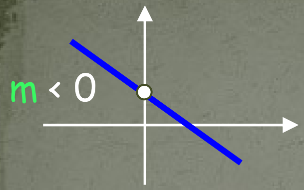
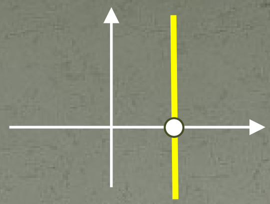
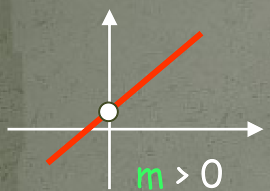
Straight Line

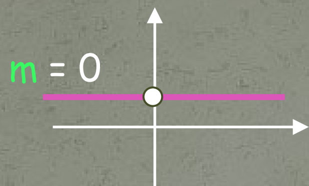
$y = mx + c$

Possible values for gradient







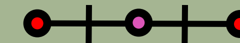


Distance between 2 points

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Terminology

$$Mid = \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$$

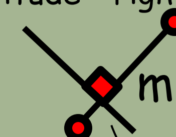


Median – midpoint

Bisector – midpoint

Perpendicular – Right Angled

Altitude – right angled



$m_1.m_2 = -1$

Form for finding line equation

$y - b = m(x + a)$

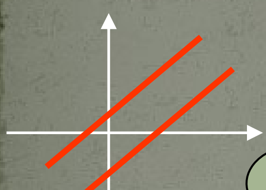
(a,b) = point on line

m = gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

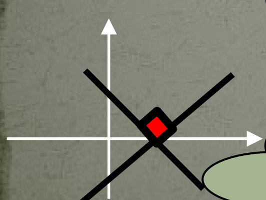
c = y intercept (0,c)

Parallel lines have same gradient

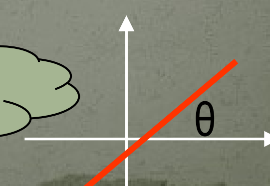


For Perpendicular lines the following is true.

