H

Mathematics
Higher
Paper 1
Practice Paper D

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4 If any of this information is wrong, tell the invigilator immediately.

5 If this information is correct, print your name and seat number in the boxes provided.

6 The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on the answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

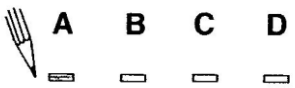
A 8

B 1

C 0

D –4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. The midpoint of the line joining $G(-1, 3, 7)$ to $H(5, -1, p)$ is $M(q, 1, 4)$.

   What are the values of p and q?

	p	q
A	1	2
B	3	2
C	1	–3
D	3	–3

2. Given that $f(x) = \frac{1}{3x^5}$, find $f'(x)$.

   A $-\frac{15}{x^4}$

   B $-\frac{1}{15x^4}$

   C $\frac{1}{15x^4}$

   D $-\frac{5}{3x^6}$

3. If $x^2 + 12x + 7$ is written in the form $(x+a)^2 + r$, find the value of r.

   A –29

   B –5

   C 1

   D 7

4. A straight line passes through the points $(4, 3)$ and $(0, -1)$.

What is the equation of the line?

A $x+y-1=0$

B $x-y-1=0$

C $x-2y-1=0$

D $3x-4y-1=0$

5. Functions f and g are defined on the set of real numbers by

$$f(x)=x^2+1 \text{ and } g(x)=3x-5$$

What is the value of $g(f(-1))$?

A -5

B -4

C 0

D 1

6. The vectors with components $\begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ t \\ -2 \end{pmatrix}$ are perpendicular.

What is the value of t?

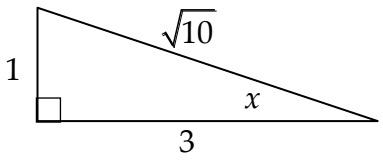
A 2

B 0

C $-\frac{1}{2}$

D -1

7. The diagram shows a right-angled triangle with sides 1, 3 and $\sqrt{10}$.



What is the value of $\cos 2x$?

A $\frac{3}{5}$

B $\frac{4}{5}$

C $\frac{1}{\sqrt{10}}$

D $\frac{2}{\sqrt{10}}$

8. Find $\int_{-2}^{0} 6x^2 \, dx$

A 12

B 16

C 24

D 96

9. For what value of k does the equation $2x^2 - 4x + k = 0$ have equal roots?

A -2

B 0

C 2

D 4

10. $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have components $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$ respectively.

Given that D has coordinates $(-2, 0, -2)$, what are the coordinates of F ?

A $(0, 1, 1)$

B $(1, 3, 0)$

C $(5, 1, 4)$

D $(9, 1, 6)$

11. What is the maximum value of $8 - 3\sin\left(x - \frac{7\pi}{9}\right)$?

A -3

B -1

C 8

D 11

12. Find $\int (2x+5)^3 \, dx$.

A $\frac{1}{2}(2x+5)^3 + c$

B $8(2x+5)^4 + c$

C $\frac{1}{8}(2x+5)^4 + c$

D $(x^2+5x)^4 + c$

13. How many solutions does the equation $(\sqrt{7}\cos x + 3)(4\tan x - 9) = 0$ have in the interval $0 \le x < 2\pi$?

A 0

B 2

C 3

D 4

14. Given that $f(x) = 4\sin 3x$, find $f'\left(\frac{\pi}{6}\right)$.

A –4

B –3

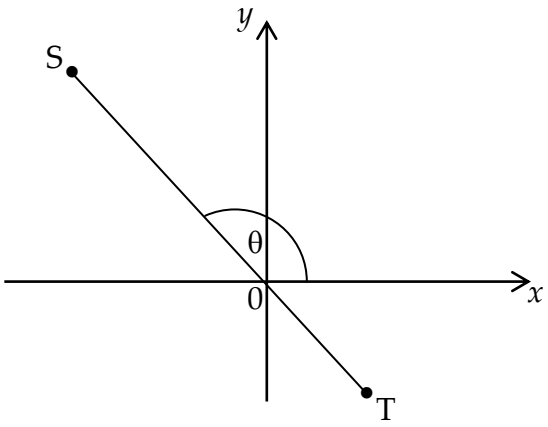
C 0

D 12

15. The diagram shows the line ST with equation $2x + y = 0$.

The angle between ST and the positive direction of the x-axis is θ.

