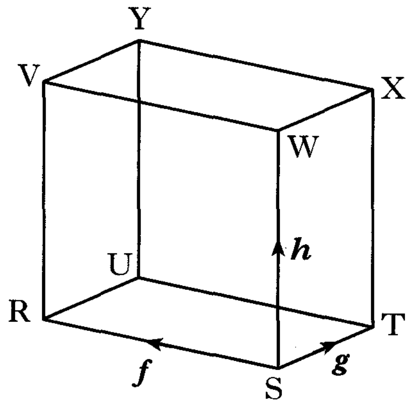
2008 P1	**3.** The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular. What is the value of k? A 0 B 3 C 4 D 5	2
Ans	C	
2008 P1	**11.** E(–2, –1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points. P lies between E and F. What is the ratio in which P divides EF? A 1:1 B 1:2 C 1:4 D 1:6	2
Ans	B	

2008 P1

12. In the diagram RSTU, VWXY represents a cuboid.

$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.

Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

2

Ans C

2008 P1

18. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.

A 0

B 14

C 26

D 28

2

Ans C

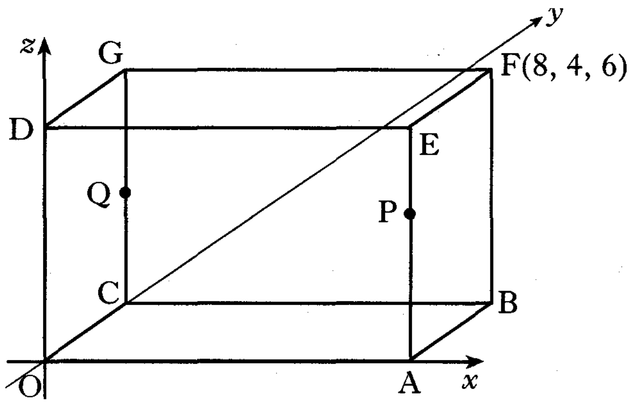
<table>
<tr><td>2008 P2</td><td>

2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.



(*a*) State the coordinates of P and Q.

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$.

(*c*) Find the size of angle QPA.

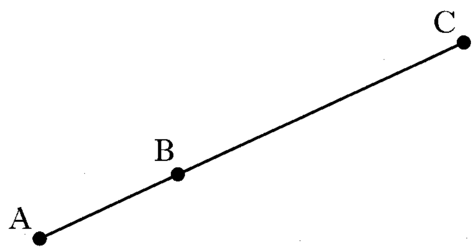
</td><td>2
2
5</td></tr>
<tr><td>*Ans*</td><td colspan="2">(a) P(8,0,4), Q(0,4,3) (b) $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix} \overrightarrow{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ (c) 83.6°</td></tr>
</table>

<table>
<tr><td>2007 P1</td><td>

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.

Find the coordinates of C.

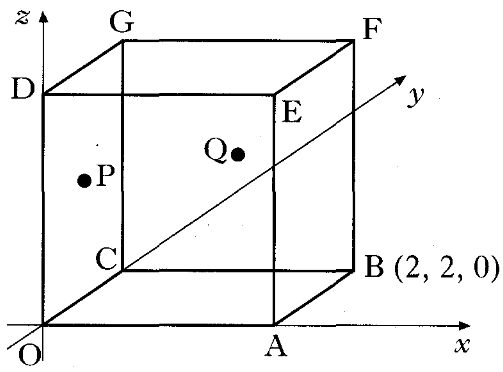


</td><td>4</td></tr>
<tr><td>*Ans*</td><td colspan="2">(7,7,8)</td></tr>
<tr><td>2007 P2</td><td>

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.

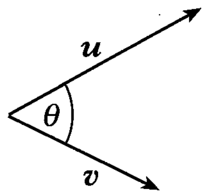


(*a*) Write down the coordinates of G.

(*b*) Find ***p*** and ***q***, the position vectors of points P and Q.

(*c*) Find the size of angle POQ.

