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Advanced Higher Maths

Unit 1.4 Second Order Differential Equations Solutions

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Ex2 General Solution Second Order

1. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$; $AE:\ D^2 - 3D + 2 = 0$

$(D-2)(D-1) = 0$; $D = 2\ and\ D = 1$; $GS:\ y = Ae^{2x} + Be^{x}$

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b. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$; $AE:\ D^2 - 4D + 3 = 0$

$(D-3)(D-1) = 0$; $D = 3\ and\ D = 1$; $GS:\ y = Ae^{3x} + Be^{x}$

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c. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$; $AE:\ D^2 - 3D = 0$

$D(D-3) = 0$; $D = 3\ and\ D = 0$; $GS:\ y = Ae^{3} + Be^{3x}$

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d. $2\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 9y = 0$; $AE:\ 2D^2 - 9D + 9 = 0$

$(2D-3)(D-3) = 0$; $D = \frac{3}{2}\ and\ D = 3$; $GS:\ y = Ae^{\frac{3}{2}3} + Be^{3x}$

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2. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$; $AE: \ D^2 + 6D + 9 = 0$

$(D+3)(D+3) = 0$; $D = -3 \ and \ D = -3$; $GS: \ y = (A + Bx)e^{-3x}$

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b. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$; $AE: \ 4D^2 + 4D + 1 = 0$

$(2D+1)(2D+1) = 0$; $D = -\frac{1}{2} \ and \ D = -\frac{1}{2}$; $GS: \ y = (A + Bx)e^{-\frac{1}{2}x}$

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c. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$; $AE: \ D^2 - 6D + 9 = 0$

$(D-3)(D-3) = 0$; $D = 3 \ and \ D = 3$; $GS: \ y = (A + Bx)e^{3x}$

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d. $\frac{d^2y}{dx^2} + 2n\frac{dy}{dx} + n^2y = 0$; $AE: \ D^2 + 2nD + n^2 = 0$

$(D-n)(D-n) = 0$; $D = n \ and \ D = n$; $GS: \ y = (A + Bx)e^{nx}$

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3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 \quad ; \quad AE:\ D^2 + 2D + 2 = 0$

$b^2 - 4ac = 4 - 4(1)(2) = -4$ *hence no real roots (complex roots)*

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = -1 \pm i \quad ; \quad y = e^{-x}(Acosx + Bsinx)$$

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b. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0 \quad ; \quad AE:\ D^2 + 4D + 8 = 0$

$b^2 - 4ac = 16 - 4(1)(8) = -16$ *hence no real roots (complex roots)*

$$x = \frac{-4 \pm \sqrt{-16}}{2} = -2 \pm 2i \quad ; \quad y = e^{-2x}(Acos2x + Bsin2x)$$

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c. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0 \quad ; \quad AE:\ D^2 + 6D + 13 = 0$

$b^2 - 4ac = 36 - 4(1)(13) = -16$ *hence no real roots (complex roots)*

$$x = \frac{-6 \pm \sqrt{-16}}{2} = -3 \pm 2i \quad ; \quad y = e^{-3x}(Acos2x + Bsin2x)$$

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d. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0 \quad ; \quad AE:\ D^2 + D + 1 = 0$

$b^2 - 4ac = 1 - 4(1)(1) = -3$ *hence no real roots (complex roots)*

$$x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\mathrm{i}}{2} \quad ; \quad y = e^{-\frac{1}{2}x}(Acos\frac{\sqrt{3}}{2}x + Bsin\frac{\sqrt{3}}{2}x)$$

Ex3 – General Solution Second Order Equations

1. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$; $AE:\ D^2 - 4D + 3 = 0$

$(D-1)(D-3) = 0$; $D = 1\ and\ D = 3$; $y = Ae^x + Be^{3x}$

$Since\ x^3\ try$ $y = Cx^3 + Dx^2 + Ex + F$

$$\frac{dy}{dx} = 3Cx^2 + 2Dx + E$$

$$\frac{d^2y}{dx^2} = 6Cx + 2D$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$$

$$(6Cx + 2D) - 4(3Cx^2 + 2Dx + E) + 3(Cx^3 + Dx^2 + Ex + F) = x^3$$

Comparing coefficients :

x^3 ; $3C = 1$; $C = \frac{1}{3}$

x^2 ; $-12C + 3D = 0$; $-4 + 3D = 0$; $D = \frac{4}{3}$

x ; $6C - 8D + 3E = 0$; $2 - \frac{32}{3} + 3E = 0$; $3E = -\frac{26}{3}$; $E = \frac{26}{9}$