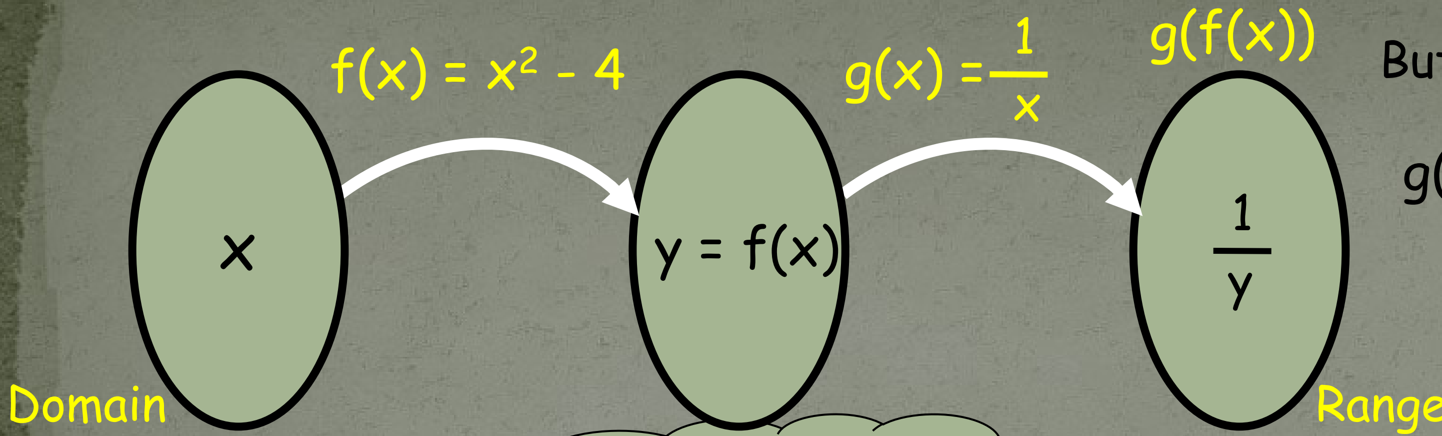
$m_1.m_2 = -1$



$m = \tan \theta$

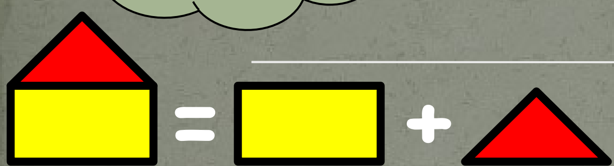


Composite Functions



A complex function made up of 2 or more simpler functions

Similar to composite Area



But $y = f(x)$ is $x^2 - 4$

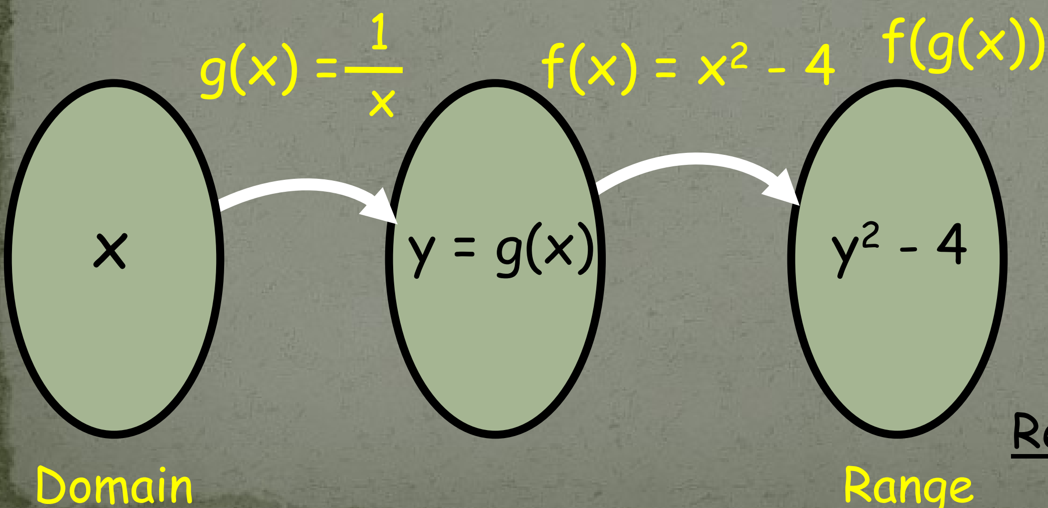
$$g(f(x)) = \frac{1}{x^2 - 4}$$

Restriction

$x^2 - 4 \neq 0$

$(x - 2)(x + 2) \neq 0$

$x \neq 2 \quad x \neq -2$



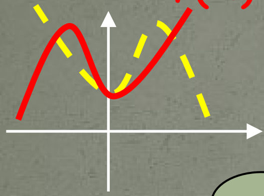
But $y = g(x)$ is $\frac{1}{x}$

$$g(f(x)) = \left(\frac{1}{x}\right)^2 - 4$$

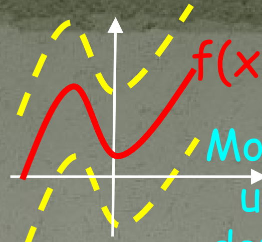
Rearranging $\frac{1}{x^2} - 4$

<u>Restriction</u> $x^2 \neq 0$

flip in
y-axis



y = f(-x)

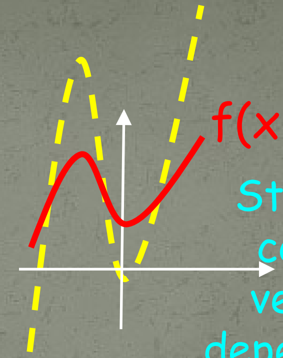


Move vertically
up or downs
depending on k

\+

\-

y = f(x) ± k



Stretch or
compress
vertically
depending on k

y = kf(x)

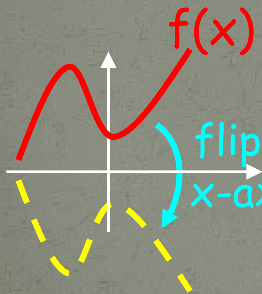
Remember we
can combine
these together !!