</td><td>1
2
5</td></tr>
<tr><td>*Ans*</td><td colspan="2">(*a*) (0,2,2) (*b*) P = $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ Q = $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ (*c*) 30°/$\frac{\pi}{6}$ radians</td></tr>
</table>

2006 P1	**9.** $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.  (*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. (*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. (*c*) Deduce the only possible value of k. (*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$.	2 5 1 3
Ans	(*a*) $k^3 + 3k^2 - k - 2 = 1$ *and complete* (*b*) $(k + 3)(k + 1)(k - 1)$ stated explicitly (*c*) $k = 1$ (*d*) $\cos\theta = \frac{1}{11}$	
2006 P2	**6.** P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$. (*a*) Write down $\overrightarrow{PQ}$ in component form. (*b*) Calculate the length of $\overrightarrow{PQ}$. (*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$.	1 1 1
Ans	(*a*) $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ (*b*) $\lvert\overrightarrow{PQ}\rvert = 5$ (*c*) $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ or $\begin{pmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$	

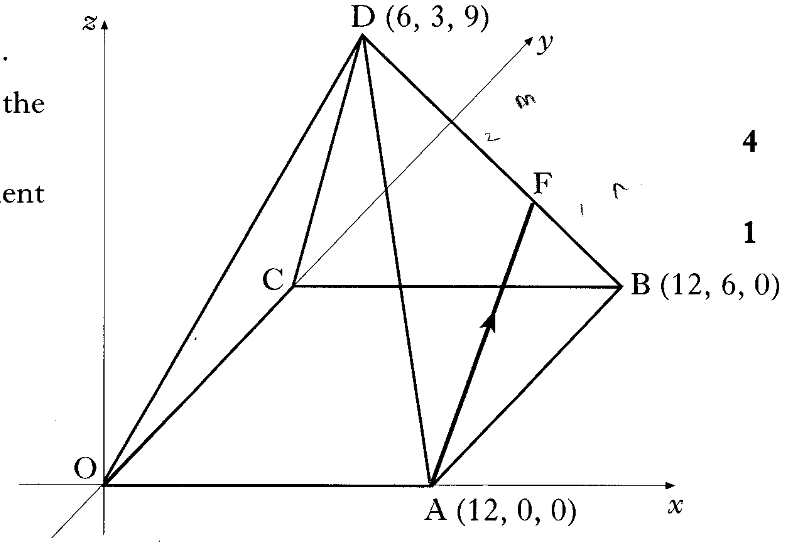
2005 P1

3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

(*a*) Find the coordinates of the point F. 4

(*b*) Express $\overrightarrow{AF}$ in component form. 1



Ans

(*a*) F = (10,5,3)

(*b*) $AF = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$

2005 P2

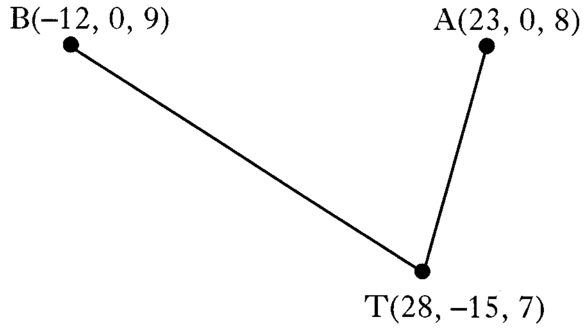
4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(−12, 0, 9) and T(28, −15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. 2

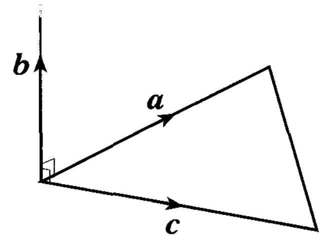
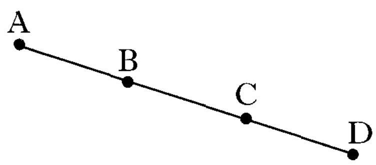
(*b*) Calculate the angle between these two beams. 5

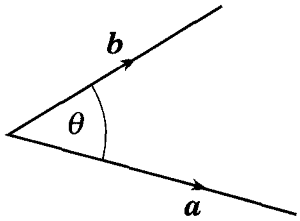
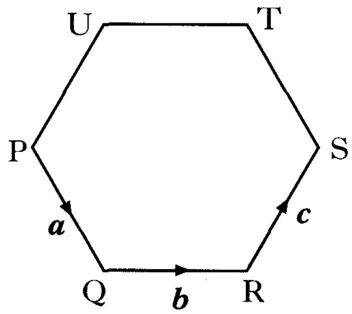


