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Advanced Higher Maths

Advanced Higher - Unit 3.3 Complex Number Theory - Solutions

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Ex 4 Expressing z = r[cosθ + isinθ]

1. $(a)(i)$ $z_1 = 2\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$; $r = |z_1| = 2$ $and\ \theta = \arg(z_1) = \frac{\pi}{4}$

(ii) $z_2 = 3\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$; $r = |z_2| = 3$ $and\ \theta = \arg(z_2) = \frac{\pi}{4}$

(iii) $z_1 z_2 = 2\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right) \times 3\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right) = 6\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$

$$= 6\left(cos\frac{\pi}{4}cos\frac{\pi}{4} - sin\frac{\pi}{4}sin\frac{\pi}{4} + i\left(sin\frac{\pi}{4}cos\frac{\pi}{4} + cos\frac{\pi}{4}sin\frac{\pi}{4}\right)\right)$$

$$= 6\left(cos\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + isin\left(\frac{\pi}{4} + \frac{\pi}{4}\right)\right)$$

$$= 6\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$$

$(b)(i)$ $z_1 = \sqrt{2}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)$; $r = |z_1| = \sqrt{2}$ $and\ \theta = \arg(z_1) = \frac{\pi}{6}$

(ii) $z_2 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$; $r = |z_2| = 2$ $and\ \theta = \arg(z_2) = \frac{\pi}{3}$

(iii) $z_1 z_2 = \sqrt{2}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right) \times 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right) = 2\sqrt{2}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$

$$= 2\sqrt{2}\left(cos\frac{\pi}{6}cos\frac{\pi}{3} - sin\frac{\pi}{6}sin\frac{\pi}{3} + i\left(sin\frac{\pi}{6}cos\frac{\pi}{3} + cos\frac{\pi}{6}sin\frac{\pi}{3}\right)\right)$$

$$= 2\sqrt{2}\left(cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) + isin\left(\frac{\pi}{6} + \frac{\pi}{3}\right)\right)$$

$$= 2\sqrt{2}\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$$

$(c)(i)$ $z_1 = r_1(cos\theta_1 + isin\theta_1)$; $r = |z_1| = r_1$ $and\ \theta = \arg(z_1) = \theta_1$

(ii) $z_2 = r_2(cos\theta_2 + isin\theta_2)$; $r = |z_2| = r_2$ $and\ \theta = \arg(z_2) = \theta_2$

(iii) $z_1 z_2 = r_1(cos\theta_1 + isin\theta_1) \times r_2(cos\theta_2 + isin\theta_2) = r_1 r_2(cos\theta_1 + isin\theta_1)(cos\theta_2 + isin\theta_2)$

$= r_1 r_2\left(cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2 + i(sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2)\right)$

$|z_1\, z_2| = r_1 r_2$; $Arg(z_1\, z_2) = \theta_1 + \theta_2$; $z_1\, z_2 = r_1 r_2\left(cos(\theta_1 + \theta_2) + i(sin(\theta_1 + \theta_2)\right)$

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2. (a) $z_1 = 3\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right)$; $z_2 = 2\left(cos\frac{5\pi}{4} + isin\frac{5\pi}{4}\right)$

$|z_1 z_2| = 3 \times 2 = 6$; $Arg(z_1 z_2) = \frac{3\pi}{4} + \frac{5\pi}{4} = 2\pi = 0\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $z_1 z_2 = 6(cos0 + isin0)$

(b) $z_1 = \frac{1}{2}\left(cos\frac{2\pi}{15} + isin\frac{2\pi}{15}\right)$; $z_2 = 2\left(cos\frac{13\pi}{15} + isin\frac{13\pi}{15}\right)$

$|z_1 z_2| = \frac{1}{2} \times 2 = 1$; $Arg(z_1 z_2) = \frac{2\pi}{15} + \frac{13\pi}{15} = \pi\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $z_1 z_2 = (cos\pi + isin\pi)$

