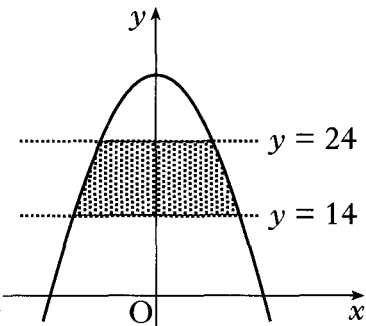
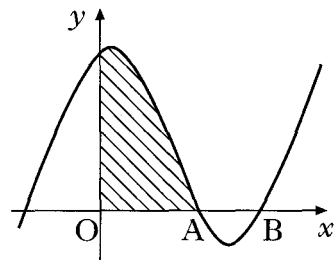
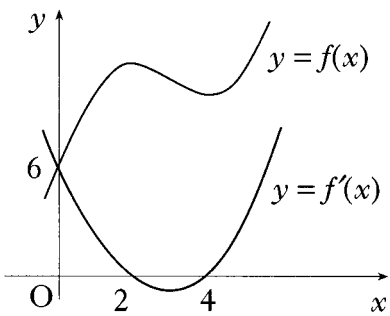
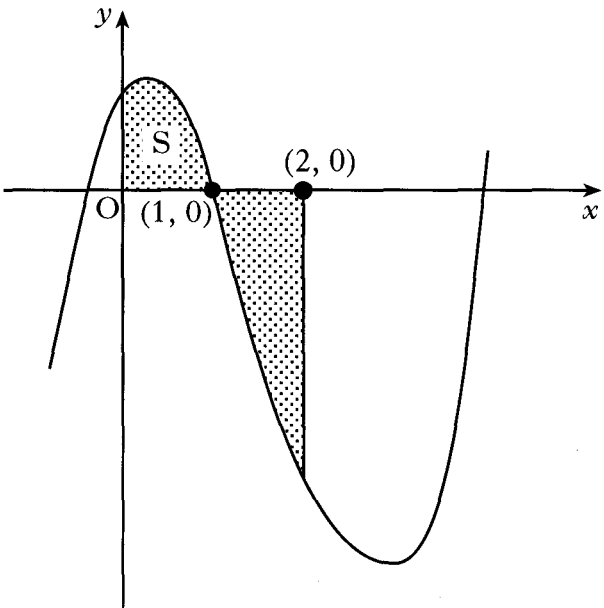
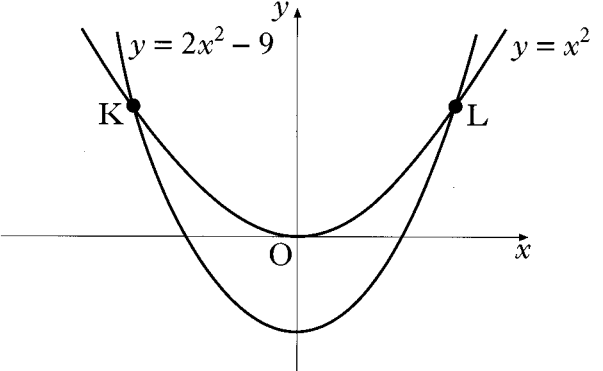
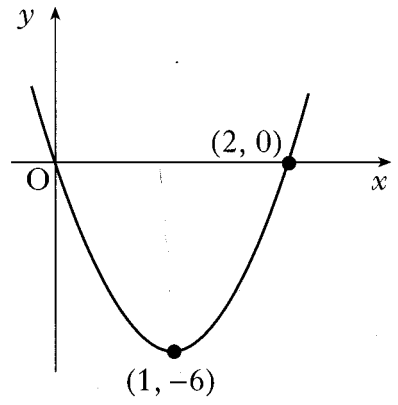
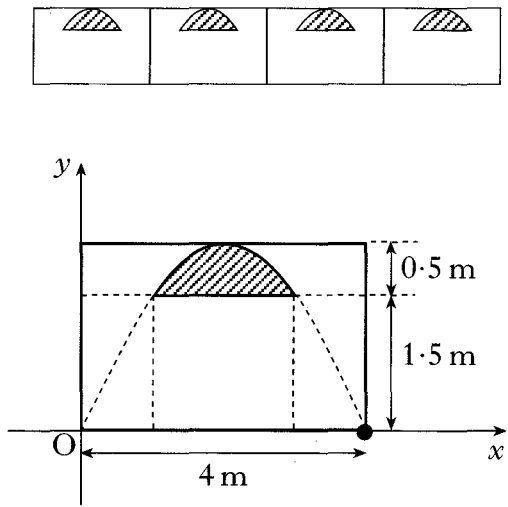
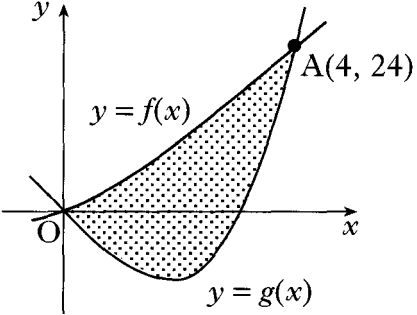


