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Advanced Higher Maths

Unit 1.3 Integration Solutions Part 1

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Ex1 Higher Revision

1. $f'(x) = 8x^3$ $\qquad f(x) = \int 8x^3 dx = \left[\frac{8x^4}{4} + c\right] = 2x^4 + c$

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2. $f'(x) = \frac{6}{x^3} = 6x^{-3}$ $\qquad f(x) = \int 6x^{-3}\, dx = \left[\frac{6x^{-2}}{-2} + c\right] = -3x^{-2} + c = -\frac{3}{x^2} + c$

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3. $f'(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\qquad f(x) = \int x^{\frac{1}{2}} dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c\right] = \frac{2}{3}x^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$

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4. $f'(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ $\qquad f(x) = \int x^{-\frac{1}{2}} dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c\right] = 2\sqrt{x} + c$

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5. $f'(x) = \frac{3x^4+6}{x^2} = \frac{3x^4}{x^2} + \frac{6}{x^2} = 3x^2 + 6x^{-2}$ $\qquad f(x) = \int 3x^2 + 6x^{-2}\, dx = \left[\frac{3x^3}{3} + \frac{3x^{-1}}{-1} + c\right]$

$$= x^3 - \frac{6}{x} + c$$

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6. $f'(x) = \frac{1-3x}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} - \frac{3x}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$ $\qquad f(x) = \int x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}\, dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c\right]$

$$= 2\sqrt{x} - 2\sqrt{x^3} + c$$

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7. $f'(x) = (3x+4)^4$ $\qquad f(x) = \int (3x+4)^4\, dx = \left[\frac{(3x+4)^5}{3\times 5} + c\right]$

$$= \frac{(3x+4)^5}{15} + c$$

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8. $f'(x) = (1-2x)^4$ $\qquad f(x) = \int (1-2x)^4\, dx = \left[\frac{(1-2x)^5}{-2\times 5} + c\right]$

$$= -\frac{(1-2x)^5}{10} + c$$

9. $f'(x) = \frac{1}{(2x-3)^2} = (2x-3)^{-2} \qquad f(x) = \int (2x-3)^{-2}\,dx = \left[\frac{(2x-3)^{-1}}{2\times(-1)} + c\right] = -\frac{1}{2(2x-3)} + c$

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10. $f'(x) = \frac{1}{\sqrt{(4x+1)}} = (4x+1)^{-\frac{1}{2}} \qquad f(x) = \int (4x+1)^{-\frac{1}{2}}\,dx = \left[\frac{(4x+1)^{\frac{1}{2}}}{4\times\frac{1}{2}} + c\right] = \frac{\sqrt{4x+1}}{2} + c$

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11. $f'(x) = \frac{3}{(3x+1)^{\frac{3}{2}}} = 3(3x+1)^{-\frac{3}{2}} \qquad f(x) = 3\int (3x+1)^{-\frac{3}{2}}\,dx = 3\left[\frac{(3x+1)^{-\frac{1}{2}}}{3\times-\frac{1}{2}} + c\right] = -\frac{2}{\sqrt{(3x+1)}} + c$

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12. $f'(x) = sin3x \qquad f(x) = \int sin3x\,dx = \left[\frac{-cos3x}{3} + c\right] = -\frac{cos3x}{3} + c$

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13. $f'(x) = cos\frac{1}{3}x \qquad f(x) = \int cos\frac{1}{3}x\,dx = \left[\frac{sin\frac{1}{3}x}{\frac{1}{3}} + c\right] = 3sin\frac{1}{3}x + c$

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14. $f'(x) = sin^2x = \frac{1}{2}(1 - cos2x) \qquad f(x) = \frac{1}{2}\int (1 - cos2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}sin2x + c\right]$

$$= \frac{1}{2}x - \frac{1}{4}sin2x + c$$

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15. $f'(x) = cos^2\frac{1}{2}x = \frac{1}{2}(1 - cosx) \qquad f(x) = \frac{1}{2}\int (1 - cosx)\,dx = \frac{1}{2}[x - sinx + c]$

$$= \frac{1}{2}(x - sinx) + c$$

Ex2 New Integrals

1. $f'(x) = e^{5x}$ $\quad f(x) = \int e^{5x}dx = \frac{1}{5}e^{5x} + c$

2. $f'(x) = e^{-2x}$ $\quad f(x) = \int e^{-2x}dx = -\frac{1}{2}e^{-2x} + c$

3. $f'(x) = 3e^{\frac{1}{2}x}$ $\quad f(x) = 3\int e^{\frac{1}{2}x}dx = 6e^{\frac{1}{2}x} + c$

4. $f'(x) = \frac{1}{3x}$ $\quad f(x) = \frac{1}{3}\int \frac{1}{x}dx = \frac{1}{3}ln3x + c$

5. $f'(x) = \frac{1}{x+5}$ $\quad f(x) = \int \frac{1}{x+5}dx = \ln(x+5) + c$

6. $f'(x) = \frac{1}{2x-3}$ $\quad f(x) = \int \frac{1}{2x-3}dx = \frac{1}{2}\ln\frac{1}{(2x-3)} + c$

7. $f'(x) = (e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$ $\quad f(x) = \int e^{2x} + 2 + e^{-2x}dx = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$

8. $f'(x) = \frac{1+e^x}{e^x} = e^{-x} + 1$ $\quad f(x) = \int e^{-x} + 1\, dx = -e^{-x} + x + c = x - e^{-x} + c$

9. $f'(x) = e^{2x} + \frac{1}{e^{2x}} = e^{2x} + e^{-2x}$ $\quad f(x) = \int e^{2x} + e^{-2x}\, dx = \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + c$

10. $f'(x) = \frac{6}{3x+2}$ $\quad f(x) = \int \frac{6}{3x+2}\, dx = 2ln(3x+2) + c$

11. $f'(x) = \frac{3}{1-2x}$ $\quad f(x) = \int \frac{6}{3x+2}\, dx = 2ln(3x+2) + c$

12. $f'(x) = \frac{5}{6-7x}$ $\quad f(x) = \int \frac{5}{6-7x}\, dx = -\frac{5}{7}ln(6-7x) + c$

13. $f'(x) = sec^2 4x$ $\quad$ $f(x) = \int sec^2 4x \, dx = \frac{1}{4} tan4x + c$

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14. $f'(x) = sec^2(\pi + 2x)$ $\quad$ $f(x) = \int sec^2(\pi + 2x) \, dx = \frac{1}{2} tan(\pi + 2x) + c$

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15. $f'(x) = 3sec^2 2x$ $\quad$ $f(x) = \int 3sec^2 2x \, dx = \frac{3}{2} tan2x + c$

