Advanced Higher Maths

Integration

2001

(a) Obtain partial fractions for

$$\frac{x}{x^2-1},\ x>1.$$

(b) Use the result of (a) to find

$$\int \frac{x^3}{x^2-1}\,dx,\ x>1.$$ **(2, 4 marks)**

2002

Use the substitution $x+2=2\tan\theta$ to obtain $\int \frac{1}{x^2+4x+8}\,dx$. **(5 marks)**

2003

Use the substitution $x=1+\sin\theta$ to evaluate $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3}\,d\theta$. **(5 marks)**

2004

① Express $\frac{1}{x^2-x-6}$ in partial fractions.

Evaluate $\int_0^1 \frac{1}{x^2-x-6}\,dx$. **(2, 4 marks)**

② A solid is formed by rotating the curve $y=e^{-2x}$ between $x=0$ and $x=1$ through $360°$ about the x-axis. Calculate the volume of the solid that is formed. **(5 marks)**

2005

① Use the substitution $u=1+x$ to evaluate $\int_0^3 \frac{x}{\sqrt{1+x}}\,dx$. **(5 marks)**

② Express $\frac{1}{x^3+x}$ in partial fractions.

Obtain a formula for $I(k)$, where $I(k)=\int_1^k \frac{1}{x^3+x}\,dx$, expressing it in the form $\ln\frac{a}{b}$, where a and b depend on k.

Write down an expression for $e^{I(k)}$ and obtain the value of $\lim_{k\to\infty} e^{I(k)}$. **(4, 4, 2 marks)**

③ (a) Given $f(x)=\sqrt{\sin x}$, where $0<x<\pi$, obtain $f'(x)$.

(b) If, in general, $f(x)=\sqrt{g(x)}$, where $g(x)>0$, show that $f'(x)=\dfrac{g'(x)}{k\sqrt{g(x)}}$,

stating the value of k.

Hence, or otherwise, find $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$. **(1, 2, 3 marks)**

2006

Find $\displaystyle\int \frac{12x^3-6x}{x^4-x^2+1}\,dx$. **(3 marks)**

2007

① Express $\dfrac{2x^2-9x-6}{x\left(x^2-x-6\right)}$ in partial fractions.

Given that $\displaystyle\int_4^6 \frac{2x^2-9x-6}{x\left(x^2-x-6\right)}\,dx=\ln\frac{m}{n}$,

determine values for the integers m and n. **(3, 3 marks)**

② Use the substitution $u=1+x^2$ to obtain $\displaystyle\int_0^1 \frac{x^3}{\left(1+x^2\right)^4}\,dx$.

A solid is formed by rotating the curve $y=\dfrac{x^{3/2}}{\left(1+x^2\right)^2}$ between $x=0$ and $x=1$ through

$360°$ about the x-axis. Write down the volume of this solid. **(5, 1 marks)**

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2008

① Express $\dfrac{12x^2+20}{x\left(x^2+5\right)}$ in partial fractions.

Hence evaluate $\displaystyle\int_1^2 \frac{12x^2+20}{x\left(x^2+5\right)}\,dx$. **(3, 3 marks)**

② Write down the derivative of $\tan x$.

Show that $1+\tan^2 x=\sec^2 x$.

Hence obtain $\displaystyle\int \tan^2 x\,dx$. **(1, 1, 2 marks)**

2009

① Show that $\displaystyle\int_{\ln 3/2}^{\ln 2} \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx=\ln\frac{9}{5}$. **(4 marks)**

② Use the substitution $x=2\sin\theta$ to obtain the exact value of $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$.

(Note that $\cos 2A=1-2\sin^2 A$.) **(6 marks)**

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2010

① Evaluate $\int_1^2 \frac{3x+5}{(x+1)(x+2)(x+3)}\,dx$

expressing your answer in the form $\ln\frac{a}{b}$, where a and b are integers. **(6 marks)**

② A new board game has been invented and the symmetrical design on the board is made from 4 identical "petal" shapes. One of these petals is the region enclosed between the curves $y=x^2$ and $y^2=8x$ as shown shaded in diagram 1 below.

Calculate the area of the complete design, as shown in diagram 2.

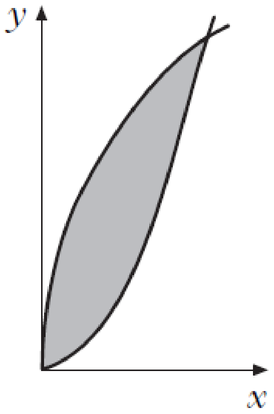

Diagram 1

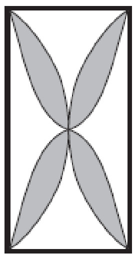
Diagram 2

The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through $360°$ about the y-axis.
Find the volume of plastic required to make one counter. **(5, 5 marks)**

2011

① Express $\frac{13-x}{x^2+4x-5}$ in partial fractions and hence obtain $\int \frac{13-x}{x^2+4x-5}\,dx$. **(5 marks)**

② Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x)\,dx$. **(3 marks)**

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2012

Use the substitution $x = 4\sin\theta$ to evaluate $\int_0^2 \sqrt{16 - x^2}\, dx$. **(6 marks)**

2013

① The velocity, v, of a particle P at time t is given by

$$v = e^{3t} + 2e^t$$

(a) Find the acceleration of P at time t.

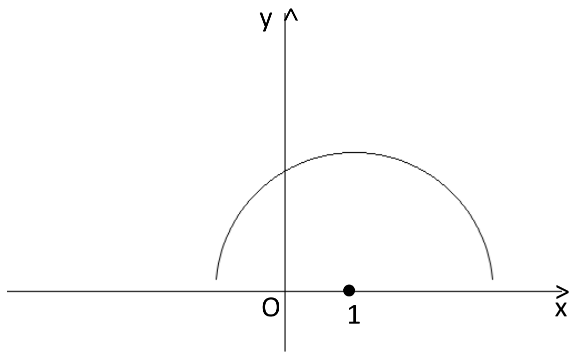
(b) Find the distance covered by P between $t = 0$ and $t = \ln 3$. **(2, 3 marks)**

② Integrate $\dfrac{\sec^2 3x}{1 + \tan 3x}$ with respect to x. **(4 marks)**

2014

① A semi-circle with centre $(1, 0)$ and radius 2, lies on the x-axis as shown.

Find the volume of the solid of revolution formed when the shaded region is rotated completely about the x-axis.



(5 marks)

② Use the substitution $x = \tan\theta$ to determine the exact value of

$$\int_0^1 \frac{dx}{\left(1 + x^2\right)^{3/2}}.$$

(6 marks)

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2015

Find $\int \frac{2x^3 - x - 1}{(x-3)(x^2+1)}\,dx,\ x > 3.$ **(9 marks)**