H

Mathematics
Higher
Paper 1
Practice Paper F

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4 If any of this information is wrong, tell the invigilator immediately.

5 If this information is correct, print your name and seat number in the boxes provided.

6 The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on the answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

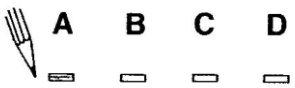
A 8

B 1

C 0

D –4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. If $f(x) = (3x+1)(x-4)$, find $f'(x)$.

   A 3

   B $6x-11$

   C $6x^2-4$

   D $2x^3-4x$

2. Vectors $\boldsymbol{p}$ is given by $3\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k}$ and $\boldsymbol{q}$ is $\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$.

   What are the components of $2\boldsymbol{p}-\boldsymbol{q}$?

   A $\begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$

   B $\begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$

   C $\begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix}$

   D $\begin{pmatrix} 6 \\ 4 \\ -4 \end{pmatrix}$

3. A circle has equation $x^2+y^2+2x+8y-2=0$.

   What is the radius of this circle?

   A $\sqrt{2}$

   B $\sqrt{8}$

   C $\sqrt{19}$

   D $\sqrt{70}$

4. The line with equation $kx-4y+1=0$ is parallel to the line with gradient 3.

What is the value of k?

A -3

B $-\frac{1}{3}$

C $\frac{1}{4}$

D 12

5. What is the derivative of $\frac{2+6x^2}{2x}$, with respect to x?

A $3-x^{-2}$

B $12-4x^{-2}$

C $6x$

D 6

6. Find $\int \sqrt[5]{x^2}\,dx$.

A $\frac{2}{5}x^{\frac{3}{5}}+c$

B $\frac{5}{7}x^{\frac{7}{5}}+c$

C $\frac{2}{7}x^{\frac{3}{2}}+c$

D $\frac{5}{2}x^{\frac{7}{2}}+c$

7. A circle centre $(3, 5)$ passes through the point $(-1, 4)$.

What is the equation of the circle?

A $(x-3)^2+(y-5)^2=17$

B $(x-3)^2+(y-5)^2=85$

C $(x+3)^2+(y+5)^2=17$

D $(x+3)^2+(y+5)^2=85$

8. What is the value of $\sin\frac{2\pi}{3}+\cos\frac{11\pi}{6}$?

A 0

B 1

C $\sqrt{2}$

D $\sqrt{3}$

9. Which of the following describes the stationary point on the curve with equation $y=4-(x-9)^2$?

A minimum at $(9, 4)$

B maximum at $(9, 4)$

C minimum at $(-9, 4)$

D maximum at $(-9, 4)$

10. Here are two statements about the equation

$$5x^2-3x-1=0$$

(1) The roots are unequal;

(2) The roots are irrational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. What is the minimum value of $11+3\cos\left(2x-\frac{\pi}{6}\right)$?

A 0

B 3

C 7

D 8

12. If $y=5\sin(7-2x)$, find $\frac{dy}{dx}$.

A $-10\cos(7-2x)$

B $35\cos(7-2x)$

C $-12\sin(7-2x)$

D $10\sin(7-2x)$

13. Find the radius of the circle with equation $x^2+y^2=8x+3$.

A $\sqrt{5}$

B $\sqrt{11}$

C $\sqrt{19}$

D $\sqrt{61}$

14. $g(x)=\frac{1}{25-x^2}$.

For what value(s) of x is $g(x)$ undefined?

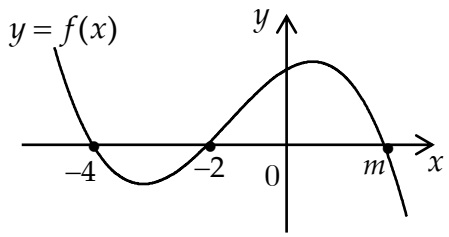
A –25 and 25

B –5 and 5

C $-\frac{1}{5}$ and $\frac{1}{5}$

D 0

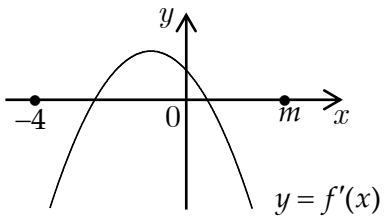
15. The diagram shows part of the graph of the cubic $y = f(x)$.



