Higher : Trigonometry Addition Formulae   Revision

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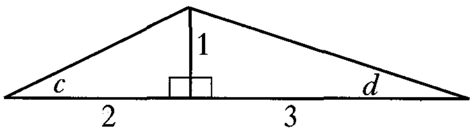
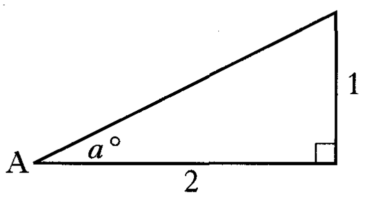
2008 P1	**9.** Given that $0 \le a \le \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$. A $\sin x + \frac{3}{5}$ B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$ C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$ D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$	2
Ans	*B*	
2008 P2	**5.** Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$.	5
Ans	*90 °, 199.5 °, 340.5 °*	
2007 P1	**6.** Solve the equation $\sin 2x° = 6\cos x°$ for $0 \le x \le 360$.	4
Ans	*x = 90, 270*	
2007 P2	**2.** The diagram shows two right-angled triangles with angles c and d marked as shown.  (a) Find the exact value of $\sin(c + d)$. (b) (i) Find the exact value of $\sin 2c$. (ii) Show that $\cos 2d$ has the same exact value.	4 4
Ans	(a) $\sqrt{\frac{1}{2}}$ (b) (i) $\frac{4}{5}$	
2006 P1	**7.** Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$.	4
Ans	$x = 0, 180, 360,\quad 60, 300$	

2006 P2

8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(*a*) Find the exact values of:

(i) $\sin a°$;

(ii) $\sin 2a°$. 4

(*b*) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$. 4

Ans

(*a*) (i) $\sin a° = \frac{1}{\sqrt{5}}$

(ii) $\sin 2a° = \frac{4}{5}$

(*b*) $\sin 3a° = \frac{11}{5\sqrt{5}}$

2005 P1

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. 4

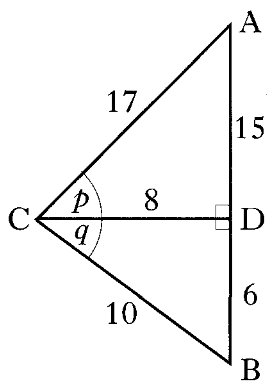
Ans

$\cos(x) = \frac{4}{5}$

$\sin(x) = \frac{3}{5}$

2005 P2

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(*a*) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$. 4

(*b*) Calculate the exact values of:

(i) $\cos(p + q)$;

(ii) $\tan(p + q)$. 3

Ans

(*a*) $\cos(p) = \frac{8}{17}$, $\sin(p) = \frac{15}{17}$

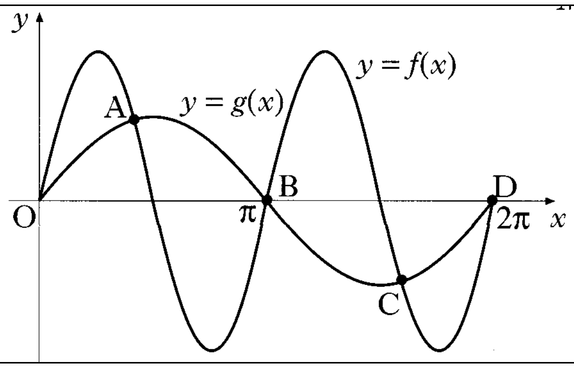
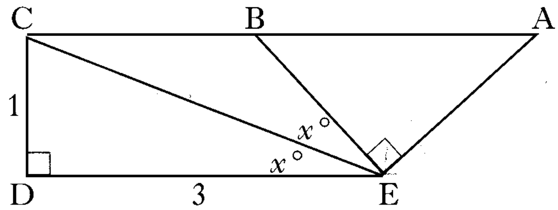
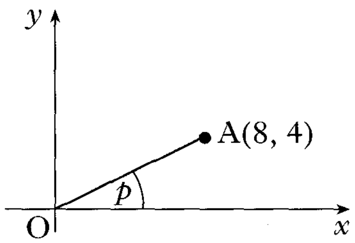
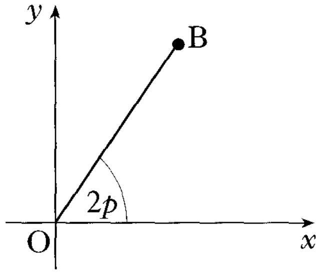
$\cos(q) = \frac{8}{10}$, $\sin(q) = \frac{6}{10}$

From sin(A+B) = sin(A)cos(B) + cos(A)sin(B)

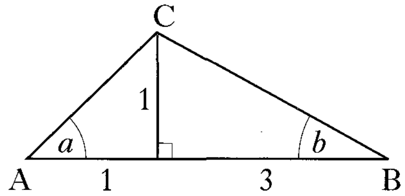
$\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \frac{84}{85}$

(*b*) (i) $-\frac{13}{85}$

(ii) $-\frac{84}{13}$

2005 P2	**8.** Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$. The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D. Show that, at A and C, $\cos x = \frac{1}{2k}$.		5
Ans	$k\sin(2x)=\sin(x)$ $k \times 2\sin(x)\cos(x)$ $\sin(x) \times (2k\cos(x) - 1) = 0$ $\sin(x) = 0$ **and** $\cos(x) = \frac{1}{2k}$ $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$ at (O), B and D **and** $\cos(x) = \frac{1}{2k}$ at A and C		
2004 P1	**10.** In the diagram angle DEC = angle CEB = $x°$ and angle CDE = angle BEA = 90°. CD = 1 unit; DE = 3 units. By writing angle DEA in terms of $x°$, find the exact value of cos(DÊA).		7
Ans	cos DÊA $= -\frac{6}{10}$		
2003 P1	**10.** A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis. (*a*) Find the exact values of: (i) $\sin(2p)$; (ii) $\cos(2p)$.		5
	The line OB is inclined at an angle $2p$ radians to the x-axis. (*b*) Write down the exact value of the gradient of OB.		1
Ans	(*a*) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (*b*) $\frac{4}{3}$		