Find an expression for θ.



A $\theta = \tan^{-1}\frac{1}{2}$

B $\theta = \pi - \tan^{-1}\frac{1}{2}$

C $\theta = \tan^{-1} 2$

D $\theta = \pi - \tan^{-1} 2$

16. What is the value of $\dfrac{\log_2 32}{\log_2 8}$?

A $\frac{5}{3}$

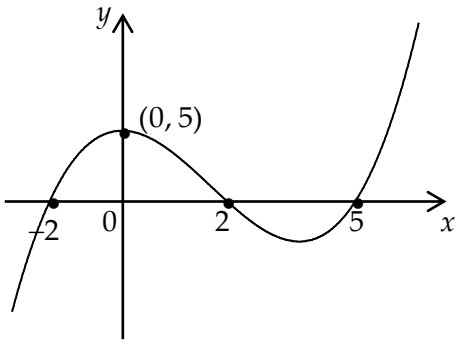
B 2

C 4

D 15

17. The diagram shows a sketch of the curve with equation

$$y = k(x+2)(x-2)(x+a)$$



What are the values of a and k?

	a	k
A	–5	$\frac{1}{4}$
B	–5	–4
C	5	$\frac{1}{4}$
D	5	–4

18. Here are two statements about the function $f(x) = \sqrt{x^2 - 4}$.

(1) The largest possible domain is $-2 \le x \le 2$.

(2) The range is $f(x) \ge 0$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

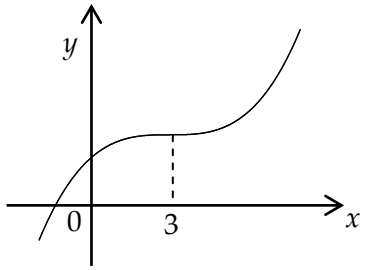
D Both statements are correct.

19. Given that

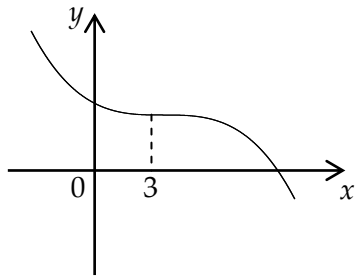
$$f'(x)\begin{cases} > 0, & \text{for } x < 3 \\ = 0, & \text{for } x = 3 \\ > 0, & \text{for } x > 3 \end{cases}$$

Which diagram shows the curve with equation $y = f(x)$?