2008 P1	<p>14. Find $\int 4 \sin (2x + 3) dx$.</p> <p>A $-4 \cos (2x + 3) + c$</p> <p>B $-2 \cos (2x + 3) + c$</p> <p>C $4 \cos (2x + 3) + c$</p> <p>D $8 \cos (2x + 3) + c$</p>	2	
Ans	B		
2008 P2	<p>7. The parabola shown in the diagram has equation</p> $y = 32 - 2x^2.$ <p>The shaded area lies between the lines $y = 14$ and $y = 24$.</p> <p>Calculate the shaded area.</p>		8
Ans	$50\frac{2}{3}$		
2007 P1	<p>8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.</p> <p>(a) Show that the graph cuts the x-axis at $(3, 0)$.</p> <p>(b) Hence or otherwise find the coordinates of A.</p> <p>(c) Find the shaded area.</p>		1 3 5
Ans	<p>(a) To cut the x-axis, $y = 0$. So</p> $0 = x^3 - 4x^2 + x + 6$ $= (x - 3)(x^2 - x - 2)$ $= (x - 3)(x - 2)(x + 1)$ <p>So graph cuts x-axis at $x = -1, 3, 2$.</p> <p>(b) $(2, 0)$</p> <p>(c) $\frac{22}{3}$</p>		

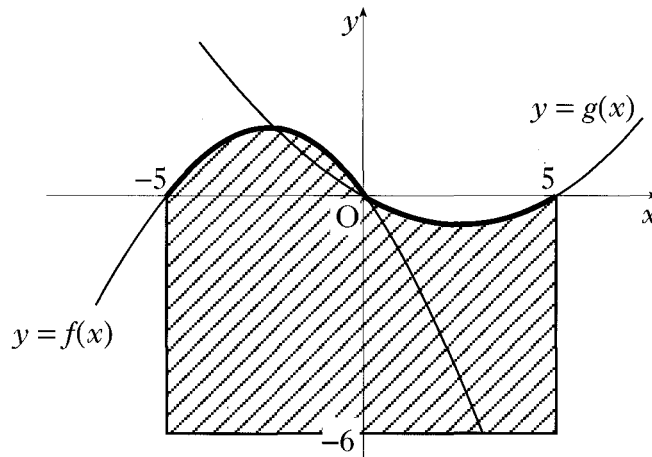
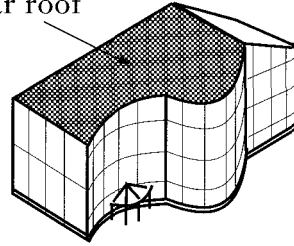
2007 P2	<p>10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.</p> <p>Both graphs pass through the point $(0, 6)$.</p> <p>The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.</p> <p>(a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:</p> <p>(i) write down the values of a and b;</p> <p>(ii) find the value of k.</p> <p>(b) Find the equation of the graph of the cubic function $y = f(x)$.</p>		3 4
Ans	<p>(a) (i) $a = 2, b = 4$</p> <p>(ii) $k = \frac{3}{4}$</p> <p>(b) $y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6$</p>		
2006 P1	<p>6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.</p> <p>The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.</p> <p>(a) Calculate the shaded area labelled S.</p> <p>(b) Hence find the total shaded area.</p>		4 3
Ans	<p>(a) $\frac{5}{4}$ or equivalent</p> <p>(b) $\frac{9}{2}$ or equivalent</p>		
2006 P2	<p>5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x.</p>		4
Ans	<p>$y = 2x^2 - 2x^3 + 5$</p>		

