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Advanced Higher Maths

Unit 1.4 First Order Differential Equations Solutions

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Ex9 General Solution First Order

1. $(1+x)\frac{dy}{dx} = xy$; $\frac{1}{y}dy = \frac{x}{1+x}dx$

$$\int \frac{1}{y}dy = \int \frac{x}{1+x}dx$$

$\int \frac{1}{y}dy = lny$

$\int \frac{x}{1+x}dx$ $\quad let\, u = 1+x$; $du = dx$; $x = u-1$

$\int \frac{u-1}{u}du = \int 1 - \frac{1}{u}du = u - lnu = (1+x) - \ln(1+x)$

$$lny = (1+x) - ln(1+x) + c$$

$$lny + ln(1+x) = (1+x) + c$$

$$\ln(y(1+x)) = (1+x) + c$$

$$y(1+x) = e^{(1+x)+c}$$

$$y = \frac{e^{(1+x)+c}}{(1+x)} = \frac{Ae^x}{(1+x)} \qquad A = e^{1+c}$$

2. $\frac{dy}{dx} = x(1-y)^2$; $\frac{1}{(1-y)^2}dy = xdx$

$$\int \frac{1}{(1-y)^2}dy = \int (1-y)^{-2}dy = \frac{1}{(1-y)}$$

$$\int xdx = \frac{x^2}{2}$$

$$\frac{1}{(1-y)} = \frac{x^2}{2} + c$$

$$\frac{1}{(1-y)} = \frac{x^2}{2} + c$$

$$1 = (1-y)\left[\frac{x^2}{2} + c\right]$$

$$\frac{1}{\frac{x^2}{2} + c} = 1 - y + c$$

$$\frac{2}{x^2 + C} = 1 - y$$

$$y = 1 - \frac{2}{x^2 + C}$$

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3. $\frac{dy}{dx} = e^x y^2$; $\frac{1}{y^2} dy = e^x dx$

$$\int \frac{1}{y^2} dy = \int y^{-2} dy = -\frac{1}{y}$$

$$\int e^x dx = e^x$$

$$-\frac{1}{y} = (e^x + c)$$

$$\frac{1}{y} = -(e^x + c)$$

$$\frac{-1}{(e^x + c)} = y$$

$$y = -\frac{1}{(e^x + c)}$$

4. $x(y-1)\frac{dy}{dx} = 2y$; $\frac{(y-1)}{y}dy = \frac{2}{x}dx$; $1 - \frac{1}{y}dy = \frac{2}{x}dx$

$$\int 1 - \frac{1}{y}dy = y - lny$$

$$\int \frac{2}{x}dx = 2lnx$$

$$y - lny = 2lnx + c$$

$$y = lnx^2 + lny + c$$

$$y = ln(yCx^2)$$

$$e^y = yCx^2$$

$$y = \frac{e^y}{Cx^2}$$

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5. $sinxcosy = sinycosx\frac{dy}{dx}$; $\frac{siny}{cosy}dy = \frac{sinx}{cosx}dx$; $tany\ dy = tanx\ dx$

$\int tany\ dy = -\ ln|cosy|$

$\int tanx\ dx = -ln|cosx|$

$$-ln|cosy| = -ln|cosx| + c$$

$$ln|cosx| - ln|cosy| = c$$

$$ln\left(\frac{|cosx|}{|cosy|}\right) = c$$

$$\frac{cosx}{cosy} = C$$

6. $y - x\frac{dy}{dx} = 1 + x^2\frac{dy}{dx}$; $(x^2 + x)\frac{dy}{dx} = (y - 1)$; $\frac{1}{(y-1)}\, dy = \frac{1}{(x^2+x)}\, dx$

$$\int \frac{1}{(y-1)}\, dy = ln(y-1)$$

$$\int \frac{1}{(x^2+x)}\, dx = \int \frac{1}{x(x+1)}\, dx$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)}$$

$$1 = A(x+1) + Bx$$

$x = 0 \quad A = 1$

$x = -1 \quad B = -1$

$$\int \frac{1}{x(x+1)}\, dx = \int \frac{1}{x} - \frac{1}{(x+1)}\, dx = lnx - \ln(x+1)$$

$$ln(y-1) = lnx - \ln(x+1) + c$$

$$ln(y-1) = ln\frac{Cx}{(x+1)}$$

$$y = \frac{Cx}{(x+1)} + 1 = \frac{Cx + (x+1)}{(x+1)}$$

$$y = \frac{x(C+1) + 1}{(x+1)}$$

Ex10 Particular Solution First Order

1. $(1 - cos2x)\frac{dy}{dx} = 2sin2x$; $when\ x = \frac{\pi}{4}$ and $y = 1$

$(1 - cos2x) = 2sin^2x \quad and\ sin2x = 2sinxcosy$

$$dy = \frac{4sinxcosx}{2sin^2x}dx = \frac{2cosx}{sinx}dx$$

$u = sinx$

$u = cosx\ dx$

$$\int dy = 2\int \frac{1}{u}du$$

$$y = 2lnu$$

$$y = 2ln(sinx) + c$$

$x = \frac{\pi}{4} \quad y = 1$

$$1 = 2ln\left(sin\left(\frac{\pi}{4}\right)\right) + c$$

$$1 = 2ln\left(sin\left(\frac{\pi}{4}\right)\right) + c$$

$$1 = ln\left(\frac{1}{2}\right) + c = ln(2)^{-1} + c$$

$$c = 1 + ln(2)$$

$$y = 2ln(sinx) + 1 + ln(2)$$

$$y = ln(2sin^2x) + 1$$

$$y = ln(1 - cos2x) + 1$$

2. $(1 + x^2)\frac{dy}{dx} = (1 + y^2)$; $when\ \ x = 0$ and $y = 1$

$$\int \frac{1}{(1 + y^2)}dy = \int \frac{1}{(1 + x^2)}dx$$

$$tan^{-1}y = tan^{-1}x + c$$

$x = 0$ $y = 1$

$$tan^{-1}(1) = tan^{-1}(0) + c$$

$$c = \frac{\pi}{4}$$

$$tan^{-1}(y) = tan^{-1}(x) + \frac{\pi}{4}$$

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3. $\frac{dy}{dx} = x(y - 2)$; $when\ \ x = 0$ and $y = 5$