Graphs & Functions

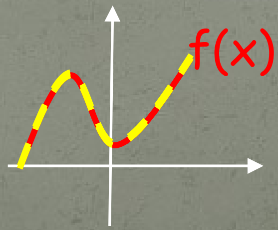
y = -f(x)

y = f(kx)

y = f(x ± k)



flip in
x-axis



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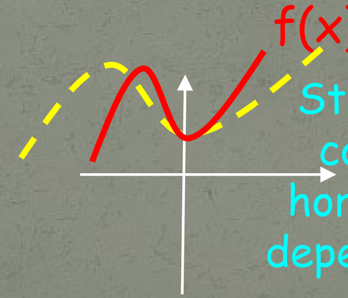
Move horizontally
left or right
depending on k



Stretch or
compress
horizontally
depending on k

Trig Formulae and Trig equations

Double Angle Formulae

$\sin 2A = 2\sin A\cos A$

$\cos 2A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

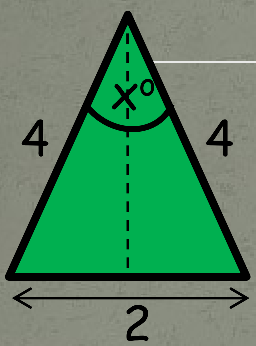
$= \cos^2 A - \sin^2 A$

Addition Formulae

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

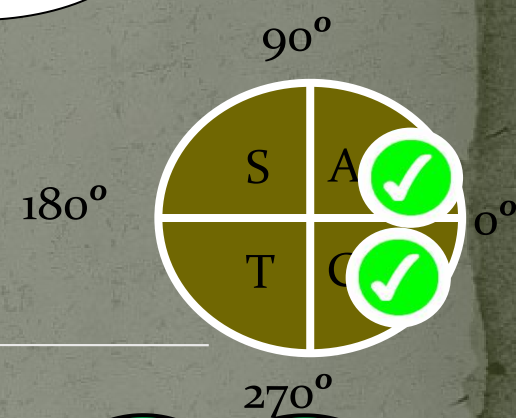
The exact value of sinx



$\sin x = 2\sin(x/2)\cos(x/2)$

$\sin x = 2\ (\frac{1}{4} + \surd(4^2 - 1^2)\ )$

$\sin x = \frac{1}{2} + 2\surd 15)$



$3\cos^2 x - 5\cos x - 2 = 0$

Let $p = \cos x$ $\quad 3p^2 - 5p - 2 = 0$

$(3p + 1)(p - 2) = 0$

$p = \cos x = 1/3$ $\qquad \cos x = 2$

$x = \cos^{-1}(1/3)$ $\qquad x =$ no sol^n

$x = 109.5^o$ and 250.5^o

Recurrence Relations

next number depends on the previous number

$U_{n+1} = aU_n + b$

Limit L is equal to

$$L = \frac{b}{(1 - a)}$$

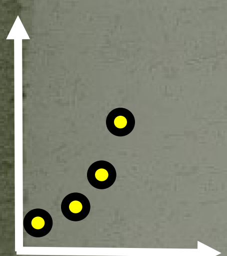
a = sets limit
b = moves limit
U_n = no effect on limit

Given three value in a sequence e.g. U_{10} , U_{11} , U_{12} we can work out recurrence relation

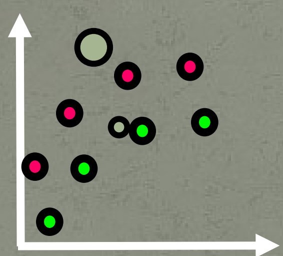
$U_{11} = aU_{10} + b$

$U_{12} = aU_{11} + b$

Use Sim. Equations



$|a| > 1$



$|a| < 1$

Limit exists when $|a| < 1$

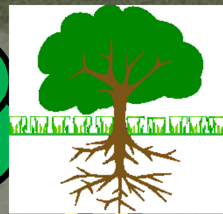
$a > 1$ then growth
$a < 1$ then decay

$+ b$ = increase
$- b$ = decrease

Basics before Differentiation

Format for differentiating

$$\sqrt[-6]{x^5} = x^{-\frac{5}{6}} = \frac{1}{x^{\frac{5}{6}}}$$



$$\sqrt[7]{x^4} = x^{\frac{4}{7}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[5]{x^{-2}} = x^{-\frac{2}{5}} = \frac{1}{x^{\frac{2}{5}}}$$