<table>
<tr><td>2003 P2</td><td>10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places.</td><td>5</td></tr>
<tr><td>Ans</td><td colspan="2">1·23 radians only</td></tr>
<tr><td>2002W P2</td><td>5. Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \leq x < 2\pi$.</td><td>4</td></tr>
<tr><td>Ans</td><td colspan="2">$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$</td></tr>
<tr><td>2002 P1</td><td>3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.
(a) Find expressions for:
(i) $f(g(x))$;
(ii) $g(f(x))$.
(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$.</td><td>2

5</td></tr>
<tr><td>Ans</td><td colspan="2">(a) (i) $\sin(2x°)$
(ii) $2\sin(x°)$
(b) $4\sin(x°)\cos(x°) - 2\sin(x°) = 0$
$2\sin(x°)(2\cos(x°) - 1) = 0$
$\sin(x°) = 0, \quad \cos(x°) =$
0°, 60°, 180°, 300°, 360°</td></tr>
<tr><td>2002 P1</td><td>5. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$.
</td><td>4</td></tr>
<tr><td>Ans</td><td colspan="2">• $AC = \sqrt{2}$ and $BC = \sqrt{10}$
• $\sin(a+b) = \sin a \cos b + \cos a \sin b$
• $= \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$
• $= \frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4} \times \sqrt{5}} = \frac{2}{\sqrt{5}}$</td></tr>
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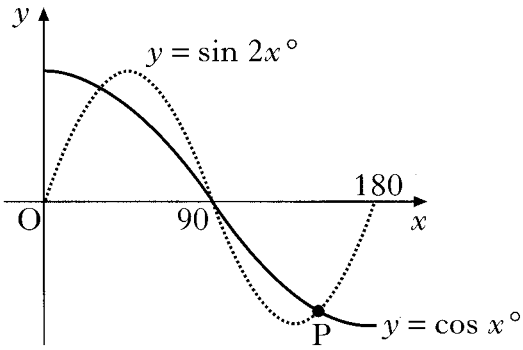
<tr><td>2001 P1</td><td>

5. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \leq x \leq 180$.

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.
Use your solutions in (*a*) to write down the coordinates of the point P.



</td><td>4

1</td></tr>
<tr><td>Ans</td><td>

(*a*) 30, 90, 150

(*b*) $\left(150, -\frac{\sqrt{3}}{2}\right)$

</td><td></td></tr>
<tr><td>2001 P1</td><td>

7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:
(i) $f(h(x))$;
(ii) $g(h(x))$.

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$.

</td><td>2

5</td></tr>
<tr><td>Ans</td><td>

(*a*) (i) $\sin\left(x + \frac{\pi}{4}\right)$;
(ii) $\cos\left(x + \frac{\pi}{4}\right)$

(*b*) (i) $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$
(ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$

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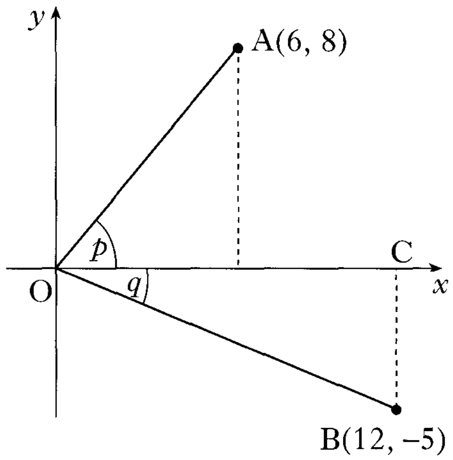
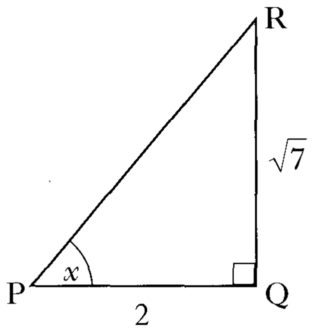
2000 P1	**1.** On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, −5). Angle AOC $= p$ and angle COB $= q$. Find the exact value of $\sin(p + q)$.  	4
Ans	$\frac{63}{65}$	
Specimen 2 P1	**7.** Using triangle PQR, as shown, find the exact value of $\cos 2x$.  	3
Ans	$\cos x = \frac{2}{\sqrt{11}}$ $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1 = -\frac{3}{11}$	
Specimen 1 P1	**4.** If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x + 30)°$ is $\frac{4\sqrt{3}+3}{10}$.	3
Ans	$\sin x° \cos 30° + \cos x° \sin 30°$ $= \frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2}$ $= \frac{4\sqrt{3}+3}{10}$	
Specimen 1 P2	**7.** (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. (*b*) **Hence** (i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$ (ii) solve this equation in the interval $0 \le x < 90$.	2 3

Ans	(*a*) $2(1 - 2\sin^2 x°) - \cos^2 x°$ $= 2 - 4\sin^2 x° - \cos^2 x°$ $= 2 - 4\sin^2 x° - (1 - \sin^2 x°)$ $= 1 - 3\sin^2 x°$ (*b*) (i) $3\sin^2 x° + 2\sin x° - 1 = 0$ (ii) $19{\cdot}5$