H

Mathematics	Time allowed	NATIONAL
Higher	1 hour 30 minutes	QUALIFICATIONS
Paper 1		
Practice Paper E		

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.

2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1   Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2   For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3   Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4   If any of this information is wrong, tell the invigilator immediately.

5   If this information is correct, print your name and seat number in the boxes provided.

6   The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7   There is **only one correct** answer to each question.

8   Rough working should **not** be done on the answer sheet.

9   At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.
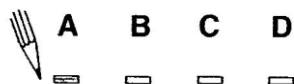
Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

       A   8

       B   1

       C   0

       D −4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).

Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

ALL questions should be attempted.

1. K and L have position vectors $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ respectively.

    What is the magnitude of $\overrightarrow{KL}$?

    A $\sqrt{5}$

    B $\sqrt{10}$

    C $\sqrt{22}$

    D $\sqrt{55}$

2. If $f(x) = x^3 - 4x + 7$, find $f'(-2)$.

    A 7

    B 8

    C 11

    D 12

3. Find $\displaystyle\int \left(x^{\frac{1}{4}} - x^{-2} \right) dx$

    A $\dfrac{1}{4} x^{-\frac{3}{4}} + 2x^{-3} + c$

    B $x^{\frac{3}{4}} + x^{-3} + c$

    C $x^{\frac{5}{4}} - x^{-1} + c$

    D $\dfrac{4}{5} x^{\frac{5}{4}} + x^{-1} + c$

4. A function f is defined on the set of real numbers by $f(x)=4x+5$.

    Find an expression for $f(f(x))$.

    A $f(f(x))=8x+10$

    B $f(f(x))=16x+25$

    C $f(f(x))=16x^2+20x+5$

    D $f(f(x))=16x^2+40x+25$

5. Evaluate $4\sqrt{2}\sin\frac{\pi}{4}\cos\frac{2\pi}{3}$.

    A -12

    B -2

    C $2\sqrt{3}$

    D $2\sqrt{6}$

6. A circle with centre $(-3,4)$ passes through the point $(-2,2)$.

    What is the equation of the circle?

    A $(x-3)^2+(y+4)^2=5$

    B $(x-3)^2+(y+4)^2=61$

    C $(x+3)^2+(y-4)^2=5$

    D $(x+3)^2+(y-4)^2=61$

7. $f(x)=2x^3-x^2-5x+4$.

    What is the remainder when $f(x)$ is divided by $(x+2)$?

    A -6
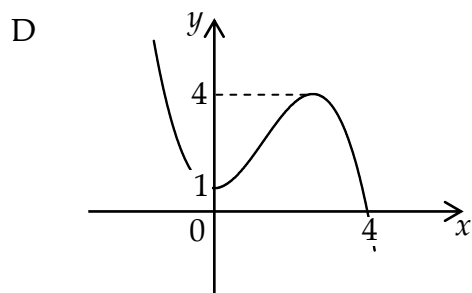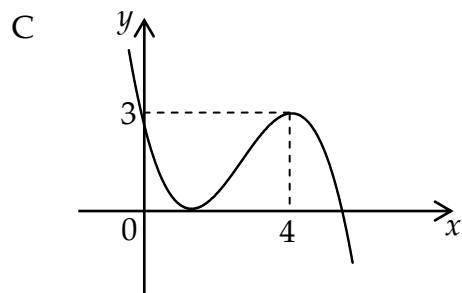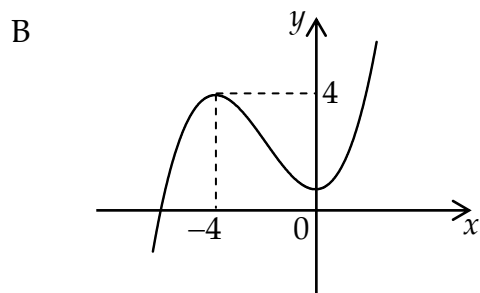
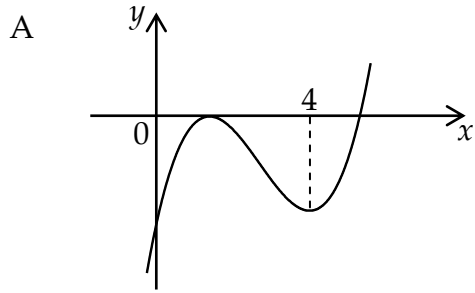   B -2