There are three roots at $x = -4$, $x = -2$ and $x = m$ as shown. There are two stationary points lying between the roots.

Which diagram shows a sketch of $y = f'(x)$?

A



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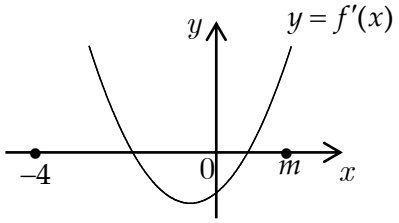
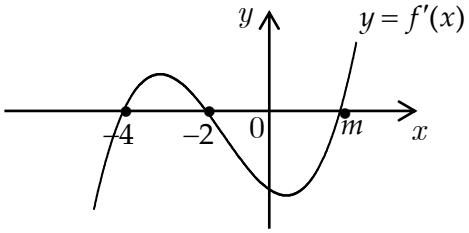
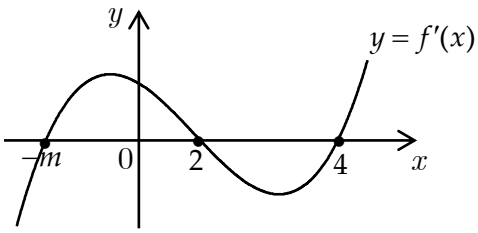


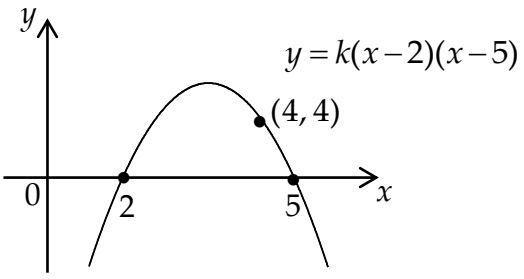
C



D



16. The equation of the parabola shown is of the form $y = k(x-2)(x-5)$.



What is the value of k?

A 2

B 1

C –1

D –2

17. What is the maximum value of $y = 2-(3x+1)(3x-1)$?

A 0

B 1

C 2

D 3

18. Given that $\boldsymbol{a}.\boldsymbol{b} = 5$ and $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = 54$, find $|\boldsymbol{a}|$.

