H

Mathematics
Higher
Paper 1
Practice Paper B

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4 If any of this information is wrong, tell the invigilator immediately.

5 If this information is correct, print your name and seat number in the boxes provided.

6 The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on the answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

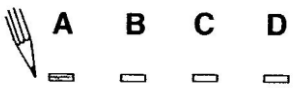
A 8

B 1

C 0

D –4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. Given that $f(x) = 2x^4 - 5x$, find $f'(2)$.

   A 6

   B 22

   C 42

   D 59

2. Find $\int (x-3)(3x+1)\,dx$.

   A $6x + c$

   B $x^3 - 4x^2 - 3x + c$

   C $6x^3 - 3x + c$

   D $3x^4 - 8x^2 - 3x + c$

3. P and Q have coordinates $(2, -3, 2)$ and $(1, 0, 5)$.

   What is the distance between P and Q?

   A $\sqrt{12}$ units

   B $\sqrt{19}$ units

   C $\sqrt{37}$ units

   D $\sqrt{67}$ units

4. If $x^2 - 8x + 3$ is expressed in the form $(x-p)^2 + q$, what is the value of q?

   A –13

   B –1

   C 3

   D 19

5. Here are two statements about the equation $3x^2 - 5x + 1 = 0$.

(1) The roots are equal.

(2) The roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

6. Find all the values of x in the interval $0 \le x < 2\pi$ for which $\cos x = -\frac{\sqrt{3}}{2}$.

A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B $\frac{5\pi}{6}$ and $\frac{5\pi}{3}$

C $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

D $\frac{\pi}{3}$ and $\frac{7\pi}{6}$

7. S is the point with coordinates $(2, -1, 1)$, $T(4, 1, 5)$ and $U(5, 2, 7)$.

Find the ratio in which T divides SU.

A $1:2$

B $1:3$

C $2:1$

D $3:1$

8. Given that $y = 4\sin(3x - 2)$, find $\frac{dy}{dx}$.

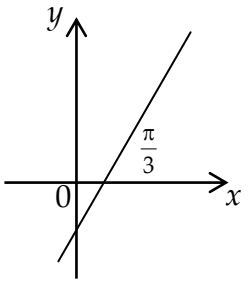
A $-\sin(3x - 2)$

B $-\cos(3x - 2)$

C $12\sin(3x - 2)$

D $12\cos(3x - 2)$

9. The angle between the line shown in the diagram and the x-axis is $\frac{\pi}{3}$.



What is the gradient of the line?

A $\sqrt{3}$

B $\frac{\sqrt{3}}{2}$

C $\frac{1}{\sqrt{3}}$

D $\frac{1}{2}$

10. Given that $\log_a 9 = \frac{2}{3}$, what is the value of a?

A 6

B $13 \cdot 5$

C 27

D 81

11. What is the maximum value of $9 - 4\sin\left(x - \frac{\pi}{5}\right)$?

A 1

B 5

C 9

D 13

12. Find $\int (3x-11)^4 \, dx$.

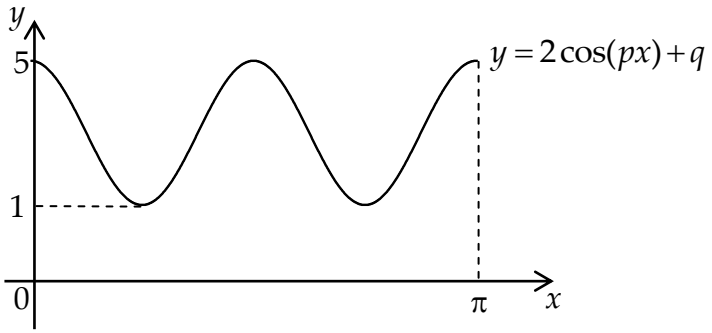
A $\frac{4}{3}(3x-11)^3 + c$

B $4(3x-11)^3 + c$

C $\frac{1}{15}(3x-11)^5 + c$

D $4(3x-11)^5 + c$

13. The graph shown in the diagram has equation of the form $y = 2\cos(px) + q$.



What are the values of p and q?

	p	q
A	2	3
B	4	3
C	2	5
D	4	5

14. Given that $h(x) = \dfrac{1}{x^2 - 16}$, what is the largest possible domain for h?

A $\mathbb{R}$, the set of real numbers.