Ans

(*a*) $\overrightarrow{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$ $\overrightarrow{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$

(*b*) 50·9° **or** 0·889^c **or** 56·6 grads

2005 P2	**10.** Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$. Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$. 	4
Ans	$13\frac{1}{2}$	
2004 P1	**5.** A, B and C have coordinates (–3, 4, 7), (–1, 8, 3) and (0, 10, 1) respectively. (*a*) Show that A, B and C are collinear. (*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$.	3 2
Ans	**5.** (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = \frac{3}{2} \times \overrightarrow{AB}$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have common direction and common point, hence A, B and C are collinear. (*b*) D = (5, 20, –9)	
2004 P2	**2.** P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively. (*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. (*b*) Hence or otherwise find the size of angle PQR.	2 5
Ans	(*a*) $\overrightarrow{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ $\overrightarrow{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ (*b*) $P\hat{Q}R = 72{\cdot}0°$	
2003 P1	**3.** Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$. Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other.	2
Ans	Vectors are perpendicular.	
2003 P1	**6.** A and B are the points (–1, –3, 2) and (2, –1, 1) respectively. B and C are the points of trisection of AD, that is AB = BC = CD. Find the coordinates of D. 	3
Ans	(8, 3, –1)	

2003 P2	**9.** The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. If $\lvert \boldsymbol{a} \rvert = 5$, $\lvert \boldsymbol{b} \rvert = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. 	4
Ans	56·6°	
2002W P1	**2.** (*a*) If $\boldsymbol{u} = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$.	2
	(*b*) Hence, or otherwise, show that $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$ are perpendicular.	2
Ans	(*a*) • $u + 3v = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ • $u - 3v = \begin{pmatrix} -2 \\ 13 \\ -5 \end{pmatrix}$ (*b*) proof • $\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 13 \\ 5 \end{pmatrix}$ $= -8 + 13 - 5 = 0$ **Alternative** • $(u + 3v).(u - 3v)$ $= \lvert u \rvert^2 - 9\lvert v \rvert^2$ $= 54 - 9 \times 6 = 0$	
2002W P1	**11.** PQRSTU is a regular hexagon of side 2 units. $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively. Find the value of $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$. 	3
Ans	0	
2002W P2	**2.** With reference to a suitable set of coordinate axes, A, B and C are the points (–8, 10, 20), (–2, 1, 8) and (0, –2, 4) respectively. Show that A, B and C are collinear and find the ratio AB : BC.	4

<table>
<tr><td>Ans</td><td colspan="2">

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix}$ **or** $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$ **or** $\overrightarrow{AC} = \begin{pmatrix} 8 \\ -12 \\ -16 \end{pmatrix}$

e.g. $\overrightarrow{AB} = 3\overrightarrow{BC}$ $\left(\textbf{or} \begin{pmatrix} 6 \\ -9 \\ -12 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} \right)$

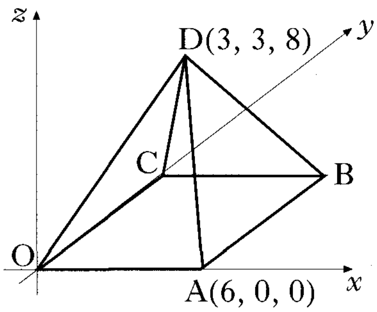
e.g. $\overrightarrow{AB}$, $3\overrightarrow{BC}$ have common direction,
B common pt. so A, B, C collinear
AB : BC = 3 : 1

</td></tr>
<tr><td>2002 P1</td><td>

2. The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1.
Find the coordinates of Q.

</td><td>3</td></tr>
<tr><td>Ans</td><td colspan="2">(3, 1, –2)</td></tr>
<tr><td>2002 P2</td><td>

2. The diagram shows a square-based pyramid of height 8 units.
Square OABC has a side length of 6 units.
The coordinates of A and D are (6, 0, 0) and (3, 3, 8).
C lies on the y-axis.



(*a*) Write down the coordinates of B.
(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$.
(*c*) Calculate the size of angle ADB.