$$\int \frac{1}{(y - 2)}dy = \int xdx$$

$$ln(y - 2) = \frac{x^2}{2} + c$$

$x = 0$ $y = 5$

$$ln3 = 0 + c$$

$$c = ln3$$

$$ln(y - 2) = \frac{x^2}{2} + ln3$$

$$ln\frac{(y - 2)}{3} = \frac{x^2}{2}$$

$$y = 3e^{\frac{x^2}{2}} + 2$$

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4. $\frac{dy}{dx} = \sqrt{(1-y^2)}$; $when\ x = \frac{\pi}{6}$ and $y = 0$

$$\int \frac{1}{\sqrt{(1-y^2)}} dy = \int dx$$

$$sin^{-1}y = x + c$$

$x = \frac{\pi}{6}$ $y = 0$

$$0 = \frac{\pi}{6} + c$$

$$c = -\frac{\pi}{6}$$

$$sin^{-1}y = x - \frac{\pi}{6}$$

$$y = sin\left(x - \frac{\pi}{6}\right)$$

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5. $\frac{dy}{dx} = ycosx$; $when\ x = 0$ and $y = 1$

$$\int \frac{1}{y} dy = \int cosx\, dx$$

$$lny = sinx + c$$

$x = 0$ $y = 1$

$$0 = 0 + c$$

$$c = 0$$

$$lny = sinx$$

$$y = e^{sinx}$$

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6. $\frac{dy}{dx} = tanxtany \quad ; \quad when \; x = \frac{\pi}{4} \quad and \quad y = \frac{\pi}{4}$

$$\int \frac{1}{tany} dy = \int tanx \, dx$$

$$\int \frac{cosy}{siny} dy = \int tanx \, dx$$

$u = siny \quad du = cosydy$

$\int \frac{cosy}{siny} dy = \int \frac{1}{u} \, du = lnu = ln(|siny|)$

$\int tanx \, dx = -\ln(|cosx|)$

$$\int \frac{cosy}{siny} dy = \int tanx \, dx$$

$$ln(|siny|) = -\ln(|cosx|) + c$$

$x = \frac{\pi}{4} \quad y = \frac{\pi}{4}$

$$ln\left(\frac{1}{\sqrt{2}}\right) = -\ln\left(\frac{1}{\sqrt{2}}\right) + c$$

$$c = ln\left(\frac{1}{2}\right)$$

$$ln(|siny|) = -\ln(|cosx|) + ln\left(\frac{1}{2}\right)$$

$$ln(|siny|) = ln\left(\frac{1}{2|cosx|}\right)$$

$$siny = \frac{1}{2cosx}$$

Ex9A Context First Order

1. $(a)\ \frac{ds}{dt} = ks \quad (b)\ \frac{ds}{dt} = \frac{k}{s} \quad (c)\ \frac{dn}{dt} = kn$

$(d)\ \frac{dV}{dt} = kr \quad (e)\ \frac{dN}{dt} = k(500 - N) \quad (f)\ \frac{dB}{dC} = \frac{k}{C}$

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2. (a)

$$\frac{dD}{dh} = \frac{k}{\sqrt{h}}$$

$$\int 1\ dD = \int kh^{-\frac{1}{2}}\ dh$$

$$\boldsymbol{D = k\sqrt{h} + c}$$

(b)

$D = 8km\ \ h = 0m$ $\qquad 8 = k\sqrt{0} + c \quad ; \quad c = 8$

$D = 55km\ \ h = 100m$ $\qquad 55 = k\sqrt{100} + 8$

$$55 = 10k + 8$$

$$47 = 10k \quad ; \quad k = 4.7$$

$$\boldsymbol{D = 4.7\sqrt{h} + 8}$$

$(c)\quad h = 850m$ $\qquad D = 4.7\sqrt{850} + 8 = \boldsymbol{145km}$

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3. (a)

$$\frac{dT}{dt} = -\frac{2}{5}t + \frac{3}{2}$$

$$\int 1\, dT = \int -\frac{2}{5}t + \frac{3}{2}\ dt$$

$$\boldsymbol{T = -\frac{1}{5}t^2 + \frac{3}{2}t\ + c}$$

$t = 0 \quad T = 98.4$

$$\boldsymbol{T = -\frac{1}{5}t^2 + \frac{3}{2}t\ + 98.4}$$

(b)

$T = 98.4$

$$98.4 = -\frac{1}{5}t^2 + \frac{3}{2}t\ + 98.4$$

$$\frac{1}{5}t^2 - \frac{3}{2}t = 0$$

$$2t^2 - 15t = 0$$

$$t(2t - 15) = 0$$

$$t = 0 \quad and \quad t = 7.5\ days$$

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4. (a)

$$\frac{dV}{dt} = ke^{-0.3t}$$

$$\int 1\ dV = k\int e^{-0.3t}\ \ dt$$

$$\boldsymbol{V = -\frac{10}{3}ke^{-0.3t} + c}$$

(b)

$t = 0 \quad V = 0$

$$0 = -\frac{10}{3}ke^{-0.3(0)} + \boldsymbol{c} \qquad ; \qquad c = \frac{10}{3}k$$

(c)

$$V = -\frac{10}{3}ke^{-0.3t} + \frac{10}{3}k = \boldsymbol{\frac{10}{3}k\left(1 - e^{-0.3t}\right)}$$

$t = 5 \quad V = 77.7\%$

$$77.7 = \frac{10}{3}k\left(1 - e^{-0.3(5)}\right)$$

$$k = \frac{77.7 \times 3}{10(1 - e^{-0.3(5)})} = 30$$

$$\boldsymbol{V = 100\left(1 - e^{-0.3t}\right)}$$

$(d) \quad t = 10 \quad V = 90\% \qquad V = 100(1 - e^{-3}) = 95\%$

The model is over estimating the true audience figures.