Surds

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

Indices

$$x^m \cdot x^n = x^{(m+n)} \qquad \frac{x^m}{x^n} = x^{(m-n)}$$

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} = x^{\frac{3}{4}}$$

$$\frac{x^{\frac{1}{3}}}{x^{-\frac{2}{3}}} = x$$

Working with fractions

Adding

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Subtracting

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Multiplication

$$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

Division

$$\frac{1}{2} \div \frac{4}{5}$$

$$\frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$

Basics before Integration

Surds

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

Indices

$$x^m \cdot x^n = x^{(m+n)} \qquad \frac{x^m}{x^n} = x^{(m-n)}$$

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} = x^{\frac{3}{4}}$$

$$\frac{x^{\frac{1}{3}}}{x^{-\frac{2}{3}}} = x$$

Format for integration

$$\sqrt[6]{x^5} = x^{-\frac{5}{6}} = \frac{1}{x^{\frac{5}{6}}}$$

$$\sqrt[7]{x^4} = x^{\frac{4}{7}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[5]{x^{-2}} = x^{-\frac{2}{5}} = \frac{1}{x^{\frac{2}{5}}}$$

Working with fractions

Adding

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Subtracting

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Multiplication

$$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

Division

$$\frac{1}{2} \div \frac{4}{5}$$

$$\frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$

Differentiation of Polynomials

Nature Table

x	-1	2	5
$f'(x)$	$+$	0	$-$
	↗	→ Max	↘

$f'(x)=0$
Stationary Pts
Max. / Mini Pts
Inflection Pt

Graphs
$f'(x)=0$

Leibniz Notation

$$\frac{dy}{dx} = f'(x)$$

Equation of tangent line

Straight Line Theory

Gradient at a point

Derivative
= gradient
= rate of change

$f(x) = ax^n$
then $f'x) = anx^{n-1}$

$$f(x) = x^{\frac{1}{2}}(2x-1)$$

$$f(x) = 2x^{\frac{3}{2}} - x^{\frac{1}{2}}$$

$$f'(x) = 3x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{-1}{2}}$$

$$f'(x) = 3x^{\frac{1}{2}} - \frac{1}{2\sqrt{x}}$$

$$f(x) = \frac{2}{3\sqrt[4]{x^5}}$$

$$f(x) = \frac{2x^{\frac{-5}{4}}}{3}$$

$$f'(x) = \frac{-\frac{5}{2}x^{\frac{-1}{4}}}{3} = \frac{-5}{6\sqrt[4]{x}}$$

Integration of Polynomials

IF $f'(x) = ax^n$

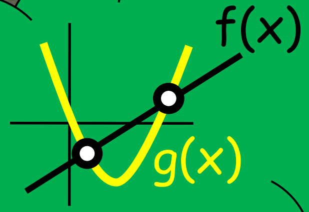
Then $I = f(x) = \dfrac{ax^{n+1}}{n+1}$

Integration is the process of finding the AREA under a curve and the x-axis

Remember to work out separately the area above and below the x-axis .

Remember to change sign to + if area is below axis.