B $\mathbb{R} - \{0\}$

C $\mathbb{R} - \{-4, 4\}$

D $\mathbb{R} - \{-16, 16\}$

15. Vector $\boldsymbol{t}$ has components $\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$. $\boldsymbol{u}$ is a unit vector such that $\boldsymbol{u} = k\boldsymbol{t}$, where $k > 0$.

Find the value of k.

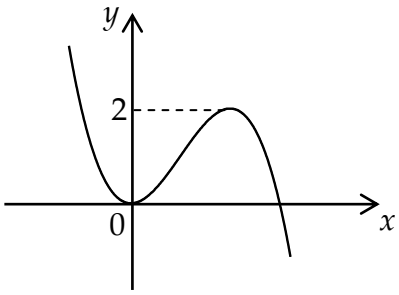
A $\frac{1}{25}$

B $\frac{1}{5}$

C 1

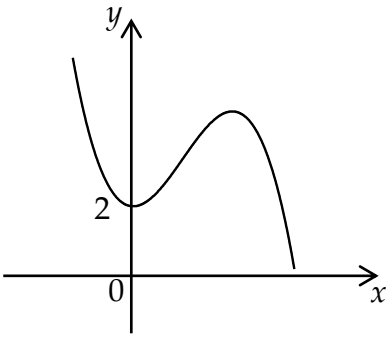
D 5

16. The diagram shows the graph of $y = f(x)$.

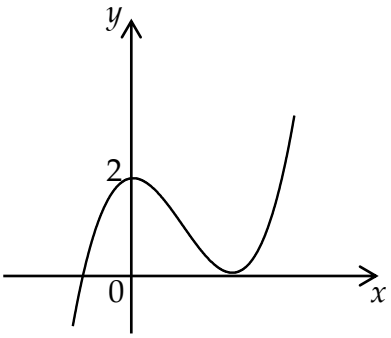


Which diagram below shows the graph of $y = 2 - f(x)$?

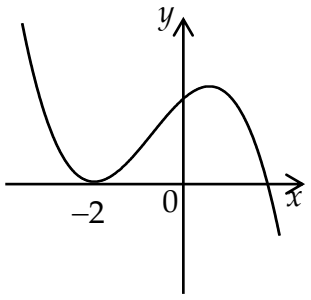
A



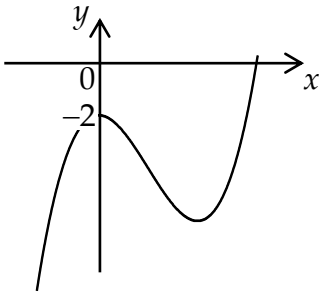
B



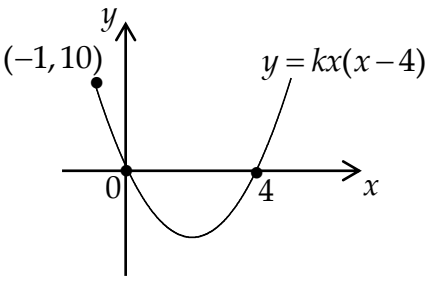
C



D



17. The equation of the parabola shown is of the form $y = kx(x-4)$.



What is the value of k?

A -4

B -1

C 1

D 2

18. Simplify $2\log_3 x - \log_3(x+1)$.

A $\log_3\left(\dfrac{x^2}{x+1}\right)$

B $\log_3\left(\dfrac{x+1}{2x}\right)$

C $\log_3(x-1)$

D $\log_3(2x^2+2x)$

19. What is the solution to $x^2+4x-5<0$?

A $-5<x<1$

B $x<-1$ or $x>5$

C $x<-4$ or $x>-1$

D $1<x<4$

20. If $v=10t^2$ and the rate of change of v with respect to t at $t=k$, $k>0$ is 160, find the value of k.

A 0

B 4

C 8

D 16

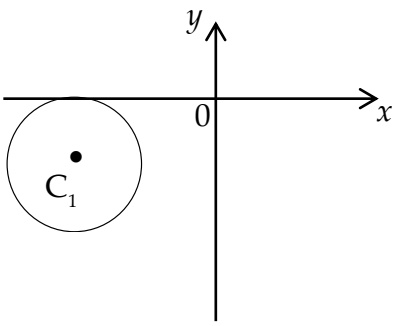
End of Section A

SECTION B

ALL questions should be attempted.