Ex11 - More Modelling

1. (a)

$$\frac{dx}{dt} = -kx$$

$$\int \frac{1}{x}\, dx = -k \int 1 \;\; dt$$

$$\ln(x) = -kt + c$$

$t = 0 \quad x = x_0$

$$\ln(x) = -kt + x_0$$

$$\boldsymbol{x = x_0 e^{-kt}}$$

$x = 50\% \quad t = 25$

$$50 = 100e^{-k(25)}$$

$$0.5 = e^{-k(25)}$$

$$l\,n(0.5) = -25k$$

$$k = \frac{l\,n(0.5)}{-25}$$

$$k = \frac{l\,n(0.5)}{-25} = 0.0277$$

$$\boldsymbol{x = x_0 e^{-0.0277t}}$$

$x = 20g \quad x_0 = 100g$

$$\boldsymbol{20 = 100e^{-0.0277t}}$$

$$\boldsymbol{0.2 = e^{-0.0277t}}$$

$$\mathrm{t} = \frac{ln(0.2)}{-0.0277}$$

$$\mathrm{t} = \frac{ln(0.2)}{-0.0277} = 58.1\;days$$

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2. (a)

$$\frac{d\emptyset}{dt} = -k(\emptyset - 15)$$

$$\int \frac{1}{(\emptyset - 15)}\, d\emptyset = -k \int 1 \;\, dt$$

$$ln(\emptyset - 15) = -kt + c$$

$$\emptyset = Ae^{-kt} + 15 \qquad ; \quad A = e^c$$

$(b)(i)$ $t = 10$ $\emptyset = 50$

$$\emptyset = Ae^{-kt} + 15$$

$$50 = 100e^{-k(10)} + 15$$

$$0.35 = e^{-10k}$$

$$k = \frac{ln(0.35)}{-10} = 0.105$$

$$\boldsymbol{\emptyset = 100e^{-0.105t} + 15}$$

$t = 5$

$$\emptyset = 100e^{-0.105t} + 15$$

$$\emptyset = 74.2\,°C$$

$(b)(ii)$ $\emptyset = 45$

$$45 = 100e^{-0.105t} + 15$$

$$0.3 = e^{-0.105t}$$

$$\text{t} = \frac{ln(0.3)}{-0.105}$$

$$\text{t} = \frac{ln(0.3)}{-0.105} = \boldsymbol{11.5\, minutes}$$

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3. (a)

$$\frac{dP}{dt} = kp$$

$$\int \frac{1}{p}\, dp = k\int 1\ \ dt$$

$$ln(p) = kt + c$$

$$P = P_0 e^{kt} \qquad ; \quad P_0 = e^c$$

$(b)(i)$ $\quad t = 10 \quad P_0 = 468 \quad P = 534$

$$P = P_0 e^{kt}$$

$$534 = 468e^{k(10)}$$

$$k = \frac{ln\left(\frac{534}{468}\right)}{10} = 0.0132$$

$$P = 468e^{0.0132t}$$

(b) $\quad t = 20$

$$P = 468e^{0.0132(20)}$$

$$P = 609\ people$$

4. (a)

$$\frac{dx}{dt} = kx$$

$$\int \frac{1}{x}\,dx = k\int 1\;\;dt$$

$$ln(x) = kt + c$$

$$x = x_0e^{kt} \qquad ; \quad x_0 = e^c$$

$(b)(i)$ $\quad t = 0 \quad x_0 = 100$ $\qquad x = 100e^{kt}$

$\quad t = 7 \quad x = 1000$ $\qquad 1000 = 100e^{k(7)}$

$$k = \frac{ln(10)}{7}$$

$$\boldsymbol{x = 100e^{\left(\frac{ln(10)}{7}\right)t}}$$

$$x = 100e^{\left(\frac{ln(10)}{7}\right)t}$$

$$ln\left(\frac{x}{100}\right) = \left(\frac{ln(10)}{7}\right)t$$

$$\boldsymbol{ln\left(\frac{x}{100}\right) = \frac{1}{7}t\,ln(10)}$$

(c)

$$x = 100e^{\left(\frac{\ln(10)}{7}\right)(10.5)} = 3162m^2$$

$$Total\ not\ covered = 10000 - 3162 = \boldsymbol{6838m^2}$$

(d) $\quad t = half - life$

$$\frac{5000}{100} = e^{\left(\frac{\ln(10)}{7}\right)t}$$

$$t = \frac{7ln(50)}{ln(10)} = \ \ 11.9\ days$$

5. (a)

$$\frac{dx}{dt} = kx$$

$$\int \frac{1}{x}\, dx = k \int 1 \ \ dt$$

$$ln(x) = kt + c$$

$t = 0 \quad x = x_0 \quad ; \quad c = ln(x_0) \qquad x = x_0 e^{kt}$

$t = 1 \quad x_0 = 2 \quad x = 1.6 \qquad 1.6 = 2e^{k(1)}$

$$ln\left(\frac{1.6}{2}\right) = k$$

$$k = -0.223$$

$$\boldsymbol{x = 2e^{-0.223t}}$$

$(b)(i) \quad t = 3 \qquad x = 2e^{-0.223\times(3)} = 1.024mg$

$(b)(ii) \quad x = 0.5 \qquad \frac{0.5}{2} = e^{-0.223t}$

$$t = \frac{ln\left(\frac{0.5}{2}\right)}{-0.223} = \ \ 6.2\ hours$$

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6. (a) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$; $V = (x^2 + 3x)$ $\frac{dV}{dx} = 2x + 3$ $\frac{dV}{dt} = x^2 + 4$

$$\frac{dx}{dt} = \frac{1}{2x+3} \times (x^2+4) = \frac{(x^2+4)}{(2x+3)}$$

$$\frac{dx}{dt} = \frac{(x^2+4)}{(2x+3)}$$

$$\int \frac{(2x+3)}{(x^2+4)}\,dx = \int 1 \; dt$$

$$\int \frac{2x}{(x^2+4)} + \frac{3}{(x^2+4)}\,dx = \int 1 \; dt$$

$$ln(x^2+4) + \frac{3}{2}tan^{-1}\left(\frac{x}{2}\right) = t + c$$

$t = 0 \quad x = 0$

$$ln(0^2+4) + \frac{3}{2}tan^{-1}\left(\frac{0}{2}\right) = 0 + c$$

$$c = ln(4)$$

$$ln(x^2+4) + \frac{3}{2}tan^{-1}\left(\frac{x}{2}\right) = t + ln(4)$$

$$ln\left(\frac{x^2+4}{4}\right) + \frac{3}{2}tan^{-1}\left(\frac{x}{2}\right) = t$$

$$t = ln\left(\frac{x^2+4}{4}\right) + \frac{3}{2}tan^{-1}\left(\frac{x}{2}\right)$$

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7. (a) $\frac{dv}{dt} = \frac{1}{2}(v - v^2)$

$$\int \frac{1}{v(1-v)}\ dv = \frac{1}{2}\int 1\ \ dt$$

$\frac{1}{v(1-v)} = \frac{A}{v} + \frac{B}{(1-v)}$; $1 = A(1-v) + Bv$

$v = 0 \quad A = 1$

$v = 1 \quad B = 1$

$$\int \frac{1}{v(1-v)}\ dv = \int \frac{1}{v} + \frac{1}{(1-v)}\ dv = ln|v| - ln|(1-v)| = ln\left|\frac{v}{(1-v)}\right|$$

