

**Higher Mathematics Booklet  
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The following items will be issued throughout the year in separate booklets. Please keep them safe and return them to you teacher at the end of the year.

- Higher Objective Test Booklet
- Additional Practice Papers
- Revision Booklets

**Higher Mathematics Booklet  
DETAILS**

Homework

All questions in the homework should be attempted.  
The questions in the box contain important revision of previous topics.

Care of booklet

- Please keep these booklets in good condition and return at the end of the year
- Please do not mark the copies with pen

**The Straight Line  
Homework 1**

1. Find the angles which the lines joining the following pairs of points make with the positive direction of the x-axes (OX)

(a)  $(0, 0), (1, 1)$     (b)  $(0, 0), (\sqrt{3}, 1)$     (c)  $(-3, 6), (5, 4)$

**(6)**

2. Find the equations of the lines through the following points, with the given gradients. Give your answers in the form  $y - b = m(x - a)$

(a)  $(2, 3), 4$     (b)  $(0, 4), -2$     (c)  $(-4, 2), \frac{1}{2}$

**(3)**

3. Find the equation of the line through  $(-3, 2)$  and parallel to the line  $2x + 3y + 4 = 0$ .

**(3)**

4. Find the equation of the line connecting points  $(1, -4)$  and  $(3, -6)$ .

**(2)**

5. P is the point  $(4, 0)$ , Q is  $(0, -3)$  and R is  $(-5, -1)$ . Given that PQRS is a parallelogram, obtain the equation of RS.

**(3)**

6. Prove that the points A $(-2, 1)$ , B $(-1, 0)$  and C $(7, -8)$  are collinear.

**(3)**

7. Solve the equation  $x(x + 29) - 9(x - 2) = 4x + 3$

**(3)**

8. Solve the following simultaneous equations

$$4x + 2y = 8$$

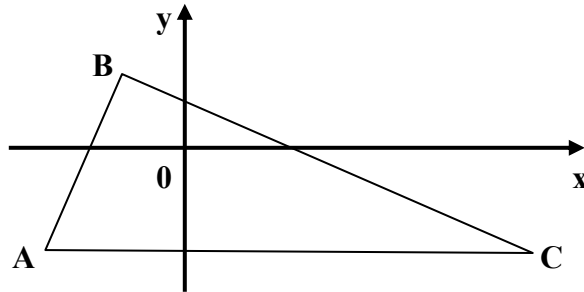
$$x + 3y = 2$$

**(4)**

**TOTAL    27**

**The Straight Line  
Homework 2**

1. Find the equation of the median AD of triangle ABC where the coordinates of A, B and C are (-2, 3), (-3, -4) and (5, 2) respectively. (3)
  
2. Find the equation of the altitude PR of triangle OPQ where the coordinates of O, P and Q are (-2, 3), (-3, -4) and (5, 2) respectively. (3)
  
3. Find the equation of the perpendicular bisector of the line joining A(2, -1) and B(8, 3). (3)
  
4. A triangle ABC has vertices A(-4, -3), B(-2, 1) and C(6, -3)



- (a) Show that the triangle ABC is right angled at B. (3)
- (b) The medians AD and BE intersect at M.
  - (i) Find the equations of AD and BE. (6)
  - (ii) Find the coordinates of M (2)

5. Solve  $x(x-5)+2(x-14)=0$  (3)
  
6. Simplify each expression
 

(a) $4b^9 \times 2b^{-6}$	(b) $(6h^5)^3$	(c) $\frac{48f^{10}}{6f^{-4}}$	(3)
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**TOTAL 26**

**Functions  
Homework 3**

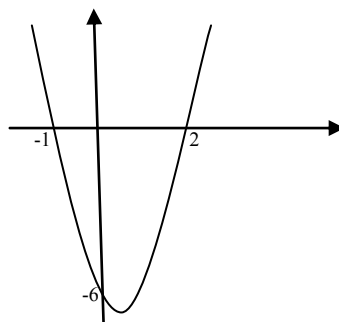
1. Convert to radians (simplify where possible)  
(a)  $60^\circ$       (b)  $225^\circ$       (c)  $330^\circ$  (3)
  