DefiniteIntegralsEx3

1. $I = \int_1^2 \left(\frac{2}{x^2} - \frac{5}{x^4}\right) dx = \int_1^2 (2x^{-2} - 5x^{-4}) dx = \left[-\frac{2}{x} + \frac{5}{3x^3}\right]_1^2 = \left[-1 + \frac{5}{24}\right] - \left[-2 + \frac{5}{3}\right] = -\frac{11}{24}$

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2. $I = \int_{\frac{1}{4}}^1 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \int_{\frac{1}{4}}^1 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx = \left[\frac{2\sqrt{x^3}}{3} - 2\sqrt{x}\right]_{\frac{1}{4}}^1 = \left[\frac{2}{3} - 2\right] - \left[\frac{1}{12} - 1\right] = -\frac{5}{12}$

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3. $I = \int_1^4 \left(\frac{2x^2+3}{\sqrt{x}}\right) dx = \int_1^4 \left(2x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}\right) dx = \left[\frac{4\sqrt{x^5}}{5} + 6\sqrt{x}\right]_1^4 = \left[\frac{128}{5} + 12\right] - \left[\frac{4}{5} + 6\right] = \frac{154}{5}$

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4. $\int_1^6 \sqrt{(x+3)}\, dx = \int_1^6 (x+3)^{\frac{1}{2}} dx = \left[\frac{2(x+3)^{\frac{3}{2}}}{3}\right]_1^6 = \left[\frac{2(6+3)^{\frac{3}{2}}}{3}\right] - \left[\frac{2(1+3)^{\frac{3}{2}}}{3}\right] = \left[18 - \frac{16}{3}\right] = \frac{38}{3}$

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5. $\int_0^1 \frac{1}{(4+5x)^2}\, dx = \int_0^1 (4+5x)^{-2}\, dx = \left[-\frac{1}{5(4+5x)}\right]_0^1$

$$= \left[-\frac{1}{5(4+5(1))}\right] - \left[-\frac{1}{5(4+5(0))}\right] = \left[-\frac{1}{45} + \frac{1}{20}\right] = \frac{1}{36}$$

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6. $\int_3^{12} (x-4)^{\frac{1}{3}}\, dx = \left[\frac{3(x-4)^{\frac{4}{3}}}{4}\right]_3^{12} = \left[\frac{3((12)-4)^{\frac{4}{3}}}{4}\right] - \left[\frac{3((3)-4)^{\frac{4}{3}}}{4}\right] = \left[12 - \frac{3}{4}\right] = \frac{45}{4} = 11\frac{1}{4}$

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7. $\int_0^{\frac{\pi}{4}} cos2x\, dx = \left[\frac{sin\,2x}{2}\right]_0^{\frac{\pi}{4}} = \left[\frac{sin\frac{\pi}{2}}{2}\right] - \left[\frac{sin\,0}{2}\right] = \left[\frac{1}{2} - 0\right] = \frac{1}{2}$

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8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cosec^2x\, dx = [-cotx]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left[-cot\frac{\pi}{2}\right] - \left[-cot\frac{\pi}{4}\right] = [0+1] = 1$ * Calculator issue *

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9. $\int_0^{2\pi} sin^2xdx = \int_0^{2\pi} \frac{1}{2}(1 - cos2x)\, dx = \frac{1}{2}\left[x - \frac{1}{2}sin2x\right]_0^{2\pi}$

$$= \frac{1}{2}\left[\left[2\pi - \frac{1}{2}sin4\pi\right] - \left[0 - \frac{1}{2}sin0\right]\right]$$

$$= \frac{1}{2}[2\pi - 0] = \pi$$

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10. $\int_0^2 e^{-3x}\, dx = \left[-\frac{1}{3}e^{-3x}\right]_0^2 = -\frac{1}{3}[[e^{-6}] - [e^0]] = -\frac{1}{3}[e^{-6} - 1] = \frac{1}{3}\left(1 - \frac{1}{e^6}\right)$

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11. $\int_0^1 e^{1-x}\, dx = [-e^{1-x}]_0^1 = [-e^0] - [-e^1] = -1 + e = e - 1$

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12. $\int_0^1 e^{\frac{x}{2}}dx = 2\left[e^{\frac{x}{2}}\right]_0^1 = 2\left[\left[e^{\frac{1}{2}}\right] - [e^0]\right] = 2\left(e^{\frac{1}{2}} - 1\right) = 2e^{\frac{1}{2}} - 2$

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13. $\int_5^9 \frac{1}{(x-3)}\, dx = [\ln(x-3)]_5^9 = \left[[\ln(6)] - [\ln(2)]\right] = \ln\left(\frac{6}{2}\right) = \ln(3)$

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14. $\int_0^1 \frac{1}{(3x+2)}\, dx = \frac{1}{3}[\ln(3x+2)]_0^1 = \frac{1}{3}\left[[\ln(5)] - [\ln(2)]\right] = \frac{1}{3}\ln\left(\frac{5}{2}\right)$

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15. $\int_{-4}^0 \frac{1}{(1-2x)}\, dx = -\frac{1}{2}[\ln(1-2x)]_{-4}^0 = -\frac{1}{2}\left[[\ln(1)] - [\ln(9)]\right] = \ln 3$

Ex 4 Integration by Substitution

1. $\int x(x^2-3)^5 dx$

$$Let \quad u = x^2 - 3 \quad \frac{du}{dx} = 2x \quad xdx = \frac{du}{2}$$

$$\int x(x^2-3)^5 dx = \frac{1}{2}u^5 du$$

$$= \left[\frac{u^6}{12} + c\right] = \frac{(x^2-3)^6}{12} + c$$

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2. $\int x^2(x^3-1)^2 dx$

$$Let \quad u = x^3 - 1 \quad \frac{du}{dx} = 3x^2 x^2 dx = \frac{du}{3}$$

$$\int x^2(x^3-1)^2 dx = \frac{1}{3}u^2 du$$

$$= \left[\frac{u^3}{9} + c\right] = \frac{(x^3-1)^3}{9} + c$$

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3. $\int x\sqrt{(1-x^2)}dx = \int x(1-x^2)^{\frac{1}{2}}dx$

$$Let \quad u = 1 - x^2 \frac{du}{dx} = -2x \quad xdx = \frac{du}{-2}$$

$$\int x\sqrt{(1-x^2)}dx = -\frac{1}{2}u^{\frac{1}{2}}du$$

$$= \left[-\frac{2u^{\frac{3}{2}}}{2 \times 3} + c\right] = -\frac{\sqrt{(1-x^2)^3}}{3} + c$$