$\frac{1}{2}\int 1\ \ dt = \frac{1}{2}t$

$$ln\left|\frac{v}{(1-v)}\right| = \frac{1}{2}t + c$$

$t = 0 \quad v = 0.2$ $\qquad c = ln\left|\frac{0.2}{0.8}\right| = ln\left(\frac{1}{4}\right)$

$$ln\left|\frac{v}{(1-v)}\right| = \frac{1}{2}t + ln\left(\frac{1}{4}\right)$$

$$\frac{v}{(1-v)} = e^{\left(\frac{1}{2}t + ln\left(\frac{1}{4}\right)\right)}$$

$$\frac{v}{(1-v)} = \frac{1}{4}e^{\left(\frac{1}{2}t\right)} \quad ; \quad 4e^{\left(-\frac{1}{2}t\right)} = \frac{1-v}{v} \quad ; \quad v4e^{\left(-\frac{1}{2}t\right)} + v = 1$$

$$v\left(1 + 4e^{\left(-\frac{1}{2}t\right)}\right) = 1$$

$$v = \frac{1}{\left(1 + 4e^{\left(-\frac{1}{2}t\right)}\right)}$$

8. $\frac{d\theta}{dt} = 40 - \frac{2}{5}(\theta - 15) = 46 - \frac{2}{5}\theta$

$$\int \frac{1}{46 - \frac{2}{5}\theta}\, d\theta = \int 1 \;\; dt$$

$$\int \frac{1}{46 - \frac{2}{5}\theta}\, d\theta = -\frac{5}{2} ln\left(46 - \frac{2}{5}\theta\right) \qquad ; \qquad \int 1 \;\; dt = 40t$$

$$-\frac{5}{2} ln\left(46 - \frac{2}{5}\theta\right) = t + c$$

$t = 0 \qquad \theta = 15$

$$-\frac{5}{2} ln\left(46 - \frac{2}{5}(15)\right) = (0) + c$$

$$c = -\frac{5}{2}\ln(40)$$

$$t = -\frac{5}{2} ln\left(46 - \frac{2}{5}\theta\right) + \frac{5}{2}\ln(40)$$

$$t = \frac{5}{2} ln\left(\frac{40}{46 - \frac{2}{5}\theta}\right)$$

$\theta = 100$

$$t = \frac{5}{2} ln\left(\frac{40}{46 - \frac{2}{5}(100)}\right)$$

$$t = \frac{5}{2} ln\left(\frac{40}{6}\right)$$

$$t = 4.74\, minutes$$

Ex1 Integrating Factor

1. $x\frac{dy}{dx} + x^2y = 3x \quad ; \quad standard\ form \quad \frac{dy}{dx} + xy = 3$

(b) $\frac{dy}{dx} + 4x = xy \quad ; \quad standard\ form \quad \frac{dy}{dx} - xy = -4x$

(c) $-5x^2\ \frac{dy}{dx} + 4xy = x^3 \quad ; \quad standard\ form \quad \frac{dy}{dx} - \frac{4}{5x}y = -\frac{x}{5}$

(d) $sinx\ \frac{dy}{dx} + ycosx = 0 \quad ; \quad standard\ form \quad \frac{dy}{dx} + ycotx = 0$

(e) $\frac{dy}{dx} - 5 = 0 \quad ; \quad standard\ form \quad \frac{dy}{dx} = 5$

(f) $t^2\ \frac{dy}{dt} + 4yt = t^3 - 1 \quad ; \quad standard\ form \quad \frac{dy}{dx} + \frac{4}{t}y = t - \frac{1}{t^2}$

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2. $\frac{dy}{dx} + xy = 1 \quad ; \quad integrating\ factor\ e^{\int x dx} = e^{\frac{1}{2}x^2}$

(b) $\frac{dy}{dx} + \frac{2}{x}y = 3x \quad ; \quad integrating\ factor\ e^{\int \frac{2}{x} dx} = e^{2lnx} = x^2$

(c) $4\frac{dy}{dx} - \frac{2}{x^2}y = 5 - x^2 \quad ; \quad \frac{dy}{dx} - \frac{1}{2x^2}y = \frac{5}{4} - \frac{x^2}{4} \quad integrating\ factor\ e^{-\frac{1}{2}\int \frac{1}{x^2} dx} = e^{\frac{1}{2x}}$

(d) $3x\frac{dy}{dx} - 6x^2y = 12x^3 \quad ; \quad \frac{dy}{dx} - 2xy = 4x^2 \quad integrating\ factor\ e^{-2\int x dx} = e^{-x^2}$

(e) $3x^2\frac{dy}{dx} - 6xy = 12x \quad ; \quad \frac{dy}{dx} - \frac{2}{x}y = \frac{4}{x} \quad integrating\ factor\ e^{-2\int \frac{1}{x} dx} = e^{-2lnx} = \frac{1}{x^2}$

(f) $9y - x^2\frac{dy}{dx} = 0 \quad ; \quad \frac{dy}{dx} - \frac{9}{x^2}y = 0 \quad integrating\ factor\ e^{-9\int \frac{1}{x^2} dx} = e^{\frac{9}{x}}$

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3. $\frac{dy}{dx} - \frac{2}{x}y = 2$; $integrating\ factor\ e^{-2\int\frac{1}{x}dx} = \frac{1}{x^2}$

$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{2}{x^2}$; $y\frac{1}{x^2} = \int \frac{2}{x^2}\,dx$

$y\frac{1}{x^2} = -\frac{2}{x} + c$

$y = -2x + cx^2$

$y = cx^2 - 2x$

(b) $x\frac{dy}{dx} - 2y = -3x$; $\frac{dy}{dx} - \frac{2}{x}y = -3$ $integrating\ factor\ e^{-2\int\frac{1}{x}dx} = \frac{1}{x^2}$

$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{-3}{x^2}$; $y\frac{1}{x^2} = -3\int \frac{1}{x^2}\,dx$