$constant$; $2D - 4E + 3F = 0$; $\frac{8}{3} - \frac{104}{9} + 3F = 0$; $3F = \frac{80}{9}$; $E = \frac{80}{27}$

$G.S. = C.F. + P.I.$; $y = Ae^x + Be^{3x} + \frac{1}{3}x^3 + \frac{4}{3}x^2 + \frac{26}{9}x + \frac{80}{27}$

b. $\frac{d^2y}{dx^2} - y = 2 - 5x \quad ; \quad AE:\ D^2 - 1 = 0$

$(D-1)(D=1) = 0 \quad ; \quad D = 1\ \ and\ \ D = -1 \quad ; \ y = Ae^{x} + Be^{-x}$

$Since\ 2 - 5x\ try \quad y = Cx + D$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{d^2y}{dx^2} - y = 2 - 5x$$

$$0 + 0 - (Cx + D) = 2 - 5x$$

Comparing coefficients:

$x\ ; \quad C = 5$

$constant\ ; \quad D = -2$

$G.S. = C.F. + P.I.\ ; \quad y = Ae^{-x} + Be^{x} + 5x - 2$

c. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 32x^2$; $AE{:}\ D^2 + 5D + 4 = 0$

$(D+1)(D+4) = 0$; $D = -1\ and\ D = -4$; $y = Ae^{-x} + Be^{-4x}$

$Since\ 32x^2\ try \quad y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 32x^2$$
$$0 + 0 - (Cx + D) = 2 - 5x$$

Comparing coefficients:

x ; $C = 5$

$constant$; $D = -2$

$G.S. = C.F. + P.I.$; $y = Ae^{-x} + Be^{x} + 5x - 2$

d. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2 + 1$; $AE:\ D^2 + 2D + 2 = 0$

$b^2 - 4ac = 4 - 4(1)(2) = -4$ *hence no real roots (complex roots)*

$x = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$; $y = e^{-x}(Acosx + Bsinx)$

Since $x^2 + 1$ *try* $y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2 + 1$$

$$2C + 2(2\,Cx + D) + 2(Cx^2 + Dx + E) = x^2 + 1$$

Comparing coefficients:

x^2 ; $2C = 1$; $C = \frac{1}{2}$

x ; $4C + 2D = 0$; $2 + 2D = 0$; $D = -1$

constant ; $2C + 2D + 2E = 1$; $1 - 2 + 2E = 1$; $E = 1$

$G.S. = C.F. + P.I.$; $y = e^{-x}(Acosx + Bsinx) + \frac{1}{2}x^2 - x + 1$

2. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$; $AE:\ D^2 + 2D - 3 = 0$

$(D-1)(D+3) = 0$; $D = 1\ and\ D = -3$; $y = Ae^{x} + Be^{-3x}$

$Since\ 10e^{2x}\ try$ $\boldsymbol{y = Ce^{2x}}$

$$\boldsymbol{\frac{dy}{dx} = 2Ce^{2x}}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = 4Ce^{2x}}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$$

$$4Ce^{2x} + 4Ce^{2x} - 3(Ce^{2x}) = 10e^{2x}$$

Comparing coefficients:

e^{2x} ; $5C = 10$; $C = 2$

$G.S. = C.F. + P.I.$; $y = Ae^{x} + Be^{-3x} + 2e^{2x}$

b. $4\frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 9y = 7e^{-2x}$; $AE:\ 4D^2 + 13D + 9 = 0$

$(4D + 9)(D + 1) = 0$; $D = -1\ and\ D = -\frac{9}{4}$; $y = Ae^{-x} + Be^{-\frac{9}{4}x}$

