H

Mathematics
Higher
Paper 1
Practice Paper C

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4 If any of this information is wrong, tell the invigilator immediately.

5 If this information is correct, print your name and seat number in the boxes provided.

6 The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on the answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

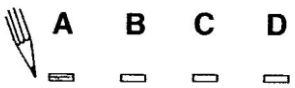
A 8

B 1

C 0

D –4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 2u_n - 5, \quad u_0 = 6$$

What is the value of u_2 ?

A 9

B 6

C –1

D –5

2. Here are two statements about the line with equation $3x + 4y - 8 = 0$.

(1) This line is parallel to a line with gradient $-\frac{3}{4}$.

(2) This line cuts the y-axis at the point $(0, 8)$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

3. Functions f and g are defined on suitable domains by

$$f(x) = 3x + 5 \text{ and } g(x) = 2 - x.$$

Find an expression for $f(g(x))$.

A $f(g(x)) = 11 - 3x$

B $f(g(x)) = 2x + 7$

C $f(g(x)) = 5 + 6x - 3x^2$

D $f(g(x)) = 10 + x - 3x^2$

4. A curve has equation $y = x^3 - 2x + 5$.

What is the gradient of the tangent at the point where $x = 2$?

A 1

B 2

C 9

D 10

5. A circle with centre $(-2, 1)$ passes through the point $(5, -2)$.

What is the equation of the circle?

A $(x+2)^2 + (y-1)^2 = 58$

B $(x+2)^2 + (y-1)^2 = 10$

C $(x-2)^2 + (y+1)^2 = 58$

D $(x-2)^2 + (y+1)^2 = 10$

6. Find $\int \frac{2}{\sqrt[3]{x}}\, dx$.

A $\frac{3}{2}x^{-\frac{4}{3}} + c$

B $3x^{\frac{2}{3}} + c$

C $\frac{2}{3}x^{\frac{4}{3}} + c$

D $\frac{2}{3}x^{\frac{5}{3}} + c$

7. $g(x) = x^3 - 2x^2 + x + 7.$

What is the remainder when $g(x)$ is divided by $(x+1)$?

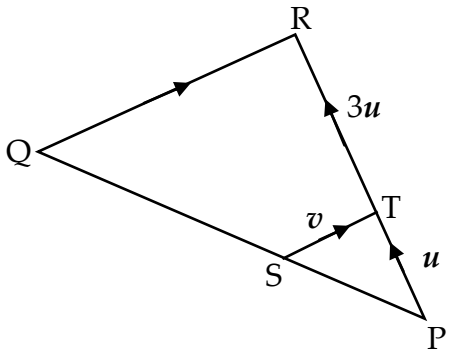
A -1

B 0

C 3

D 7

8. Vectors $\boldsymbol{u}$ and v are shown in the diagram below.



$|\mathrm{QR}| = 3|\mathrm{ST}|$

Find $\overrightarrow{\mathrm{PQ}}$ in terms of **u** and **v**.

A $3\boldsymbol{u} + \boldsymbol{v}$

B $3\boldsymbol{u} - 4\boldsymbol{v}$

C $4\boldsymbol{u} + 4\boldsymbol{v}$

D $4\boldsymbol{u} - 3\boldsymbol{v}$

9. P and Q are the points with coordinates $(-1, 0, 5)$ and $(2, 3, 3)$ respectively.

If $\overrightarrow{\mathrm{PR}} = 2\overrightarrow{\mathrm{PQ}}$, find the coordinates of R.