Area between 2 curves



$A = \int_a^b f(x) - g(x)\, dx$

Finding where curve and line intersect $f(x)=g(x)$ gives the limits a and b

$$I = \int x^{\frac{1}{2}}(2x-1)dx$$

$$I = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}}\right)dx$$

$$I = \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$$

$$I = \int_1^2 \frac{1}{2\sqrt{x}}dx$$

$$I = \int_1^2 \frac{x^{-\frac{1}{2}}}{2}dx$$

$$I = \left[x^{\frac{1}{2}}\right]_1^2$$

$$= \sqrt{2} - 1$$

Easy to graph functions & graphs

Completing the square

$f(x) = a(x + b)^2 + c$

$f(x) = 2x^2 + 4x + 3$

$f(x) = 2(x + 1)^2 - 2 + 3$

$f(x) = 2(x + 1)^2 + 1$

-2	1	4	5	2
	↓	-2	-4	-2
	1	2	1	0

(x+2) is a factor since no remainder

Factor Theorem

x = a is a factor of f(x) if f(a) = 0

If finding coefficients Sim. Equations

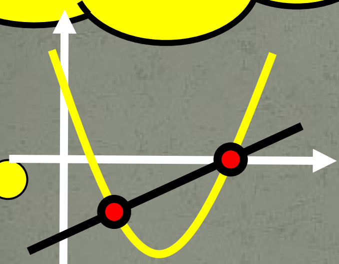
Polynomials

Functions of the type

$f(x) = 3x^4 + 2x^3 + 2x + x + 5$

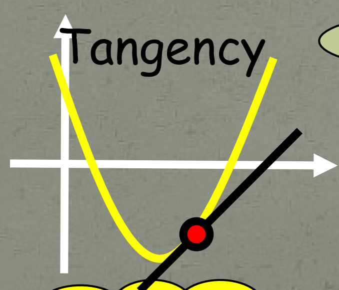
Discriminant of a quadratic is $b^2 - 4ac$