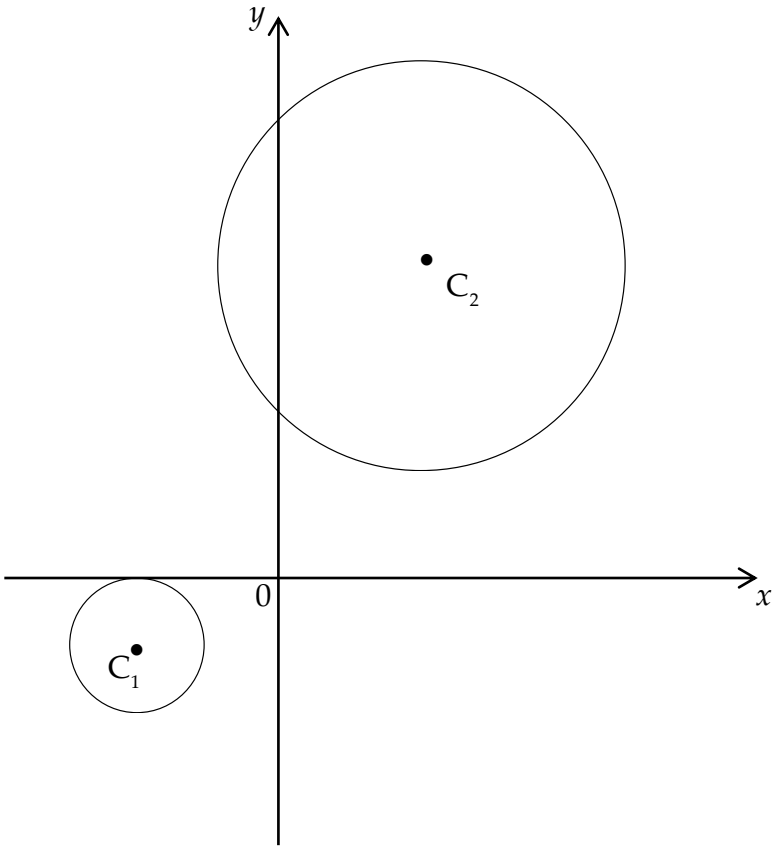
Marks

21. A circle with equation $x^2 + y^2 + 6x + 2y + 9 = 0$ has centre C_1.



(*a*) Write down the coordinates of the centre C_1 and find the length of the radius of this circle. **2**

A second circle with equation $(x-3)^2 + (y-7)^2 = 36$ has centre C_2.



(*b*) (i) Find the distance between the centres C_1 and C_2.

(ii) Hence find the minimum distance between the circumferences of the two circles. **4**

Marks

22. A is the point with coordinates $(1, -1, 2)$, B$(3, 0, 3)$ and C$(-2, 3, 4)$.

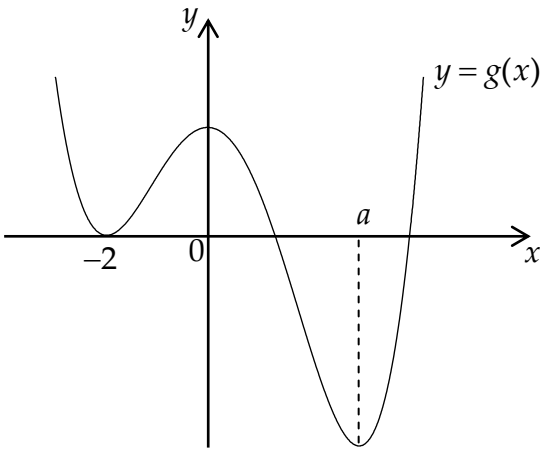
(*a*) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2**

(*b*) Find the size of angle BAC. **3**

23. Solve $2\sin 2x = 5\cos x$ for $0 \le x < 2\pi$. **5**

24. The diagram shows part of the quartic with equation $y = g(x)$.

There are stationary points at $x = -2$, $x = 0$ and $x = a$.



On separate diagrams sketch the graph of

(a) $y = g'(x)$. **3**

(b) $y = g'(x-3)$. **2**

25. Find the values of x for which the function $f(x) = 5 + 24x + 3x^2 - x^3$ is decreasing. **5**

26. P is the point with coordinates $(-1, -6)$ and Q is $(3, 10)$.

Find the locus of points which are equidistant from both P and Q. **4**

End of question paper

H

Mathematics
Higher
Paper 2
Practice Paper B

Time allowed
1 hour 10 minutes

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ALL questions should be attempted.