$Since\ 7e^{-2x}\ try$ $\boldsymbol{y = Ce^{-2x}}$

$$\boldsymbol{\frac{dy}{dx} = -2Ce^{-2x}}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = 4Ce^{-2x}}$$

$$4\frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 9y = 7e^{-2x}$$

$$16Ce^{-2x} - 26Ce^{-2x} + 9Ce^{-2x} = 7e^{-2x}$$

$$-Ce^{-2x} = 7e^{-2x}$$

Comparing coefficients:

e^{-2x} ; $-C = 7$; $C = -7$

$G.S. = C.F. + P.I.$; $y = Ae^{-x} + Be^{-\frac{9}{4}x} - 7e^{-2x}$

c. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x$; $AE: \ D^2 - D - 2 = 0$

$(D-2)(D+1) = 0$; $D = 2 \ and \ D = -1$; $y = Ae^{-x} + Be^{2x}$

$Since\ Ce^x\ try$ $\boldsymbol{y = Ce^x}$

$$\boldsymbol{\frac{dy}{dx} = Ce^x}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = Ce^x}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x$$

$$Ce^x - Ce^x - 2Ce^x = e^x$$

$$-2Ce^x = e^x$$

Comparing coefficients:

e^{-2x} ; $-2C = 1$; $C = -\frac{1}{2}$

$G.S. = C.F. + P.I.$; $y = Ae^{-x} + Be^{2x} - \frac{1}{2}e^x$

d. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{-2x}$; $AE:\ D^2 - 4D + 4 = 0$

$(D-2)(D-2) = 0$; $D = 2\ and\ D = 2$; $y = e^{2x}(A + Bx)$

$Since\ Ce^x\ try$ $\boldsymbol{y = Ce^{-2x}}$

$$\boldsymbol{\frac{dy}{dx} = -2Ce^{-2x}}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = 4Ce^{-2x}}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{-2x}$$

$$4Ce^{-2x} + 8Ce^{-2x} + 4Ce^x = 2e^{-2x}$$

$$16Ce^{-2x} = 2e^{-2x}$$

Comparing coefficients:

e^{-2x} ; $16C = 2$; $C = \frac{1}{8}$

$G.S. = C.F. + P.I.$; $y = e^{2x}(A + Bx) + \frac{1}{8}e^{-2x}$

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3. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = sinx$; $AE\!: \; D^2 - 3D + 2 = 0$

$(D-1)(D-2) = 0$; $D = 1 \;\; and \;\; D = 2$; $y = Ae^x + Be^{2x}$

$Since\ sinx\ try$ $\boldsymbol{y = Csinx + Dcosx}$

$$\boldsymbol{\frac{dy}{dx} = Ccosx - Dsinx}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = -Csinx - Dcosx = -(Csinx + Dcosx)}$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = sinx$$

$$-(Csinx + Dcosx) - 3(Ccosx - Dsinx) + 2(Csinx + Dcosx) = sinx$$

Comparing coefficients:

$sinx$; $-C + 3D + 2C = 1$; $C + 3D = 1$

$cosx$; $-D - 3C + 2D = 0$; $-3C + D = 0$; $D = 3C$

$Sim.Eqn.:$ $C + 9C = 1$; $10C = 1$; $C = \frac{1}{10}$ $D = \frac{3}{10}$

$G.S. = C.F. + P.I.$; $y = e^{2x}(A + Bx) + \frac{1}{10}sinx + \frac{3}{10}cosx$

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b. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 10cos2x$; $AE: D^2 - 1D + 1 = 0$

$(D-1)(D-1) = 0$; $D = 1$ and $D = 1$; $\boldsymbol{y = (A + Bx)e^{2x}}$

Since $10cos2x$ try $\boldsymbol{y = Csin2x + Dcos2x}$

$$\boldsymbol{\frac{dy}{dx} = 2Ccos2x - 2Dsin2x}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = -4Csin2x - 4Dcos2x = -4(Csin2x + Dcos2x)}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 10cos2x$$

$$-4(Csin2x + Dcos2x) - 2(2Ccos2x - 2Dsin2x) + (Csin2x + Dcos2x) = 10cos2x$$

Comparing coefficients:

$sinx$; $-4C + 4D + C = 0$; $-3C + 4D = 0$

$cosx$; $-4D - 4C + D = 10$; $-4C - 3D = 10$; $D = \frac{10 + 4C}{-3}$

$Sim. Eqn.$: $-3C + 4\left(\frac{10 + 4C}{-3}\right) = 0$; $9C + 40 + 16C = 0$; $25C = -40$ $C = -\frac{40}{25} = -\boldsymbol{\frac{8}{5}}$