4. $\int cosxsin^4xdx$

$$Let \qquad u = sinx\frac{du}{dx} = cosx \qquad cosxdx = du$$

$$\int u^4dx = \frac{1}{5}u^5du$$

$$= \frac{sin^5x}{5} + c$$

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5. $\int \frac{x}{(1-x^2)^3}dx = \int x(1-x^2)^{-3}dx$

$$Let \qquad u = 1 - x^2\frac{du}{dx} = -2x \qquad xdx = \frac{du}{-2}$$

$$\int x(1-x^2)^{-3}dx = \int -\frac{u^{-3}}{2}du = \frac{1}{4}u^{-2} + c$$

$$= \frac{1}{4(1-x^2)^2} + c$$

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6. $\int \frac{x^3}{1+x^4}dx$

$$Let \qquad u = 1 + x^4\frac{du}{dx} = 4x^3 \qquad xdx = \frac{du}{4}$$

$$\int \frac{x^3}{1+x^4}dx = \int \frac{1}{4u}du = \frac{1}{4}\ln u + c$$

$$= \frac{1}{4}\ln(1 + x^4) + c$$

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7. $\int \frac{sinx}{cos^3x}dx = \int sinxcos^{-3}xdx$

$$Let \qquad u = cosx \frac{du}{dx} = -sinx \qquad sinxdx = -du$$

$$\int sinxcos^{-3}xdx = \int -u^{-3}du = \frac{u^{-2}}{2} + c$$

$$= \frac{1}{2cos^2x} + c = \frac{1}{2}sec^2x + c$$

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8. $\int \frac{cosx}{sin^6x}dx = \int cosxsin^{-6}xdx$

$$Let \qquad u = sinx \frac{du}{dx} = cosx \qquad cosxdx = du$$

$$\int cosxsin^{-6}xdx = \int u^{-6}du = \frac{u^{-5}}{-5} + c$$

$$= -\frac{1}{5sin^5x} + c$$

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9. $\int \frac{e^x}{3e^x - 1}dx$

$$Let \qquad u = 3e^x - 1 \frac{du}{dx} = 3e^x e^x dx = \frac{du}{3}$$

$$\int \frac{e^x}{3e^x - 1}dx = \int \frac{1}{3u}du = \frac{lnu}{3} + c$$

$$= \frac{ln(3e^x - 1)}{36} + c$$

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10. $\int \frac{sec^2x}{tanx} dx$

$$Let \qquad u = tanx \qquad \frac{du}{dx} = sec^2x \qquad sec^2xdx = du$$

$$\int \frac{sec^2x}{tanx} dx = \int \frac{du}{u} = lnu + c = \ln(tanx) + c$$

Ex5HarderSubstitution

1. $y = \int 9x(3x+2)^3 dx$ $\quad u = 3x+2 \quad \frac{du}{dx} = 3 \quad dx = \frac{du}{3} \quad x = \frac{u-2}{3}$

$$\int 9x(3x+2)^3 dx = \int 9\left(\frac{u-2}{3}\right) \times u^3 \times \frac{1}{3} du$$

$$= \int u^3(u-2)du = \int u^4 - 2u^3 du = \frac{u^5}{5} - \frac{u^4}{2} + c = \frac{(x+2)^5}{5} - \frac{(x+2)^4}{2} + c$$

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2. $y = \int 7x(2x+3)^5 dx$ $\quad u = 2x+3 \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2} \quad x = \frac{u-3}{2}$

$$\int 7x(2x+3)^5 dx = \int 7\left(\frac{u-3}{2}\right) \times u^5 \times \frac{1}{2} du$$

$$= \int \frac{7}{4}u^5(u-3)du = \frac{7}{4}\int u^6 - 3u^5 du = \frac{7}{4}\left(\frac{u^7}{7} - \frac{u^6}{2}\right) + c = \frac{u^7}{4} - \frac{7u^6}{8} + c = \frac{(2x+3)^7}{4} - \frac{7(2x+3)^6}{8} + c$$

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3. $y = \int 3x\sqrt{(1+x^2)}dx = \int 3x(1+x^2)^{\frac{1}{2}}dx$

$$u = 1 + x^2 \frac{du}{dx} = 2x \quad x\,dx = \frac{du}{2} \quad x = \sqrt{(u-1)} = (u-1)^{\frac{1}{2}}$$

$$\int 3x(1+x^2)^{\frac{1}{2}}dx = \int 3 \times u^{\frac{1}{2}} \times \frac{1}{2} du$$

$$= \frac{3}{2}\int u^{\frac{1}{2}}du = \frac{3}{2}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + c = u^{\frac{3}{2}} + c = \left(1+x^2\right)^{\frac{3}{2}} + c = \sqrt{(1+x^2)^3} + c$$

4. $y = \int \frac{3x}{\sqrt{(2x+3)}}dx = \int 3x(2x+3)^{-\frac{1}{2}}dx$

$$u = 2x + 3 \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2} \quad x = \frac{u-3}{2}$$

$$\int 3\left(\frac{u-3}{2}\right) \times (u)^{-\frac{1}{2}} \times \frac{1}{2}du = \frac{3}{4}\int u^{-\frac{1}{2}}(u-3)du = \frac{3}{4}\int u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}du$$

$$= \frac{3}{4}\left[\frac{2u^{\frac{3}{2}}}{3} - 6u^{\frac{1}{2}} + c\right] = \frac{1}{2}u^{\frac{3}{2}} - \frac{9}{2}u^{\frac{1}{2}} + c = \frac{1}{2}\sqrt{(2x+3)^3} - \frac{9}{2}\sqrt{(2x+3)} + c$$

Ex6 Trig Substitutions

1. $y = \int \frac{x}{\sqrt{(1-x^2)}} dx$

$$x = sin\theta \quad \frac{dx}{d\theta} = cos\theta \quad dx = cos\theta d\theta$$

$$\int \frac{sin\theta cos\theta}{\sqrt{(1-sin^2\theta)}} d\theta = \int \frac{sin\theta cos\theta}{cos\theta} d\theta = \int sin\theta d\theta = -cos\theta + c = -\sqrt{(1-x^2)} + c$$

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2. $y = \int \sqrt{(4-x^2)} dx \quad Check$!!!!

$$x = 2sin\theta \quad \frac{dx}{d\theta} = 2cos\theta \quad dx = 2cos\theta d\theta$$

$$\int \sqrt{(4-x^2)} dx = \int \sqrt{(4-4sin^2\theta)} \times 2cos\theta d\theta = \int 2cos\theta \times 2cos\theta d\theta = \int 4cos^2\theta d\theta$$

$$\int 4cos^2\theta d\theta = 4\int \frac{1}{2}(1+cos2\theta) d\theta = 2\int (1+cos2\theta) d\theta$$

$$= 2\left[\theta + \frac{1}{2}sin2\theta + c\right] = 2\theta + sin2\theta + c = 2\theta + 2sin\theta cos\theta + c$$

$$2\theta + 2sin\theta cos\theta + c = 2sin^{-1}\left(\frac{x}{2}\right) + x \times \frac{1}{2}\left(\sqrt{(4-x^2)}\right) + c$$

$$2\theta + sin2\theta + c = 2sin^{-1}\left(\frac{x}{2}\right) + \frac{x}{2}\sqrt{(4-x^2)} + c$$