</td><td>1
2
4</td></tr>
<tr><td>Ans</td><td colspan="2">

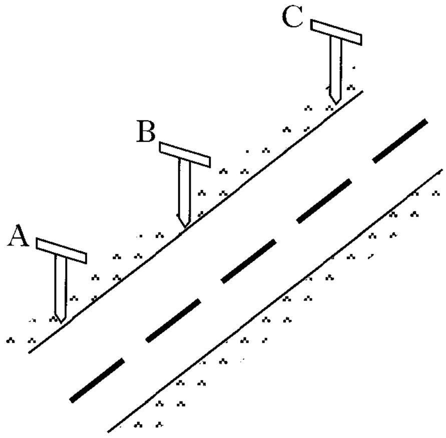
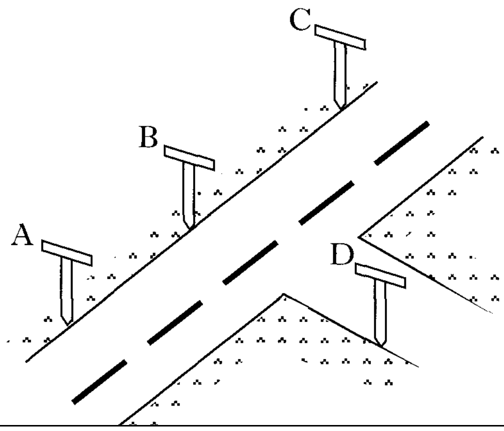
(*a*) (6, 6, 0)

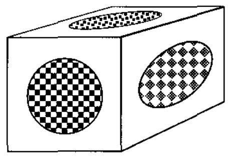
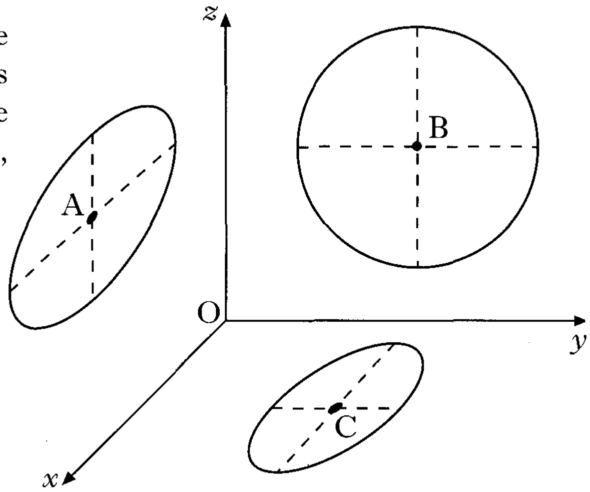
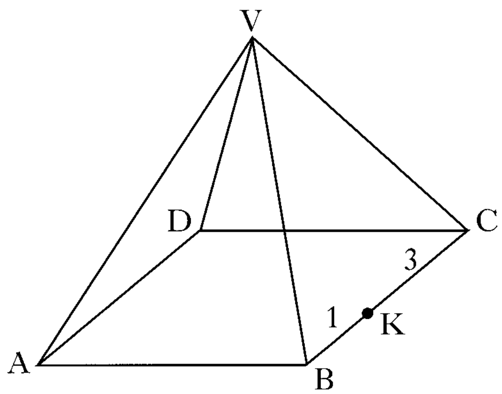
(*b*) $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

(*c*) $\cos A\hat{D}B = \dfrac{\overrightarrow{DA}.\overrightarrow{DB}}{\left|\overrightarrow{DA}\right|\left|\overrightarrow{DB}\right|}$