(c) $z_1 = 10\left(cos\frac{5\pi}{9} + isin\frac{5\pi}{9}\right)$; $z_2 = 10\left(cos\frac{13\pi}{9} + isin\frac{13\pi}{9}\right)$

$|z_1 z_2| = 10 \times 10 = 100$; $Arg(z_1 z_2) = \frac{5\pi}{9} + \frac{13\pi}{9} = 2\pi = 0\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $z_1 z_2$
$= 100(cos0 + isin0)$

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3. $(a)(i)$ $z_1 = 2\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$; $r = |z_1| = 2$ $and\ \theta = \arg(z_1) = \frac{\pi}{2}$

(ii) $z_2 = 3\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$; $r = |z_2| = 3$ $and\ \theta = \arg(z_2) = \frac{\pi}{4}$

(iii)
$$\frac{z_1}{z_2} = \frac{2\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)}{3\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)} = \frac{2}{3}\left[\frac{\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)}{\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)} \times \frac{\left(cos\frac{\pi}{4} - isin\frac{\pi}{4}\right)}{\left(cos\frac{\pi}{4} - isin\frac{\pi}{4}\right)}\right]$$

$$= \frac{2}{3}\left(\frac{cos\frac{\pi}{2}cos\frac{\pi}{4} + sin\frac{\pi}{2}sin\frac{\pi}{4} + i\left(sin\frac{\pi}{2}cos\frac{\pi}{4} - cos\frac{\pi}{2}sin\frac{\pi}{4}\right)}{cos^2\frac{\pi}{4} + sin^2\frac{\pi}{4}}\right)$$

$$= \frac{2}{3}\left(cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + isin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right)$$

$$= \frac{2}{3}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$$

$\left|\frac{z_1}{z_2}\right| = \frac{2}{3}$; $Arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{4}$; $\frac{z_1}{z_2} = \frac{2}{3}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$

$3.(b)(i)\quad z_1 = \sqrt{2}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right) \quad ; \; r = |z_1| = \sqrt{2} \quad and\ \theta = \arg(z_1) = \frac{\pi}{6}$

$(ii)\quad z_2 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right) \quad ; \; r = |z_2| = 2 \quad and\ \theta = \arg(z_2) = \frac{\pi}{3}$

$$(iii)\qquad \frac{z_1}{z_2} = \frac{\sqrt{2}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)}{2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)} = \frac{\sqrt{2}}{2}\left[\frac{\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)}{\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)} \times \frac{\left(cos\frac{\pi}{3} - isin\frac{\pi}{3}\right)}{\left(cos\frac{\pi}{3} - isin\frac{\pi}{3}\right)}\right]$$

$$= \frac{\sqrt{2}}{2}\left(\frac{cos\frac{\pi}{6}cos\frac{\pi}{3} + sin\frac{\pi}{6}sin\frac{\pi}{3} + i\left(sin\frac{\pi}{6}cos\frac{\pi}{3} - cos\frac{\pi}{3}sin\frac{\pi}{3}\right)}{cos^2\frac{\pi}{3} + sin^2\frac{\pi}{3}}\right)$$

$$= \frac{\sqrt{2}}{2}\left(cos\left(\frac{\pi}{6} - \frac{\pi}{3}\right) + isin\left(\frac{\pi}{6} - \frac{\pi}{3}\right)\right)$$

$$= \frac{\sqrt{2}}{2}\left(cos\left(-\frac{\pi}{6}\right) + isin\left(-\frac{\pi}{6}\right)\right)$$

$$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad ; \quad Arg\left(\frac{z_1}{z_2}\right) = -\frac{\pi}{6} \quad ; \quad \frac{z_1}{z_2} = \frac{\sqrt{2}}{2}\left(cos\left(-\frac{\pi}{6}\right) + isin\left(-\frac{\pi}{6}\right)\right) = \frac{1}{\sqrt{2}}\left(cos\left(-\frac{\pi}{6}\right) + isin\left(-\frac{\pi}{6}\right)\right)$$