    C 0

    D 18

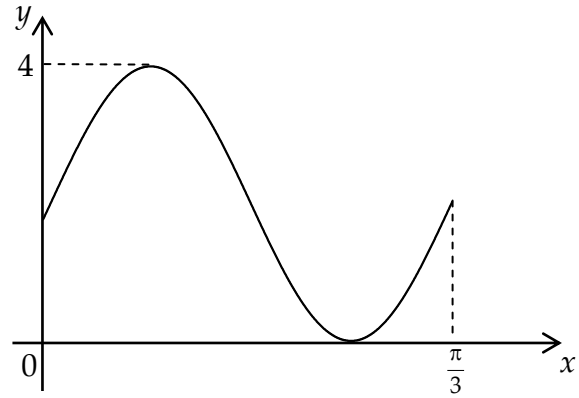8. The diagram shows the part of the graph of the cubic $y = f(x)$.
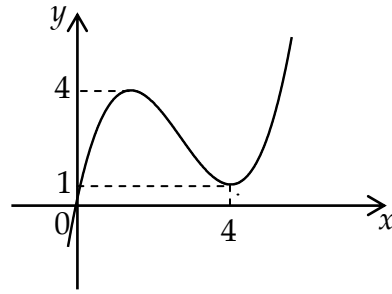
Which diagram below shows the graph of $y = 4 - f(x)$?

A

B

C

D

9. The graph shown in the diagram has equation $y = p + 2\sin(qx)$.

What are the values of p and q?

	p	q
A	2	3
B	4	3
C	2	6
D	4	6

10. A sequence is generated by the recurrence relation $u_{n+1} = 7 - 2u_n$.

If $u_2 = 5$, what is the value of u_0?

A   3

B   5

C   7

D   13

11. For what value of k does the equation $kx^2 - 6x + 1 = 0$ have equal roots?

A   0

B   1

C   9

D   12

12. Find $\int (2x+7)^4 \, dx$.

A $\frac{1}{10}(2x+7)^5 + c$

B $\frac{1}{5}(x^2 + 7x)^5 + c$

C $2(2x+7)^3 + c$

D $4(x^2 + 7x)^3 + c$

13. Given that $f'(x) = 6x^2$ and $f(1) = 5$, find a formula for $f(x)$ in terms of x.

A $f(x) = 12x - 7$

B $f(x) = 6x^2 - 1$

C $f(x) = 2x^3$

D $f(x) = 2x^3 + 3$

14. What are the coordinates of the centre of the circle with equation
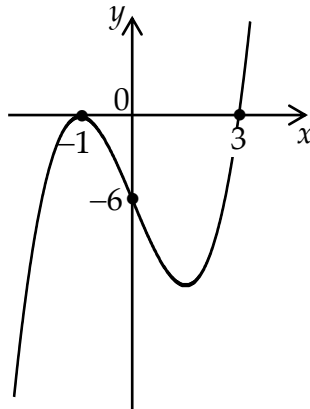
$$3x^2 + 3y^2 - 6x + 18y - 5 = 0?$$

A $(1, -3)$

B $(3, -9)$

C $(-2, 6)$

D $(-6, 18)$

15. The diagram shows part of the graph of a cubic function.

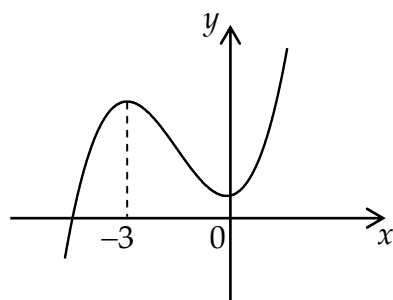What is the equation of this graph?