A $(1, 6, 21)$

B $(2, 6, 16)$

C $(4, 3, -7)$

D $(5, 6, 1)$

10. What is the exact value of $\sin\frac{5\pi}{4} + \cos\frac{\pi}{4}$?

A -2

B 0

C 1

D 2

11. Find $\int 5\cos(2x-1)\,dx$.

A $\frac{5}{2}\sin(2x-1)+c$

B $-10\sin(2x-1)+c$

C $\frac{5}{2}\cos(2x-1)+c$

D $-10\cos(2x-1)+c$

12. Given that $\log_2 y = 3\log_2 x + \log_2 8$, express y in terms of x.

A $y = 3x+3$

B $y = 3x+8$

C $y = 8x^3$

D $y = x^3+8$

13. Given that $y = \sin^4 x$, find $\frac{dy}{dx}$.

A $\sin^3 x$

B $4\cos^3 x$

C $4\sin^3 x\cos x$

D $4\cos^3 x\sin x$

14. If $5-6x-x^2$ is written in the form $p-(x+q)^2$, what is the value of p?

A -4

B -1

C 5

D 14

15. Solve $\tan^2 x = \frac{1}{3}$ for $\frac{\pi}{2} < x < \pi$.

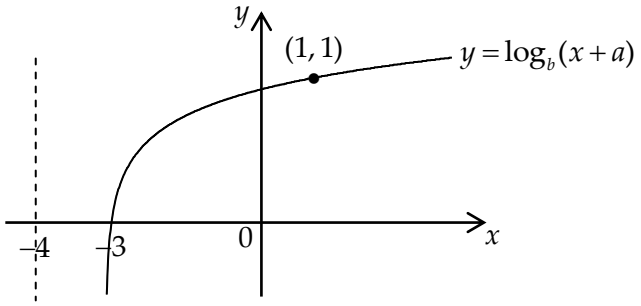
A $\frac{3\pi}{4}$

B $\frac{4\pi}{5}$

C $\frac{5\pi}{6}$

D $\frac{6\pi}{7}$

16. The diagram shows the graph with equation $y = \log_b(x + a)$.



What are the values of a and b?

	a	b
A	3	4
B	4	5
C	3	5
D	4	4

17. What is the nature of the roots of the quadratic equation $x^2 + 10x = 25$?

A Two real equal roots

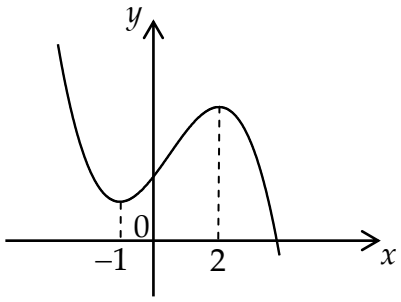
B Two real distinct roots

C No real roots

D Three real distinct roots

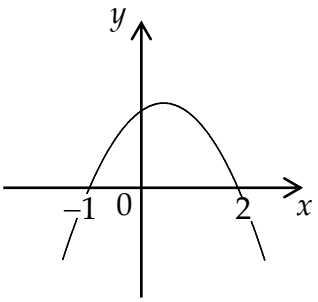
18. The diagram shows part of the graph of cubic with equation $y = g(x)$.

The graph has turning points at $x = -1$ and $x = 2$.

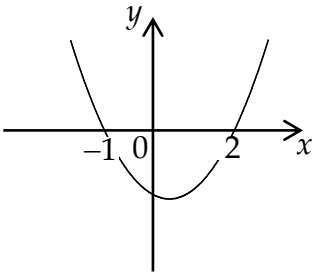


Which diagram below shows the graph of $y = g'(x)$?

