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Advanced Higher Maths

Advanced Higher - Unit 3.4 Number Theory - Solutions

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Ex6 Page 86 – The Division Algorithm

1. $a = 75 \;\; and \;\; b = 12$; $\frac{75}{12} = 6.25$; $12 \times 0.25 = 3$ $q = 12$ $r = 3$

   $75 = 6 \times 12 + 3$ $q = 12$ $r = 3$

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2. $a = 327 \;\; and \;\; b = 13$; $\frac{327}{13} = 25.153$; $13 \times 0.153 = 2$ $q = 13$ $r = 2$

   $327 = 25 \times 13 + 2$ $q = 13$ $r = 2$

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3. $a = 392 \;\; and \;\; b = 19$; $\frac{392}{19} = 20.63$; $19 \times 0.63 = 12$ $q = 20$ $r = 12$

   $327 = 19 \times 20 + 12$ $q = 20$ $r = 12$

Ex 7 – Euclidean Algorithm

1. (i) $G.C.D.$ $(15,27)$

$27 = 1 \times 15 + 12$
$15 = 1 \times 12 + 3$
$12 = 4 \times 3 + 0$ $\quad G.C.D = 3$

(ii) $G.C.D.$ $(16,42)$

$42 = 2 \times 16 + 10$
$16 = 1 \times 10 + 6$
$10 = 1 \times 6 + 4$
$6 = 1 \times 4 + 2$
$4 = 2 \times 2 + 0$ $\quad G.C.D = 2$

(iii) $G.C.D.$ $(72,108)$

$108 = 1 \times 72 + 36$
$72 = 2 \times 36 + 0$ $\quad G.C.D = 36$

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2. (i) $G.C.D.$ $(1219,901)$

$1219 = 1 \times 901 + 318$
$901 = 2 \times 318 + 265$
$318 = 1 \times 265 + 53$
$265 = 5 \times 53 + 0$ $\quad G.C.D = 53$

(ii) $G.C.D.$ $(4277,2821)$

$4277 = 1 \times 2821 + 1456$
$2821 = 1 \times 1456 + 1365$
$1456 = 1 \times 1365 + 91$
$1365 = 15 \times 91 + 0$ $\quad G.C.D = 91$

(iii) $G.C.D.$ (5213,2867)

$5213 = 1 \times 2867 + 2346$
$2867 = 1 \times 2346 + 521$
$2346 = 4 \times 521 + 262$
$521 = 1 \times 262 + 259$
$262 = 1 \times 259 + 3$
$259 = 86 \times 3 + 1$
$3 = 3 \times 1 + 0$ $G.C.D = 1$

Ex8 – Linear Combination of two Integers

From Ex 7 Q2

1. (i) $G.C.D.$ $(x, y) \rightarrow (1219,901)$

$1219 = 1 \times 901 + 318$
$901 = 2 \times 318 + 265$
$318 = 1 \times 265 + 53$
$265 = 5 \times 53 + 0$ $G.C.D = 53$

So $53 = 318 - 1 \times 265$
$53 = 318 - 1 \times (901 - 2 \times 318)$
$53 = 3 \times 318 - 1 \times 901)$
$53 = 3 \times (1219 - 1 \times 901) - 1 \times 901)$
$53 = 3 \times 1219 - 4 \times 901$

$x = 3$ $y = -4$ $d = xa + yb$; $53 = 3 \times 1219 - 4 \times 901$

(ii) $G.C.D.$ $(x, y) \rightarrow (4277,2821)$

$4277 = 1 \times 2821 + 1456$
$2821 = 1 \times 1456 + 1365$
$1456 = 1 \times 1365 + 91$
$1365 = 15 \times 91 + 0$ $G.C.D = 91$

So $91 = 1456 - 1 \times 1365$
$91 = 1456 - 1 \times (2821 - 1 \times 1456)$
$91 = 2 \times 1456 - 1 \times 2821$
$91 = 2 \times (4277 - 1 \times 2821) - 1 \times 2821)$
$91 = 2 \times 4277 - 3 \times 2821$