A $y = -2(x-1)^2(x+3)$

B $y = -(x+1)^2(x-3)$

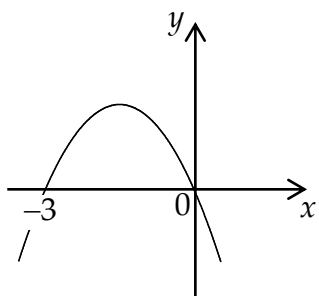C $y = (x-1)^2(x+3)$

D $y = 2(x+1)^2(x-3)$

16. The diagram shows part of the graph of the cubic $y = f(x)$.
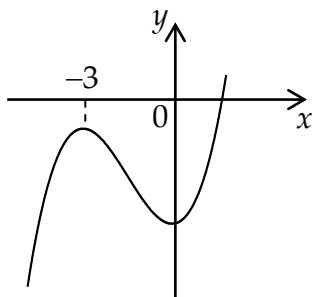
There are stationary points at $x = 0$ and $x = -3$.
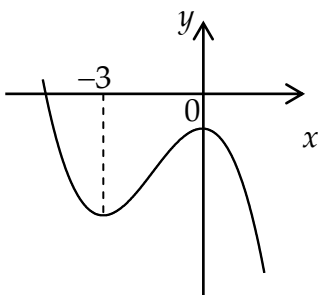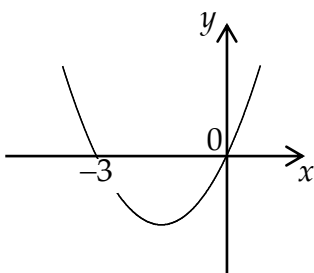
Which diagram below shows the graph of $y = f'(x)$?

A

B

C

D

17. If $4x^2 + 8x - 1$ is expressed in the form $4(x+p)^2 + q$, what is the value of q?

    A   –5

    B   –2

    C   –1

    D   0

18. If $3\log_2 t - \log_2 5 = 3$, find the value of t.

    A $\dfrac{8}{3}$

    B $\sqrt[3]{40}$

    C $\dfrac{13}{3}$

    D   5

19. If $p = 4x^{-3}$ find the rate of change of p with respect to x when $x = 2$.

    A   –12

    B   –3

    C $-\dfrac{3}{4}$

    D $\dfrac{1}{2}$

20. What is the solution of $8 - 2x - x^2 < 0$?

    A $-2 < x < 4$

    B $x < -2$ or $x > 4$

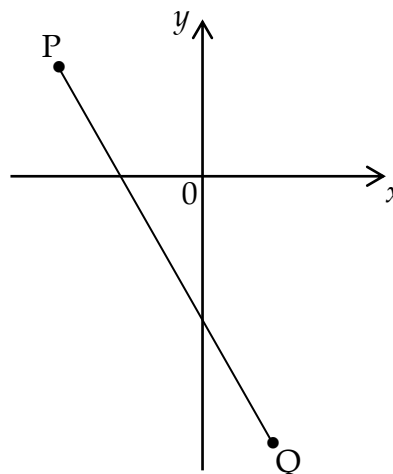   C $x < -4$ or $x > 2$

    D $-4 < x < 2$

End of Section A

ALL questions should be attempted.

Marks

21. A line joins the points $P(-4, 3)$ and $Q(2, -7)$.

Find the equation of the perpendicular bisector of PQ.

4

22. Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact of the tangent and circle.