2. Express  $160^\circ$  in radians correct to 2 decimal places. (1)
  
3. Convert to degrees  
(a)  $\frac{\pi}{4}$  radians      (b)  $\frac{4\pi}{3}$  radians      (c)  $\frac{9\pi}{5}$  radians (3)
  
4. Sketch the graph of  $y = 2\sin(3x + 60)^\circ$ ,  $0^\circ < x < 180^\circ$  (3)
  
5.  $f(x) = 2x - 1$ ,  $g(x) = \frac{x^2 + 1}{x^2 - 1}$  and  $h(x) = g(f(x))$   
(a) Find a formula for  $h(x)$ . (2)  
(b) For what values of  $x$  is  $h$  undefined? (2)
  
6.  $f(x) = x^2 + x$  and  $g(x) = 3x - 1$   
Evaluate  $g(f(2))$  and  $f(g(-1))$  (4)

7. Express the following as exact values  
(a)  $\sin 225^\circ$       (b)  $-\tan 150^\circ$  (2)
  
8. Solve  $2\cos 3x = 1$ , for  $0^\circ \leq x \leq 360^\circ$  (4)

**TOTAL    24**

**Functions  
Homework 4**

1. The following sketch shows the graph of  $f(x)$ .



Make separate sketches of the following graphs:

- (a)  $y = -f(x)$
- (b)  $y = f(-x)$
- (c)  $y = 3f(x)$  (3)

2.  $g(x) = 2^x$ . Make separate sketches of the following graphs:

- (a)  $y = g(x)$
- (b)  $y = 2^x + 3$
- (c)  $y = 2^{(x-4)}$  (3)

3. (a) Show that the function  $f(x) = 2x^2 + 8x - 3$  can be written in the form  $f(x) = a(x+b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. (3)

- (b) Hence, or otherwise, find the coordinates of the turning point of the function  $f$ . (1)

4. (a) Express  $7 - 2x - x^2$  in the form  $a - (x+b)^2$  and write down the values of  $a$  and  $b$ . (3)

- (b) State the minimum value of  $\frac{1}{7 - 2x - x^2}$  and justify your answer. (2)

5. Find the coordinates of the points of intersection of the curve  $y = 3x^2 - 5x + 2$  and the line  $y = 4x + 2$ . (4)

6. Evaluate  $64^{\frac{4}{3}}$  (2)

**TOTAL 21**

**Recurrence Relations  
Homework 5**

1. (a) Write down the next 3 terms of the sequence given by the recurrence relation

$$T_{n+1} = 0.9T_n + 2, \quad T_0 = 40 \quad (2)$$

- (b) What is the limit of this sequence? (2)

2. A car designer has calculated that water escapes from an engine's cooling system at a rate of 20% per month. The system was initially filled with 36 litres of coolant and each month she adds 4 litres of coolant to the system.

- (a) Find a recurrence relation to describe this. (2)

- (b) Calculate the volume of coolant in the system after 6 months. (2)

- (c) If the coolant drops below 18 litres the engine will overheat.  
Is the engine in danger of overheating? (explain fully) (2)

3. A recurrence relation is defined by  $U_{n+1} = aU_n + b$  for some constants  $a$  and  $b$ .

- (a) If  $U_2 = 190, U_3 = 430$  and  $U_4 = 910$  calculate the values of  $a$  and  $b$ . (3)

- (b) What is the initial value,  $U_0$ , of this sequence? (2)

4. (a) Given that  $f(x) = x^2 - 2$  and  $g(x) = 3x + 1$ , find functions

$$k(x) = f(g(x)) \text{ and } h(x) = g(f(x)).$$

- (b) If  $f(g(p)) = f(g(p)) + 16$  for some number  $p$ , find the value(s) of  $p$ . (6)

**TOTAL    21**

**Differentiation  
Homework 6**

1. Differentiate the following functions with respect to  $x$ .

(a)  $f(x) = 3x^2 - 6x$       (b)  $f(x) = (2x + 3)^2$       (3)