3. $(c)(i)$ $z_1 = r_1(cos\theta_1 + isin\theta_1)$; $r = |z_1| = r_1$ $and\ \theta = \arg(z_1) = \theta_1$

(ii) $z_2 = r_2(cos\theta_2 + isin\theta_2)$; $r = |z_2| = r_2$ $and\ \theta = \arg(z_2) = \theta_2$

(iii) $$\frac{z_1}{z_2} = \frac{r_1(cos\theta_1 + isin\theta_1)}{r_2(cos\theta_2 + isin\theta_2)} = \frac{r_1}{r_2}\left[\frac{(cos\theta_1 + isin\theta_1)}{(cos\theta_2 + isin\theta_2)} \times \frac{(cos\theta_2 - isin\theta_2)}{(cos\theta_2 - isin\theta_2)}\right]$$

$$= \frac{r_1}{r_2}\left(\frac{cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2 + i(sin\theta_1 cos\theta_2 - cos\theta_1 sin\theta_2)}{cos^2\theta_2 + sin^2\theta_2}\right)$$

$$= \frac{r_1}{r_2}\big(cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)\big)$$

$\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$; $Arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2$; $\frac{z_1}{z_2} = \frac{r_1}{r_2}\big(cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)\big)$

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4. (a) $z_1 = 3\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right)$; $z_2 = 2\left(cos\frac{5\pi}{4} + isin\frac{5\pi}{4}\right)$

$\left|\frac{z_1}{z_2}\right| = \frac{3}{2}$; $Arg\left(\frac{z_1}{z_2}\right) = \frac{3\pi}{4} - \frac{5\pi}{4} = -\frac{\pi}{2} = 0\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $\frac{z_1}{z_2} = \frac{3}{2}\left(cos\left(-\frac{\pi}{2}\right) + isin\left(-\frac{\pi}{2}\right)\right)$

(b) $z_1 = 2\left(cos\frac{13\pi}{15} + isin\frac{13\pi}{15}\right)$; $z_2 = \frac{1}{2}\left(cos\frac{2\pi}{15} + isin\frac{2\pi}{15}\right)$

$\left|\frac{z_1}{z_2}\right| = \frac{2}{\frac{1}{2}} = 4$; $Arg\left(\frac{z_1}{z_2}\right) = \frac{13\pi}{15} - \frac{2\pi}{15} = \frac{11\pi}{15} = 0\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $\frac{z_1}{z_2} = 4\left(cos\left(\frac{11\pi}{15}\right) + isin\left(\frac{11\pi}{15}\right)\right)$

(c) $z_1 = 10\left(cos\frac{13\pi}{9} + isin\frac{13\pi}{9}\right)$; $z_2 = 10\left(cos\frac{5\pi}{9} + isin\frac{5\pi}{9}\right)$

$\left|\frac{z_1}{z_2}\right| = \frac{10}{10} = 1$; $Arg\left(\frac{z_1}{z_2}\right) = \frac{13\pi}{9} - \frac{5\pi}{9} = \frac{8\pi}{9} = 0\ (-\boldsymbol{\pi} \leq \boldsymbol{\theta} \leq \boldsymbol{\pi})$; $\frac{z_1}{z_2} = \left(cos\left(\frac{8\pi}{9}\right) + isin\left(\frac{8\pi}{9}\right)\right)$

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5. (a) $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \times \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \times \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \times \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(b) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \times 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(c) $20\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \div \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 20\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

(d) $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \div 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= \frac{1}{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

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6. $|z_1\, z_2| = r_1 r_2$; $Arg(z_1\, z_2) = \theta_1 + \theta_2$; $z_1\, z_2 = r_1 r_2\big(\cos(\theta_1 + \theta_2) + i(\sin(\theta_1 + \theta_2)\big)$

$\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$; $Arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2$; $\frac{z_1}{z_2} = \frac{r_1}{r_2}\big(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\big)$

