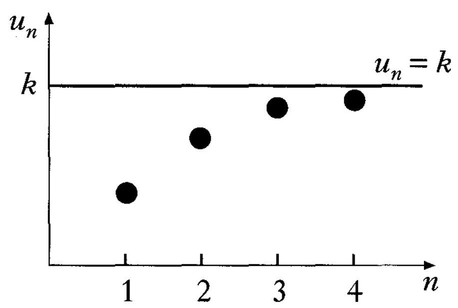
2008 P1	**4.** A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$. What is the limit of this sequence as $n \to \infty$? A -800 B -400 C 200 D 400	2
Ans	B	
2008 P1	**1.** A sequence is defined by the recurrence relation $$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$ What is the value of u_{12}? A $6{\cdot}6$ B $7{\cdot}8$ C $8{\cdot}7$ D $9{\cdot}6$	2
Ans	C	
2007 P1	**7.** A sequence is defined by the recurrence relation $$u_{n+1} = \frac{1}{4}u_n + 16, \ u_0 = 0.$$ (*a*) Calculate the values of u_1, u_2 and u_3. Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph. As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.  (*b*) (i) Give a reason why this sequence has a limit. (ii) Find the exact value of k.	3 3
Ans	(*a*) $u_1 = 16$ $u_2 = 20$ $u_3 = 21$ (*b*) (i) $-1 < \frac{1}{4} < 1$ (ii) $\frac{64}{3}$	

2006 P1	**4.** A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$. (*a*) State why this sequence has a limit. (*b*) Find this limit.	 1 2
Ans	(*a*) sequence has limit since $-1 < 0{\cdot}8 < 1$ (*b*) limit = 60	
2005 P1	**6.** (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. (*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$. (i) Express u_1 and u_2 in terms of m. (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit.	2 5
Ans	(*a*) $k = -\frac{1}{4}$ (*b*) (i) $u_1 = 3m + 5$, $u_2 = m(3m + 5) + 5$ (ii) $m = -2$	
2004 P2	**4.** A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$. (*a*) Write down the condition on k for this sequence to have a limit. (*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k.	 1 3
Ans	(*a*) $-1 < k < 1$ (*b*) $k = \frac{2}{5}$	
2003 P1	**4.** A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$. (*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. (*b*) Find the limit of this recurrence relation as $n \to \infty$.	 2 2
Ans	(*a*) $p = \frac{1}{3}$, $q = 11$ (*b*) 16½	
2002W P2	**3.** (*a*) Calculate the limit as $n \to \infty$ of the sequence defined by $u_{n+1} = 0{\cdot}9u_n + 10$, $u_0 = 1$. (*b*) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.	3 2
Ans	(*a*) 100 (*b*) $n = 7$, $u_7 = 52{\cdot}65$	

2002 P2

4. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? 3

(*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? 3

Ans

(*a*) $-1 < 0{\cdot}8 < 1$ and limit = 2·5 metres

(*b*) trim 25%

2001 P2

3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1·5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(*a*) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. 2

(*b*) Find the date and the amount of the final payment. 4

Ans

(*a*) $u_{n+1} = 1{\cdot}015u_n - 300,\ u_0 = 2500$

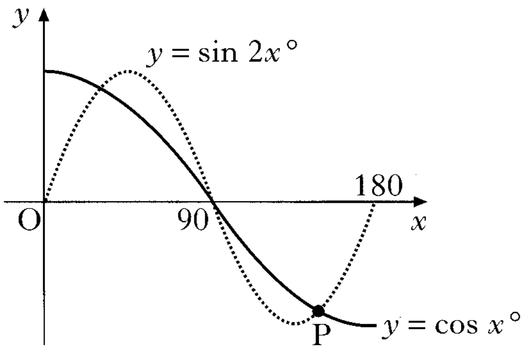
(*b*) Dec 1st, £290·68

2000 P1

5. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. 4

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. 1



Ans	$a = \frac{3}{5}$, $L = 25$	
Specimen 2 P2	**4.** Two sequences are defined by the recurrence relations $$u_{n+1} = 0{\cdot}2u_n + p, \quad u_0 = 1 \quad \text{and}$$ $$v_{n+1} = 0{\cdot}6v_n + q, \quad v_0 = 1.$$ (*a*) Explain why each of these sequences has a limit. (*b*) If both sequences have the same limit, express p in terms of q.	3
Ans	(*a*) $-1 < 0{\cdot}2 < 1$ and $-1 < 0{\cdot}6 < 1$ (*b*) $\text{Limit} = \frac{p}{0 \cdot 8}$ and $\text{Limit} = \frac{q}{0 \cdot 4} \Rightarrow p = 2q$	
Specimen 1 P1	**2.** A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1. (*a*) Explain why this sequence has a limit as n tends to infinity. (*b*) Find the **exact** value of this limit.	1 2
Ans	(*a*) $-1 < 0{\cdot}3 < 1$ (*b*) $\frac{50}{7}$	
Specimen 1 P2	**2.** Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees. A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees. Which pesticide will be more effective in the long term?	5
Ans	Pestkill	