2. Differentiate each of the following functions with respect to the relevant variable

(a)  $f(x) = x^3(x - x^2)$       (b)  $f(t) = t^{\frac{1}{2}}(t^2 + t^{\frac{3}{2}})$       (c)  $g(p) = \frac{x^5 + 2x^2}{x^4}$       (6)

3. Calculate the rate of change of  $f(x) = x^3 - 2x^2$  at  $x = 2$ .      (2)

4. Find the equation of the tangent to the curve with equation  $y = 4x^2 + 3$  at the point where  $x = 1$ .      (4)

5. The gradient of the tangent to the curve  $y = ax^2 + b$  is equal to 30 at the point (3, 1). Find the values of  $a$  and  $b$ .      (4)

6. Triangle PQR has vertices (2, 3), (-3, -2) and (3, 0) respectively.

- (a) Find the equations of the perpendicular bisectors of sides RQ and PR.  
(b) Find the coordinates of the point T, the point of intersection of these two lines.      (6)

**TOTAL      25**

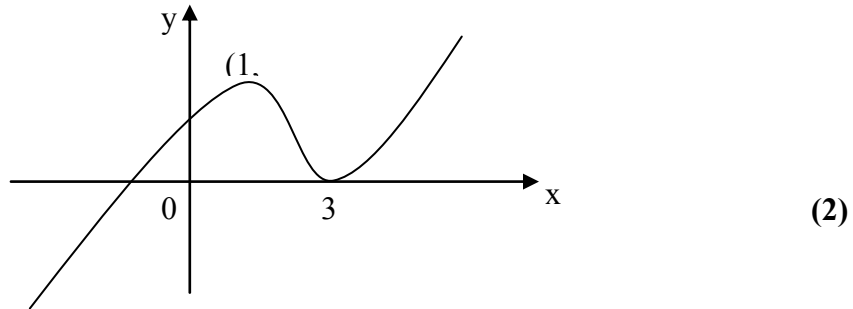


**Differentiation  
Homework 7**

1. Differentiate the following functions with respect to  $x$ .

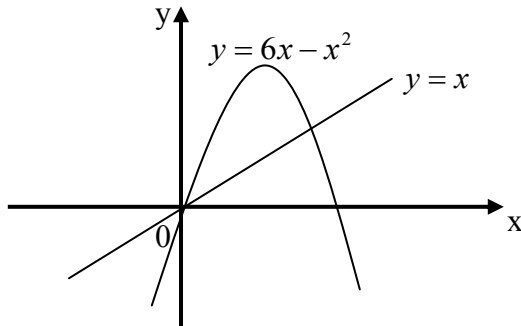
(a)  $y = \frac{5x^4 - 3x^2 - 14}{x^2}$                       (b)  $y = \frac{(x-3)^2}{3\sqrt{x}}$                       (6)

2. For the following graph of  $f(x)$ , sketch the graph of  $f'(x)$ .



3. For what values of  $x$  is the function  $h(x) = 2x^3 + 3x^2 - 12x + 1$  decreasing?                      (4)

4. The diagram shows a sketch of the parabola  $y = 6x - x^2$  and the line  $y = x$ .



(a) Find the gradient of the tangent to the parabola at the point  $(0, 0)$ .                      (2)

(b) Hence or otherwise calculate the size of the angle between the line  $y = x$  and the tangent to the parabola at the point  $(0, 0)$ .                      (3)

5. Sketch the graph of the curve  $y = 2x(x - 4)^2$ , indicating all important points.                      (5)

6. Sketch the graph of  $y = 3\cos(2x - 30)^\circ$ ,  $0^\circ < x < 360^\circ$                       (3)

**TOTAL      25**

**Polynomials**  
**Homework 8**

1. (a) Show that  $x = 2$  is a root of the equation  $2x^3 + x^2 - 13x + 6 = 0$  (1)  
(b) Hence find the other roots. (3)
  
2. Given that  $(x - 4)$  is a factor of  $h(x) = x^3 - 5x^2 - ax + 80$ , find  
(a) The value of  $a$  (2)  
(b) Hence solve the equation  $x^3 - 5x^2 - ax + 80 = 0$  when  $a$  takes this value. (2)
  