Ex 5 - De Moivre's Theorem - $(cos\theta + isin\theta)^n = (\cos(n\theta) + isin(n\theta))$ *for n positive*

1. $n = 0$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^0 = 1$; $n = 1$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^1 = \left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$n = 2$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^2 = \left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$n = 3$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^3 = \left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right) = i$

$n = 4$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^4 = \left(cos\frac{2\pi}{3} + isin\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$n = 5$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^5 = \left(cos\frac{5\pi}{6} + isin\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$n = 6$; $\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^6 = (cos\pi + isin\pi) = -1$

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2. $n = 0$; $\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^0 = 1$; $n = 1$; $\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^1 = \left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$n = 2$; $\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^2 = \left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right) = i$

$n = 3$; $\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^3 = \left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$n = 4$; $\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^4 = (cos\pi + isin\pi) = -1$

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3. $\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)^{-2} = \left(cos\left(-\frac{2\pi}{3}\right) + isin\left(-\frac{2\pi}{3}\right)\right) = \left(cos\left(\frac{2\pi}{3}\right) - isin\left(\frac{2\pi}{3}\right)\right) = -\frac{1}{2} - \frac{\sqrt{3}}{\sqrt{2}}i$; NO

$$(cos\theta + isin\theta)^{-n} = (\cos(-n\theta) - isin(n\theta)) \quad \textit{for n negative}$$

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4. (a) $\left(cos\frac{5\pi}{24} + isin\frac{5\pi}{24}\right)^4 = \left(cos\frac{5\pi}{6} + isin\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3} + \frac{1}{2}i$

(b) $\left(cos\frac{2\pi}{5} + isin\frac{2\pi}{5}\right)^5 = (cos2\pi + isin2\pi) = 1$

(c) $\frac{(cos2\theta + isin2\theta)^5}{(cos3\theta + isin3\theta)^3} = \frac{(cos10\theta + isin10\theta)^5}{(cos3\theta + isin3\theta)^3} = \frac{(cos10\theta + isin10\theta)}{(cos9\theta + isin9\theta)} = \frac{(cos\theta + isin\theta)^{10}}{(cos\theta + isin\theta)^9}$

$= \frac{(cos\theta + isin\theta)^{10}}{(cos\theta + isin\theta)^9} = (cos\theta + isin\theta)$

(d) $(cos\theta + isin\theta)^8(cos\theta - isin\theta)^4 = (cos\theta + isin\theta)^8(cos\theta + isin\theta)^{-4}$

$= (cos\theta + isin\theta)^4$

$= (cos4\theta + isin4\theta)$

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5. $(cos\theta + isin\theta)^3 = cos3\theta + isin3\theta = cos^3\theta + 3cos^2\theta sin\theta i - 3cos\theta sin^2\theta - sin^3\theta i$ $(Binomial$!!!)

$comparing\ real\ part$

$cos3\theta = cos^3\theta - 3cos^2\theta sin^2\theta = cos^3\theta - 3cos\theta(1 - cos^2\theta)$
$= cos^3\theta - 3cos\theta + 3cos^3\theta$
$= 4cos^3\theta - 3cos\theta$

$comparing\ imaginary\ \ part$
$sin3\vartheta = 3cos^2\theta sin\theta - sin^3\theta = 3sin\theta(1 - sin^2\theta) - sin^3\theta$
$= 3sin\theta - 3sin^3\theta - sin^3\theta$
$= 3sin\theta - 4sin^3\theta$

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6. $(i) \quad z = (cos\theta + isin\theta)$

$(i) \quad z + \frac{1}{z} = z + z^{-1} = (cos\theta + isin\theta) + (cos\theta + isin\theta)^{-1}$

$= (cos\theta + isin\theta) + (cos\theta - isin\theta)$

$= 2cos\theta$

$(ii) \quad z^2 + \frac{1}{z^2} = z^2 + z^{-2} = (cos\theta + isin\theta)^2 + (cos\theta + isin\theta)^{-2}$