A 7

B 9

C $10 \cdot 8$

D 270

19. If $\log_6 y = 2\log_6 x + \log_6 12$ express y in terms of x.

A $y = 2x + 12$

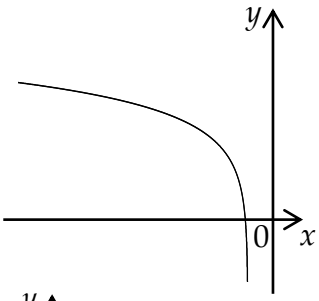
B $y = 12x^2$

C $y = x^2 + 12$

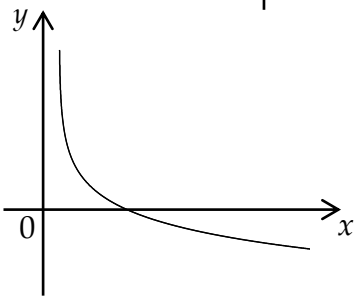
D $y = 12 \times 2^x$

20. Which diagram below shows the graph of $y = \log_5\left(\frac{1}{x}\right)$?

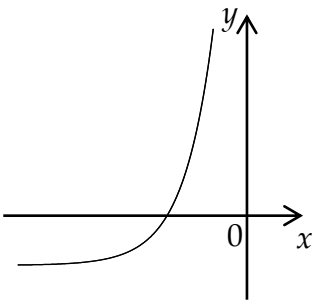
A



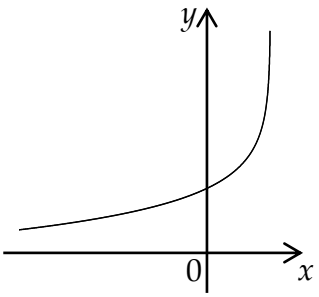
B



C



D



End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. (*a*) (i) Show that $(x+1)$ is a factor of $f(x)=2x^3+3x^2-5x-6$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Given that $\int_0^p \left(6x^2+6x-5\right)dx=6,\ p>0,$ find the value of p. **5**

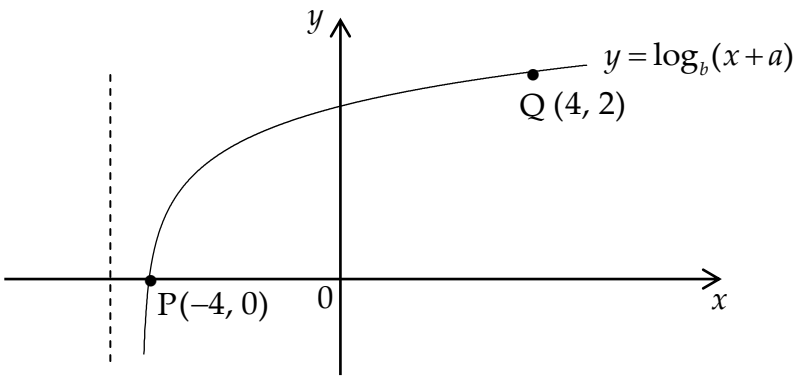
22. (*a*) Write $x^2-6x+13$ in the form $(x+a)^2+b$. **2**

(*b*) (i) Sketch the graph of $y=x^2-6x+13$.

(ii) State the range of values of y. **4**

(*c*) Write down the maximum value of $\dfrac{1}{x^2-6x+13}$. **1**

23. The diagram shows part of the curve with equation $y=\log_b(x+a)$.



The curve passes through the points P$(-4, 0)$ and Q$(4, 2)$.

Find the values of a and b. **4**

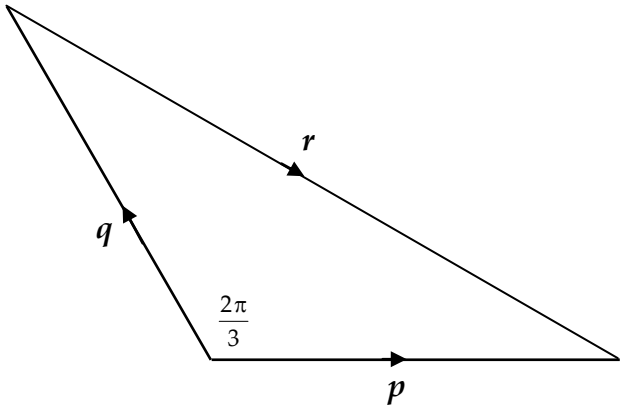
24. (*a*) Write $\cos^2 x$ in terms of $\cos 2x$. **1**

(*b*) Find $\int 4\cos^2 x\ dx$. **3**

Marks

25. In the triangle shown

$|\boldsymbol{p}| = |\boldsymbol{q}| = 2$ and $|\boldsymbol{r}| = 2\sqrt{3}$.



Find $\boldsymbol{p}.(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})$ **5**

End of question paper

H

Mathematics
Higher
Paper 2
Practice Paper F

Time allowed
1 hour 10 minutes

NATIONAL QUALIFICATIONS

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ALL questions should be attempted.