$y\frac{1}{x^2} = \frac{3}{x} + c$

$y = 3x + cx^2$

$y = cx^2 - 2x$

(c) $3x\frac{dy}{dx} + 3y = 2x \quad ; \ \frac{dy}{dx} + \frac{1}{x}y = \frac{2}{3} \ \ integrating\ factor\ e^{\int \frac{1}{x}dx} \ = \ x$

$$x\frac{dy}{dx} - y = \frac{2}{3}x \qquad ; \quad xy = \frac{2}{3}\int x\,dx$$

$$xy = \frac{1}{3}x^2 + c$$

$$y = \frac{1}{3}x + \frac{c}{x}$$

(d) $x\frac{dy}{dx} + 2y = \frac{4}{x} \quad ; \ \frac{dy}{dx} + \frac{2}{x}y = \frac{4}{x^2} \ \ integrating\ factor\ e^{2\int \frac{1}{x}dx} \ = x^2$

$$x^2\frac{dy}{dx} - 2xy = 4 \qquad ; \quad x^2y = \int 4\,dx$$

$$x^2y = 4x + c$$

$$y = \frac{4}{x} + \frac{c}{x^2}$$

(e) $x\frac{dy}{dx} - 2y = 6x^4 \quad ; \ \frac{dy}{dx} - \frac{2}{x}y = 6x^3 \ \ integrating\ factor\ e^{-2\int \frac{1}{x}dx} \ = \frac{1}{x^2}$

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = 6x \qquad ; \quad y\frac{1}{x^2} = \int 6x\,dx$$

$$\frac{y}{x^2} = 3x^2 + c$$

$$y = 3x^4 + cx^2$$

(f) $\frac{dy}{dx} + 2y = 1$; $integrating\ factor\ e^{2\int 1dx} = e^{2x}$

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = e^{2x} \quad ; \quad y\,e^{2x} = \int e^{2x}\,dx$$

$$y\,e^{2x} = \frac{1}{2}\,e^{2x} + c$$

$$y = \frac{1}{2} + \frac{c}{e^{2x}}$$

$$y = \frac{1}{2} + ce^{-2x}$$

(g) $\frac{dy}{dx} - y = -2e^{-x}$; $integrating\ factor\ e^{-\int 1dx} = e^{-x}$

$$e^{-x}\frac{dy}{dx} - e^{-x}y = -2e^{-2x} \quad ; \quad y\,e^{-x} = -2\int e^{-2x}\,dx$$

$$y\,e^{-x} = e^{-2x} + c$$

$$y = e^{-x} + ce^{x}$$

(h) $\frac{dy}{dx} + y = 2e^{x}$; $integrating\ factor\ e^{\int 1dx} = e^{x}$

$$e^{x}\frac{dy}{dx} + e^{x}y = 2e^{2x} \quad ; \quad y\,e^{x} = 2\int e^{2x}\,dx$$

$$y\,e^{x} = e^{2x} + c$$

$$y = e^{x} + ce^{-x}$$

(i) $\frac{dy}{dx} + 2x = 1 + 2y$; $\frac{dy}{dx} - 2y = 1 - 2x$ $\quad$ *integrating factor* $e^{-2\int 1dx} = e^{-2x}$

$$e^{-2x}\frac{dy}{dx} - 2e^{-2x}y = e^{-2x} - 2xe^{-2x} \quad ; \quad y\,e^{-2x} = \int e^{-2x} - 2xe^{-2x}\,dx$$

$$\int e^{-2x} - 2xe^{-2x}\,dx = -\boldsymbol{\frac{1}{2}e^{-2x} - 2}\int xe^{-2x}dx$$

$$\int xe^{-2x}dx \quad ; \quad u = x \quad du = dx \quad dv = e^{-2x}dx \quad v = -\frac{1}{2}e^{-2x}$$

$$\int udv = uv - \int vdu \quad ; \quad -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

$$y\,e^{-2x} = -\frac{1}{2}e^{-2x} - 2\left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right] + c$$

$$y\,e^{-2x} = -\frac{1}{2}e^{-2x} + xe^{-2x} + \frac{1}{2}e^{-2x} + c$$

$$y\,e^{-2x} = xe^{-2x} + c$$

$$y = x + ce^{2x}$$

(j) $\frac{dy}{dx} + ycotx = cotx$; $\frac{dy}{dx} + y\frac{cosx}{sinx} = cotx \; integrating \; factor \; e^{\int \frac{cosx}{sinx}dx}$

$$\int \frac{cosx}{sinx}dx \quad ; \quad u = sinx \quad du = cosxdx \quad \int \frac{1}{u}du = lnu = ln(sinx)$$

$$e^{\int \frac{cosx}{sinx}dx} = sinx$$

$$sinx\frac{dy}{dx} - sinxy = sinxcotx \quad ; \quad y\,sinx = \int sinxcotx\,dx = \int cosxdx = sinx$$

$$y\,sinx = sinx + c$$

$$y = 1 + \frac{c}{sinx}$$

$$y = 1 + c(cosecx)$$

(k) $cosx\,\frac{dy}{dx}+ysinx=2xcosx+x^2sinx$; $\frac{dy}{dx}+ytanx=2x+x^2tanx$??????/

$$integrating\ factor\ e^{\int tanxdx} = e^{-ln|cosx|}=\frac{1}{cosx}=secx$$

$secx\,\frac{dy}{dx}+ysecxtanx=2xsecx+x^2secxtanx$; $\boldsymbol{y\,secx=\int 2xsecx+x^2secxtanx\,dx}$

$2\int xsecx\,dx$; $u=x \quad du=dx \quad dv=secx\,dx \quad v=ln(secx+tanx)$

$2\int xsecx\,dx=uv-\int v\,du=\ x \quad du=dx \quad dv=secx\,dx \quad v=-\frac{1}{2}e^{-2x}$

error !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! I think

(l) $x\frac{dy}{dx} - 2y = x$; $\frac{dy}{dx} - \frac{2}{x}y = 1$ $integrating\ factor\ e^{-2\int\frac{1}{x}dx} = \frac{1}{x^2}$

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{1}{x^2} \quad ; \quad \frac{y}{x^2} = \int \frac{1}{x^2}\,dx = -\frac{1}{x}$$

$$\frac{y}{x^2} = -\frac{1}{x} + c$$

$$\frac{y}{x} = -x + cx^2$$

(m) $\frac{dy}{dx} + ysecx = secx$; $integrating\ factor$ $e^{\int secx\ dx} = e^{ln|secx\ +tanx\ |} = (secx + tanx)$

$$(secx + tanx)\frac{dy}{dx} + ysecx(secx + tanx) = secx(secx + tanx)$$

$$y(secx + tanx) = \int sec^2x + secxtanx\,dx$$

$$y = \frac{(tanx + secx) + c}{(secx + tanx)}$$

$$y = 1 + \frac{c}{(secx + tanx)}$$

Ex1 Integrating Factor TJ Sheets

1. $(x+1)\frac{dy}{dx} - y = (x+1)^2$; $\frac{dy}{dx} - \frac{y}{(x+1)} = (x+1)$; $I.F. = e^{\int \frac{-1}{(x+1)} dx} = e^{-ln|x+1|} = \frac{\mathbf{1}}{(\boldsymbol{x}+\mathbf{1})}$