38·7°

</td></tr>
</table>

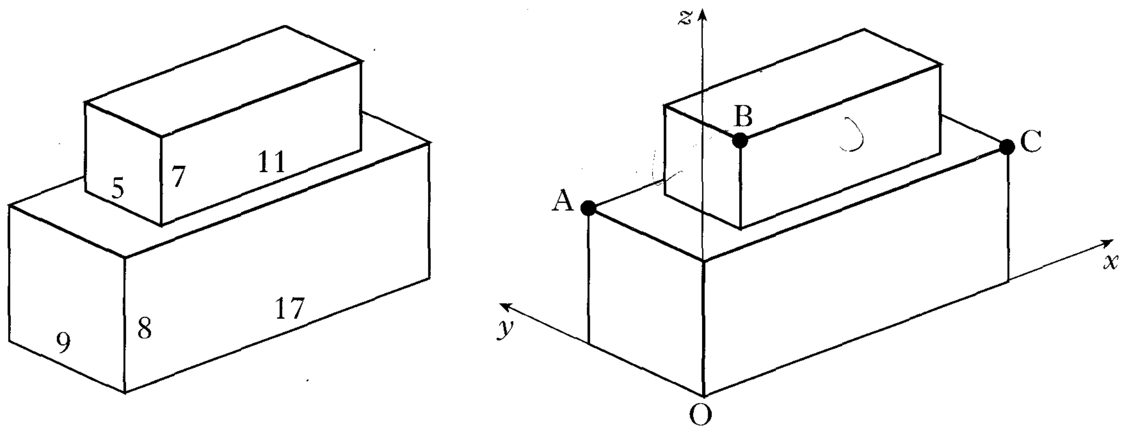
2001 P1	**3.** (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B(–2, –1, 1) and C(6, 11, 5). Determine whether or not the section of road ABC has been built in a straight line. 	3
	(*b*) A further T-rod is placed such that D has coordinates (1, –4, 4). Show that DB is perpendicular to AB. 	3
Ans	(*a*) the road ABC is straight (*b*) $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$ $\overrightarrow{AB}.\overrightarrow{BD} = 0$ $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$	

2001 P2	**4.** A box in the shape of a cuboid is designed with **circles** of different sizes on each face.  The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0). Find the size of angle ABC.  	7
Ans	71·5°	
2000 P1	**7.** VABCD is a pyramid with a rectangular base ABCD. Relative to some appropriate axes, $\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$ $\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$ $\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$. K divides BC in the ratio 1:3. Find $\overrightarrow{VK}$ in component form.  	3
Ans	$\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$	
2000 P2	**7.** For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?	2
Ans	$t = 4$	

2000 P2

9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.



(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

Ans

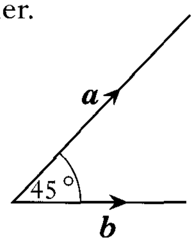
(*a*) B = (3, 2, 15)

(*b*) 92·5°

Specimen 2 P1

10. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$.

These vectors are inclined at an angle of 45 ° to each other.



(*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$

(ii) $\boldsymbol{b}.\boldsymbol{b}$

(iii) $\boldsymbol{a}.\boldsymbol{b}$ **2**

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.

Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $|\boldsymbol{p}|$. **4**

Ans

(*a*) $\boldsymbol{a}.\boldsymbol{a} = 9$, $\boldsymbol{b}.\boldsymbol{b} = 8$, $\boldsymbol{a}.\boldsymbol{b} = 6$

(*b*) $\boldsymbol{p}.\boldsymbol{p} = (2\boldsymbol{a} + 3\boldsymbol{b}).(2\boldsymbol{a} + 3\boldsymbol{b})$

$|\boldsymbol{p}|^2 = 4\boldsymbol{a}.\boldsymbol{a} + 9\boldsymbol{b}.\boldsymbol{b} + 12\boldsymbol{a}.\boldsymbol{b}$

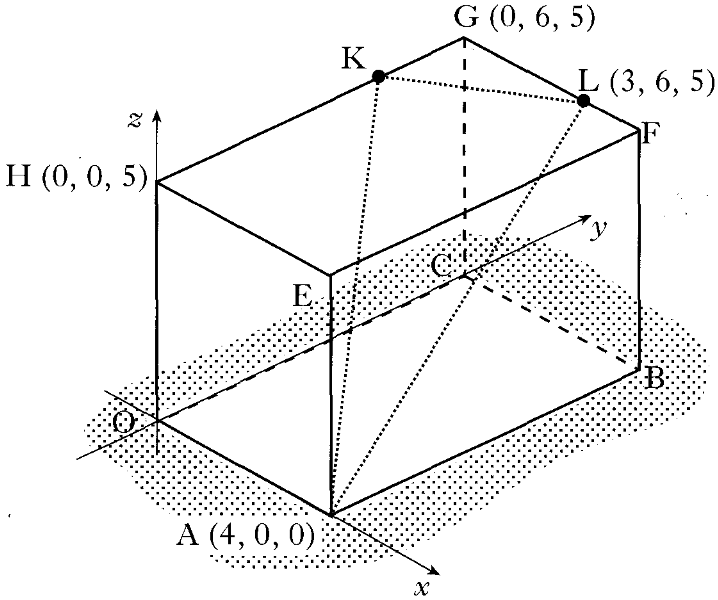
$|\boldsymbol{p}| = \sqrt{180}$

Specimen 2 P2

2. OABCEFGH is a cuboid.

With axes as shown, O is the origin and the coordinates of A, H, G and L are (4, 0, 0), (0, 0, 5), (0, 6, 5) and (3, 6, 5) respectively.