4. When  $x^4 - x^3 + x^2 + ax + b$  is divided by  $x - 1$  the remainder is 0, and when divided by  $x - 2$  the remainder is 11. Find  $a$  and  $b$ . (6)

4. Find the equation of the perpendicular bisector of CD where the coordinates of C and D are (4,3) and (2,-3) respectively. (4)
  
5. Find the equation of the tangent to the curve  $y = x^3 - 3x + 1$ , at the point  $x = -1$ . (4)

**TOTAL    22**

**Quadratics  
Homework 9**

1. Decide the nature of the roots in the following equation:

$$9x^2 + 8x - 1 = 0 \quad (2)$$

2. Prove that  $y = 6 + 2x$  is a tangent to  $y = 5 - x^2$  and find the point of contact. (4)

3. Find  $p$  if the roots of  $(p + 1)x^2 + 2px + (p - 2) = 0$  are equal. (3)

4. Express  $x^2 + 2x + 7$  in the form  $(x + a)^2 + b$

(a) Hence state the minimum value of  $x^2 + 2x + 7$ .

(b) State the maximum value of  $\frac{1}{x^2 + 2x + 7}$  and the corresponding value of  $x$ . (5)

5. A statistician claims that purchasing 15 new street cleaners will reduce the amount of litter by 65% after cleaning. However, after testing it is found that a further 1400kg of litter are dropped between cleaning times. What is the minimum weight of litter found at any point after cleaning? (5)

6. Differentiate  $\frac{x^3 - 4x}{\sqrt{x}}$  (2)

**TOTAL 21**

**Integration  
Homework 10**

1. Integrate the following:

a)  $\int \frac{dx}{3x^2}$                       b)  $\int (4 - 3x)^2 dx$                       (5)

2. Evaluate  $\int_1^9 \frac{r+1}{\sqrt{r}} dr$                       (4)

3. If  $\int_0^k x^{\frac{1}{3}} dx = 12$ , find the value of k.                      (4)

4. The function  $g$ , defined on a suitable domain, is given by  $g(x) = \frac{5}{2x+1}$ .  
(a) Describe any restriction on the domain of  $g$ .                      (1)

(b) Find an expression for  $h(x)$  where  $h(x) = g(g(x))$ , giving your answer as a fraction in its simplest form.                      (3)

5. Find the rate of change of the function  $f(t) = 5t(3t - 4)$ , when  $t = -2$                       (3)

**TOTAL    20**

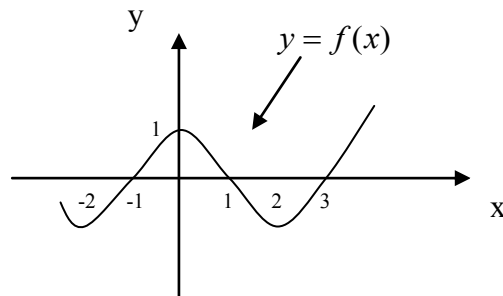
**Integration  
Homework 11**

1. Find the area between the curve  $y = 6x - 5 - x^2$  and the x axis, between the limits  $x = 2$  and  $x = 4$ . (3)

2. Find the area between the curves  $y = x^2 + 1$  and  $y = 3x + 1$  (5)

3. Given that  $\frac{dy}{dx} = 4x^3 + 1$  and that  $y = 2$  when  $x = -1$  find  $y$  in terms of  $x$ . (4)

4.