$\frac{1}{(x+1)}\frac{dy}{dx} - \frac{y}{(x+1)^2} = 1$; $\frac{\boldsymbol{y}}{(\boldsymbol{x}+\mathbf{1})} = \int \mathbf{1}\, \boldsymbol{dx}$

$$\frac{y}{(x+1)} = \int 1\, dx$$

$$\frac{y}{(x+1)} = x + c$$

$$y = x(x+1) + c(x+1) = (\boldsymbol{x}+\mathbf{1})(\boldsymbol{x}+\boldsymbol{c})$$

(b) $\frac{dy}{dx} - ytanx = sinxcosx$; $I.F. = e^{-\int tanx\ dx} = e^{-ln|cosx|} = cosx$

$cosx\frac{dy}{dx} - ycosxtanx = sinxcos^2x$; $\boldsymbol{ycosx} = \int \boldsymbol{sinxcos^2x\, dx}$

$\int sinxcos^2x\, dx$; $u = cosx$ $du = -sinxdx$; $-\int u^2\, du = -\frac{u^3}{3} = -\frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{cos^3x}$

$$ycosx = -\frac{1}{3}cos^3x + c$$

$$\boldsymbol{y} = -\frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{cos^2x} + \boldsymbol{c}(\boldsymbol{secx})$$

(c) $tanx\,\frac{dy}{dx} + 2y = xcosecx$; $\frac{dy}{dx} + \frac{2cosx}{sinx}y = \frac{xcosx}{sin^2x}$ $I.F. = e^{2\int\frac{cosx}{sinx}dx}$

$\int \frac{cosx}{sinx}\,dx$; $u = sinx$; $du = cosx\,dx$; $\int \frac{1}{u}\,du = ln(u) = ln(sinx)$

$I.F. = e^{2\int\frac{cosx}{sinx}dx} = e^{2ln(sinx)} = \boldsymbol{sin^2x}$

$sin^2x\frac{dy}{dx} + 2sinxcosx\,y = xcosx$

$$ysin^2x = \int xcosx\;dx$$

$\int xcosx\,dx$; $u = x$ $du = dx$ $dv = cosxdx$ $v = sinx$

$\int xcosx\,dx = uv - \int v\,du = xsinx - \int sinx\;dx = \boldsymbol{xsinx + cosx + c}$

$$ysin^2x = xsinx + cosx + c$$

$$y = xcosecx + cosecx\,cotx + c\,cosec^2x$$

$$\boldsymbol{y = cosecx(x + cotx + c\,cosecx)}$$

(d) $\frac{dy}{dx} + \frac{2y}{(1-x^2)} = 1 - x \quad ; \qquad I.F. = e^{\int \frac{2}{(1-x^2)} dx}$

$$\frac{2}{(1-x^2)} = \frac{A}{(1-x)} + \frac{B}{(1+x)} \qquad ; \qquad 2 = A(1+x) + B(1-x)$$

$x = 1 \;\; ; \;\; A = 1$

$x = -1 \;\; ; \;\; B = 1$

$$\int \frac{-2}{(1-x^2)}\, dx = \frac{1}{(1-x)} + \frac{1}{(1+x)} = ln(1+x) - ln(1-x) = ln\left(\frac{1+x}{1-x}\right)$$

$$I.F. = e^{\int \frac{-2}{(1-x^2)} dx} = e^{ln\left(\frac{1-x}{1+x}\right)} = \left(\boldsymbol{\frac{1+x}{1-x}}\right)$$

$$\frac{dy}{dx} - \frac{2y}{(1-x^2)} = 1 - x \qquad ; \qquad \left(\frac{1+x}{1-x}\right)\frac{dy}{dx} + \left(\frac{1+x}{1-x}\right)\frac{2y}{(1-x^2)} = \left(\frac{1+x}{1-x}\right)(1-x) = 1 + x$$

$$\boldsymbol{y\left(\frac{1-x}{1+x}\right) = \int (1+x)\, dx}$$

$$\int (1+x)\, dx = \boldsymbol{x + \frac{1}{2}x^2 + c}$$

$$y\left(\frac{1-x}{1+x}\right) = x + \frac{1}{2}x^2 + c$$

$$\boldsymbol{y = \left(\frac{1-x}{1+x}\right)\left(\frac{1}{2}x^2 + x + c\right)}$$

(e) $x(x+1)\frac{dy}{dx} - y = x^3e^x$; $\frac{dy}{dx} - \frac{y}{x(x+1)} = \frac{x^2}{(x+1)}e^x$ $\quad I.F. = e^{\int \frac{-1}{x(x+1)}dx}$

$\frac{-1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)}$; $-1 = A(x+1) + Bx$

$x = 0$; $A = -1$

$x = -1$; $B = 1$

$\frac{-1}{x(x+1)} = \frac{-1}{x} + \frac{1}{(x+1)} = ln(x+1) - ln(x) = ln\left(\frac{x+1}{x}\right)$; $I.F. = e^{\int ln\left(\frac{1+x}{x}\right)dx} = \left(\frac{\boldsymbol{x+1}}{\boldsymbol{x}}\right)$

$\frac{dy}{dx} - \frac{y}{x(x+1)} = \frac{x^2}{(x+1)}e^x$; $\left(\frac{x+1}{x}\right)\frac{dy}{dx} - \left(\frac{x+1}{x}\right)\frac{y}{x(x+1)} = \left(\frac{x+1}{x}\right)\frac{x^2}{(x+1)}e^x = \boldsymbol{xe^x}$

$$\boldsymbol{y\left(\frac{x+1}{x}\right) = \int xe^x\, dx}$$

$\int xe^x\, dx$; $u = x \quad du = dx \quad dv = e^x dx \quad v = e^x$

$\int xe^x\, dx = uv - \int v\, du = xe^x - \int e^x\, dx = \boldsymbol{xe^x - e^x}$

$$y\left(\frac{x+1}{x}\right) = xe^x - e^x + c$$

$$\boldsymbol{y = \left(\frac{x}{x+1}\right)(xe^x - e^x + c)}$$

(f) $\frac{dy}{dx} + y = 5cos2x$; $I.F. = e^{\int 1\,dx} = e^x$

$$e^x \frac{dy}{dx} + e^x y = 5e^x cos2x$$

$\boldsymbol{ye^x = 5\int e^x cos2x\,dx}$; $u = cos2x$ $du = -2sin2x\,dx$ $dv = e^x dx$ $v = e^x$