$x = 2$ $y = -3$ $d = xa + yb$; $91 = 2 \times 4277 - 3 \times 2821$

(iii) $G.C.D.$ $(x, y) \rightarrow (5213,2867)$

$5213 = 1 \times 2867 + 2346$ (1)
$2867 = 1 \times 2346 + 521$ (2)
$2346 = 4 \times 521 + 262$ (3)
$521 = 1 \times 262 + 259$ (4)
$262 = 1 \times 259 + 3$ (5)
$259 = 86 \times 3 + 1$ (6)
$3 = 3 \times 1 + 0$ $G.C.D = 1$

So $1 = 259 - 86 \times 3$ $from$ (6)
$1 = 259 - 86 \times (262 - 1 \times 259)$ $from$ (5)
$1 = 87 \times 259 - 86 \times 262$
$1 = 87 \times (521 - 1 \times 262) - 86 \times 262$ $from$ (4)
$1 = 87 \times 521 - 173 \times 262$
$1 = 87 \times 521 - 173 \times (2346 - 4 \times 521)$ $from$ (3)
$1 = 779 \times 521 - 173 \times 2346$
$1 = 779 \times (2867 - 1 \times 2346) - 173 \times 2346$ $from$ (2)
$1 = 779 \times 2867 - 952 \times 2346$
$1 = 779 \times 2867 - 952 \times (5213 - 1 \times 2867)$ $from$ (1)
$1 = 1731 \times 2867 - 952 \times 5213$

$x = -952$ $y = 1731$ $d = xa + yb$; $1 = -952 \times 5213 + 1731 \times 2867$

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2. (iii) $G.C.D.$ $(x, y) \rightarrow (7293,798)$

$7293 = 9 \times 798 + 111$ (1)
$798 = 7 \times 111 + 21$ (2)
$111 = 5 \times 21 + 6$ (3)
$21 = 3 \times 6 + 3$ (4)
$3 = 1 \times 3 + 0$ $G.C.D = 3$

So $3 = 21 - 3 \times 6$ $from$ (4)
$3 = 21 - 3(111 - 5 \times 21)$ $from$ (3)
$3 = 16 \times 21 - 3 \times 111$
$3 = 16 \times (798 - 7 \times 111) - 3 \times 111$ $from$ (2)
$3 = 16 \times 798 - 115 \times 111$
$3 = 16 \times 798 - 115 \times (7293 - 9 \times 798)$ $from$ (1)
$1 = 1051 \times 798 - 115 \times 7293$

$x = -115$ $y = 1051$ $d = xa + yb$; $1 = -115 \times 7293 + 1051 \times 798$

Ex 9 Expressing Base 10 in other Bases

1. $81 = 2 \times 40 + 1$
   $40 = 2 \times 20 + 0$
   $20 = 2 \times 10 + 0$
   $10 = 2 \times 5 + 0$
   $5 = 2 \times 2 + 1$
   $2 = 2 \times 1 + 0$
   $1 = 2 \times 0 + 1$ $\quad base\ 2 \quad 1010001_2$

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2. $579 = 5 \times 115 + 4$
   $115 = 5 \times 23 + 0$
   $23 = 5 \times 4 + 3$
   $4 = 5 \times 0 + 4$ $\quad base\ 5 \quad 4304_5$

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3. $1064 = 7 \times 152 + 0$
   $152 = 7 \times 21 + 5$
   $21 = 7 \times 3 + 0$
   $3 = 7 \times 0 + 3$ $\quad base\ 7 \quad 3050_7$

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4. $15287 = 9 \times 1698 + 5$
   $1698 = 9 \times 188 + 6$
   $188 = 9 \times 20 + 8$
   $20 = 9 \times 2 + 2$
   $2 = 9 \times 0 + 2$ $\quad base\ 9 \quad 22865_9$

Ex 6 – Number Theory

1. $490 \div 2 = 245 \div 5 = 49 \div 7 = 7$; $Product\ of\ primes\ 490 = 2 \times 5 \times 7 \times 7$

2. $1125 \div 5 = 225 \div 5 = 45 \div 5 = 9 \div 3 = 3$; $Product\ of\ primes\ 1125 = 3 \times 3 \times 5 \times 5 \times 5$

3. $2728 \div 2 = 1364 \div 2 = 682 \div 2 = 341 \div 11 = 31$; $Product\ of\ primes\ 2728 = 2 \times 2 \times 2 \times 11 \times 31$