Marks

1. (*a*) Find the equation of the tangent to the curve $y = x^3 - x - 10$ at the point where $x = 1$. **5**

(*b*) Show that this line is also a tangent to the circle with equation

$$x^2 + y^2 - 6x - 8y + 5 = 0$$

and state the coordinates of the point of contact. **6**

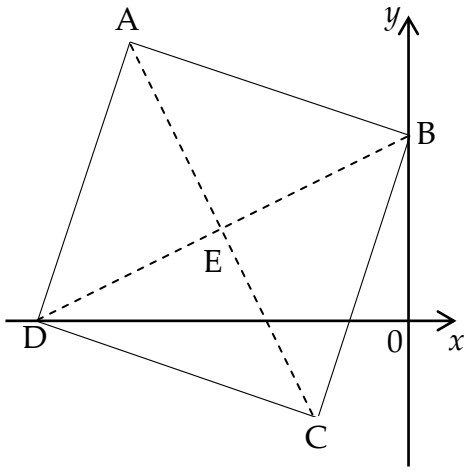
2. The diagram opposite shows a rhombus ABCD.

AC and BD are diagonals of the rhombus.

Diagonal AC has equation $2x + y + 3 = 0$.

D is point with coordinates $(-4, 0)$.

E is the point of intersection of the diagonals.



(*a*) Find the equation of diagonal BD. **4**

(*b*) Hence find the coordinates of E. **2**

3. Given that $\boldsymbol{p} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$, find the angle between $\boldsymbol{p}$ and $\boldsymbol{q}$. **5**

4. If $\cos 2x = \frac{7}{15}$, find the exact value of :

(*a*) $\cos^2 x$. **2**

(*b*) $\tan^2 x$. **3**

Marks

5. A sequence is defined by the recurrence relation $u_{n+1} = au_n + b,\ u_0 = 0.$

(*a*) If $u_1 = 25,$ write down the value of *b*. **1**

(*b*) Given that $u_3 = 31,$ find the possible values of *a*. **4**

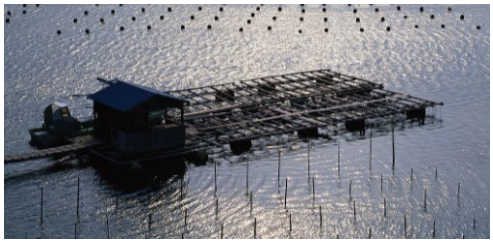
This sequence tends to a limit as $n \to \infty.$

(*c*) Find the limit of the sequence. **2**

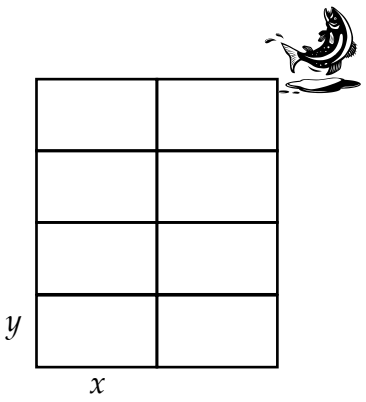
6. (*a*) Express $4\cos x^\circ - 7\sin x^\circ$ in the form $k\sin(x-a)^\circ,$ where $k > 0$ and $0 \le a < 360.$ **4**

(*b*) Hence solve $4\cos x^\circ = 7\sin x^\circ + 3$ for $0 \le x < 360.$ **4**

7. A new fish farm is being established consisting of a number of rectangular enclosures.



Each enclosure is made up from eight identical rectangular cages.



Each cage measures *x* metres by *y* metres.

The total length of edging around the top of all of the caging is 480 metres.

