Higher: Polynomials

Revision



21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

2008 PI

- (a) Find the coordinates of the stationary points on the curve y = f(x) and determine their nature.
- (b) (i) Show that (x-1) is a factor of $x^3 3x + 2$.
 - (ii) Hence or otherwise factorise $x^3 3x + 2$ fully.
- (c) State the coordinates of the points where the curve with equation y = f(x) meets both the axes and hence sketch the curve.

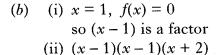
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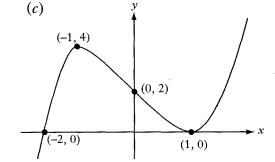
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Ans

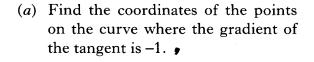
(a) (-1, 4) maximum (1, 0) minimum

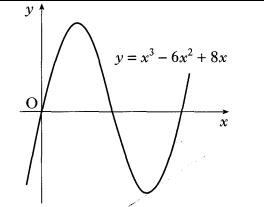




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22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.





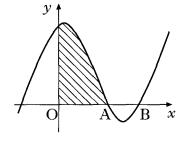
(b) The line y = 4 - x is a tangent to this curve at a point A. Find the coordinates of A.

Ans (a) (1,3), (3,-3) (b) (1,3)

2007 PI

2008 PI

- 8. The diagram shows a sketch of the graph of $y = x^3 4x^2 + x + 6$.
 - (a) Show that the graph cuts the x-axis at (3, 0).
 - (b) Hence or otherwise find the coordinates of A.



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	(a) To cut the x-axis, $y = 0$. So	
Ans	$0 = x^3 - 4x^2 + x + 6$	
	$= (x-3)(x^2-x-2)$ = $(x-3)(x-2)(x+1)$	
	So graph cuts x-axis at $x = -1$, 3, 2.	
	(b) (2,0)	
	10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.	
	Both graphs pass through the point $(0, 6)$.	
2007 P2	The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).	
2	O 2 4 x	
	(a) Given that $f'(x)$ is of the form $k(x-a)(x-b)$:	
	(i) write down the values of a and b ;	
	(ii) find the value of k .	3
	(a) (i) $a = 2$, $b = 4$	•
Ans	(ii) $k = \frac{3}{4}$	
	8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.	
. PI	(a) Show that $(x-3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.	5
2005 P.I	(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.	2
	(c) Find the greatest and least values of f in the interval $-2 \le x \le 2$.	5
	(a)(x-3)(2x-3)(x+1)	
Ans	$(b)(-1,0), (\frac{3}{2},0), (3,0)$	
Ans	(c) greatest value = 9 least value = -35	
P2	11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.	1
2005 P2	(b) Hence find the range of values of p for which all the roots of the cubic equation are real.	7
	(a)f(-1) = -1 + p - p + 1 = 0	
Ans	$(b)p \leq -1, p \geq 3$	

2004 PI	 2. f(x) = x³ - x² - 5x - 3. (a) (i) Show that (x + 1) is a factor of f(x). (ii) Hence or otherwise factorise f(x) fully. (b) One of the turning points of the graph of y = f(x) lies on the x-axis. Write down the coordinates of this turning point. 	5
Ans	(x+1)(x+1)(x-3) (-1, 0)	
2003 P2	 f(x) = 6x³ - 5x² - 17x + 6. (a) Show that (x - 2) is a factor of f(x). (b) Express f(x) in its fully factorised form. 	4
Ans	(b) $(x-2)(2x+3)(3x-1)$	
2002W PI	5. Given that $(x-2)$ and $(x+3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d .	5
Ans	c = -19, d = 6	
2001 P2	 (a) Given that x + 2 is a factor of 2x³ + x² + kx + 2, find the value of k. (b) Hence solve the equation 2x³ + x² + kx + 2 = 0 when k takes this value. 	3 2
Ans	(a) $k = -5$ (b) $x = -2, \frac{1}{2}, 1$	
2000 P2	 1. The diagram shows a sketch of the graph of y = x³ - 3x² + 2x. (a) Find the equation of the tangent to this curve at the point where x = 1. (b) The tangent at the point (2, 0) has equation y = 2x - 4. Find the coordinates of the point where this tangent meets the curve again. 	5
Ans	(a) $x + y = 1$ (b) $(-1, -6)$	

Specimen 2 P1	1. Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4
Ans	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
Specimen 1 P1	 (a) Show that (x - 1) is a factor of f(x) = x³ - 6x² + 9x - 4 and find the other factors. (b) Write down the coordinates of the points at which the graph of y = f(x) meets the axes. 	3
Ans	(a) $f(1) = 0$, $(x - 4)$, $(x - 1)$ (b) $(1,0)$, $(4,0)$, $(0, -4)$,