$\int e^x cos2x\,dx = uv - \int v\,du = \boldsymbol{cos2x\,e^x + 2}\int e^x\,sin2x\,dx$

$\int e^x sin2x\,dx$; $u = sin2x$ $du = 2cos2x\,dx$ $dv = e^x dx$ $v = e^x$

$\int e^x sin2x\,dx = uv - \int v\,du = sin2x\,e^x - 2\int e^x\,cos2x\,dx$

$\int e^x cos2x\,dx = cos2x\,e^x + 2\left[sin2x\,e^x - 2\int e^x\,cos2x\,dx\right]$

$\int e^x cos2x\,dx = \frac{1}{5}cos2x\,e^x + \frac{2}{5}sin2x\,e^x$

$$ye^x = 5\left(\frac{1}{5}cos2x\,e^x + \frac{2}{5}sin2x\,e^x\right) + c$$

$$y = (cos2x + 2sin2x) + ce^x$$

(g) $(1-x)\frac{dy}{dx}+xy=(1-x)^2e^{-x}$; $\frac{dy}{dx}+\frac{xy}{(1-x)}=(1-x)e^{-x}$ $I.F.=\ e^{\int\frac{x}{(1-x)}dx}$

$\int\frac{x}{(1-x)}\,dx$; $u=1-x$ $x=1-u$ $du=-dx$; $=\int\frac{(u-1)}{u}\,du=\int 1-\frac{1}{u}\,du$

$\int 1-\frac{1}{u}\,du=u-ln(u)=(1-x)-ln(1-x)$

$I.F.=\ e^{\int\frac{x}{(1-x)}dx}=e^{(1-x)-ln(1-x)}=e^{(1-x)}e^{-ln(1-x)}=\frac{1}{(1-x)}e^{(1-x)}$

$$\frac{1}{(1-x)}e^{(1-x)}\frac{dy}{dx}+\frac{1}{(1-x)}e^{(1-x)}\frac{xy}{(1-x)}=\frac{1}{(1-x)}e^{(1-x)}(1-x)e^{-x}$$

$$\frac{e^{(1-x)}}{(1-x)}\frac{dy}{dx}+\frac{xe^{(1-x)}y}{(1-x)^2}=e^{(1-2x)}$$

$$y\frac{e^{(1-x)}}{(1-x)}=\int e^{(1-2x)}\,dx$$

$$y\frac{e^{(1-x)}}{(1-x)}=e\int e^{(-2x)}\,dx$$

$$y\frac{e^{(1-x)}}{(1-x)}=e\left[-\frac{1}{2}e^{-2x}+c\right]$$

$$y=\frac{(1-x)}{e^{(1-x)}}e\left[-\frac{1}{2}e^{-2x}+c\right]$$

$$y=\frac{(1-x)}{e^{(-x)}}\left[-\frac{1}{2}e^{-2x}+c\right]$$

$$y=ce^{x}(1-x)-\frac{1}{2}e^{-x}(1-x)$$

(h) $\frac{dy}{dx} + \frac{(x+1)}{x}y = e^{-x}$; $I.F. = e^{\int \frac{(x+1)}{x} dx} = e^{\int 1+\frac{1}{x} dx} = e^{x+ln(x)} = xe^x$

$$xe^x \frac{dy}{dx} + xe^x y = xe^x e^{-x}$$

$$xe^x \frac{dy}{dx} + xe^x y = x$$

$$yxe^x = \int x\, dx$$

$$yxe^x = \frac{x^2}{2} + c$$

$$y = \frac{xe^{-x}}{2} + \frac{ce^{-x}}{x}$$

$$y = e^{-x}\left(\frac{x}{2} + \frac{c}{x}\right)$$

(i) $x(x+1)\frac{dy}{dx} + y = x(x+1)^2e^{-x}$; $\frac{dy}{dx} + \frac{y}{x(x+1)} = (x+1)e^{-x}$ $I.F. = e^{\int \frac{1}{x(x+1)}dx}$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)} \quad ; \quad 1 = A(x+1) + Bx$$

$x = 0 \quad A = 1$

$x = -1 \quad B = -1$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{(x+1)}$$

$$\int \frac{1}{x(x+1)}\,dx = \int \frac{1}{x} - \frac{1}{(x+1)}\,dx = ln(x) - ln(x+1) = ln\left(\frac{x}{x+1}\right)$$

$$I.F. = e^{\int \frac{1}{x(x+1)}dx} = e^{ln\left(\frac{x}{x+1}\right)} = \left(\boldsymbol{\frac{x}{x+1}}\right)$$

$$\left(\frac{x}{x+1}\right)\frac{dy}{dx} + \frac{y}{(x+1)^2} = xe^{-x}$$

$$y\left(\frac{x}{x+1}\right) = \int xe^{-x}\,dx$$

$\int xe^{-x}\,dx$; $u = x \quad du = dx \quad dv = e^{-x}dx \quad v = -e^{-x}$

$$\int xe^{-x}\,dx = uv - \int v\,du = -xe^{-x} + \int e^{-x}\,dx = \boldsymbol{-xe^{-x} - e^{-x}}$$

$$y\left(\frac{x}{x+1}\right) = -xe^{-x} - e^{-x} + c$$

$$y = -(x+1)e^{-x}\left(1 - \frac{1}{x}\right) + c\,\frac{(x+1)}{x}$$

$$y = -(x+1)e^{-x}\left(\frac{x-1}{x}\right) + \frac{c(x+1)}{x} = \boldsymbol{\frac{c(x+1)}{x} - \frac{(x+1)^2}{x}e^{-x}}$$

(j) $\frac{dy}{dx} + ycotx = cosx$; $I.F. = e^{\int cotx\ dx}$

$\int cotx\, dx = \int \frac{cosx}{sinx} dx$; $u = sinx \quad du = cosx\, dx$; $\int \frac{1}{u} du = ln(u) = ln(sinx)$