(*a*) Show that the total surface area, in square metres, of the top of the eight cages is given by $A(x) = 320x - \frac{20}{3}x^2.$ **3**

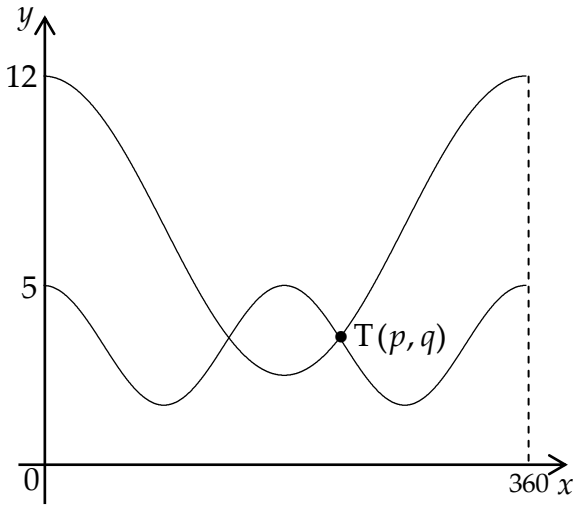
(*b*) (i) Find the value of *x* which maximises this surface area.

(ii) Hence find the dimensions of each enclosure. **7**

Marks

8. The diagram shows the graphs of $y = 2\cos(2x)^\circ + 3$ and $y = 7 + 5\cos(x)^\circ$ for $0 \le x \le 360$.

The two graphs intersect at T, which has coordinates (p, q).



(*a*) Find the exact value of $\cos p$. **6**

(*b*) Determine the value of p. **2**

End of Question Paper

Paper 1

Section A

1.	B	11.	D
2.	C	12.	A
3.	C	13.	C
4.	D	14.	B
5.	A	15.	A
6.	B	16.	D
7.	A	17.	D
8.	D	18.	A
9.	B	19.	B
10.	D	20.	B

Section B

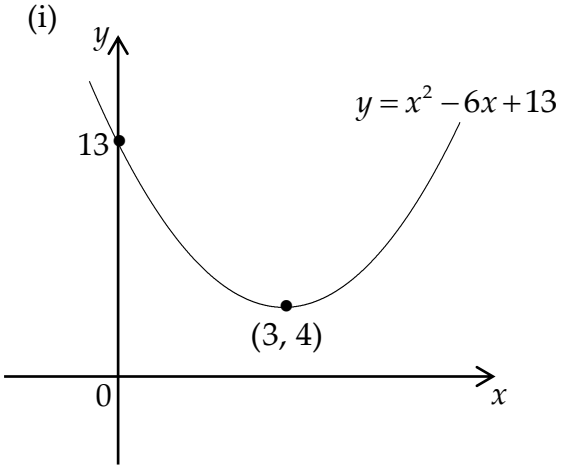
21. (*a*) (i) Show that $f(-1)=0$

(ii) $(x+1)(x+2)(2x-3)$

(*b*) $p=\frac{3}{2}$

22. (*a*) $(x-3)^2+4$

(*b*) (i)



(ii) $y \geq 4$

(*c*) $\frac{1}{4}$

23. $a=5,\ b=3$

24. (*a*) $\frac{1}{2}(1+\cos 2x)$ or $\frac{1}{2}+\frac{1}{2}\cos 2x$ (*b*) $2x+\sin 2x+c$

25. 8

Paper 2

1. (*a*) $2x - y - 12 = 0$
   (*b*) (7, 2)

2. (*a*) $x - 2y + 4 = 0$
   (*b*) E(–2, 1)

3. $108 \cdot 8^\circ$ or $1 \cdot 899$ radians

4. (*a*) $\frac{11}{15}$
   (*b*) $\frac{4}{11}$

5. (*a*) $b = 25$
   (*b*) $a = \frac{1}{5}$ or $-\frac{6}{5}$
   (*c*) $\frac{125}{4}$

6. (*a*) $\sqrt{65}\sin(x - 209 \cdot 7)^\circ$
   (*b*) $\{7 \cdot 9,\ 231 \cdot 5\}$

7. (*a*) Proof
   (*b*) (i) 24 (ii) $24\,m \times 20\,m$

8. (*a*) $\cos p = -\frac{3}{4}$
   (*b*) $221 \cdot 4$