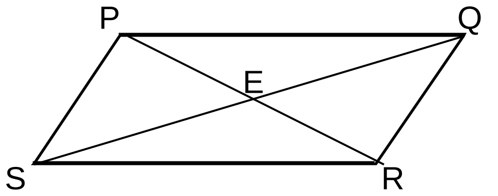
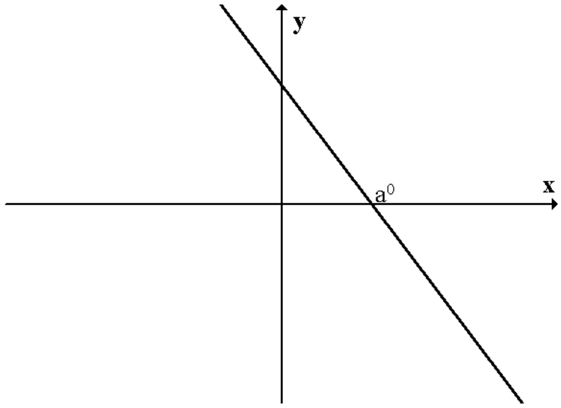
Higher Mathematics
Unit 1

1. PQRS is a parallelogram whose diagonals meet at E.
   P is the point (-2,2), Q is (0,8) and E is (2,4).
   Find the equation of the line RS.



2. The diagram shows part of the line
   $\sqrt{3}y = -3x + 6$.

   Angle a^0 is equal to

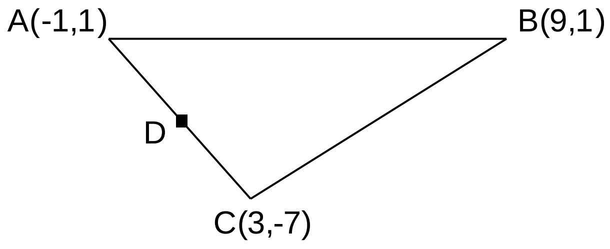


3. A line AB has equation 3x – 2y – 5 = 0. Find the equation of the line perpendicular to AB which passes through the point (-4,2).

4. A triangle has vertices P(-3,1), Q(1,13) and R(7,-4). Find the equation of the altitude drawn from R.

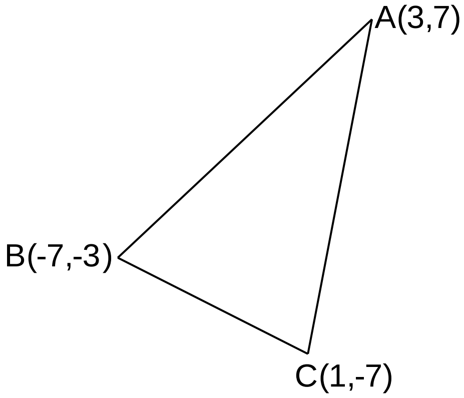
5. A triangle ABC has vertices A(-1,1), B(9,1) and C(3,-7).

   (a) Find the equation of the median BD.
   (b) Write down the equation of the perpendicular bisector of AB.
   (c) Find the coordinates of the point of intersection of these two points.



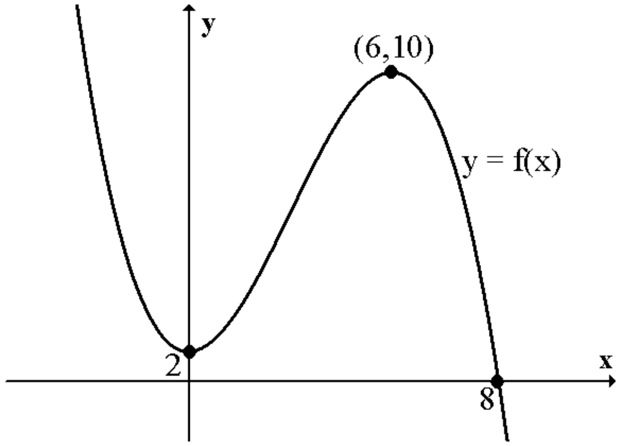
6. Triangle ABC has vertices A(3,7), B(-7,-3) and C(1,-7). Find

   (a) the equation of the median from C.
   (b) the equation of the perpendicular bisector of BC.
   (c) the coordinates of the point of intersection of these lines.