2005 P2	<p>1. Find $\int \frac{4x^3 - 1}{x^2} dx, x \neq 0$.</p>	4
Ans	$2x^2 + x^{-1} + c$	
2005 P2	<p>5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.</p> <p>Calculate the area enclosed between the curves.</p>	 <p>8</p>
Ans	36	
2004 P1	<p>11. The diagram shows a parabola passing through the points $(0, 0)$, $(1, -6)$ and $(2, 0)$.</p> <p>(a) The equation of the parabola is of the form $y = ax(x - b)$. Find the values of a and b.</p> <p>(b) This parabola is the graph of $y = f'(x)$. Given that $f(1) = 4$, find the formula for $f(x)$.</p>	 <p>3</p> <p>5</p>
Ans	<p>(a) $a = 6, b = 2$</p> <p>(b) $f(x) = 2x^3 - 6x^2 + 8$</p>	
2004 P2	<p>11. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.</p> <p>The second diagram shows one such window. The shaded part represents the glass.</p> <p>The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.</p> <p>Find the area in square metres of the glass in one window.</p>	 <p>8</p>
Ans	$\frac{2}{3}$	

2003 P2	<p>3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at $A(4, 24)$ and the origin.</p> <p>Find the shaded area enclosed between the curves.</p>		5
Ans	$A = \int_0^4 [(x^2 + 2x) - (x^3 - x^2 - 6x)] dx$ $42\frac{2}{3}$		
2002W P1	<p>7. Find $\int \left(\sqrt[3]{x} - \frac{1}{\sqrt{x}} \right) dx$.</p>		4
Ans	$\frac{3}{4}x^{\frac{4}{3}} - 2x^{\frac{1}{2}} + c$		

11. An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.

Solar roof



The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x) = \frac{1}{4}(-x^2 - 5x)$, $g(x) = \frac{1}{12}(x^2 - 5x)$ and the lines $x = -5$, $x = 5$ and $y = -6$.

(a) Find the area of the solar roof.

(b) Ten square units of solar cells generate a maximum of 1 kilowatt.

What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?

7

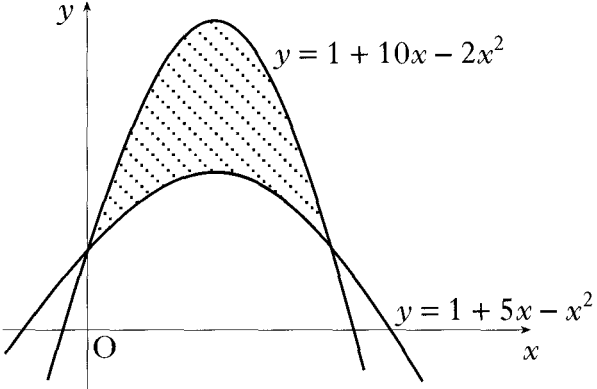
1

Ans

$$\int_{-5}^0 (f(x) - (-6))dx + \int_0^5 (g(x) - (-6))dx$$

$$63\frac{17}{36}$$

(b) 6 kilowatts

2002 P2	<p>5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.</p>		6
Ans	$\text{Area} = \int_0^5 ((1 + 10x - 2x^2) - (1 + 5x - x^2)) dx$ $20\frac{5}{6}$		
2002 P2	<p>8. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$.</p>		4
Ans	$v = \int (2(4 - t)^{\frac{1}{2}}) dt$ <p>use $v = 0$, $t = 0$ to find c</p> $v = -\frac{4}{3}(4 - t)^{\frac{3}{2}} + \frac{32}{3}$		
2001 P2	<p>6. Find $\int \frac{(x^2 - 2)(x^2 + 2)}{x^2} dx$, $x \neq 0$</p>		4
Ans	$\frac{1}{3}x^3 + 4x^{-1} + c$		