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3. $y = \int \frac{x}{\sqrt{(9-x^2)}} dx$

$$x = 3sin\theta \qquad \frac{dx}{d\theta} = 3cos\theta \qquad dx = 3cos\theta d\theta$$

$$\int \frac{9sin\theta cos\theta}{\sqrt{(9-9sin^2\theta)}} d\theta = \int \frac{3sin\theta cos\theta}{cos\theta} d\theta = 3\int sin\theta d\theta = -3cos\theta + c \ = -\sqrt{(9-x^2)} + c$$

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4. $y = \int \frac{x^2}{\sqrt{(4-x^2)}} dx$

$$x = 2sin\theta \qquad \frac{dx}{d\theta} = 2cos\theta \qquad dx = 2cos\theta d\theta$$

$$\int \frac{4sin^2\theta \times 2cos\theta}{\sqrt{(4-4sin^2\theta)}} d\theta = \int \frac{4sin^2\theta cos\theta}{cos\theta} d\theta = 4\int sin^2\theta d\theta = -2\int (1 + cos2\theta)\, d\theta + c$$

$$= -2\left(\theta + \frac{1}{2}sin2\theta\right) + c = -2\theta - 2sin\theta cos\theta + c$$

$$= -2sin^{-1}\left(\frac{x}{2}\right) - x \times \frac{1}{2}\sqrt{(4-x^2)} + c$$

$$= -2sin^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\sqrt{(4-x^2)} + c$$

Ex5A Definite Integrals

1. $y = \int_0^1 x^3(2+x^4)^5 dx$

$$u = 2 + x^4 \quad \frac{du}{dx} = 4x^3 \quad \frac{du}{4} = x^3 dx \quad ; \quad x = 0 \;\; u = 2 \quad x = 1 \;\; u = 3$$

$$\int_0^1 x^3(2+x^4)^5 dx = \int_2^3 \frac{u^5}{4} du = \left[\frac{u^6}{24}\right]_2^3 = \left[\frac{3^6}{24}\right] - \left[\frac{2^6}{24}\right] = \frac{665}{24}$$

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2. $y = \int_4^9 \frac{x+1}{x^2+2x-7} dx$

$$u = x^2 + 2x - 7 \quad \frac{du}{dx} = 2(x+1) \quad \frac{du}{2} = (x+1)dx \;;\; x = 9 \;\; u = 92 \;;\; x = 4 \;\; u = 17$$

$$\int_{17}^{92} \frac{1}{2u} du = \frac{1}{2}[\ln(u)]_{17}^{92} = \frac{1}{2}\left[[ln(92)] - [ln(17)]\right] = \frac{1}{2}\left[ln\left(\frac{92}{17}\right)\right]$$

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3. $y = \int_0^4 \frac{\sqrt{x}}{2+\sqrt{x}} dx = \int_0^4 \frac{x^{\frac{1}{2}}}{2+x^{\frac{1}{2}}} dx$

$$u = 2 + x^{\frac{1}{2}} \quad \frac{du}{dx} = \frac{1}{2x^{\frac{1}{2}}} \qquad 2(u-2)du = dx \;;\; x^{\frac{1}{2}} = (u-2) \;\; x = 4 \;\; u = 4 \;;\; x = 0 \;\; u = 2$$

$$\int_2^4 \frac{1}{u} \times (u-2) \times 2(u-2) du = 2\int_2^4 \left(\frac{u^2 - 4u + 4}{u}\right) du = 2\int_2^4 \left(u - 4 + \frac{4}{u}\right) du$$

$$= 2\left[\frac{u^2}{2} - 4u + 4lnu\right]_2^4 = 2\left[[8 - 16 + 4ln4] - [2 - 8 + 4ln2]\right] = 2[-2 + 4(ln2)]$$

$$= -4 + 8ln2 = 8ln2 - 4$$

4. $y = \int_0^{\frac{1}{2}} \frac{x}{\sqrt{(1-x^2)}} dx$

$$u = (1 - x^2)\ \frac{du}{dx} = -2x \qquad -\frac{du}{2} = xdx\ ;\quad x = \frac{1}{2}\ \ u = \frac{3}{4}\ ;\ x = 0\ \ u = 1$$

$$\int_0^{\frac{1}{2}} \frac{x}{\sqrt{(1-x^2)}} dx = \int_1^{\frac{3}{4}} \left(\frac{u^{-\frac{1}{2}}}{-2}\right) du = -\frac{1}{2}\left[2u^{\frac{1}{2}}\right]_1^{\frac{3}{4}}$$

$$= -\frac{1}{2}\left[[\sqrt{3}] - [2]\right] = 1 - \frac{\sqrt{3}}{2}$$

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5. $y = \int_0^1 x\sqrt{2x^2+3}dx$

$$u = 2x^2 + 3 \quad \frac{du}{dx} = 4x \qquad \frac{du}{4} = xdx \quad ; \quad x = 1\ \ u = 5 \quad x = 0\ \ u = 3$$

$$\int_0^1 x\sqrt{2x^2+3}dx = \frac{1}{4}\int_3^5 u^{\frac{1}{2}}du = \frac{1}{4}\left|\frac{2u^{\frac{3}{2}}}{3}\right|_3^5 = \frac{1}{12}\left[\left[10\sqrt{5}\right] - \left[6\sqrt{3}\right]\right] = \frac{1}{6}\left(5\sqrt{5} - 3\sqrt{3}\right)$$

---

6. $y = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{sinx}{1+cosx} dx$

$$u = 1 + cosx \quad \frac{du}{dx} = -sinx \qquad -du = sinxdx \quad ; \quad x = \frac{\pi}{2}\ \ u = 1 \quad x = \frac{\pi}{3}\ \ u = \frac{3}{2}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{sinx}{1+cosx} dx = -\int_{\frac{3}{2}}^{1} \frac{1}{u} du = -[lnu]_{\frac{3}{2}}^{1} = -\left[[ln1] - ln\left(\frac{3}{2}\right)\right] = ln\left(\frac{3}{2}\right)$$

---

7. $y = \int_0^{\frac{\pi}{2}} \frac{1-2sinx}{x+2cosx} dx$

$$u = x + 2cosx \quad \frac{du}{dx} = 1 - 2sinx \quad du = 1 - 2sinxdx \quad ; \quad x = \frac{\pi}{2} \;\; u = \frac{\pi}{2} \quad x = 0 \;\; u = 2$$

$$\int_0^{\frac{\pi}{2}} \frac{1 - 2sinx}{x + 2cosx} dx = \int_2^{\frac{\pi}{2}} \frac{1}{u} du = [lnu]_2^{\frac{\pi}{2}} = \left[\left[ln\left(\frac{\pi}{2}\right)\right] - ln2\right] = ln\left(\frac{\pi}{4}\right)$$