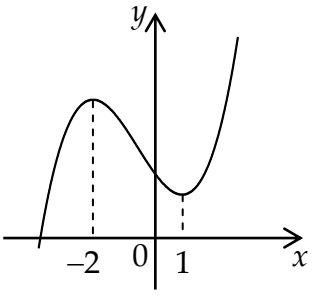
A



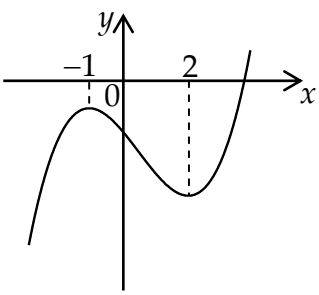
B



C



D



19. Solve $x^2 - 8x + 15 \geq 0$.

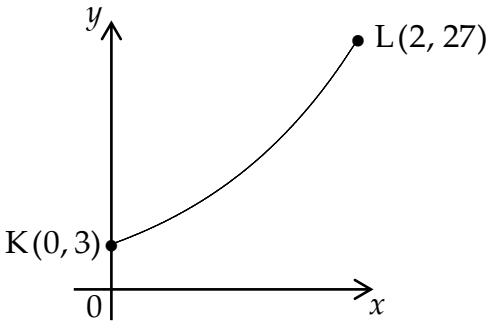
A $-5 \leq x \leq -3$

B $x \leq -5$ or $x \geq -3$

C $3 \leq x \leq 5$

D $x \leq 3$ or $x \geq 5$

20. The diagram shows part of the curve $y = f(x)$.



The curve passes through the points $K(0, 3)$ and $L(2, 27)$.

Which of the following represents the equation of the curve?

A $y = x^2 + 3$

B $y = 3^{x+1}$

C $y = e^{x+3}$

D $y = 3^x + 24$

End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. A function f is defined by $f(x) = 2x^3 - 3x^2$, where x is a real number.

(a) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x and y-axes. **3**

(b) Find the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(c) (i) Sketch the curve $y = f(x)$.

(ii) Hence solve $2x^3 > 3x^2$. **3**

22. Two sequences are generated by the recurrence relations

$$u_{n+1} = 0 \cdot 4u_n + 8 \cdot 4$$
$$v_{n+1} = kv_n + 2$$

The two sequences approach the same limit as $n \to \infty$.

(a) Evaluate this limit. **2**

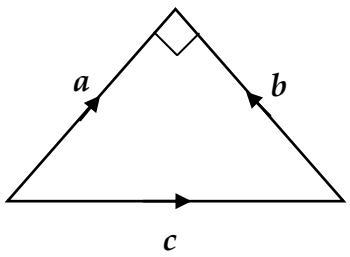
(b) Hence determine the value of k. **2**

23. Given that $\sin a = \frac{4}{5}$ and $\sin b = \frac{2}{\sqrt{5}}$, where $0 \le a < \frac{\pi}{2}$ and $0 \le b < \frac{\pi}{2}$, find the exact values of :

(a) $\sin(a+b)$; **4**

(b) $\tan(a+b)$. **4**

24. In the triangle opposite $|\boldsymbol{a}| = |\boldsymbol{b}| = 2$ units



Find $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$ **6**

End of question paper

H

Mathematics
Higher
Paper 2
Practice Paper C

Time allowed
1 hour 10 minutes

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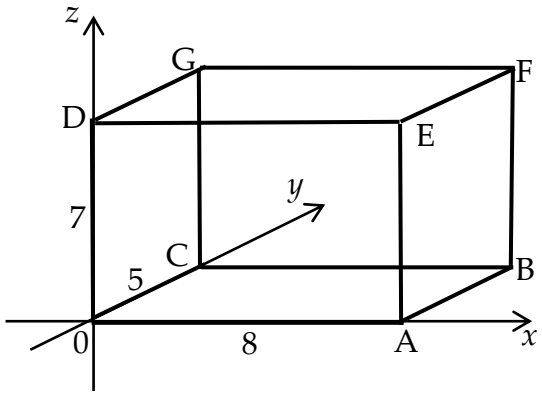
ALL questions should be attempted.

Marks

1. (*a*) Given that $(x+1)$ is a factor of $2x^3+3x^2+kx-6,$ find the value of k. **3**

(*b*) Hence, or otherwise, solve $2x^3+3x^2+kx-6=0.$ **4**

2. OABC,DEFG is a rectangular prism as show.



OA is 8 units long, OC is 5 units and OD is 7 units.

(*a*) Write down the coordinates of B and G. **2**

(*b*) Calculate the size of angle BEG. **6**

3. A circle, centre C, has equation $x^2+y^2-4x-2y-20=0.$

(*a*) Find the centre C and radius of this circle. **2**

(*b*) (i) Show that the point $P(5,-3)$ lies on the circumference of the circle.