Sketch the graph of (a)  $f(-x)$   
(b)  $-f(x)$  (2)

5. Find the equation of the curve from question 4 in the form  $y = k(x - a)(x - b)(x - c)$  (4)

6. Given that  $y = 5\sqrt{x} - \frac{3}{x}$ , find  $\frac{dy}{dx}$ . (3)

**TOTAL 21**

**Further Trigonometry  
Homework 12**

1. Sketch the graph of the function  $f(x) = 2\sin(x - 30)$ . (2)

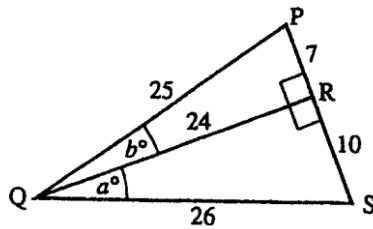
2. If  $\tan x^\circ = \frac{1}{3}$ , find the exact value of  $\sin x^\circ$ . (2)

3. (a) Write  $\cos \frac{\pi}{6} \cos x - \sin \frac{\pi}{6} \sin x$  in the form  $\cos(A + B)$  (1)

(b) Hence solve the equation  $\cos \frac{\pi}{6} \cos x - \sin \frac{\pi}{6} \sin x = \frac{\sqrt{3}}{2}$ ,  
for  $0 \leq x \leq 2\pi$  (3)

4. The diagram shows the cross-section of an adjustable ramp which is made from two right-angled triangles, PQR and RQS. Angle RQS =  $a^\circ$  and PQR =  $b^\circ$ .

Find the exact value of  $\sin(a+b)^\circ$ .



(4)

5. (a) Given that  $(-1,0)$  is a point of intersection of the curve  $y = 3x^3 + 3x^2 + x + 1$  on the x-axis. Find the other points of intersection on this axis.  
 (b) State the point of intersection on the y-axis.  
 (c) Find the coordinates of the stationary point(s) and justify the(ir) nature.  
 (d) Sketch the graph of the function. (9)

**TOTAL 21**

**Further Trigonometry  
Homework 13**

1. Solve the following equations:

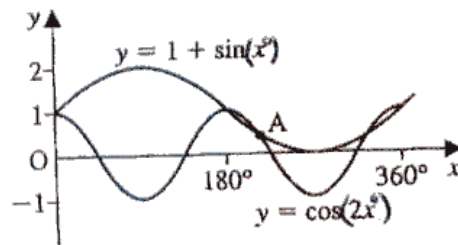
a)  $\sin 2x - \sin x = 0 \quad (0 \leq x \leq 2\pi)$

b)  $4 \cos 2x^\circ - 6 \cos x^\circ - 1 = 0 \quad (0 \leq x \leq 360).$

**(10)**

2. The diagram shows two curves  $y = \cos 2x^\circ$  and  $y = 1 + \sin x^\circ$  where  $0 \leq x \leq 360$ .

Find the x-coordinate of the point of intersection at A.



**(5)**

3. A rectangular sheet of cardboard is to be folded to form a closed cuboid with a length of  $2x$  cm, breadth  $x$  cm and a volume of  $72\text{cm}^3$

(a) Show that the height is  $\frac{36}{x^2}$ .

(b) Show that the total surface area  $A$  is given by the formula  $A(x) = 4x^2 + \frac{216}{x} \text{cm}^2$

(c) Find the dimensions of the cuboid so that the total surface area is minimum.

**(7)**

**TOTAL 22**

**The Circle  
Homework 14**

1. Find the equations of the following circles with:
  - (a) centre (0,0) and radius 3
  - (b) centre (-2,3) and radius 5

**(2)**
  
2. State the centre and radius of each of the following circles
  - (a)  $x^2 + y^2 + 4x + 8y + 16 = 0$
  - (b)  $x^2 + y^2 - 6y + 2 = 0$
  - (c)  $2x^2 + 2y^2 + 6x - 4y - 3 = 0$

**(4)**
  
3. Prove whether point (-2, 1) lies inside or outside the circle
$$x^2 + y^2 + 4x + 5y - 8 = 0$$

**(2)**
  
4. The end points of the diameter of a circle are A(-6, 4) and B(2, -2). Find the equation of the circle.

**(3)**
  
5. Find the value of k if point (-2, k) lies on the circle
$$x^2 + y^2 + 2x - 5y - 6 = 0$$

**(3)**

6.  $\triangle ABC$  has vertices A(-1, -6), B(3, 4) and C(-3, 7). Find the:
    - (a) Equation of the altitude from A
    - (b) Equation of the median from C
    - (c) Coordinates of the point of intersection between the altitude and median above.