$Sim. Eqn.$: $-3\left(-\frac{8}{5}\right) + 4D = 0$; $4D = -\frac{24}{5}$ $D = -\frac{24}{20} = -\boldsymbol{\frac{6}{5}}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = (A + Bx)e^{2x} - \frac{8}{5}sinx - \frac{6}{5}cosx}$

c. $4\frac{d^2y}{dx^2} + y = 4sinx$; $AE:\ 4D^2 + 1 = 0$

$b^2 - 4ac = 0 - 4(4)(1) = -16$ *hence no real roots (complex roots)*

$x = \frac{-0 \pm \sqrt{-16}}{8} = \pm\frac{1}{2}i$; $\boldsymbol{y = \left(Asin\frac{1}{2}x + Bcos\frac{1}{2}x\right)}$

Since $4sinx$ *try* $\boldsymbol{y = Csinx + Dcosx}$

$$\boldsymbol{\frac{dy}{dx} = Ccosx - Dsinx}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = -Csinx - Dcosx = -(Csinx + Dcosx)}$$

$$4\frac{d^2y}{dx^2} + y = 4sinx$$

$$-4(Csinx + Dcosx) + (Csinx + Dcosx) = 4sinx$$

Comparing coefficients:

$sinx$; $-4C + C = 4$; $C = -\boldsymbol{\frac{4}{3}}$

$cosx$; $-4D + D = 0$; $D = \boldsymbol{0}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = \left(Asin\frac{1}{2}x + Bcos\frac{1}{2}x\right) - \frac{4}{3}sinx}$

d. $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 26cos3x$; $AE:\ D^2 + 8D + 25 = 0$

$b^2 - 4ac = 64 - 4(1)(25) = -36$ $hence\ no\ real\ roots\ (complex\ roots)$

$x = \frac{-8 \pm \sqrt{-36}}{2} = -4 \pm 3i$; $\boldsymbol{y = e^{-4x}(Asin3x + Bcos3x)}$

$Since\ 26cos3x\ \ try$ $\boldsymbol{y = Csin3x + Dcos3x}$

$$\boldsymbol{\frac{dy}{dx} = 3Ccos3x - 3Dsin3x}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = -9Csin3x - 9Dcos3x = -9(Csin3x + Dcos3x)}$$

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 26cos3x$$

$$-9(Csin3x + Dcos3x) + 8(3Ccos3x - 3Dsin3x) + 25(Csin3x + Dcos3x) = 26cos3x$$

Comparing coefficients:

$sinx$; $-9C - 24D + 25C = 0$; $16C - 24D = 0$; $2C - 3D = 0$; $C = \frac{3}{2}D$

$cosx$; $-9D + 24C + 25D = 26$; $24C + 16D = 26$; $12C + 8D = 13$

$Sim.Eqn.:$ $18D + 8D = 13$; $26D = 13$; $\boldsymbol{D = \frac{1}{2}}$

$Sim.Eqn.:$ $2C - \frac{3}{2} = 0$; $C = \boldsymbol{\frac{3}{4}}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = e^{-4x}(Asin3x + Bcos3x) + \frac{3}{4}sin3x + \frac{1}{2}cos3x}$

4. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x - e^{2x}$; $AE: \ D^2 + 4D + 4 = 0$

$(D+2)(D+2) = 0$; $D = -2 \ and \ D = -2$; $\boldsymbol{y = (A + Bx)e^{-2x}}$

$Since \ x - e^{2x} \ try$ $\boldsymbol{y = Cx + D + Fe^{2x}}$

$$\boldsymbol{\frac{dy}{dx} = C + 2Fe^{2x}}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = 4Fe^{2x}}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x - e^{2x}$$

$$4Fe^{2x} + 4(C + 2Fe^{2x}) + 4(Cx + D + Fe^{2x}) = x - e^{2x}$$

Comparing coefficients:

e^{2x}; $4F + 8F + 4F = -1$; $F = -\boldsymbol{\frac{1}{16}}$

x ; $4C = 1$; $C = \boldsymbol{\frac{1}{4}}$

$constant$; $4C + 4D = 0$; $D = -\boldsymbol{\frac{1}{4}}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = (A + Bx)e^{-2x} + \frac{1}{4}x - \frac{1}{4} - \frac{1}{16}e^{2x}}$

b. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^x + e^{-x}$; $AE:\ D^2 + D - 6 = 0$