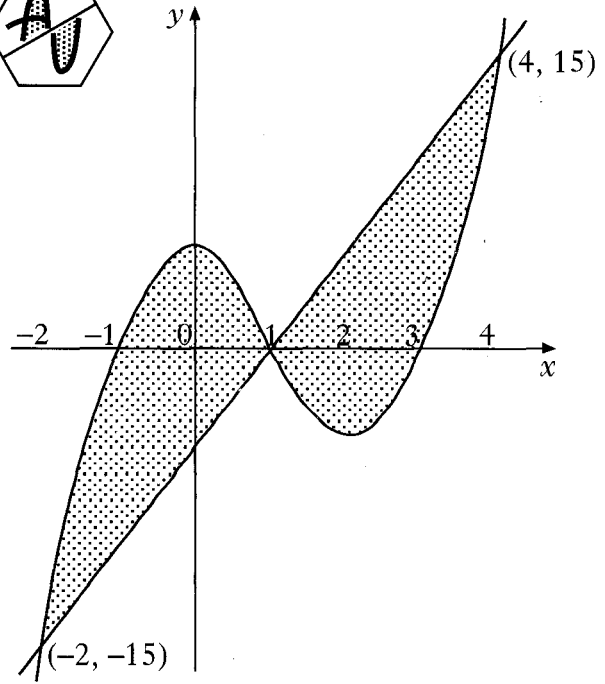
8. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.



A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

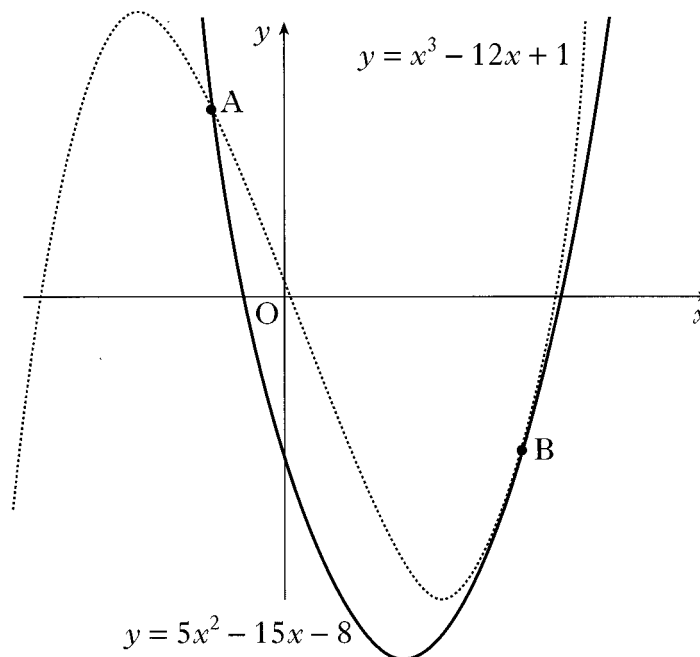
Calculate the total shaded area.



Ans

 $40\frac{1}{2}$ units²

4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.

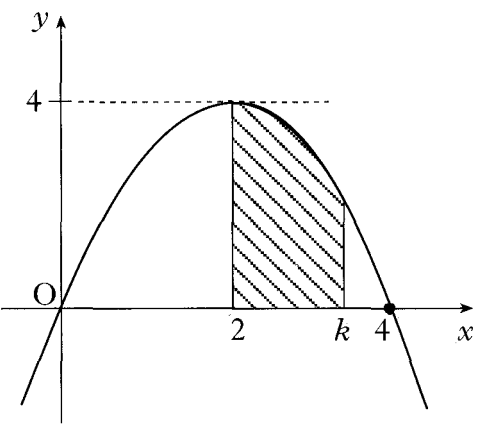


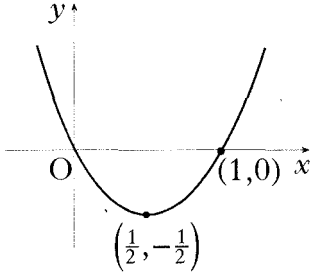
- (a) (i) Find the x -coordinates of the points on the curves where the gradients are equal.
- (ii) By considering the corresponding y -coordinates, or otherwise, distinguish geometrically between the two cases found in part (i).
- (b) The point A is $(-1, 12)$ and B is $(3, -8)$.
Find the area enclosed between the two curves.