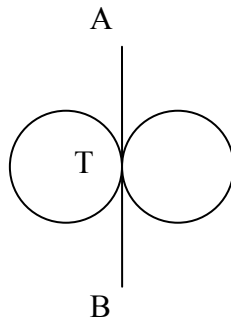
**(11)**

**TOTAL    25**



**The Circle  
Homework 15**

1. Prove that  $T(5, 1)$  lies on the circle  $(x - 1)^2 + (y + 2)^2 = 25$ , and find the equation of the tangent at  $T$ . (5)
  
2. Show that  $y = x + 1$  is a tangent to the circle  $x^2 + y^2 + 8x - 2y + 9 = 0$  and find the point of contact. (4)
  
3. The diagram shows two identically sized circles that have line  $AB$  as a common tangent with  $T$  as the point of contact. The equation of  $AB$  is  $x = -2$ . One of the circles has equation  $x^2 + y^2 - 2x + 2y - 7 = 0$ . Find the equation of the other circle. (3)



4. Calculate the length of the tangent from  $A(11, -4)$  to the circle with equation  $(x - 5)^2 + (y - 4)^2 = 51$ . (6)

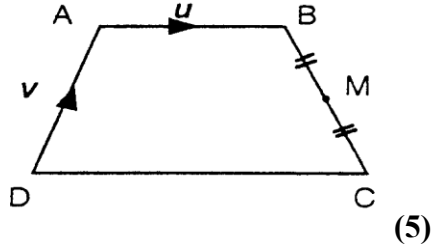
5. If  $f$  and  $g$  are functions defined on set  $\mathbf{R}$  by  $f(x) = 3 - 2x$  and  $g(x) = 4x^2 - 3$ 
  - (a) Find  $g(f(x))$
  - (b) Find  $f(f(x))$(4)

**TOTAL    22**

**Vectors**  
**Homework 16**

1. R divides  $\overrightarrow{EF}$  in the ratio 3:2. Find the coordinates of R given that the coordinates of E and F are (5, 0, 10) and (0, 10, -5) respectively. (4)

2. In this diagram  $\overrightarrow{AB} = \mathbf{u}$  and  $\overrightarrow{DA} = \mathbf{v}$ .  
 $DC = 2AB$ , and  $DC$  is parallel to  $AB$ .  
 Write down, in terms of  $\mathbf{u}$  and  $\mathbf{v}$ , the vectors:  
 (a)  $\overrightarrow{DC}$  (b)  $\overrightarrow{BC}$  (c)  $\overrightarrow{BM}$  (d)  $\overrightarrow{DM}$



3. A(1,3,-2), B(5,4,1) and C(-2,6,3) are vertices of a parallelogram ABCD.  
 Find the coordinates of the point D. (3)
4. A, B, and C are the points (3, -4, 4), (5, 2, 0) and (8, 11, -6) respectively. Show that A, B and C are collinear and find the ratio in which C divides AB. (5)

5. Find the value of k if  $(x+3)$  is a factor of  $2x^3 - 5x^2 + kx + 18$ . (3)

6. Evaluate  $\int_1^8 \sqrt[3]{x^2} dx$  (4)

**TOTAL**      **24**

**Vectors**  
**Homework 17**

1. The position vectors of the points P and Q are  $\mathbf{p} = 3\mathbf{i} + 4\mathbf{j}$  and  $\mathbf{q} = -3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$  respectively. Calculate the size of angle POQ, where O is the origin. (9)

2. (a) Show that  $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2$ .