---

8. $y = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} cotxdx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{cosx}{sinx} dx$

$$u = sinx \quad \frac{du}{dx} = cosx \quad du = cosxdx \quad ; \quad x = \frac{\pi}{2} \;\; u = 1 \quad x = \frac{\pi}{3} \;\; u = \frac{\sqrt{3}}{2}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{cosx}{sinx} dx = \int_{\frac{\sqrt{3}}{2}}^{1} \frac{1}{u} du = [lnu]_{\frac{\sqrt{3}}{2}}^{1} = \left[[ln(1)] - ln\left(\frac{\sqrt{3}}{2}\right)\right] = -ln\left(\frac{\sqrt{3}}{2}\right)$$

$$-ln\left(\frac{\sqrt{3}}{2}\right) = -\left(ln\sqrt{3} - ln2\right) = \left(ln2 - ln\sqrt{3}\right) = ln\left(\frac{2}{\sqrt{3}}\right)$$

---

9. $y = \int_0^{\frac{\pi}{3}} sinxcos^4xdx$

$$u = cosx \quad \frac{du}{dx} = -sinx \quad -du = sinxdx \quad ; \quad x = \frac{\pi}{3} \;\; u = \frac{1}{2} \quad x = 0 \;\; u = 1$$

$$\int_0^{\frac{\pi}{3}} sinxcos^4xdx = -\int_{\frac{1}{2}}^{1} u^4 du = \left[\frac{u^5}{5}\right]_{\frac{1}{2}}^{1} = \left[\left(\frac{1}{5}\right) - \left(\frac{1}{160}\right)\right] = \frac{128}{160} = \frac{31}{160}$$

---

10. $y = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{sinx}{cos^2x} dx$

$$u = cosx \quad \frac{du}{dx} = -sinx \quad -du = sinxdx \quad ; \quad x = \frac{\pi}{4} \;\; u = \frac{1}{\sqrt{2}} \quad x = \frac{\pi}{6} \;\; u = \frac{\sqrt{3}}{2}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{sinx}{cos^2x} dx = -\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{\sqrt{2}}} u^{-2} du = -\left[-\frac{1}{u}\right]_{\frac{\sqrt{3}}{2}}^{\frac{1}{\sqrt{2}}} = \left[(\sqrt{2}) - \left(\frac{2}{\sqrt{3}}\right)\right] = \sqrt{2} - \frac{2}{\sqrt{3}}$$

---

11. $y = \int_{e}^{e^2} \frac{1}{xlnx} dx$

$$t = lnx \quad \frac{dt}{dx} = \frac{1}{x} \quad dt = \frac{1}{x} dx \quad ; \quad x = e^2 \;\; t = 2 \quad x = e \;\; t = 1$$

$$\int_{e}^{e^2} \frac{1}{xlnx} dx = \int_{1}^{2} \frac{1}{t} dt = [lnt]_1^2 = [(ln2) - (ln1)] = ln2$$

---

12. $y = \int_{1}^{2} \frac{e^{2x}}{(e^{2x}-1)^2} dx$

$$t = e^{2x} - 1 \quad \frac{dt}{dx} = 2e^{2x} \; \frac{dt}{2} = e^{2x} dx \quad ; \quad x = 2 \;\; t = e^4 - 1 \quad x = 1 \;\; t = e^2 - 1$$

$$\int_{1}^{2} \frac{e^{2x}}{(e^{2x}-1)^2} dx = \int_{e^2-1}^{e^4-1} \frac{1}{2t^2} dt = \frac{1}{2}\left[-\frac{1}{t}\right]_{e^2-1}^{e^4-1} = \frac{1}{2}\left[-\left(\frac{1}{e^4-1}\right) - \left(-\frac{1}{(e^2-1)}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{(e^2-1)}\right) - \left(\frac{1}{(e^4-1)}\right)\right] = \frac{1}{2}\left[\frac{(e^4-1)-(e^2-1)}{(e^2-1)(e^4-1)}\right] = \frac{(e^4-e^2)}{2(e^2-1)(e^4-1)}$$

$$= \frac{e^2(e^2-1)}{2(e^2-1)(e^4-1)} = \frac{e^2}{2(e^4-1)}$$

Ex5B Definite Integrals

1. $y = \int_0^{\frac{\pi}{2}} sin^5 x dx$

$$u = cosx \quad \frac{du}{dx} = -sinx \quad -du = sinxdx \quad ; \quad x = \frac{\pi}{2} \quad u = 0 \quad x = 0 \quad u = 1$$

$$sin^2x = 1 - cos^2x = (1-u^2) sin^4x = (1-u^2)(1-u^2) = u^4 - 2u^2 + 1$$

$$\int_0^{\frac{\pi}{2}} sin^5 xdx = \int_0^{\frac{\pi}{2}} sin^4x\, sinxdx = -\int_1^0 (u^4 - 2u^2 + 1)du = -\left[\frac{u^5}{5} - \frac{2u^3}{3} + u\right]_1^0$$

$$= -\left[\left(\frac{0^5}{5} - \frac{2(0)^3}{3} + 0\right) - \left(\frac{1^5}{5} - \frac{2(1)^3}{3} + 1\right)\right]$$

$$= \left[\frac{1}{5} - \frac{2}{3} + 1\right] = \frac{8}{15}$$

---

2. $y = \int_0^{\frac{\pi}{4}} sin^3 cos^2 xdx$ *check* !!!!!

$$u = cosx \quad \frac{du}{dx} = -sinx \quad -du = sinxdx \quad ; \quad x = \frac{\pi}{4} \quad u = \frac{1}{\sqrt{2}} \quad x = 0 \quad u = 1$$

$$sin^2x = 1 - cos^2x = (1-u^2)$$

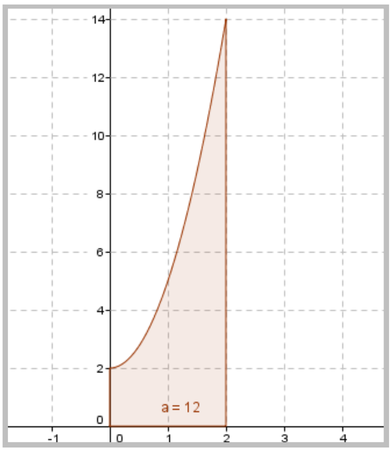
$$\int_0^{\frac{\pi}{4}} sin^3 cos^2 xdx = \int_0^{\frac{\pi}{2}} sin^2x\, cos^2x\, sinxdx = -\int_1^{\frac{1}{\sqrt{2}}} u^2(1-u^2)du = \int_1^{\frac{1}{\sqrt{2}}} u^4 - u^2 du = \left[\frac{u^5}{5} - \frac{u^3}{3}\right]_1^{\frac{1}{\sqrt{2}}}$$

$$= \left[\left(\frac{\left(\frac{1}{\sqrt{2}}\right)^5}{5} - \frac{\left(\frac{1}{\sqrt{2}}\right)^3}{3}\right) - \left(\frac{1^5}{5} - \frac{1^3}{3}\right)\right]$$