$(D-2)(D+3) = 0$; $D = 2\ and\ D = -3$; $\boldsymbol{y = Ae^{2x} + Be^{-3x}}$

$Since\ e^x + e^{-x}\ try$ $\boldsymbol{y = Ce^x + De^{-x}}$

$$\boldsymbol{\frac{dy}{dx} = Ce^x - De^{-x}}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = Ce^x + De^{-x}}$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^x + e^{-x}$$

$$(\boldsymbol{Ce^x + De^{-x}}) + (\boldsymbol{Ce^x - De^{-x}}) - 6(\boldsymbol{Ce^x + De^{-x}}) = e^x + e^{-x}$$

Comparing coefficients:

$e^x;$ $C + C - 6C = 1$; $C = -\boldsymbol{\frac{1}{4}}$

$x\,;$ $D - D - 6D = 1$; $D = -\boldsymbol{\frac{1}{6}}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = Ae^{2x} + Be^{-3x}} - \boldsymbol{\frac{1}{4}}e^x - \boldsymbol{\frac{1}{6}}e^{-x}$

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c. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x - e^x$; $AE:\ D^2 + 4D + 8 = 0$

$b^2 - 4ac = 16 - 4(1)(8) = -16$ *hence no real roots (complex roots)*

$x = \frac{-4 \pm \sqrt{-16}}{2} = -2 \pm 2i$; $\boldsymbol{y = e^{-2x}(Asin2x + Bcos2x)}$

Since $x - e^x$ *try* $\boldsymbol{y = Cx + D + Ee^x}$

$$\boldsymbol{\frac{dy}{dx} = C + Ee^x}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = Ee^x}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x - e^x$$

$$(\boldsymbol{Ee^x}) + 4(\boldsymbol{C + Ee^x}) + 8(\boldsymbol{Cx + D + Ee^x}) = x - e^x$$

Comparing coefficients:

e^x; $E + 4E + 8E = -1$; $E = -\boldsymbol{\frac{1}{13}}$

x ; $8C = 1$; $C = \boldsymbol{\frac{1}{8}}$

constant ; $4C + 8D = 0$; $D = -\boldsymbol{\frac{1}{16}}$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = e^{-2x}(Asin2x + Bcos2x)} + \boldsymbol{\frac{1}{8}}x - \boldsymbol{\frac{1}{16}} - \boldsymbol{\frac{1}{13}}e^x$

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d. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x - 2cos2x \quad ; \quad AE: \ 4D^2 + 4D + 1 = 0$

$(2D+1)(2D+1) = 0 \quad ; \quad D = -\frac{1}{2} \ and \ D = -\frac{1}{2} \quad ; \ \boldsymbol{y = (A + Bx)e^{-\frac{1}{2}x}}$

$Since \ e^x - 2cos2x \ try \quad \boldsymbol{y = Ce^x + (Dsin2x + Ecos2x)}$

$$\boldsymbol{\frac{dy}{dx} = Ce^x + 2Dcos2x - 2Esin2x}$$

$$\boldsymbol{\frac{d^2y}{dx^2} = Ce^x - 4(Dsin2x + Ecos2x)}$$

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x - 2cos2x$$

$$4[Ce^x - 4(Dsin2x + Ecos2x)] + 4[Ce^x + 2Dcos2x - 2Esin2x] + Ce^x + (Dsin2x + Ecos2x) = e^x - 2cos2x$$

Comparing coefficients:

$e^x; \quad 4C + 4C + C = 1 \quad ; \quad \boldsymbol{C = \frac{1}{9}}$

$sin2x; \quad -16D - 8E + D = 0 \quad ; \quad -15D - 8E = 0 \quad ; \quad E = \frac{15D}{-8}$

$cos2x; \quad -16E + 8D + E = 0 \quad ; \quad -15E + 8D = -2 \quad ; \quad -15\left(\frac{15D}{-8}\right) + 8D = -2 \quad ; \quad \boldsymbol{D = -\frac{16}{289}}$

$$\boldsymbol{E} = \frac{15\left(-\frac{16}{289}\right)}{-8} = \boldsymbol{\frac{30}{289}}$$