K lies two thirds of the way along HG, (ie HK:KG = 2:1).



(*a*) Determine the coordinates of K. 2

(*b*) Write down the components of $\overrightarrow{AK}$ and $\overrightarrow{AL}$. 2

(*c*) Calculate the size of angle KAL. 5

Ans

(*a*)

$$\overrightarrow{HG} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \overrightarrow{HK} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \Rightarrow K = (0,4,5)$$

(*b*)

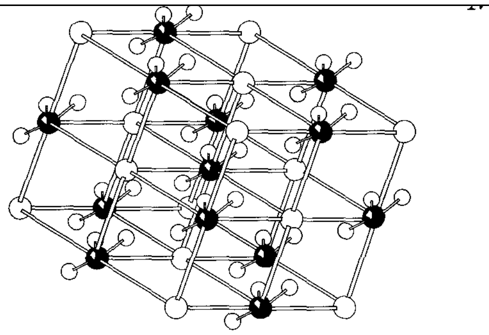
$$\overrightarrow{AK} = \begin{pmatrix} -4 \\ 4 \\ 5 \end{pmatrix}, \quad \overrightarrow{AL} = \begin{pmatrix} -1 \\ 6 \\ 5 \end{pmatrix}$$

(*c*) $\cos K\hat{A}L = \dfrac{4+24+25}{\sqrt{57}\sqrt{62}}$

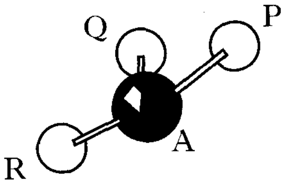
$K\hat{A}L = 26 \cdot 9°$

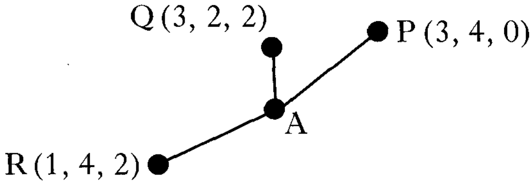
Specimen 1 P1

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. 5

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2 : 1.

(i) Find the coordinates of T. 6

(ii) Show that P, Q and R are equidistant from T.

Ans

(*a*) $\cos PQR = \dfrac{\overrightarrow{PQ}.\overrightarrow{RQ}}{|\overrightarrow{PQ}||\overrightarrow{RQ}|}$

$= \dfrac{4}{\sqrt{8}\sqrt{8}}$

$= \dfrac{4}{8}$

(*b*) (i) M(2,3,2) $T\left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$

(ii) substitution into

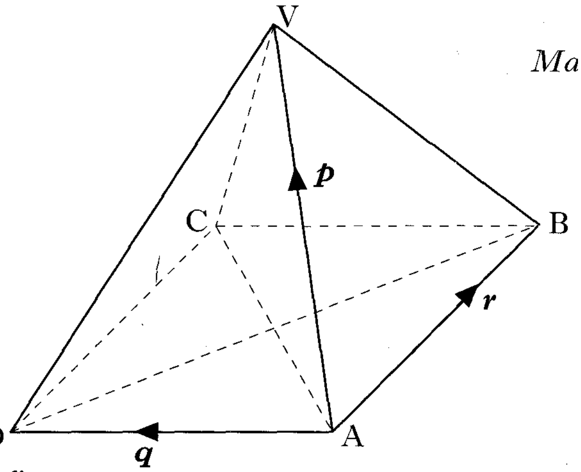
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

gives $PT = QT = RT = 2\sqrt{\frac{2}{3}}$

Specimen 1 P2

Ma

5. VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.



$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$. 3

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$.

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$. 4

(ii) Hence show that angle CVA is 90°.

Ans	(*a*) (i) 4·5 (ii) 9 (*b*) (i) $\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r}$ (ii) $\overrightarrow{CV}.\overrightarrow{AV} = \boldsymbol{p}.(\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r})$ $= \boldsymbol{p}.\boldsymbol{p} - \boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ $= 0$