4
1
5

Ans

- (a) (i) $x = \frac{1}{3}$ and $x = 3$
- (ii) parallel and coincident
- (b) $21\frac{1}{3}$

2000 P2	<p>4. The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.</p> <p>The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.</p> <p>(a) Find the equation of the parabola.</p> <p>(b) Hence show that the shaded area, A, is given by</p> $A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$		2 3
Ans	<p>(a) $y = 4x - x^2$</p> <p>(b) $\int_2^k (\text{function from (a)})$</p> $= -\frac{1}{3}x^3 + 2x^2$ $= -\frac{1}{3}k^3 + 2k^2 - \left(\frac{-8}{3} + 8\right)$		
Specimen 2 P1	<p>8. Functions f and g are defined on the set of real numbers by</p> $f(x) = x - 1$ $g(x) = x^2.$ <p>(a) Find formulae for</p> <p>(i) $f(g(x))$</p> <p>(ii) $g(f(x))$.</p> <p>(b) The function h is defined by $h(x) = f(g(x)) + g(f(x))$. Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h.</p> <p>(c) Find the area enclosed between this graph and the x-axis.</p>		3 3 4

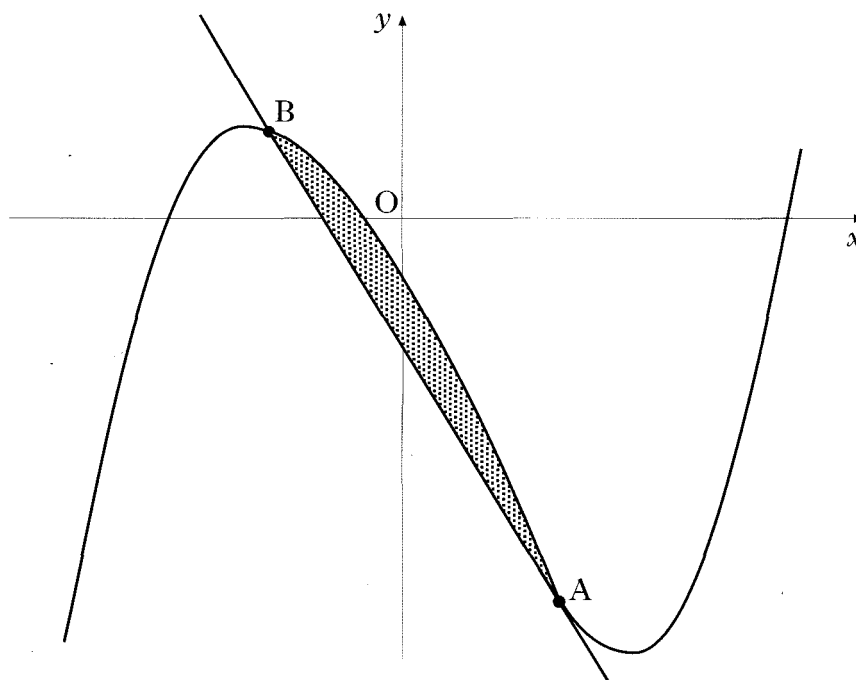
Ans	<p>(a) $f(g(x)) = x^2 - 1$, $g(f(x)) = (x - 1)^2$</p> <p>(b) $h(x) = x^2 - 1 + x^2 - 2x + 1 = 2x^2 - 2x$</p>  <p>(c) $Area = \int_0^1 (2x^2 - 2x) dx = \frac{1}{3}$</p>	
Specimen 2 PI	<p>9. Find $\int \frac{x^2 - 5}{x\sqrt{x}} dx$.</p>	4
Ans	$\int (x^{\frac{1}{2}} - 5x^{-\frac{3}{2}}) dx = \frac{2}{3}x^{\frac{3}{2}} + 10x^{-\frac{1}{2}} + C$	
Specimen 1 PI	<p>7. Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2} du$.</p>	5
Ans	1	

4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, -8).

(a) Find the equation of the tangent at A.

(b) Hence find the coordinates of B.

(c) Find the area of the shaded part which represents the land bounded by the river and the road.



(a) $y = -5x - 3$

(b) $B = (-1, 2)$

(c) $\text{area} = 1\frac{1}{3}$

Ans

3

4

3