Marks

1. (*a*) A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 4u_n + 6,\ u_0 = 0.$

Determine the values of u_1, u_2 and u_3. **2**

(*b*) Why does this sequence have a limit as $n \to \infty$? **1**

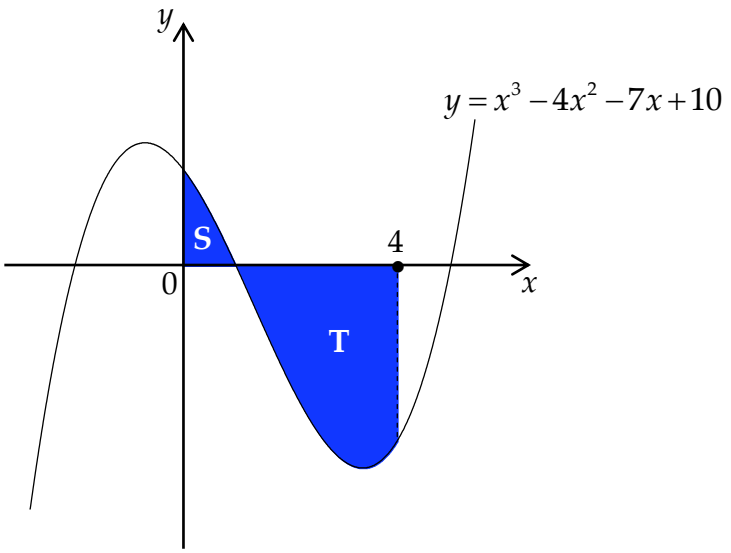
(*c*) A second sequence, generated by $v_{n+1} = pv_n + 4$, has the same limit as the sequence in (*a*).

Find the value of p. **3**

2. A function f is defined on the set of real numbers by $f(x) = x^3 - 4x^2 - 7x + 10.$

(*a*) Show that $(x-1)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **4**

The graph shown has equation of the form $y = x^3 - 4x^2 - 7x + 10.$



(*b*) Calculate the shaded area labelled S. **4**

(*c*) Find the total shaded area. **3**

3. D has coordinates $(7, -2, 1)$ and F is $(-1, 2, 5)$.

(*a*) Find the coordinates of E which divides DF in the ratio $1:3$. **3**

G has coordinates $(6, -2, 5)$.

(*b*) Show that EG is perpendicular to DF. **3**

4. P, Q and R have coordinates $(-4, 6)$, $(8, 10)$ and $(2, 28)$ respectively.

(*a*) Show that PQ is perpendicular to QR. **2**

(*b*) Hence find the equation of the circle which passes through P, Q and R. **3**

5. Two functions *f* and g are defined on the set of real numbers by

$$f(x) = 2x + k \text{ and } g(x) = x^2 - 2k, \text{ where } k \neq 0$$

(*a*) Find (i) $f(g(x))$;

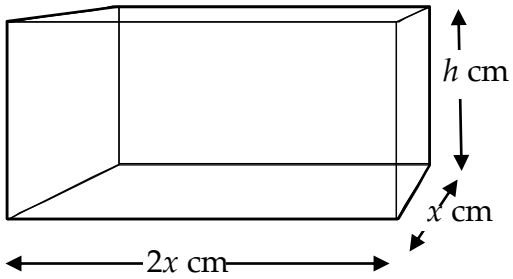
(ii) $g(f(x))$. **3**

(*b*) Find the value of *k* for which $f(g(x)) = g(f(x))$ has equal roots. **5**

6. A closed wooden box, in the shape of a cuboid, is constructed from a sheet of wood of area $600\ \text{cm}^2$.

The base of the box measures $2x$ cm by x cm.

The height of the box is h cm.



(*a*) Assuming the thickness of the sides of the box are negligible, show that the volume (in cubic centimetres) of the box is given by

$$V(x) = 200x - \tfrac{4}{3}x^3$$ **3**

(*b*) (i) Calculate the value of *x* for which this volume is a maximum.