$= (cos2\theta + isin2\theta) + (cos\theta - isin2\theta)$

$= 2cos2\theta$

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7. $(i) \quad z = (cos\theta + isin\theta) \qquad cos(n\theta) = \frac{1}{2}(z^n + z^{-n})$

$RHS = \frac{1}{2}(z^n + z^{-n}) = \frac{1}{2}[(cos\theta + isin\theta)^n + (cos\theta + isin\theta)^{-n}]$

$= \frac{1}{2}[(\cos(n\theta) + isin(n\theta)) + (\cos(-n\theta) + isin(-n\theta))]$

$= \frac{1}{2}[\cos(n\theta) + isin(n\theta) + \cos(n\theta) - isin(n\theta)] \qquad [NB \;\; cos(-n\theta) = cos(n\theta)]$

$= cos(n\theta) = LHS$

$(ii) \quad sin(n\theta) = \frac{1}{2i}(z^n - z^{-n})$

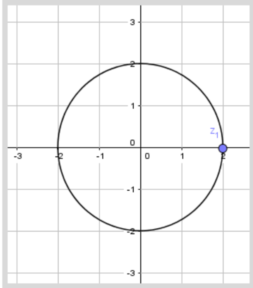
$RHS = \frac{1}{2i}(z^n - z^{-n}) = \frac{1}{2i}[(cos\theta + isin\theta)^n - (cos\theta + isin\theta)^{-n}]$

$= \frac{1}{2}\left[\left(\cos(n\theta) + isin(n\theta)\right) - (\cos(-n\theta) + isin(-n\theta))\right]$

$= \frac{1}{2i}[\cos(n\theta) + isin(n\theta) - \cos(n\theta) + isin(n\theta)] \qquad [NB \;\; cos(-n\theta) = cos(n\theta)]$

$= \frac{1}{2i}\left(2isin(n\theta)\right) = sin(n\theta) = LHS$

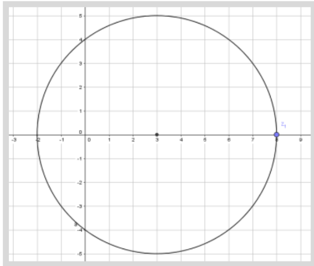
Ex 7 – Geometric Interpretation of Equations and Inequalities in the Complex Plane



1. $|z| = 2$ $\qquad |x + yi| = 2$

$$\sqrt{x^2 + y^2} = 2$$

$$x^2 + y^2 = 4 \qquad a\ circle\ centre\ (0,0)\ and\ radius\ 2$$



2. $|z - 3| = 5$ $\qquad |x + yi - 3| = 5$

$$|(x - 3) + yi| = 5$$

$$\sqrt{(x - 3)^2 + y^2} = 5$$

$$(x - 3)^2 + y^2 = 25 \qquad a\ circle\ centre\ (3,0)\ and\ radius\ 5$$

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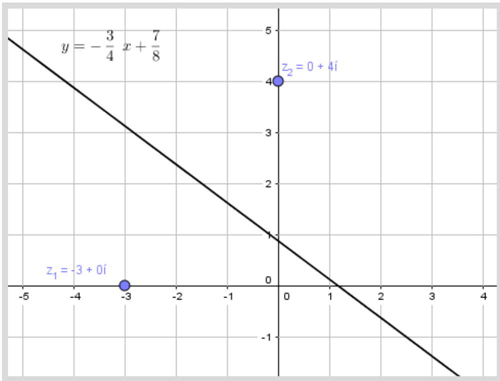
3. $|z + 3| = |z - 4i|$ $\qquad |x + yi + 3| = |x + yi - 4i|$



$$|(x + 3) + yi| = |x + (y - 4)i|$$

$$\sqrt{(x + 3)^2 + y^2} = \sqrt{x^2 + (y - 4)^2}$$

$$(x + 3)^2 + y^2 = x^2 + (y - 4)^2$$

$$x^2 + 6x + 9 + y^2 = x^2 + y^2 - 8y + 16$$

$$6x + 9 = -8y + 16$$

$$8y = -6x + 7$$

$$y = -\frac{3}{4}x + 7$$

Since -3 *is represented by* $(-3,0)$ *and* $4i$ *is represented by* $(0,4)$, *all points representing complex numbers* z *for which* $|z + 3| = |z - 4i|$ *lie on the line perpendicular bisector of line joining* $(-3,0)$ *and* $(0,4)$.