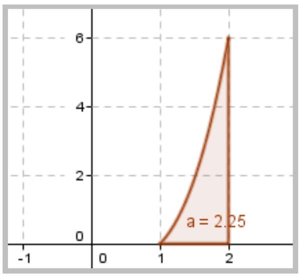
$$= \left[\left(\frac{1}{20\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)\right] = -\frac{14\sqrt{2}}{240} + \frac{2}{15} = -\frac{14\sqrt{2}}{240} + \frac{2}{15} = -\frac{7\sqrt{2}}{120} + \frac{2}{15}$$

<u>Ex7 Area under a Curve</u>

1. $\int_0^2 3x^2 + 2dx = [x^3 + 2x]_0^2 = \left[[8+4] - [0]\right] = 12$

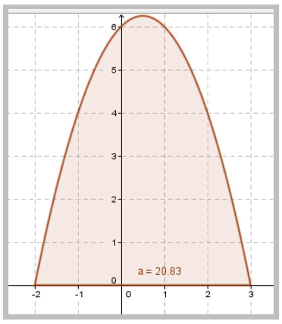


1. (b) $\int_1^2 x^3 - xdx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2 = \left[[4-2] - \left[\frac{1}{4} - \frac{1}{2}\right]\right] = 2\frac{1}{4}$



Q2. (a) $y = 6 + x - x^2 = (3-x)(2+x)$ $\qquad$ *roots are* $x = -2$ *and* $x = 3$

$$\int_{-2}^{3} (6 + x - x^2)\, dx = \left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{3} = \left[\left[18 + \frac{9}{2} - 9\right] - \left[-12 + 2 + \frac{8}{3}\right]\right] = \frac{125}{4} = 20\frac{5}{6}$$

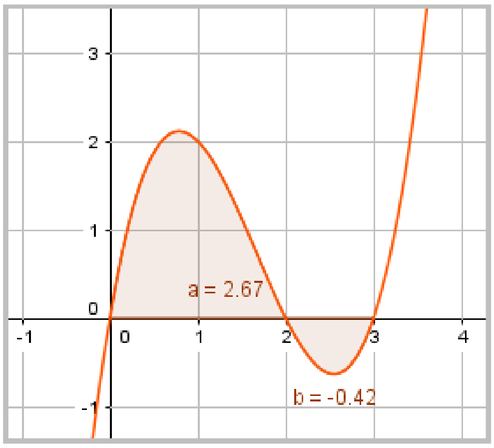


Q2. (b) $y = x(x-2)(x-3) = x(x^2-5x+6) = x^3-5x^2+6x \qquad roots\ are\ x=0 \quad x=2\ and\ x=3$

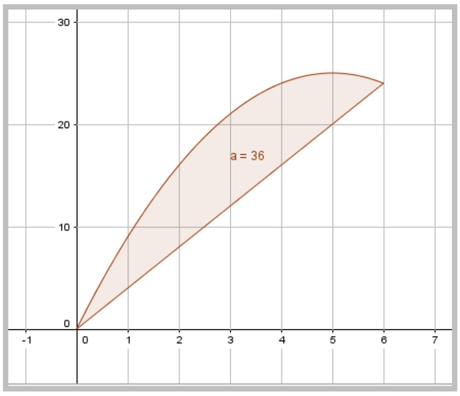
$$A_1 = \int_0^2 (x^3-5x^2+6x)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_0^2 = \left[\left[4 - \frac{40}{3} + 12\right] - [0]\right] = \frac{125}{4} = \mathbf{\frac{8}{3}}$$

$$A_2 = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_2^3 = \left[\left[\frac{81}{4} - 45 + 27\right] - \left[4 - \frac{40}{3} + 12\right]\right] = -\frac{5}{12} = \mathbf{\frac{5}{12}}$$

$$A_T = A_1 + A_2 = \frac{8}{3} + \frac{5}{12} = \frac{37}{12} = \mathbf{3\frac{1}{12}}$$



Ex8 Area between two curves

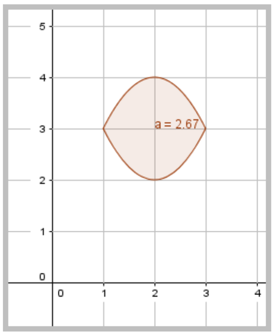


1. $y = x(10 - x) \qquad and \;\; y = 4x$

$10x - x^2 = 4x$
$x^2 - 6x \;\; = 0$

$x(x - 6) = 0 \qquad roots \;\; x = 0 \; and \;\; x = 6$

$\int_0^6 (10x - x^2) - 4x \, dx = \int_0^6 -x^2 + 6x \, dx \left[-\frac{x^3}{3} + 3x^2\right]_0^6 = \left[\left[-\frac{216}{3} + 108\right] - [0]\right] = 36$



2. $y = 4x - x^2 \qquad and \;\; y = x^2 - 4x + 6$

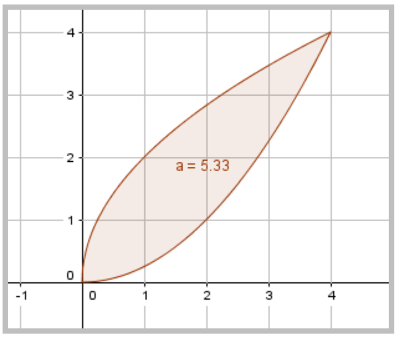
$x^2 - 4x + 6 = 4x - x^2$
$2x^2 - 8x + 6 \;\; = 0$
$x^2 - 4x + 3 \;\; = 0$
$(x - 1)(x - 3) \;\; = 0 \qquad roots \;\; x = 1 \; and \;\; x = 3$

$\int_1^3 (4x - x^2) - (x^2 - 4x + 6) \, dx = \int_1^3 -2x^2 + 8x - 6 \, dx = \left[-\frac{2x^3}{3} + 4x^2 - 6x\right]_1^3$

$$= \left[[-18 + 36 - 18] - \left[-\frac{2}{3} + 4 - 6\right]\right] = 2\frac{2}{3}$$