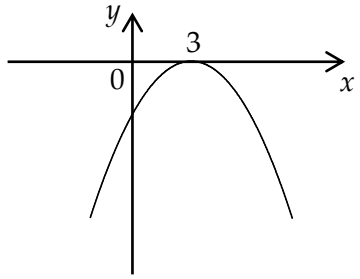
A



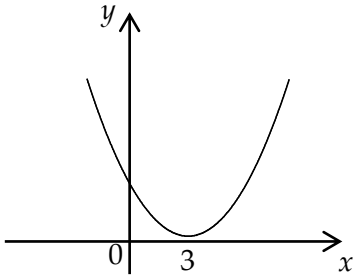
B



C



D



20. If $5^x = a^2$, find an expression for x.

A $x = \frac{a^2}{5}$

B $x = \sqrt[5]{a^2}$

C $x = \frac{2}{\log_a 5}$

D $x = \frac{5}{\log_2 a}$

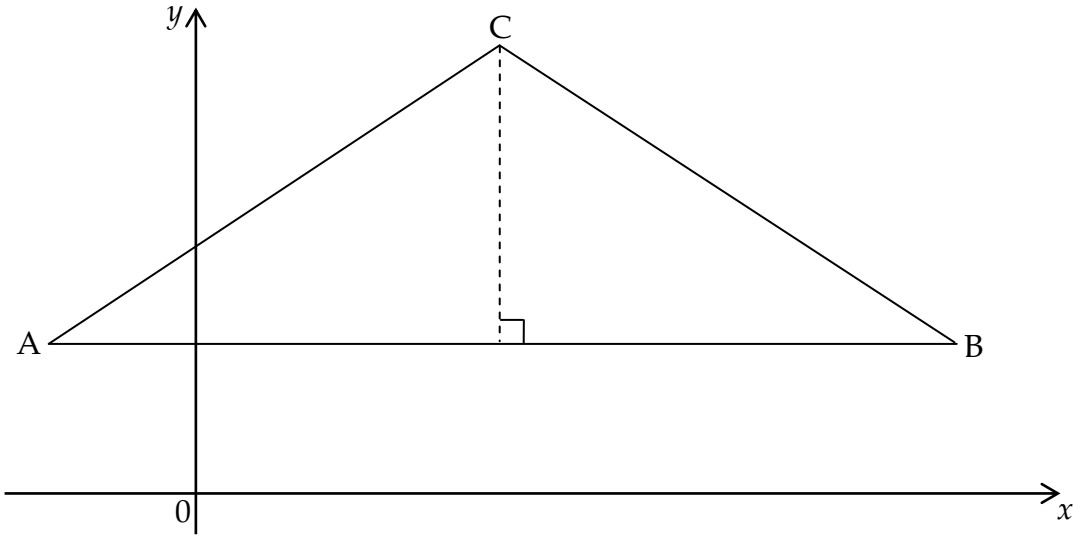
End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. A(−2, 4), B(10, 4) and C(4, 8) are the vertices of triangle ABC shown in the diagram.



(*a*) Write down the equation of the altitude from C. **1**

(*b*) Find the equation of the perpendicular bisector of BC. **4**

(*c*) Find the point of intersection of the lines found in (*a*) and (*b*). **2**

22. P is the point (4, 1, −2), Q is (5, 2, 0) and R is (7, 4, 4).

(*a*) Show that P, Q and R are collinear. **3**

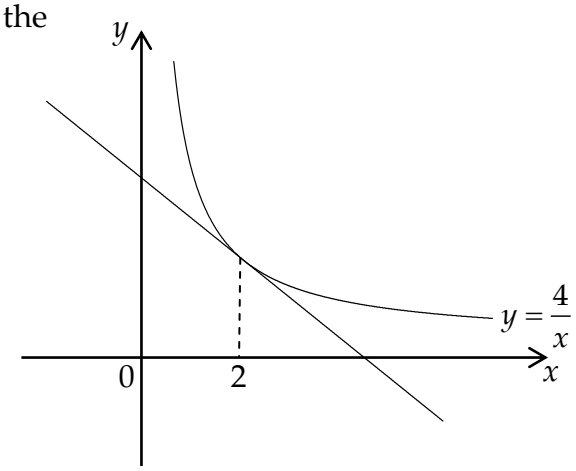
(*b*) Find the ratio in which Q divides PR. **1**

Marks

23. Find the equation of the tangent to the curve with equation

$$y = \frac{4}{x}$$

at the point where $x = 2$.



6

24. (*a*) Given that $f'(x) = 3x^2 + 2x - 10$ and $(x-2)$ is a factor of $f(x)$, find a formula for $f(x)$. **4**

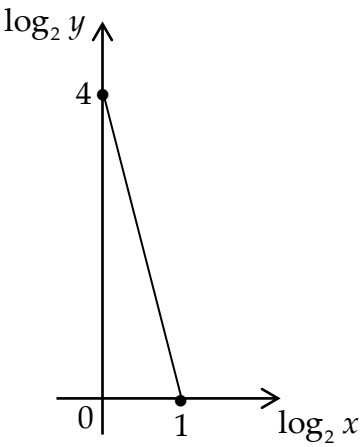
(*b*) Hence factorise $f(x)$ fully. **1**

(*c*) Solve $f(x) = 0$. **1**

25. The graph illustrates the law $y = ax^b$.

The straight line joins the points (0, 4) and (1, 0).

Find the values of a and b.



4

End of question paper

H

Mathematics
Higher
Paper 2
Practice Paper D

Time allowed
1 hour 10 minutes

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ALL questions should be attempted.