(b) Hence evaluate this expression when  $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$ . (5)

3. If  $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} p \\ 6 \\ -2 \end{pmatrix}$

Find the value of  $p$  given that  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular. (3)

4. For what real values of  $c$  will the equation  $3x^2 - 5x + c = 0$  have real roots. (2)

5. Evaluate  $\int_1^2 \frac{u^2 + 2}{2u^4} du$  (4)

**TOTAL 23**

**Further Calculus  
Homework 18**

1. Differentiate (a)  $2x - \cos x$   
(b)  $4p^2 - 3\sin p$  (2)

2. Find  
(a)  $\int (5x^3 + x + \sin x) dx$   
(b)  $\int (2 \cos y - \frac{4}{5y^2}) dy$  (3)

3. Let  $f(x) = (2x - 3)^4$ . Find  $f'(x)$ . (2)

4. Find the area between the curve  $y = 4 \sin x$  and the x axis from  $x = \frac{\pi}{4}$  and  
 $x = \frac{2\pi}{3}$ . (4)

5. Find the equation of the tangent to the curve  $y = \frac{5}{(1-x)^2}$ , at the point  $x = -2$ . (6)

6. If  $\tan(A) = \frac{3}{4}$  and  $\tan(B) = \frac{1}{7}$ , where  $A$  and  $B$  are acute angles, show  
without a calculator that  $A + B = \frac{\pi}{4}$ . (4)

7. Prove that  $\frac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A$  (2)

**TOTAL 23**

**Further Calculus  
Homework 19**

1. Find  $f'(x)$  given that  $f(x) = \sin(2x + 3)$  (1)

2. Differentiate the following

(a)  $f(x) = \cos^2 x$       (b)  $f(x) = \frac{1}{\sin x}$  (6)

3. Find the derivative with respect to  $x$  of :-

$$f(x) = \frac{1}{\sqrt{3x-1}} + \cos(3x-1) \quad (4)$$

4. Find  $\int \sqrt{1+3x} dx$  and hence find the **exact** value of  $\int_0^1 \sqrt{1+3x} dx$  (4)

5. Integrate  $\int 6\sin(2x-1) dx$  (2)

6. (a) Show that  $3x^3 - 2x + 5$  has a root between -2 and -1.  
(b) Find this root correct to 1 decimal place. (4)

7. Find the equation of the tangent to the circle  $x^2 + y^2 = 25$  at the point (-4, 3). (4)

**TOTAL    25**

**Logarithms and Exponentials**  
**Homework 20**

1. Make sketches to show  $y = \log_3 x$  and  $y = \log_3(x - 2)$  (2)

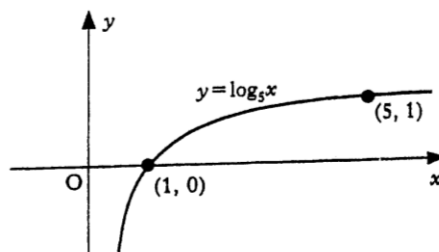
2. Find a)  $\log_3 27$    b)  $\log_9 1$    c)  $\log_5 \frac{1}{25}$  (3)

3. Simplify  $\log_4 32 - \log_4 2$  (3)

4. Solve for x:-  $\log(2x - 3) - \log x = 0$  (4)

5. Find x, if  $7^x = 65$  (3)

6. The diagram shows a sketch of part of the graph of  $y = \log_5 x$ .



a) Make a copy of the graph of  $y = \log_5 x$ .

On your copy, sketch the graph of  $y = 1 + \log_5 x$  (2)

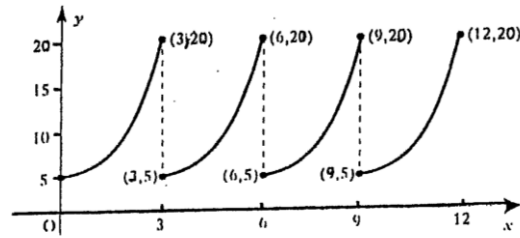
b) Find the coordinates of the point where it crosses the x-axis. (2)

7. Solve  $\tan^2 2x^\circ = 1$ , given  $0 \leq x \leq 180$  (4)

**TOTAL    23**

**Logarithms and Exponentials**  
**Homework 21**

1. A medical technician obtains this print-out of a wave generated by an oscilloscope.



The technician knows that the equation of the first branch of the graph for  $0 \leq x \leq 3$  should be of the form  $y = ae^{kx}$ .

