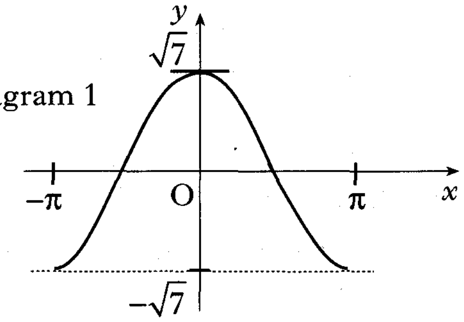
Higher : Wave Function Revision

2008 P2

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1

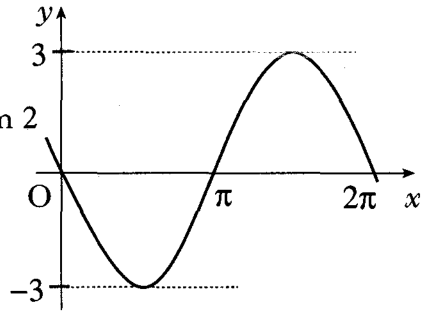


(2)

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. (4)

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. (2)

Ans (a) $p = \sqrt{7}$, $q=-3$ (b) $4Cos(x+0.848)$ (c) $-4Sin(x+0.848)$

2007 P2

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. (4)

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. (4)

Ans

(*a*) $a = \frac{1}{2}$

(*b*) $b = \frac{3}{2}$

(*c*) $y = 3 \times 4^x$

$\log_{10}y = \log_{10}3 + \log_{10}(4^x)$

$= \log_{10}3 + x\log_{10}(4)$

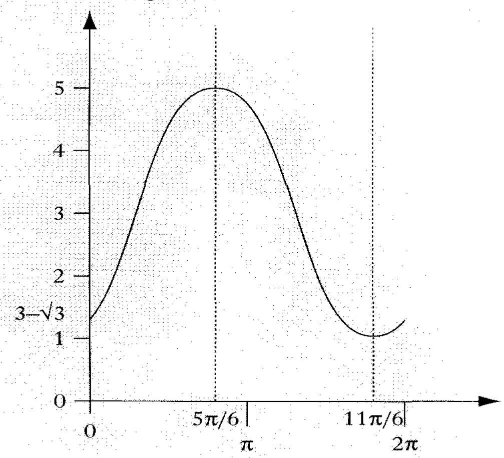
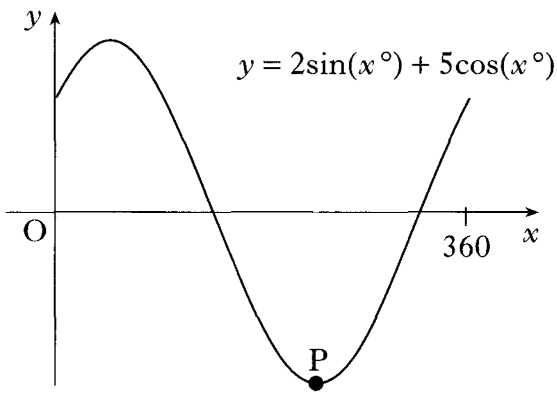
So gradient of line $= \log_{10}(4)$

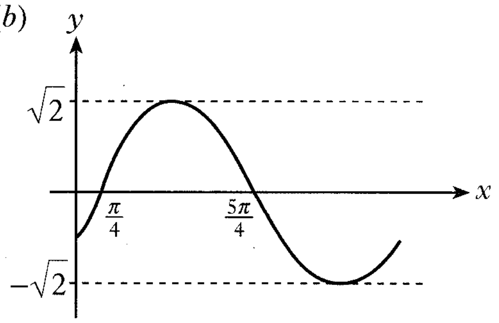
2006 P2

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. (4)

(*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. (3)

Ans	(*a*) $k = 25$ $a = 1{\cdot}29$ (*b*) $x = 2{\cdot}82$	
2005 P1	**10.** (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a \le 2\pi$. (*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$.	4 5
Ans	(*a*) $2\sin(x - \frac{\pi}{3})$ (*b*) 	
2004 P2	**6.** (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \le a \le 90$. (*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \le x \le 90$.	4 3
Ans	(*a*) $\sqrt{34}\cos(x - 59)°$ (*b*) $x = 12{\cdot}3$	
2003 P2	**7.** Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram. (*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \le a < 360$. (*b*) Find the coordinates of the minimum turning point P. 	4 3
Ans	(*a*) $\sqrt{29}\sin(x + 68{\cdot}2)°$ (*b*) $(201{\cdot}8°, -\sqrt{29})$	
2002W P2	**4.** (*a*) Write $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a < 360$. (*b*) Hence find the maximum value of $5 + \sqrt{3}\sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \le x \le 360$.	4 2

Ans	(a) $2\sin(x+30)°$ (b) max = 7 when $x = 60$	
2002 P1	**9.** (a) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \le a \le 2\pi$. (b) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.	4 3
Ans	(a) $\sqrt{2}\sin(x - \frac{\pi}{4})$ (b) 	
2001 P2	**5.** Express $8\cos x° - 6\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 < a < 360$.	4
Ans	$10\cos(x + 36{\cdot}9)°$	
2000 P1	**10.** Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$.	6
Ans	max value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$	
Specimen 2 P1	**6.** $f(x) = \sqrt{3}\sin x° - \cos x°$ (a) Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a < 360$. (b) Hence solve the equation $f(x) = \sqrt{2}$ in the interval $0 \le a < 360$.	4 3
Ans	(a) compare $\sqrt{3}\sin x° - \cos x°$ with $k\sin x°\cos a° - k\cos x°\sin a°$ $k\cos a° = \sqrt{3}$, $k\sin a° = 1$ $k = 2$, $\tan a° = \frac{1}{\sqrt{3}} \Rightarrow a = 30$ (b) $2\sin(x - 30)° = \sqrt{2}$ $x - 30 = 45, 135$ $x = 75, 165$	

Specimen 1 P2	**6.** $f(x) = 2\cos x° + 3\sin x°$. (*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. (*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \le x < 360$. (*c*) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \le x < 360$.	4 3 2
Ans	(*a*) $\sqrt{13}\cos(x - 56\cdot3)°$ (*b*) 138·8, 334·3 (*c*) 146·3°	