Marks

1. A sequence is defined by recurrence relation $u_{n+1} = ku_n - 6,\ u_0 = 0.$

(*a*) Given that $u_2 = -8,$ find the value of k. **2**

(*b*) (i) Why does this sequence tend to a limit as $n \to \infty$?

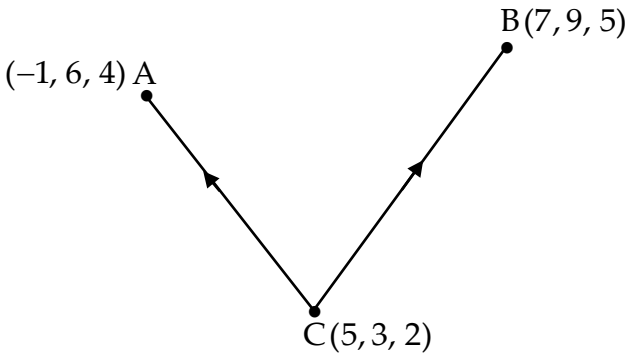
(ii) Find the value of this limit. **3**

2. $f(x) = 2x^3 + px^2 + qx + 4.$

Given that $(x-2)$ is a factor of $f(x)$, and the remainder when $f(x)$ is divided by $(x+1)$ is 9, find the values of p and q. **5**

3. Security guards are watching a parked car, via two CCTV cameras, in a supermarket car park.

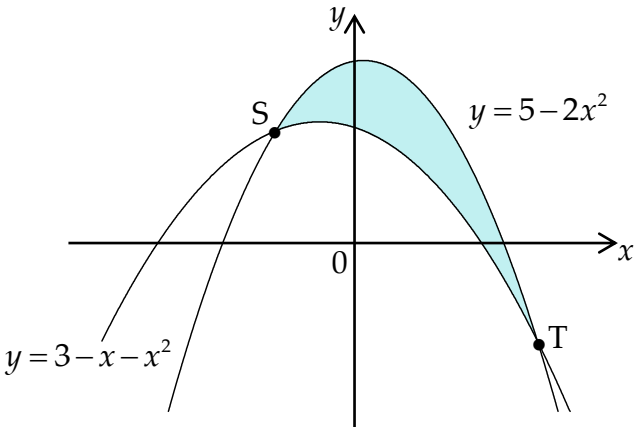
With reference to a suitable set of axes, the car is at C(5, 3, 2) and the cameras are at positions A(–1, 6, 4) and B(7, 9, 5) as shown.



Calculate the size of angle ACB. **7**

4. Part of the graphs of $y = 3 - x - x^2$ and $y = 5 - 2x^2$ are shown opposite.

The curves intersect at the points S and T.

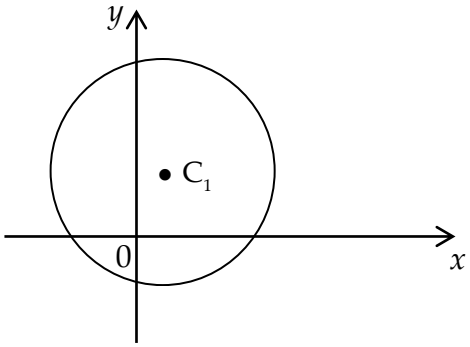


(*a*) Find the coordinates of S and T. **4**

(*b*) Find the shaded area enclosed between the two curves. **5**

5. A circle with centre C_1 has equation $x^2 + y^2 - 2x - 6y - 15 = 0$.

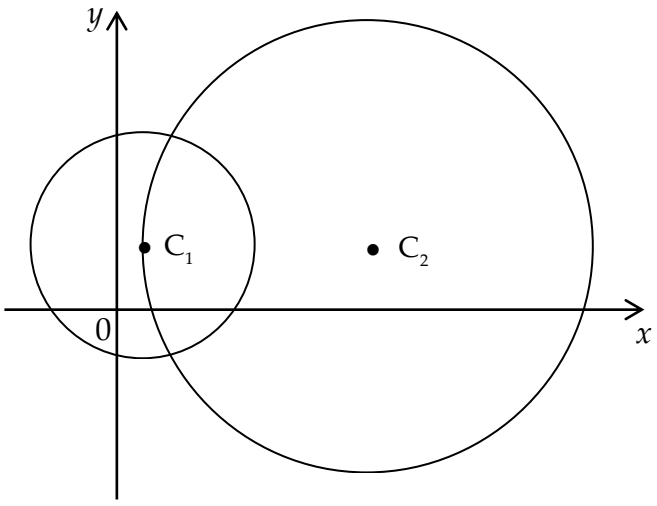
(*a*) Write down the coordinates of the centre and calculate the length of the radius of this circle. **2**



A second circle with centre C_2 has a diameter twice that of the circle with centre C_1.

