34}\cos(x-49)^\circ$,
solve $5\sin x^\circ + 3\cos x^\circ = 3\cdot 6$, for 0 < x < 180.

[#2.1+3]

Practice test 2

1 Solve $2\cos 2x = \sqrt{3}$, for $0^\circ \le x \le 180^\circ$.

[3]

2 Solve $2\sin 2w - \cos w = 0$ for $0^\circ \le t \le 180^\circ$.

[4]

3 Given $3\sin x + 5\cos x = \sqrt{34}\cos(x - 31.0)^\circ$,
solve $3\sin 2x + 5\cos 2x = 3.5$, for $0^\circ < x < 90^\circ$.

[#2.1+3]

Unit title: **Relationships and Calculus**

Assessment standard: 1.2 Applying trigonometric skills to solve equations

Answers to Practice Tests

Practice test 1

1 $x = 22{\cdot}5^0$ and 157.5^0

2 $t = 7{\cdot}2^0, 90^0, 172{\cdot}8^0$

3 $x = 100{\cdot}9^0$out of range. No solutions within range.

Practice test 2

1 $x = 15^0$ and 165^0

2 $w = 14{\cdot}5^0, 90^0, 165{\cdot}5^0$

3 $x = 42.06^0$, ~~168.95~~^{0} (solution out of range)

Unit title: **Relationships and Calculus**

Assessment standard: 1.3 Applying calculus skills of differentiation

Sub-skills	♦ differentiating an algebraic function which is, or can be simplified to, an expression in powers of x ♦ differentiating $k\sin x,\ k\cos x$ ♦ determining the equation of a tangent to a curve at a given point by differentiation

Practice test 1

1 Find $f'(x)$, given that $f(x) = 5\sqrt{x} - \frac{7}{x^3}$, $x > 0$.

[3]

2 A bowler throws a cricket ball vertically upwards. The height (in metres) of the ball above the ground, t seconds after it is thrown, can be represented by the formula $h(t) = 16t - 4t^2$.

The velocity, v ms^{-1}, of the ball at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the cricket ball three seconds after it is thrown.
Explain what this means in the context of the question.

[#2.2 + 2]

3 Differentiate the function $f(x) = 6\sin x$ with respect to x.

[1]

4 A curve has equation $y = 7x^2 + 5x - 3$.
Find the equation of the tangent to the curve at the point where $x = 1$.

[4]

Practice test 2

1 Find $f'(x)$, given that $f(x) = x\sqrt[3]{x} + \frac{6}{\sqrt[4]{x^3}}$, $x > 0$.

[3]

2 Differentiate $-3cosx$ With respect to x.

[1]

3 A particle moves in a horizontal line. The distance x (in metres) of the particle after t seconds can be represented by the formula $x = 4t^2 - 24t$.

The velocity of the particle at time t is given by $v = \frac{dx}{dt}$.

(a) Find the velocity of the particle after three seconds.

[2]

(b) Explain your answer in terms of the particle's movement.

[#2.2]

4 A curve has equation $y = 3x^2 + 2x - 5$.
Find the equation of the tangent to the curve at the point where $x = -2$.

[4]

Unit title: **Relationships and Calculus**

Assessment standard: 1.3 Applying calculus skills of differentiation

Answers to Practice Tests

Practice test 1

1 $f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 21x^{-4} = \frac{5}{2\sqrt{x}} + \frac{21}{x^4}$

2 $v = \frac{dh}{dt} = 16 - 8t$ when t = 3, v = -8m/s

e.g. The cricket ball is has turned and is now falling downwards with an instantaneous speed of 8m/s.

3 $\frac{d}{dx}(6\sin x) = 6\cos x$

4 $y = 19x - 10$

Practice test 2

1 $f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{9}{2}x^{-\frac{7}{4}} = \frac{4\sqrt[3]{x}}{3} + \frac{9}{2\sqrt[4]{x^7}}$

2 $\frac{d}{dx}(-3\cos x) = 3\sin x$

3 $v = \frac{dx}{dt} = 8t - 24$ when t = 3, v = 0m/s

e.g. The particle is instantaneously at rest.

4 $y = -10x - 17$

Unit title: **Relationships and Calculus**

Assessment standard: 1.4 Applying calculus skills of integration

Sub-skills	♦ integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ integrating functions of the form $f(x)=(x+q)^n$, $n\neq -1$ ♦ integrating functions of the form $f(x)=p\cos x$ and $f(x)=p\sin x$ ♦ calculating definite integrals of polynomial functions with integer limits

Practice test 1

1 Find $\int\left(4x^{\frac{1}{3}}+\frac{1}{x^3}\right)dx,\ x>0.$

[4]

2 $h'(x)=(x+5)^{-4}$ find $h(x)$, $x\neq -5$.

[2]

3 Find $\int 4\cos\theta\, d\theta.$

[1]

4 Find $\int_{-3}^{2}\left(x^2-8x+16\right)dx$.

[3]

Practice test 2

1 Find $\int(2x^{\frac{1}{3}}+\frac{1}{x^4})\ dx,\ x\neq 0.$

[4]

2 $h'(x)=(x-3)^{-2}$, find $h(x)$, $x\neq 3.$

[2]

3 Find $\int 2\sin\theta\, d\theta..$

[1]

4 Find $\int_{-3}^{1}(x+2)^3\,dx.$

[3]

Unit title: <u>Relationships and Calculus</u>

Assessment standard: 1.4 Applying calculus skills of integration

Answers to Practice Tests

Practice test 1

1 $3x^{\frac{4}{3}} - \frac{1}{2x^2} + c$

2 $h(x) = -\frac{1}{3}(x+5)^{-3} + C$

3 $4\sin\theta + C$

4 $\frac{335}{3} or 111\frac{2}{3}$

Practice test 2

1 $\frac{3}{2}x^{\frac{4}{3}} - \frac{1}{3}x^{-3} + c$

2 $h(x) = -(x-3)^{-1} + c$

3 $-2cos\theta + c$

4 20

BANCHORY ACADEMY REVISION GUIDES

Block 1 Test

You will find more basic questions as well as extended questions to practise from your Higher Maths textbook Chapters 1 and 5. The "Mixed Question" exercises at the end of each chapter are recommended revision for **extension tests**.

Block 2 Test

You will find more basic questions as well as extended questions to practise from your Higher Maths textbook Chapters 2, 3, 4 and 6. Note that from Chapter 6 we will only have covered some of the exercises. The "Mixed Question" exercises at the end of each chapter are recommended revision for **extension tests**.

Block 3 Test

You will find more basic questions as well as extended questions to practise from your Higher Maths textbook Chapters 11, 16, 7, 8, 9 and 12. Note that from Chapter 9 we will only have covered some of the exercises. The "Mixed Question" exercises at the end of each chapter are recommended revision for **extension tests**.

Block 4 Test

You will find more basic questions as well as extended questions to practise from your Higher Maths textbook Chapters 15, 6, 9, 14 and 13. Note that from Chapter 6 and 9 some of the exercises were covered earlier in the Blocks 2 and 3. The "Mixed Question" exercises at the end of each chapter are recommended revision for **extension tests**.