---

3. $y = 2\sqrt{x} \qquad and \;\; y = \frac{x^2}{4}$



$2\sqrt{x} = \frac{x^2}{4}$

$4x = \frac{x^4}{16}$

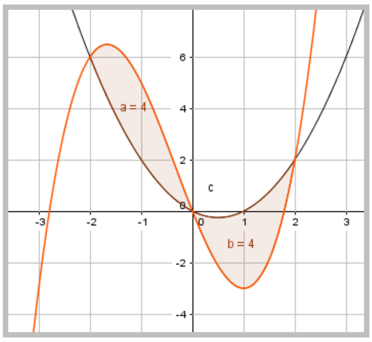
$x^4 - 64x = 0$

$x(x^3 - 64) = 0 \qquad roots \;\; x = 0 \;\; ; \;\; x = 4$

$$A = \int_0^4 \left(2x^{\frac{1}{2}}\right) - \frac{x^2}{4}\, dx = \left[\frac{4x^{\frac{3}{2}}}{3} - \frac{x^3}{12}\right]_0^4 = \left[\left[\frac{32}{3} - \frac{64}{12}\right] - [0]\right] = 5\,\frac{1}{3}$$

---

4. $y = x^3 + x^2 - 5x \qquad and \;\; y = x^2 - x$



$x^3 + x^2 - 5x = x^2 - x$

$x^3 - 4x \;\; = 0$

$x(x^2 - 4) \;\; = 0$

$x(x+2)(x-2) \;\; = 0 \qquad roots \;\; x = -2\, ; \;\; x = 0 \;\; ; \;\; x = 2$

$$A_1 = \int_{-2}^0 (x^3 + x^2 - 5x) - (x^2 - x)\, dx = \int_1^3 (x^3 - 4x)\, dx = \left[-\frac{x^4}{4} - 2x^2\right]_{-2}^0$$

$$= \left[[0] - [4 - 8]\right] = 4$$

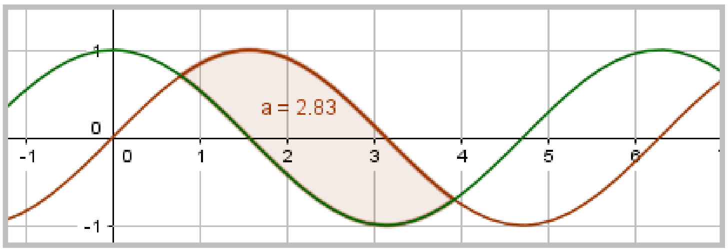
$$A_2 = \int_0^2 (x^2 - x) - (x^3 + x^2 - 5x)\, dx = \int_1^3 (4x - x^3)\, dx = \left[2x^2 - \frac{x^4}{4}\right]_0^2$$

$$= \left[[8 - 4] - [0]\right] = 4$$

$$A_T = 4 + 4 = 8$$

---

5. $y = sinx \qquad and \;\; y = cosx$



$sinx = cosx$

$tanx = 1$

$roots\; x = \frac{\pi}{4} \qquad roots\;\; x = \frac{5\pi}{4}$

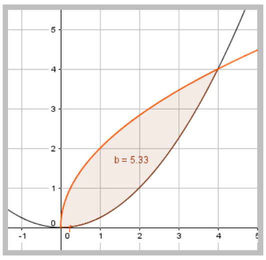
$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (sinx - cosx)\, dx = \left[[-cosx] - [sinx]\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= \left[\left[\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right)\right] - \left[\left(-\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\right]$$

$$= \left[\left(\frac{2}{\sqrt{2}}\right) + \left(\frac{2}{\sqrt{2}}\right)\right] = 2\sqrt{2}$$

---

6. $y^2 = 4ax \qquad and \;\; x^2 = 4ay$



Example

when a = 1

$y = 2\sqrt{ax} \qquad and \;\; y = \frac{x^2}{4a}$

$2\sqrt{ax} = \frac{x^2}{4a} \qquad ; \qquad 4ax = \frac{x^4}{16a^2} \qquad ; \qquad 64a^3x = x^4$

$x^4 - 64a^3x = 0 \qquad ; \qquad x(x^3 - 64a^3) = 0 \qquad ; \qquad roots\;\; are\;\; x = 0 \;\; ; \;\; x = 4a$

$$A = \int_0^{4a} \left(2\sqrt{a}x^{\frac{1}{2}} - \frac{1}{4a}x^2\right) dx = \left[\left[\frac{4\sqrt{a}x^{\frac{3}{2}}}{3}\right] - \left[\frac{1}{12a}x^3\right]\right]_0^{4a}$$

$$= \left[\left[\frac{4\sqrt{a}(4a)^{\frac{3}{2}}}{3}\right] - \left[\frac{1}{12a}(4a)^3\right]\right] - [0] = \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3}$$

Ex 9 Area enclosed by curve and the y-axis

1. $x = y^2 \quad : y = 3$

$$A = \int_0^3 y^2 \, dy = \left[\frac{y^3}{3}\right]_0^3 = [[9] - [0]] = 9$$

---

2. $y = x^3 \quad x = y^{\frac{1}{3}}; \quad y = 1 \; and \; y = 8$

$$A = \int_1^8 y^{\frac{1}{3}} \, dy = \left[\frac{3y^{\frac{4}{3}}}{4}\right]_0^3 = \left[\left[\frac{3(8)^{\frac{4}{3}}}{4}\right] - \left[\frac{3}{4}\right]\right] = \left[[12] - \left[\frac{3}{4}\right]\right] = 11\frac{1}{4}$$

---

3. $y = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}} \quad y = 2 \; and \; y = 3$

$$A = \int_2^3 y^{-\frac{1}{3}} \, dy = \left[2y^{\frac{1}{3}}\right]_2^3 = \left[\left[2(3)^{\frac{1}{3}}\right] - \left[2(2)^{\frac{1}{3}}\right]\right] = [[2\sqrt{3}] - [2\sqrt{2}]] = 2\sqrt{3} - 2\sqrt{2}$$

---

4. $y^2 = 1 - x \quad ; \; x = 1 - y^2 \quad y = 0 \; and \; y = 1$

$$A = \int_0^1 1 - y^2 \, dy = \left[y - \frac{y^3}{3}\right]_0^1 = \left[\left[1 - \frac{(1)^3}{3}\right] - [0]\right] = \frac{2}{3}$$

---

5. $y = \frac{1}{x^3} \quad ; \; x = \frac{1}{y^{\frac{1}{3}}} \; ; \; x = y^{-\frac{1}{3}} \quad y = 8 \; and \; y = 27$

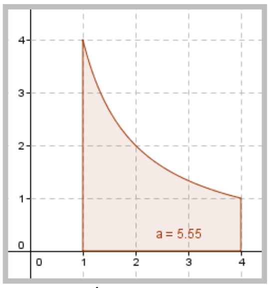
$$A = \int_8^{27} y^{-\frac{1}{3}} \, dy = \left[\frac{3y^{\frac{2}{3}}}{2}\right]_8^{27} = \left[\left[\frac{27}{2}\right] - [6]\right] = \frac{15}{2} = 7\frac{1}{2}$$