$\boldsymbol{G.S. = C.F. + P.I.} \ ; \quad \boldsymbol{y = (A + Bx)e^{-\frac{1}{2}x}} + \boldsymbol{\frac{1}{9}}e^x - \boldsymbol{\frac{16}{289}sin2x} + \boldsymbol{\frac{30}{289}cos2x}$

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5. $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=3e^{-2x}$; $AE:\ D^2+5D+6=0$

$$(D+2)(D+3)=0 \quad ; \quad D=-2 \ and \ D=-3 \quad ; \ \boldsymbol{y=Ae^{-2x}+Be^{-3x}}$$

$Since\ 3e^{-2x}\ try$ $\boldsymbol{y=Cxe^{-2x}}$

$$u=Cx \quad \frac{du}{dx}=C \quad v=e^{-2x} \quad \frac{dv}{dx}=-2e^{-2x}$$

$$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}=\boldsymbol{Ce^{-2x}-2Cxe^{-2x}}$$

$\boldsymbol{-2Cxe^{-2x}}$; $u=-2Cx \quad \frac{du}{dx}=-2C \quad v=e^{-2x} \quad \frac{dv}{dx}=-2e^{-2x}$

$$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}=-2Ce^{-2x}+4Cxe^{-2x}$$

$$\frac{d^2y}{dx^2}=-2Ce^{x}-2Ce^{-2x}+4Cxe^{-2x}=\boldsymbol{-4Ce^{-2x}+4Cxe^{-2x}}$$

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=3e^{-2x}$$

$$(-4Ce^{-2x}+4Cxe^{-2x})+5(Ce^{-2x}-2Cxe^{-2x})+6(Cxe^{-2x})=3e^{-2x}$$

Comparing coefficients:

e^{-2x} ; $-4C+5C=3$; $C=3$

$\boldsymbol{G.S.=C.F.+P.I.}$; $\boldsymbol{y=Ae^{-2x}+Be^{-3x}+3xe^{-2x}}$

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b. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^x$; $AE: \ D^2 - 4D + 3 = 0$

$$(D-3)(D-1) = 0 \quad ; \quad D = 3 \ \ and \ \ D = 1 \quad ; \ \boldsymbol{y = Ae^x + Be^{3x}}$$

$Since \ 2e^x \ \ try$ $\boldsymbol{y = Cxe^x}$

$$u = Cx \quad \frac{du}{dx} = C \quad v = e^x \quad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = \boldsymbol{Ce^x + Cxe^x}$$

$\boldsymbol{Cxe^x}$; $u = Cx \quad \frac{du}{dx} = C \quad v = e^x \quad \frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = Ce^x + Cxe^x$$

$$\frac{d^2y}{dx^2} = Ce^x + Ce^x + Cxe^x = \boldsymbol{2Ce^x + Cxe^x}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^x$$

$$2Ce^x + Cxe^x - 4(Ce^x + Cxe^x) + 3(Cxe^x) = 2e^x$$

Comparing coefficients:

e^x ; $2C - 4C = 2$; $C = -1$

$\boldsymbol{G.S. = C.F. + P.I.}$; $\boldsymbol{y = Ae^x + Be^{3x} - xe^x}$

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6. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = sinx$; $AE:\ D^2 + 2D + 2 = 0$

$b^2 - 4ac = 4 - 4(1)(2) = -4$ *hence no real roots (complex roots)*

$$x = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i \quad ; \quad \boldsymbol{y = e^{-x}(Asinx + Bcosx)}$$

Since sinx try $\boldsymbol{y = Csinx + Dcosx}$

$$\boldsymbol{\frac{dy}{dx} = Ccosx - Dsinx}$$

$$\boldsymbol{\frac{d^2y}{dx^2}} = -(\boldsymbol{Csinx + Dcosx})$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = sinx$$

$$-(Csinx + Dcosx) + 2(Ccosx - Dsinx) + 2(Csinx + Dcosx) = sinx$$

Comparing coefficients:

$sinx$; $-C - 2D + 2C = 1$; $C - 2D = 1$; $C = 1 + 2D$

$cosx$; $-D + 2C + 2D = 0$; $D + 2C = 0$; $D + 2 + 4D = 0$; $D = -\frac{2}{5}$

$C = 1 - \frac{4}{5} = \frac{1}{5}$

$$\boldsymbol{G.S. = C.F. + P.I.}\ ;\quad \boldsymbol{y = e^{-x}(Asinx + Bcosx) + \frac{1}{5}sinx - \frac{2}{5}cosx}$$

$$\boldsymbol{y = e^{-x}(A\sin x + B\cos x) + \frac{1}{5}\sin x - \frac{2}{5}\cos x}$$