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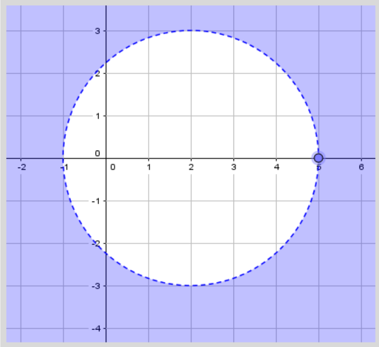
4. $|z-2|>3$ $\qquad |x+yi-2|>3$

$$(x-2)^2+y^2>9$$

$$(x-2)^2+y^2>9$$

$Outside\ circle\ centre\ (2{,}0) and\ radius > 3$



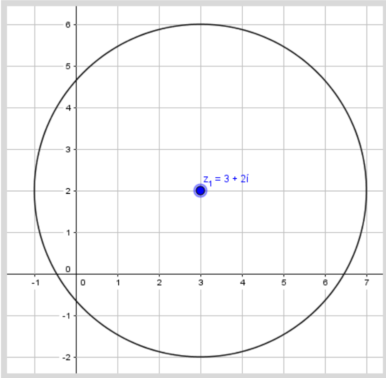
5.

$|z-(3+2i)|=4$ $\qquad |x+yi-(3+2i)|=4$

$$|(x-3)+(y+2)i|=4$$

$$(x-3)^2+(y+2)^2=16$$

$Circle\ centre\ (3,-2) and\ radius = 4$



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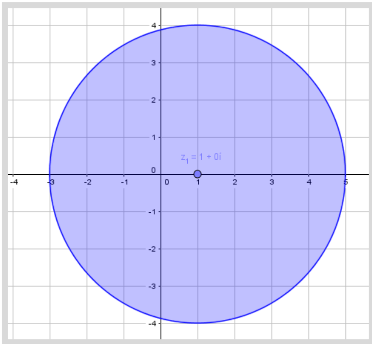
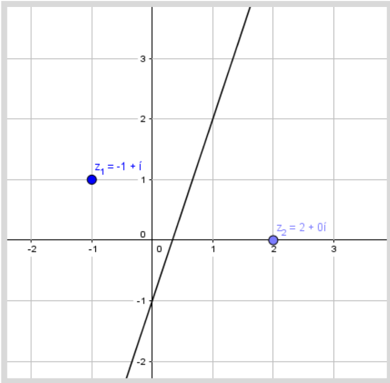
6. $|z-1|\leq 4$ $\qquad |x+yi-1|\leq 4$

$$(x-1)^2+y^2\leq 16$$

$$(x-1)^2+y^2\leq 16$$

$Inside\ and\ on\ circle\ centre\ (1{,}0) and\ radius > 4$





7. $|z-2| = |z+1-i|$ $\quad\quad$ $|x+yi-2| = |x+yi+1-i|$

$$|(x-2)+yi| = |(x+1)+(y-1)i|$$

$$\sqrt{(x-2)^2+y^2} = \sqrt{(x+1)^2+(y-1)^2}$$

$$(x-2)^2+y^2 = (x+1)^2+(y-1)^2$$

$$x^2-4x+4+y^2 = x^2+2x+1+y^2-2y+1$$

$$2y = 6x-2$$

$$y = 3x-1$$

Since -2 *is represented by* $(2,0)$ *and* $1-i$ *is represented by* $(-1,1)$, *all points representing complex numbers* z *for which* $|z+3| = |z-4i|$ *lie on the line perpendicular bisector of line joining* $(2,0)$ *and* $(-1,1)$.