6

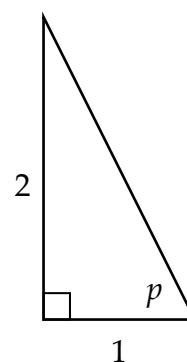**23.** The diagram shows a right-angled triangle with height 2 units, base 1 unit and an angle of p.

(*a*) Find the exact values of:

(i) $\cos p$;

(ii) $\cos 2p$.

3

(*b*) By writing $3p = 2p + p$, find the exact value of $\cos 3p$.

4

24. A function f is defined by $f(x) = x^3 - 2x^2 - 4x + 1$, where $0 \leq x \leq 3$.

Find the maximum and minimum values of f.

5

25. (*a*) Express $2\sqrt{2}\cos x° - 2\sqrt{2}\sin x°$ in the form $k\cos(x-a)°$, where $k>0$

and $0 \le a < 360$. **4**

(*b*) Find:

(i) the maximum value of $3 + 2\sqrt{2}\sin x° - 2\sqrt{2}\cos x°$;

(ii) a value of x where this maximum value occurs in the interval $0 \le x < 360$. **4**

End of question paper

Mathematics	Time allowed	NATIONAL
Higher	1 hour 10 minutes	QUALIFICATIONS
Paper 2		
Practice Paper E		

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Circle:

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Scalar Product : $\quad \boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$\text{or} \quad \boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3, \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Trigonometric formulae:
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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Table of standard derivatives :

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Table of standard integrals :

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$\cos ax$	$\dfrac{1}{a}\sin ax + C$

Marks

1. (a) A line, l_1, passes through the points $A(-3, 0)$ and $B(5, 4)$.

   The line makes an angle of $a°$ with the positive direction on the x-axis.
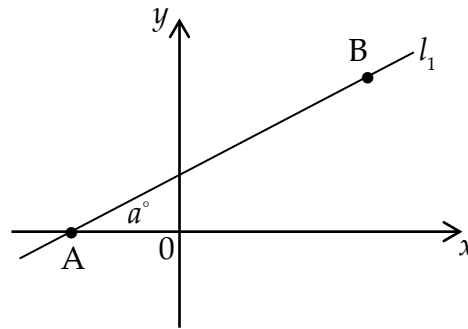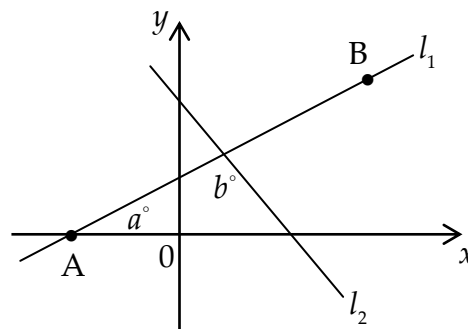
   Find the value of a.   **3**

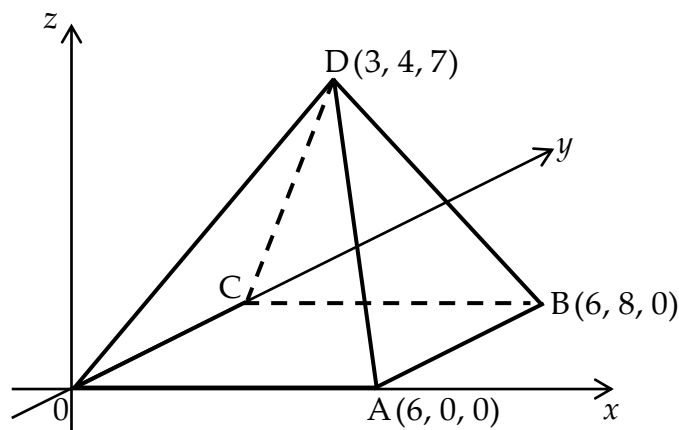   (b) A second line, l_2, with equation $4x + 3y = 12$, crosses the line in (a).

   The angle between the two lines is $b°$, as shown.