(ii) Find the equation of radius CP. **4**

(*c*) Find the equation of the chord which passes through $(7,1)$ and is perpendicular to radius CP. **3**

4. Solve $3\cos 2x = 11\cos x - 6$ for $0 \le x < 2\pi.$ **6**

Marks

5. (*a*) Diagram 1 shows part of the graph with equation $y = x^3 - 5x^2 + 2x + 8$.

Calculate the shaded area.

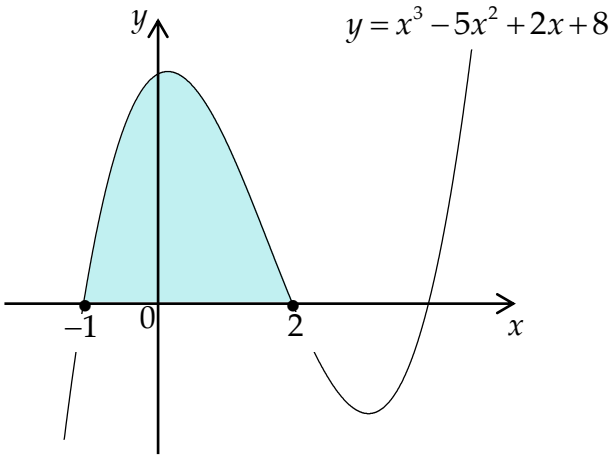


Diagram 1

5

(*b*) Given that

$$\int_{-1}^{p} (x^3 - 5x^2 + 2x + 8)\, dx = 12 \cdot 13$$

find the total shaded area in diagram 2.

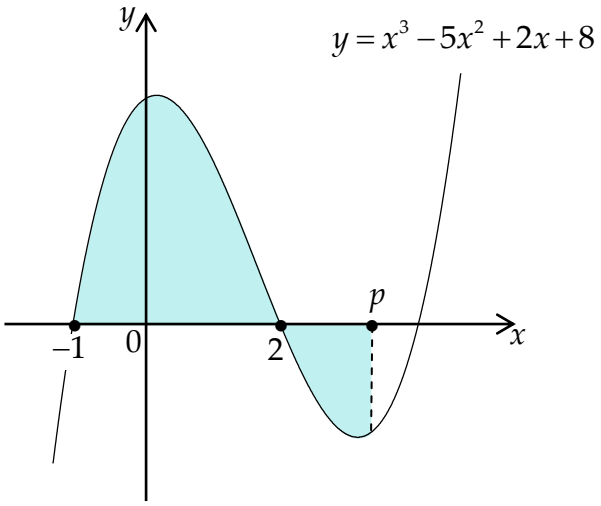


Diagram 2

2

6. Find the smallest integer value of c for which

$$g(x) = (x-2)(x^2 - 2x + c)$$

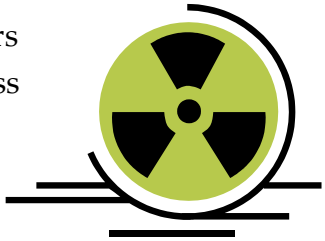
has only one real root. **5**

7. (*a*) Write $2\sin x + \sqrt{5}\cos x$ in the form $k\sin(x+a)$, where $k > 0$ and $0 \le a < \frac{\pi}{2}$. **4**

(*b*) Sketch the graph of $y = 4\sin x + 2\sqrt{5}\cos x$ for $0 \le x \le 2\pi$. **4**

Marks

8. For a particular radioactive isotope, the mass of the original isotope remaining, m grams, after time t years is given by $m = m_0 e^{-0{\cdot}18t}$ where m_0 is the original mass of the isotope.



(*a*) If the original mass is 1000g, find the mass of the original isotope remaining after 20 years. **2**

The half-life of the isotope is the time taken for half the original mass to decay.