(ii) Find the maximum volume of the box. **7**

Marks

7. Whilst carrying out an experiment a scientist gathered some data.

The table shows the data collected by the scientist.

x	4·3	4·7	5·2	6·1
y	0·027	0·018	0·011	0·004

The variables x and y, in the table, are connected by a relationship of the form $y = ae^{bx}$.

Find the values of a and b. **6**

8. Solve $2 - 3\cos x - 4\sin x = 0$ for $0 \leq x \leq 2\pi$. **8**

End of Question Paper

Paper 1

Section A

1.	D	11.	D
2.	B	12.	C
3.	B	13.	B
4.	A	14.	C
5.	A	15.	B
6.	C	16.	B
7.	C	17.	D
8.	D	18.	A
9.	A	19.	A
10.	C	20.	C

Section B

21. (*a*) Centre $(-3, -1)$ and radius 1 unit

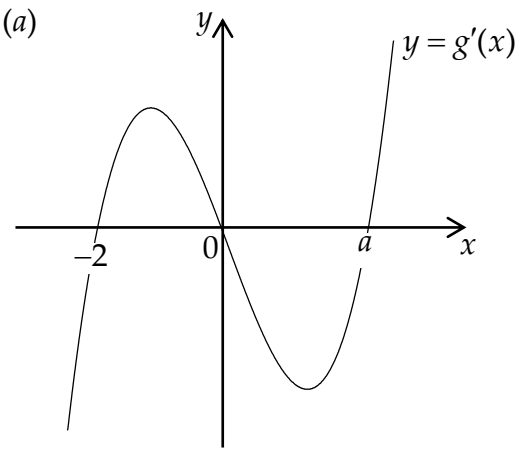
(*b*) (i) 10 units (ii) 3 units

22. (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$

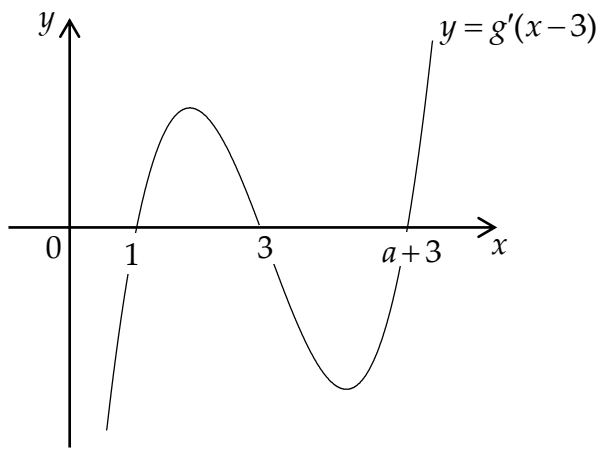
(*b*) Angle BAC 90° or $\frac{\pi}{2}$

23. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

24. (*a*)



(*b*)



25. $\{x : x < -2 \cup x > 4, x \in \mathbb{R}\}$

26. $x + 4y - 9 = 0$

Paper 2

1. (*a*) $u_1 = 6,\ u_2 = 8 \cdot 4$ and $u_3 = 9 \cdot 36$

   (*b*) $-1 < 0 \cdot 4 < 1$

   (*c*) $p = \frac{3}{5}$ (or $0 \cdot 6$)

2. (*a*) Show that $f(1) = 0;\quad f(x) = (x-1)(x-5)(x+2)$

   (*b*) $\frac{65}{12}$ (or $5 \cdot 41\dot{6}$)

   (*c*) $\frac{289}{6}$ (or $48 \cdot 1\dot{6}$)

3. (*a*) E(5, −1, 2)

   (*b*) Show that $\overrightarrow{\text{EG}} . \overrightarrow{\text{DF}} = 0$

4. (*a*) Show that $m_{\text{PQ}} \times m_{\text{QR}} = -1$

   (*b*) $(x+1)^2 + (y-17)^2 = 130$

5. (*a*) (i) $f(g(x)) = 2x^2 - 3k$ (ii) $g(f(x)) = 4x^2 + 4kx + k^2 - 2k$

   (*b*) $k = 1$

6. (*a*) Proof

   (*b*) (i) $x = 5\sqrt{2} \approx 7 \cdot 071\, cm$ (ii) Maximum volume : $\frac{2000\sqrt{2}}{3} \approx 942 \cdot 8\, cm^3$

7. $a = e^{0 \cdot 7} \approx 2 \cdot 0,\ b = -1$

8. $\{2 \cdot 086,\ 6 \cdot 051\}$