   Find the value of b.   **4**

2. The rectangular based pyramid D,OABC has vertices $A(6, 0, 0)$, $B(6, 8, 0)$ and $D(3, 4, 7)$.

   (a) (i) Write down the coordinates of C.

   (ii) Express $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in component form.   **3**

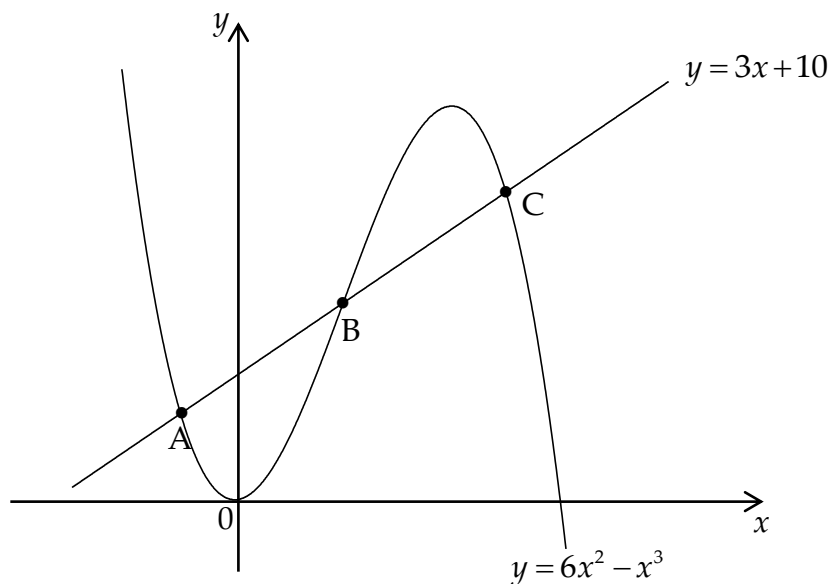   (b) Calculate the size of angle CAD.   **5**

3. (*a*) (i) Show that $(x-2)$ is a factor of $x^3 - 6x^2 + 3x + 10$.

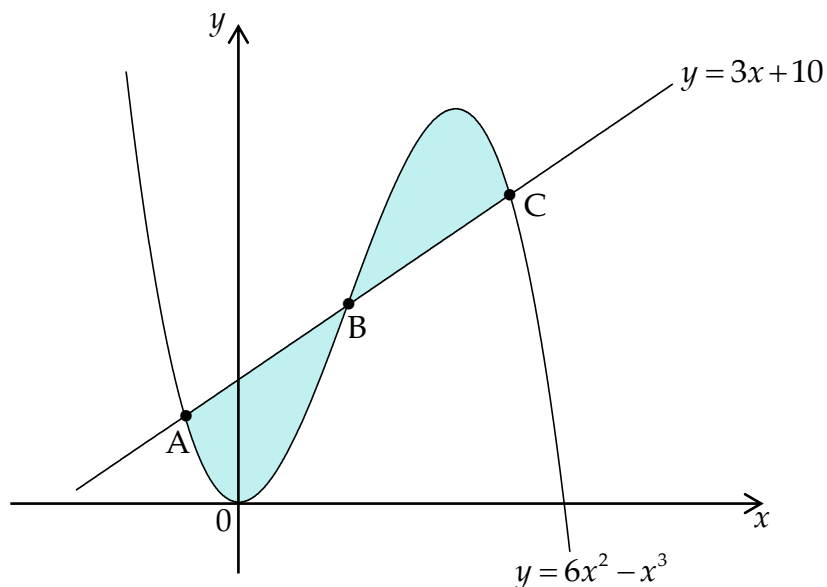(ii) Hence factorise $x^3 - 6x^2 + 3x + 10$ fully. **4**

The line with equation $y = 3x + 10$ intersects the curve with equation $y = 6x^2 - x^3$ at the points A, B and C.