$y = 0 \quad x = 0$ $\qquad 0 = B - \frac{2}{5} \qquad ; \qquad \boldsymbol{B = \frac{2}{5}}$

$\frac{dy}{dx} = 0 \quad x = 0$

$$u = e^{-x} \quad \frac{du}{dx} = -e^{-x} \quad v = (A\sin x + B\cos x) \quad \frac{dv}{dx} = (A\cos x - B\sin x)$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = -e^{-x}(A\sin x + B\cos x) + e^{-x}(A\cos x - B\sin x)$$

$$\boldsymbol{\frac{dy}{dx}} = -e^{-x}\boldsymbol{(A\sin x + B\cos x) + e^{-x}(A\cos x - B\sin x) + \frac{1}{5}\cos x + \frac{2}{5}\sin x}$$

$\frac{dy}{dx} = 0 \quad x = 0$

$$\boldsymbol{\frac{dy}{dx} = -B + A + \frac{1}{5} = 0}$$

$$\boldsymbol{-\frac{2}{5} + A + \frac{1}{5} = 0} \qquad ; \qquad \boldsymbol{A = \frac{1}{5}}$$

$$\boldsymbol{y = e^{-x}\left(\frac{1}{5}\sin x - \frac{2}{5}\cos x\right) + \frac{1}{5}\sin x - \frac{2}{5}\cos x}$$

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7. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 5e^{3x}$; $AE:\ D^2 - D - 6 = 0$

$(D-3)(D+2) = 0$; $D = 3\ and\ D = -2$; $\boldsymbol{y = Ae^{3x} + Be^{-2x}}$

$Since\ 5e^{3x}\ try$ $\boldsymbol{y = Cxe^{3x}}$

$$u = Cx \quad \frac{du}{dx} = C \quad v = e^{3x} \quad \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = Ce^{3x} + 3Cxe^{3x}$$

$$\boldsymbol{\frac{dy}{dx} = Ce^{3x} + 3Cxe^{3x}}$$

$\boldsymbol{3Cxe^{3x}}$

$$u = 3Cx \quad \frac{du}{dx} = 3C \quad v = e^{3x} \quad \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = 3Ce^{3x} + 9Cxe^{3x}$$

$$\boldsymbol{\frac{d^2y}{dx^2}} = 3Ce^{3x} + 3Ce^{3x} + 9Cxe^{3x} = \boldsymbol{6Ce^{3x} + 9Cxe^{3x}}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 5e^{3x}$$

$$(6Ce^{3x} + 9Cxe^{3x}) - (Ce^{3x} + 3Cxe^{3x}) - 6(Cxe^{3x}) = 5e^{3x}$$

Comparing coefficients:

e^{3x} ; $6C - C = 5$; $C = 1$

$$\boldsymbol{G.S. = C.F. + P.I.}\ ; \quad \boldsymbol{y = Ae^{3x} + Be^{-2x} + xe^{3x}}$$

$$y = Ae^{3x} + Be^{-2x} + xe^{3x}$$

$y = 1 \quad x = 0$ $\quad$ $\mathbf{1 = A + B}$ $\quad ; \quad$ $\mathbf{A = 1 - B}$

xe^{3x}

$$u = x \qquad \frac{du}{dx} = 1 \qquad v = e^x \qquad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = e^x \times 1 + xe^x = e^x + xe^x$$

$$\frac{dy}{dx} = 3Ae^{3x} - 2Be^{-2x} + e^x + xe^x$$

$\frac{dy}{dx} = -6 \quad x = 0$

$$-6 = 3A - 2B + 1$$

$$3A - 2B = -7$$

$3(1 - B) - 2B = -7 \quad ; \quad 3 - 5B = -7 \quad ; \quad \mathbf{B = 2} \quad ; \quad \mathbf{A = 1 - B} \quad ; \quad \mathbf{A = -1}$

$$y = 2e^{-2x} - e^{3x} + xe^{3x}$$