7. The diagram shows the graph of $y = f(x)$.

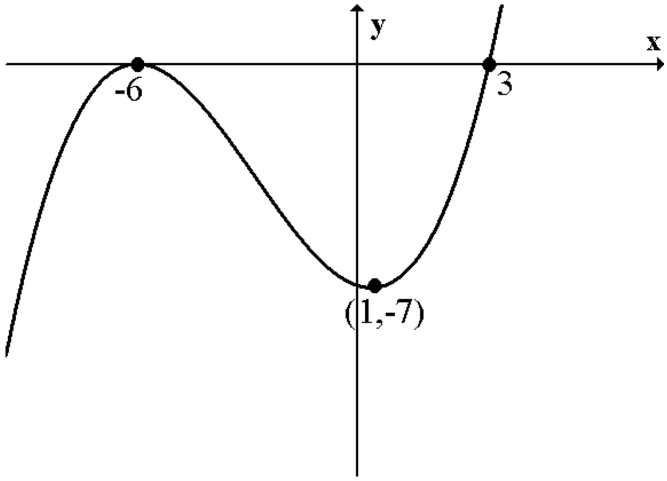
Sketch the graph of $y = 5 - f(x)$.



8. Part of the graph of $y = g(x)$ is shown.

On separate diagrams sketch the graphs of

(i) $y = -3g(x)$
(ii) $y = g(x - 6)$
(iii) $y = g'(x)$



9. The functions f and g are defined on suitable domains with

$$f(x) = \frac{1}{x^2 - 1} \qquad \text{and} \qquad g(x) = x + 1$$

(a) $h(x) = g(f(x))$. Find an expression for $h(x)$. Give your answer as a single fraction.
(b) State a suitable domain for $h(x)$.

10. $f(x) = 2x - 6$ $\quad g(x) = 4 - 3x$ $\quad h(x) = \frac{1}{6}(2 - x)$

(a) $k(x) = f(g(x))$. Find $k(x)$.
(b) Find a formula for $h(k(x))$.
(c) What is the connection between h and k?

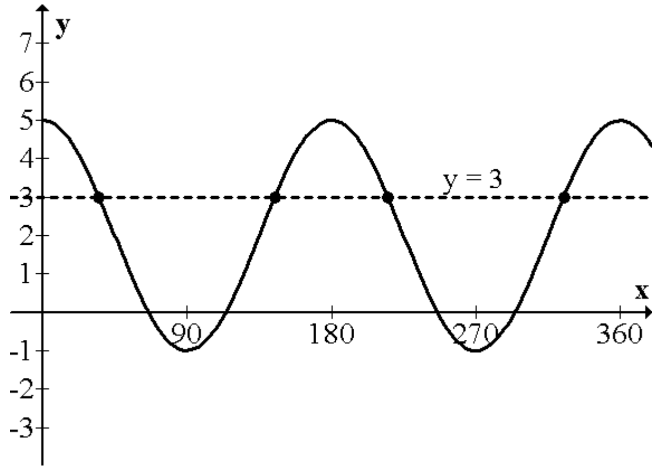
11. Solve the following equations.

(a) $2\sin 2x - 1 = 0$ $\quad 0 \le x \le 360$ $\qquad$ (b) $2\cos 2x + \sqrt{3} = 2\sqrt{3}$ $\quad 0 \le x \le 2\pi$

(c) $4\cos^2 x - 1 = 0$ $\quad 0 \le x \le 2\pi$ $\qquad$ (d) $5\cos^2 x - 2\cos x - 3 = 0$ $\quad 0 \le x \le 360$

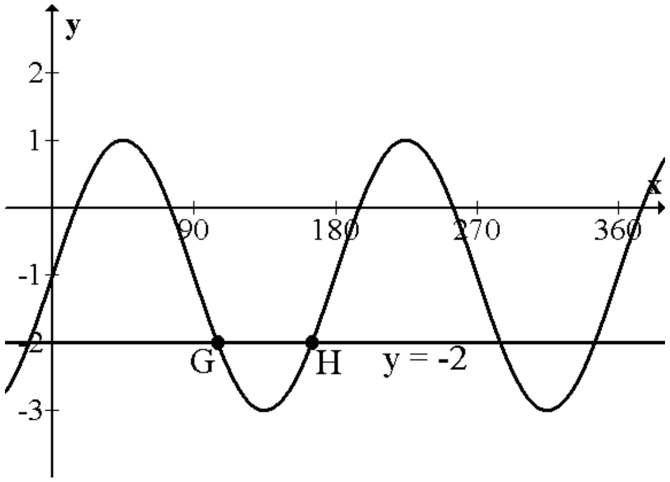
12. (a) The diagram shows the graph of
$y = a\cos bx + c$.
Write down the values of a, b and c.

(b) Find the points of intersection of the line $y = 3$ and this curve.



13. (a) The diagram shows the graph of
$y = a\sin bx + c$.
Write down the values of a, b and c.