(*b*) Find the *x*-coordinates of the points A and C. **3**

The area between the curve and the line from A to C is shaded in the diagram below.

(*c*) Calculate the total shaded area shown in the diagram. **7**

Marks

4. Solve $2\cos 2x - \sin x + 1 = 0$ for $0 \le x < 2\pi$. **6**

5. A new '24 hour anti-biotic' is being tested on a patient in hospital.

It is know, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.

On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(*a*) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? **1**

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(*b*) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.

Write down the values of *a* and *b*. **2**

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(*c*) Is it safe to administer this anti-biotic over an extended period of time? **4**

6. The diagram shows part of the graph of $y = 3\cos(2x) - 1$.

Find the equation of the tangent at the point T, where $x = \frac{\pi}{4}$.

$y = 3\cos(2x) - 1$
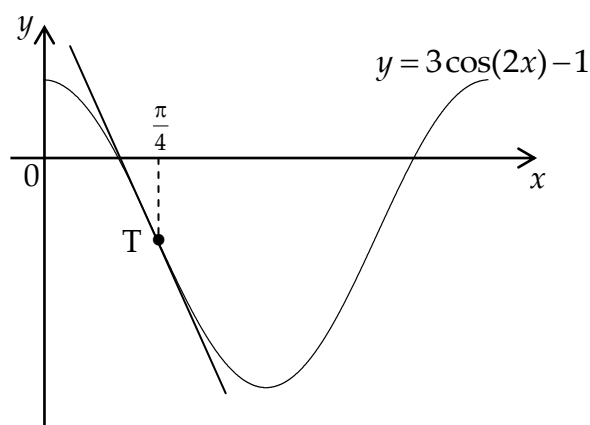
7

7. Solve $\log_x(x+2) + \log_x(2x-3) = 2$, $x > \frac{3}{2}$. **5**

Page 17

8. A circle has the following properties:

- The *x*-axis and the line $y = 20$ are tangents to the circle.

- The circle passes through the points $(0,2)$ and $(0,18)$.

- The centre lies in the first quadrant.

Find the equation of this circle. **6**

End of Question Paper

Paper 1

Section A

1.	C	11.	C
2.	B	12.	A
3.	D	13.	D
4.	B	14.	A
5.	B	15.	D
6.	C	16.	D
7.	A	17.	A
8.	C	18.	B
9.	C	19.	C
10.	A	20.	C

Section B

21. $3x - 5y - 7 = 0$

22. Point of contact $(-3, 4)$

23. (*a*) (i) $\dfrac{1}{\sqrt{5}}$ (ii) $-\dfrac{3}{5}$

   (*b*) $-\dfrac{11}{5\sqrt{5}}$

24. Maximum value 1, minimum value -7

25. (*a*) $4\cos(x - 315)°$

   (*b*) Maximum value 7 at $x = 315$

Paper 2

1.　(a)　$a = 26 \cdot 6$
　　(b)　$b = 100 \cdot 3$

2.　(a)　(i)　C(0, 8, 0)　　(ii)　$\overrightarrow{AC} = \begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix}$

　　(b)　$54 \cdot 5°$ or $0 \cdot 951$ radians

3.　(a)　(i)　Proof　　(ii)　$(x-2)(x-5)(x+1)$
　　(b)　$A : x = -1$　$C : x = 5$
　　(c)　$\dfrac{81}{2}$ square units

4.　$\left\{ 0 \cdot 848, \ 2 \cdot 294, \ 4 \cdot 712 \left(\dfrac{3\pi}{2} \right) \right\}$

5.　(a)　$50 \ mg$
　　(b)　$u_{n+1} = 0 \cdot 2 u_n + 250$
　　(c)　$312 \cdot 5 < 350 \Rightarrow$ safe to administer long term

6.　$6x + y - \dfrac{3\pi}{2} + 1 = 0$

7.　$x = 2$

8.　$(x-6)^2 + (y-10)^2 = 100$