C_1 lies on the circumference of this second circle.

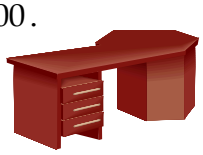
The line joining C_1 and C_2 is parallel to the *x*-axis.



(b) Find the equation of the circle with centre C_2. **4**

Marks

6. A manufacturer of executive desks estimates that the weekly cost, in £, of making x desks is given by $C(x) = x^3 - 6x^2 + 560x + 800$.



Each executive desk sells for £2000.

(*a*) Show that the weekly profit made from making x desks is given by

$$P(x) = -x^3 + 6x^2 + 1440x - 800$$ **2**

(*b*) (i) How many desks would the manufacturer have to make each week in order to maximise his profit?

(ii) What would his annual profit be? **8**

7. The number of bacteria, b, in a culture after t hours is given by $b = b_0 e^{kt}$ where b_0 is the original number of bacteria present.

(*a*) The number of bacteria in a culture increases from 800 to 2400 in 2 hours.

Find the value of k correct to 3 significant figures. **3**

(*b*) How many bacteria, to the nearest hundred, are present after a further 4 hours? **3**

8. (*a*) Express $2\cos x^\circ - 5\sin x^\circ$ in the form $k\cos(x+a)^\circ$, where $k > 0$ and $0 < a < 90$. **4**

(*b*) (i) Hence write $2\cos 2x^\circ - 5\sin 2x^\circ$ in the form $R\cos(2x+b)^\circ$, where $R > 0$ and $0 < b < 90$. **2**

(ii) Solve $2\cos 2x^\circ - 5\sin 2x^\circ = 5$ in the interval $0 \le x < 360$. **4**

End of Question Paper

Paper 1

Section A

1.	A	11.	D
2.	D	12.	C
3.	A	13.	B
4.	B	14.	C
5.	D	15.	D
6.	A	16.	A
7.	B	17.	A
8.	B	18.	C
9.	C	19.	A
10.	B	20.	C

Section B

21. (*a*) $x = 4$

(*b*) $3x - 2y + 9 = 0$

(*c*) $\left(4, \frac{3}{2}\right)$

22. (*a*) Proof e.g. show that $\overrightarrow{QR} = 2\overrightarrow{PQ}$

(*b*) $1 : 2$

23. $x + y - 4 = 0$

24. (*a*) $f(x) = x^3 + x^2 - 10x + 8$

(*b*) $f(x) = (x-2)(x-1)(x+4)$

(*c*) $\{-4, 1, 2\}$

25. $a = 16$ and $b = -4$

Paper 2

1. (*a*) $k = \frac{1}{3}$

   (*b*) $-1 < \frac{1}{3} < 1$

   (*c*) -9

2. $p = -1,\ \ q = -8$

3. $75 \cdot 8^\circ$ or $1 \cdot 323$ radians

4. (*a*) S$(-1, 3)$ and T$(2, -3)$

   (*b*) $\frac{9}{2}$ square units

5. (*a*) $C_1(1, 3)$ and radius 5 units

   (*b*) $(x-11)^2 + (y-3)^2 = 100$

6. (*a*) Proof

   (*b*) (i) 24 (ii) £1 216 384

7. (*a*) $0 \cdot 549$

   (*b*) 21 600

8. (*a*) $\sqrt{29}\cos(x + 68 \cdot 2)^\circ$

   (*b*) (i) $\sqrt{29}\cos(2x + 68 \cdot 2)^\circ$ (ii) $\{135, 156 \cdot 8, 315, 336 \cdot 8\}$