Degree of a polynomial = highest power



$b^2 - 4ac > 0$

Real and distinct roots

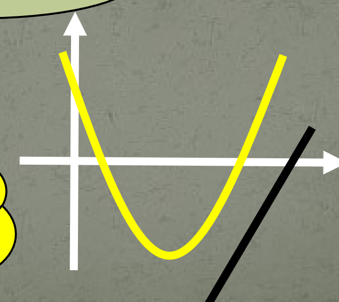


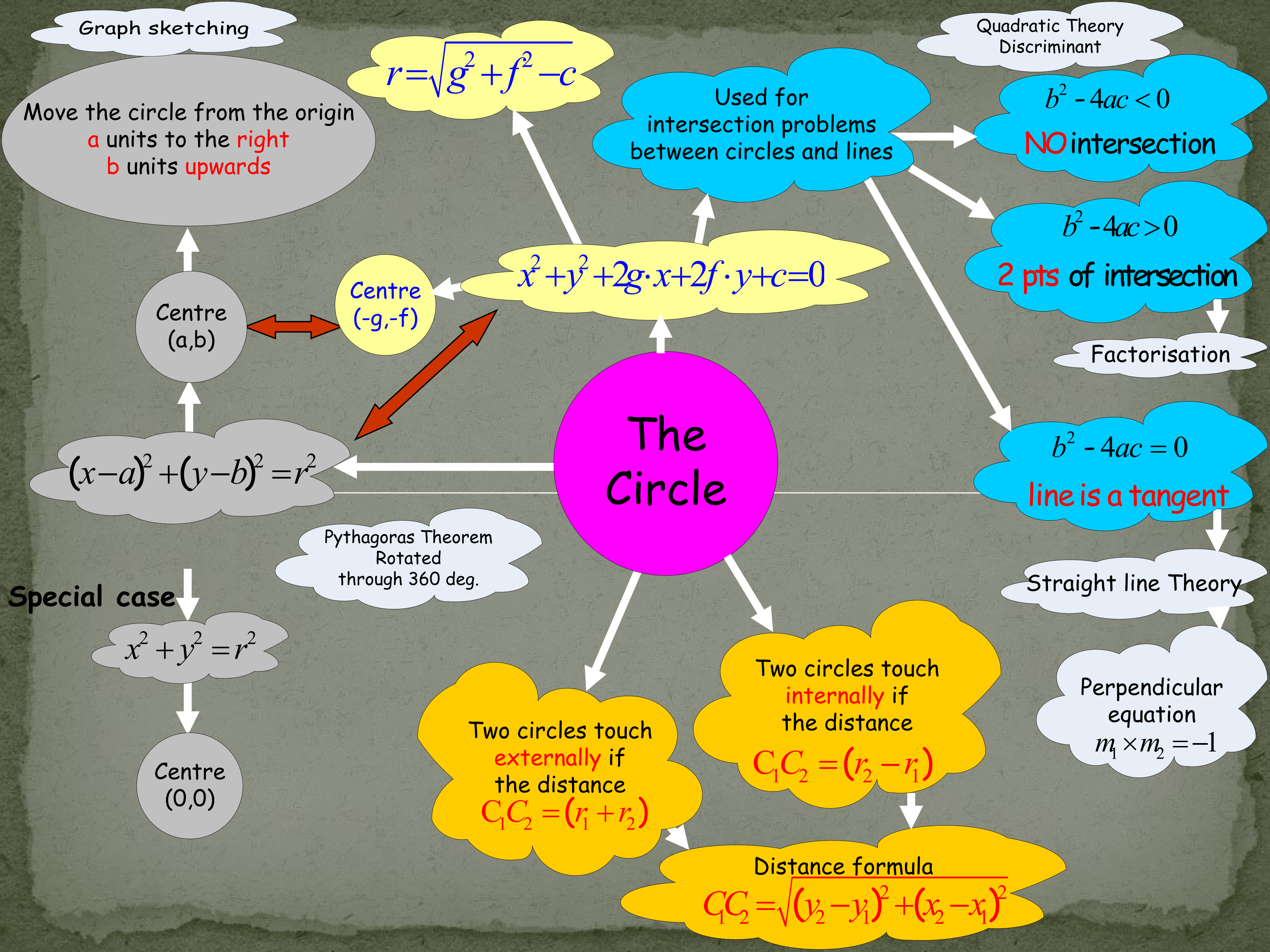
$b^2 - 4ac = 0$

Equal roots

$b^2 - 4ac < 0$

No real roots



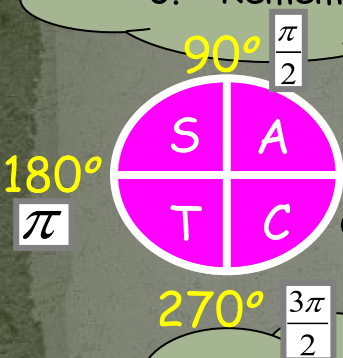


The Circle
Graph sketching
Move the circle from the origin
a units to the right
b units upwards
$r=\sqrt{g^2+f^2-c}$
Used for intersection problems between circles and lines
Quadratic Theory Discriminant
$b^2-4ac<0$
NO intersection
$b^2-4ac>0$
2 pts of intersection
Factorisation
$b^2-4ac=0$
line is a tangent
Straight line Theory
Perpendicular equation
$m_1 \times m_2 = -1$
$x^2+y^2+2g\cdot x+2f\cdot y+c=0$
Centre (-g,-f)
Centre (a,b)
$(x-a)^2+(y-b)^2=r^2$
Pythagoras Theorem Rotated through 360 deg.
Special case
$x^2+y^2=r^2$
Centre (0,0)
Two circles touch externally if the distance
$C_1C_2=(r_1+r_2)$
Two circles touch internally if the distance
$C_1C_2=(r_2-r_1)$
Distance formula
$C_1C_2=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

Trigonometry
sin, cos , tan

Basic Strategy for Solving Trig Equations

1. Rearrange into sin =
2. Find solution in Basic Quads
3. Remember Multiple solutions



90°

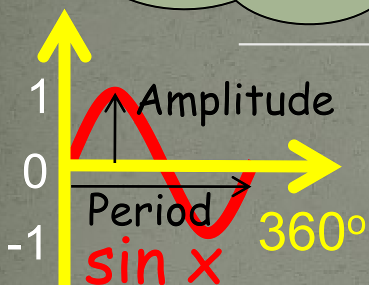
degrees → ÷180 then × π → radians $\frac{\pi}{2}$