(*b*) Find the half life of this isotope. **3**

9. Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\sin 4x}{\sin 2x} \right) dx$. **5**

End of Question Paper

Paper 1

Section A

1.	A	11.	A
2.	B	12.	C
3.	A	13.	C
4.	D	14.	D
5.	A	15.	C
6.	B	16.	B
7.	C	17.	B
8.	D	18.	A
9.	D	19.	D
10.	B	20.	B

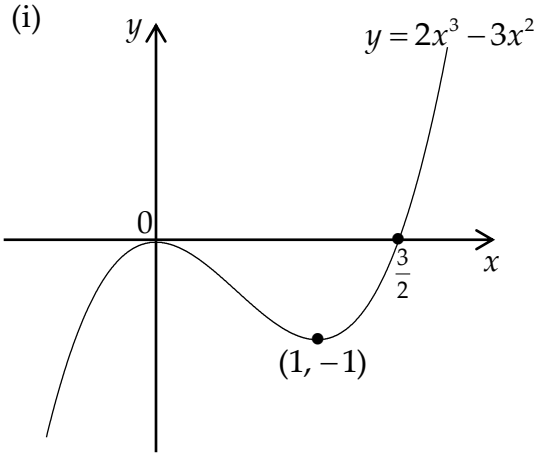
Section B

21. (*a*) $(0, 0)$ and $\left(\frac{3}{2}, 0\right)$

(*b*) Maximum turning point at $(0, 0)$

Minimum turning point at $(1, -1)$

(*c*) (i)



(ii) $x > \frac{3}{2}$

22. (*a*) Limit : 14

(*b*) $k = \frac{6}{7}$

23. (*a*) $\sin(a+b) = \frac{2}{\sqrt{5}}$

(*b*) $\tan(a+b) = -2$

24. 8

Paper 2

1. (*a*) $k = -5$

   (*b*) $\left\{-2, -1, \frac{3}{2}\right\}$

2. (*a*) B(8, 5, 0) and G(0, 5, 7)

   (*b*) $72 \cdot 1^{\circ}$ (or $1 \cdot 258$ rads)

3. (*a*) Centre : (2, 1) Radius : 5 units

   (*b*) (i) Proof

   (ii) $4x + 3y - 11 = 0$

   (*c*) $3x - 4y - 17 = 0$

4. $\{1 \cdot 231, 5 \cdot 052\}$

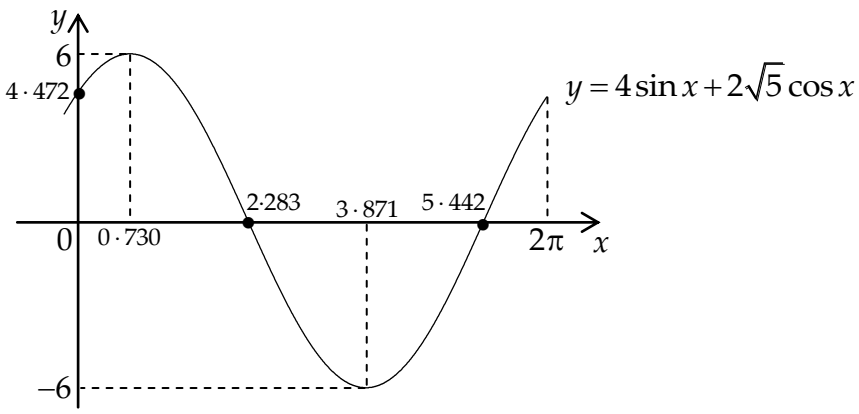
5. (*a*) $\frac{63}{4}$ square units (or $15 \cdot 75$)

   (*b*) $19 \cdot 37$

6. $c = 2$

7. (*a*) $3\sin(x + 0 \cdot 841)$

   (*b*)



8. (*a*) $27 \cdot 3g$

   (*b*) $3 \cdot 85$ years

9. $1 - \frac{\sqrt{3}}{2} \approx 0 \cdot 134$