(b) The line $y = -2$ is also drawn on the graph.
Find the coordinates of G and H.



14. $f(x) = \dfrac{x^3 - 3x}{\sqrt{x}}$ find $f'(1)$

15. (a) Show that the function $f(x) = x^3 + 3x^2 + 3x - 15$ is never decreasing.
(b) Find the coordinates of the stationary point of f(x).

16. The distance a rocket travels is calculated using the formula $d(t) = 2t^3$, where t is the time in seconds after lift-off.

(a) How far has the rocket travelled after 5 seconds?
(b) Calculate the speed of the rocket after 10 seconds.

17. Find the intervals in which $y = x^3 - 6x^2 + 1$ is increasing.

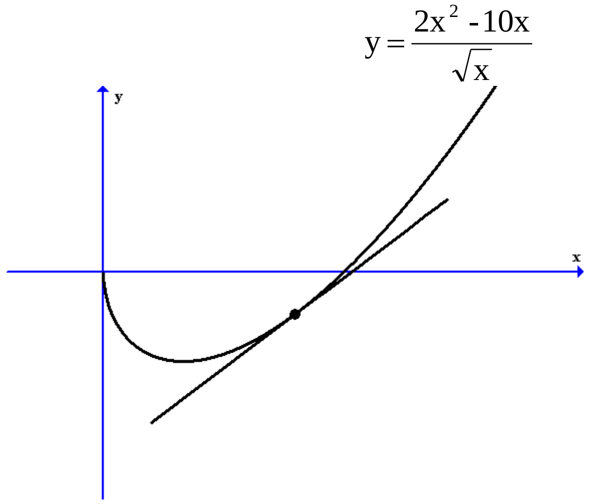
18. A curve has equation $f(x) = 8x^3 - 3x^2$
(a) Find the stationary points of f(x) and determine their nature.
(b) Find the maximum and minimum values of f(x) in the interval $-2 \le x \le 1$.

19. Find the equation of the tangent to the curve $f(x) = 3\sqrt[3]{x^2}$ at the point where $x = 8$.

20. Find the equation of the tangent to the curve

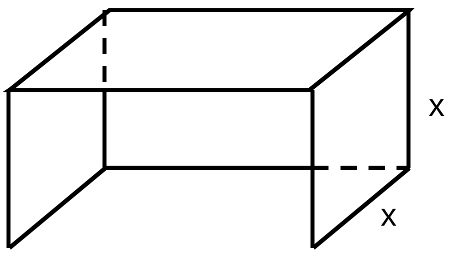
$$y = \frac{2x^2 - 10x}{\sqrt{x}}$$

at the point where x = 4.



21. Find the equation of the tangent to the curve $y = \frac{1}{4}x^4 - 7x + 10$ which makes an angle of 45^0 with the positive direction of the x-axis.

22. A wind shelter, as shown opposite, has a back, top and two square sides. The total amount of canvas used in the shelter is 96 m^2 and the length of each square side is x metres.



(a) If the volume of the shelter is V cm^3, show that $V = x(48 - x^2)$.

(b) Find the exact value of x for which the shelter has a maximum volume.

23. A recurrence relation is defined as $u_{n+1} = 0.6u_n + 18$, $u_1 = 30$

(a) Find the value of u_o and u_2.
(b) State why this relation has a limit and calculate this limit.

24. The recurrence relations

$$u_{n+1} = 0.8u_n + 12 \text{ and } v_{n+1} = av_n + 18$$

have the same limit. Find the value of a.

25. A recurrence relation is defined as $u_{n+1} = pu_n + 12$.

(a) The limit of this relation is 18. Find the value of p.
(b) Given $u_o = 72$, find u_3.

26. A recurrence relation is defined as $u_n = 0.85u_{n-1} + 30$, $u_o = 40$.

(a) Find the smallest value of n such that $u_n > 110$.
(b) Find the limit of this recurrence relation, stating why a limit exists.

27. A recurrence relation is defined as $u_{n+1} = au_n + b$.

(a) Given $u_1 = 32$, $u_2 = 20$ and $u_3 = 17$, find the values of a and b.

(b) The limit of the recurrence relation in part (a) is the same as the limit of $v_{n+1} = pv_n + 10$. Find the value of p.

28. A patient is injected with 50 ml of an antibiotic drug. Every 6 hours 60% of the drug passes out of her bloodstream. To compensate for this an extra 15ml of antibiotic is given every 6 hours.

(a) Find a recurrence relation for the amount of drug in the patient's bloodstream.
(b) Calculate the amount of antibiotic remaining in the bloodstream after one day.