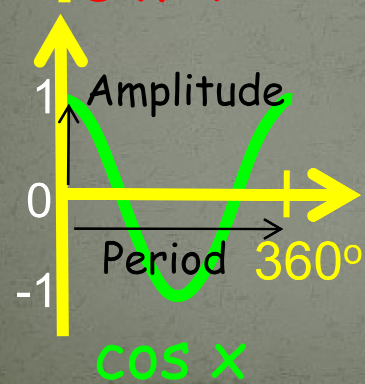
radians → then × 180 ÷ π → degrees

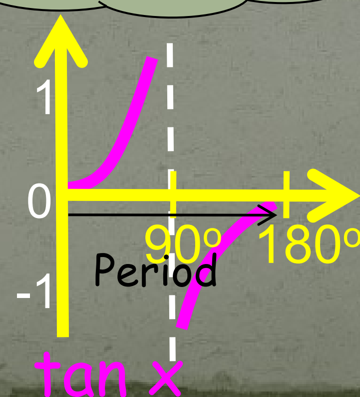
Exact Value Table

	sin	cos	tan
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

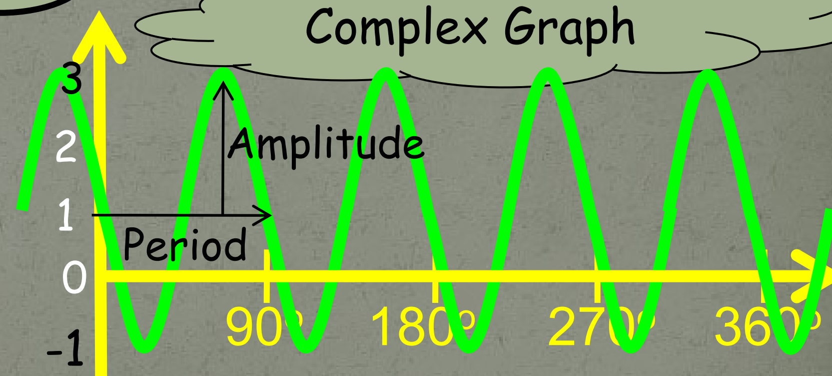
Basic Graphs







Complex Graph



$y = 2\sin(4x + 45^{\circ}) + 1$

Max. Value = 2+1 = 3 Period = 360 ÷ 4 = 90°

Mini. Value = -2+1 -1 Amplitude = 2

Vector Theory

Magnitude & Direction

Basic properties

Addition $\overrightarrow{PQ} + \underline{a} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} u_1 + a_1 \\ u_2 + a_2 \\ u_3 + a_3 \end{pmatrix}$

same for subtraction

Scalar product $k\underline{a} = \begin{pmatrix} ku_1 \\ ku_2 \\ ku_3 \end{pmatrix}$

Magnitude $|\underline{a}| = \sqrt{a_1 + a_2 + a_3}$

scalar product

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

Component form $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$

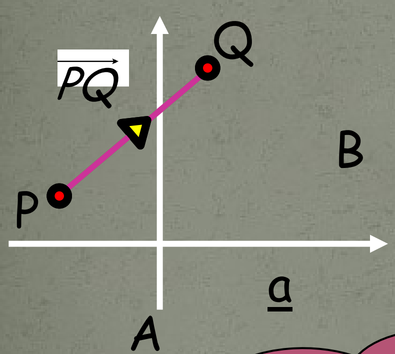
2 vectors perpendicular if

$\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3 = 0$

Notation

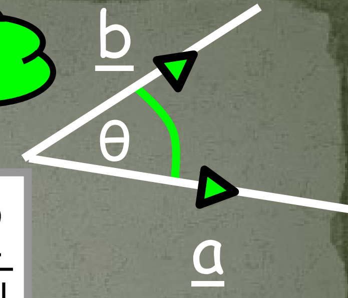
Component form $\overrightarrow{PQ} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$

Unit vector form $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$



Vectors are equal if they have the same magnitude & direction

Tail to tail



$$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|}$$

$$\underline{a}\cdot(\underline{b}+\underline{c}) = \underline{a}\cdot\underline{b}+\underline{a}\cdot\underline{c}$$

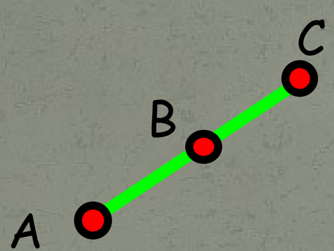
$$\underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a}$$

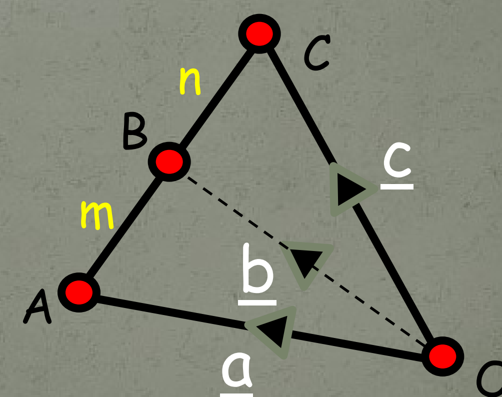
properties

Angle between two vectors

Vector Theory
Magnitude & Direction

Section formula





Points A, B and C are said to be Collinear if

$$\overrightarrow{AB} - k\overrightarrow{BC}$$

B is a point in common.

$$\underline{b} = \frac{n}{m+n}\underline{a} + \frac{m}{m+n}\underline{c}$$