---

6. $y = lnx \quad ; \quad x = e^y \qquad y = 2\ and\ y = 5$

$$A = \int_2^5 e^y\ dy = [e^y]_2^5 = [[e^5]—[e^2]] = e^5 - e^2 = e^2(e^3 - 1)$$

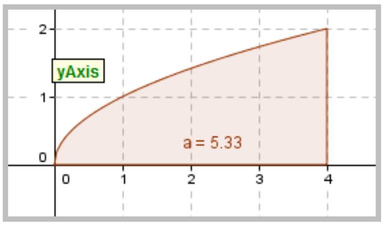
Ex10 - The Volume of Revolution



1.(a) $y = \frac{4}{x}$ $\quad\quad x = 1 \text{ and } x = 4$

$$A = \int_1^4 \pi\left(\frac{4}{x}\right)^2 \; dx = A = \pi\int_1^4 \frac{16}{x^2} \; dx = \pi\int_1^4 16x^{-2} \; dx = \pi\left[-\frac{16}{x}\right]_1^4 = \pi[[-4]—[-16]] = 12\pi$$

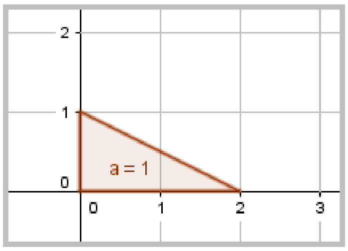
(b) $y = \sqrt{x}$; $\quad y = x^{\frac{1}{2}}$ $\quad\quad x = 0 \text{ and } x = 4$



$$A = \int_0^4 \pi\left(x^{\frac{1}{2}}\right)^2 \; dx = A = \pi\int_0^4 x \; dx = \pi\left[\frac{x^2}{2}\right]_0^4$$

$$= \pi[[8]—[0]] = 8\pi$$

(c) $y = x + 2y = 2$; $\quad y = -\frac{1}{2}x + 1$ $\quad\quad x = 0 \text{ and } x = 2$

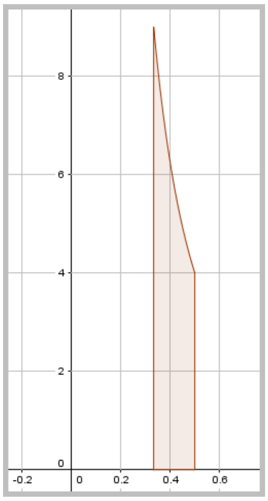


$$A = \int_0^2 \pi\left(-\frac{1}{2}x + 1\right)^2 \; dx = A$$

$$= \pi\int_0^2 \left(\frac{1}{4}x^2 - x + 1\right) \; dx = \pi\left[\frac{1}{12}x^3 - \frac{x^2}{2} + x\right]_0^2$$

$$= \pi\left[\left[\frac{2}{3}\right] — [0]\right] = \frac{2}{3}\pi$$

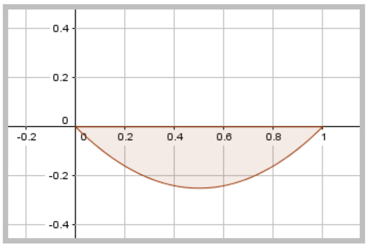
(d) $y = \frac{1}{x^2}$; $\quad y = x^{-2}$ $\quad\quad x = \frac{1}{3} \text{ and } x = \frac{1}{2}$



$$A = \int_{\frac{1}{3}}^{\frac{1}{2}} \pi(x^{-2})^2 \; dx = A = \pi\int_{\frac{1}{3}}^{\frac{1}{2}} (x^{-4}) \; dx = \pi\left[-\frac{1}{3x^3}\right]_{\frac{1}{3}}^{\frac{1}{2}}$$

$$= \pi\left[\left[-\frac{8}{3}\right] — [-9]\right] = \frac{19}{3}\pi$$

(e) $y = x(x-1);$; $y = x^2 - x$ $x = 0$ *and* $x = 1$



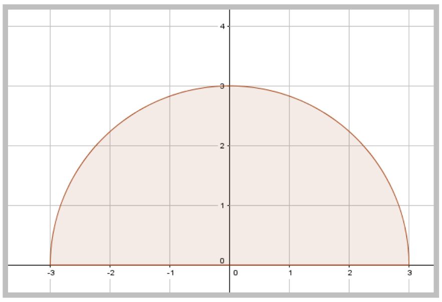
$$A = \int_0^1 \pi\,(x^2 - x)^2\ dx = \pi \int_0^1 x^4 - 2x^3 + x^2\,dx = \pi\left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right]_0^1$$

$$= \pi\left[\left[\frac{(1)^5}{5} - \frac{(1)^4}{2} + \frac{(1)^3}{3}\right] - [0]\right] = \frac{\pi}{30}$$

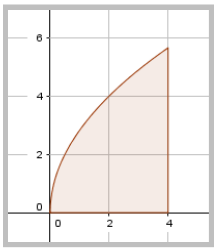
(f) $y = \sqrt{(9 - x^2)};$; $y = (9 - x^2)^{\frac{1}{2}}$ $x = -3$ *and* $x = 3$

$$A = \int_{-3}^{3} \pi\left((9 - x^2)^{\frac{1}{2}}\right)^2\ dx = \pi \int_{-3}^{3} (9 - x^2)\,dx = \pi\left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

$$= \pi[[27 - 9] - [-27 + 9]] = \pi(54 - 18) = 36\pi$$



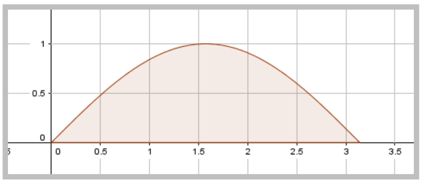
(g) $y^2 = 8x$; $y = \sqrt{8x}$ $x = 0$ *and* $x = 4$



$$A = \int_0^4 \pi\left((8x)^{\frac{1}{2}}\right)^2\ dx = \pi \int_0^4 8x\,dx = \pi[4x^2]_0^4$$

$$= \pi[[64] - [0]] = 64\pi$$

(h) $y = sinx$ $x = 0$ *and* $x = \pi$



$$A = \int_0^4 \pi(sinx)^2\ dx$$

$$= \pi \int_0^4 \frac{1}{2}(1 - cos2x)\,dx = \frac{\pi}{2}\int_0^4 (1 - cos2x)\,dx$$

$$= \frac{\pi}{2}\left[x - \frac{1}{2}sin2x\right]_0^{\pi}$$

$$= \frac{\pi}{2}[[\pi - 0] - [0]] = \frac{\pi^2}{2}$$