- (a) Find the values of  $a$  and  $k$ . (4)  
 (b) Find the equation of the second branch of the curve (i.e. for  $3 \leq x \leq 6$ ). (1)

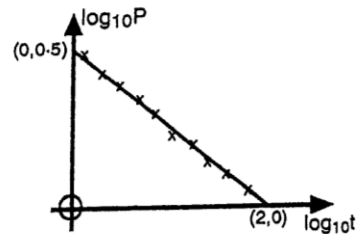
2. A lake is polluted by a cleaning agent from a dye-works. Every day, the pollutant is broken down by natural processes, (rain, the fish in the lake etc.)  
 The formula for the **percentage** of the pollutant is :-

$$P(d) = 100e^{-0.02d}$$

where  $P(d)$  = the percentage of pollutant after  $d$  days.

- (a) What percentage of the pollutant will remain in the lake after 10 days? (2)  
 (b) How long will it take for **half** the pollutant to be removed from the lake? (4)

3. A scientific experiment is carried out to find a connection between two variables  $P$  and  $t$ . Results are recorded, and when the graph of  $\log_{10} P$  against  $\log_{10} t$  is plotted, the best fitting line is drawn in as shown in the diagram.



It is thought that the relationship between  $P$  and  $t$  is of the form  $P = kt^n$

- (a) Show that  $\log_{10} P = n \log_{10} t + \log_{10} k$   
 (b) Find  $k$  and  $n$ . (5)

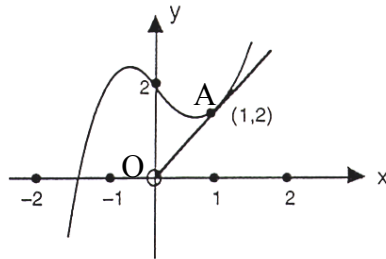
4. Find the points of intersection of the line  $3y + x = -1$  and the circle  $x^2 + y^2 + 10x + 4y + 19 = 0$ . (5)

**TOTAL 21**

**The Wave Function  
Homework 22**

1. Express  $3\sin x - 3\cos x$  in the form  $k \cos(x - \alpha)$  where  $k > 0$  and  $0 \leq \alpha \leq 360$ .  
(4)
  
2. Express  $\sqrt{3}\cos x - \sin x$  in the form  $k \sin(x + \alpha)$  where  $k > 0$  and  $0 \leq \alpha \leq 360$ .  
(4)
  
3. Express  $5\cos 8x + \sin 8x$  in the form  $r \sin(8x + \alpha)$  where  $r > 0$  and  $0 \leq \alpha \leq 2\pi$ .  
(4)
  
4. (a) Write down the maximum value of  $f(x) = 3\cos x + 4\sin x$ . (5)  
(b) Find the value of  $x$  for which this maximum occurs, for  $0 \leq x \leq 360$ . (1)

5. The straight line  $OA$  is a tangent to the curve  $y = x^3 - x + 2$  at point  $A(1,2)$



- (a) Find the equation of the line  $OA$ .
- (b) Calculate the finite area bounded by the curve,  $y$  axis and the line  $OA$ . (6)

**TOTAL 24**



**The Wave Function  
Homework 23**

1. Solve this equation algebraically for  $0 \leq \theta \leq 2\pi$ . (4)  
$$\sqrt{3} \sin \theta - \cos \theta = 1$$

2. (a) Find the minimum value of  $7 + 3 \sin x + 4 \cos x$ . (5)

(b) Hence or otherwise find the maximum value of  $f(x) = \frac{6}{7 + 3 \sin x + 4 \cos x}$   
and the value of  $x$  at which this occurs. (2)

3. The expression  $20 \sin 10T + 40 \cos 10T$  represents the displacement of a wave after  $T$  seconds and can be written in the form  $20\sqrt{5} \sin(10T + 63.4^\circ)$ .

a) Write down the amplitude and period of the wave. (2)

b) Use your values of  $R$  and  $\alpha$  to sketch the graph of  $R \sin(10T + \alpha)$  against  $T$  for  $0 \leq T \leq 36$ , showing clearly the points where the graph cuts the x-axis and any stationary points. (6)

4. For what values of  $x$  is function  $f(x) = 2x^3 - 24x - 9$  decreasing? (4)

**TOTAL    23**