$I.F. = e^{\int cotx\ dx} = e^{ln(sinx)} = \boldsymbol{sinx}$

$$sinx\frac{dy}{dx} + ysinxcotx = sinxcosx$$

$$ysinx = \int sinxcosx\, dx = \frac{1}{2}\int sin2x\, dx$$

$$ysinx = -\frac{1}{4}cos2x + c$$

$$y = -\frac{cos2x}{4sinx} + \frac{c}{sinx}$$

$$\boldsymbol{y = c\ cosecx - \frac{cos2x}{4sinx}}$$

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2. $x\frac{dy}{dx} + 2y = x^3$; $\frac{dy}{dx} + \frac{2}{x}y = x^2$; $I.F. = e^{\int \frac{2}{x}dx} = e^{2ln(x)} = x^2$

$$x^2\frac{dy}{dx} + 2xy = x^4$$

$$yx^2 = \int x^4\, dx$$

$$yx^2 = \frac{x^5}{5} + C$$

$$\boldsymbol{y = \frac{x^3}{5} + \frac{C}{x^2}}$$

$x = 1 \quad y = 2$ $\qquad 2 = \frac{(1)^3}{5} + \frac{C}{(1)^2}$; $C = \frac{9}{5}$

$$y = \frac{x^3}{5} + \frac{9}{5x^2}$$

$$\boldsymbol{y = \frac{1}{5x^2}\left(x^5 + 9\right)}$$

(b) $(1+x)\frac{dy}{dx}+2y=x^2$; $\frac{dy}{dx}+\frac{2}{(1+x)}y=\frac{x^2}{(1+x)}$; $I.F.=e^{\int\frac{2}{(1+x)}dx} = e^{2\ln(1+x)}=(1+x)^2$

$$(1+x)^2\frac{dy}{dx}+2(1+x)y=x^2(1+x)$$

$$(1+x)^2\frac{dy}{dx}+2(1+x)y=x^2+x^3$$

$$y(1+x)^2=\int x^2+x^3\,dx$$

$$y(1+x)^2=\frac{x^3}{3}+\frac{x^4}{4}+C$$

$$y=\frac{x^3}{3(1+x)^2}+\frac{x^4}{4(1+x)^2}+\frac{C}{(1+x)^2}$$

$x=0 \quad y=0$ $\qquad 0=0++C$; $C=0$

$$y=\frac{x^3}{3(1+x)^2}+\frac{x^4}{4(1+x)^2}=\frac{x^3}{(1+x)^2}\left(\frac{1}{3}+\frac{x}{4}\right)$$

$$y=\frac{x^3}{(1+x)^2}\left(\frac{4+3x}{12}\right)$$

$$\boldsymbol{y=\frac{x^3(3x+4)}{12(1+x)^2}}$$

Error !!!! Check !

(c) $sinx\frac{dy}{dx} - ycosx = 1$; $\frac{dy}{dx} - \frac{cosx}{sinx}y = \frac{1}{sinx}$; $I.F. = e^{-\int\frac{cosx}{sinx}dx}$

$\int \frac{cosx}{sinx}\,dx$; $u = sinx$ $du = cosx\,du$; $\int \frac{cosx}{sinx}\,dx = \int \frac{1}{u}\,du = ln(u) = ln(sinx)$

$I.F. = e^{-\int\frac{cosx}{sinx}dx} = e^{-ln(sinx)} = \mathbf{\frac{1}{sinx}}$

$$\frac{1}{sinx}\frac{dy}{dx} - \frac{cotx}{sinx}\,y = \frac{1}{sin^2x}$$

$$y\frac{1}{sinx} = \int \frac{1}{sin^2x}dx$$

$$y\frac{1}{sinx} = \int cosec^2x\,dx$$

$$y\frac{1}{sinx} = -cotx + c$$

$$y = -sinxcotx + 3\,sinx$$

$x = \frac{\pi}{2}$ $y = 3$ $3 = 0 + C$; $C = 3$

$$y = -sinxcotx + 3\,sinx$$

(d) $x(x+1)\frac{dy}{dx} + y = (x+1)^2 e^x$; $\frac{dy}{dx} + \frac{y}{x(x+1)} = \frac{(x+1)}{x}e^x$; $I.F. = e^{\int \frac{1}{x(x+1)}dx}$

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)}$; $1 = A(x+1) + Bx$

$x = 0 \quad A = 1$

$x = -1 \quad B = -1$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{(x+1)}$$

$$I.F. = e^{\int \frac{1}{x} - \frac{1}{(x+1)}dx} = e^{ln(x) - ln(x+1)} = e^{ln\left(\frac{x}{x+1}\right)} = \left(\boldsymbol{\frac{x}{x+1}}\right)$$

$$\left(\frac{x}{x+1}\right)\frac{dy}{dx} + \frac{y}{(x+1)^2} = e^x$$

$$y\left(\frac{x}{x+1}\right) = \int e^x\, dx$$

$$y = \left(\frac{x+1}{x}\right)(e^x + c)$$

$x = 1 \quad y = 0$ $\qquad 0 = \left(\frac{1+1}{1}\right)(e^x + c)$; $c = -e$

$$y = \left(\frac{x+1}{x}\right)(e^x - e)$$

(e) $x\frac{dy}{dx} = y + x^2(sinx + cosx)$; $\frac{dy}{dx} - \frac{y}{x} = x^2(sinx + cosx)$; $I.F. = e^{-\int\frac{1}{x}dx} = e^{-ln(x)} = \mathbf{\frac{1}{x}}$

$$\left(\frac{1}{x}\right)\frac{dy}{dx} - \frac{y}{x^2} = x(sinx + cosx)$$

$$\frac{y}{x} = \int x(sinx + cosx)\,dx$$

$\int xsinx\,dx$; $u = x$ $du = dx$ $dv = sinx\,dx$ $v = -cosx$

$\int xsinx\,dx = uv - \int v\,du = -xcosx + \int cosx\,dx = \mathbf{-xcosx + sinx}$

$\int xcosx\,dx$; $u = x$ $du = dx$ $dv = cosx\,dx$ $v = sinx$

$\int xcosx\,dx = uv - \int v\,du = xsinx - \int sinx\,dx = \mathbf{xsinx + cosx}$

$$\frac{y}{x} = \mathbf{-xcosx + sinx + xsinx + cosx + c}$$

$x = \frac{\pi}{2}$ $y = 0$ $0 = 0 + 1 + \frac{\pi}{2} + 0 + c$; $c = -\frac{\pi}{2} - 1$

$$\boldsymbol{y = x^2(sinx - cosx) + x(sinx + cosx) - x\left(\frac{\pi}{2} + 1\right)}$$
