Advanced Higher Maths

Matrices

2001

Let $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & -1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0 & 1\\ 4 & -2 & -2\\ -3 & 2 & 1\end{pmatrix}$

Show that $AB=kI$ for some constant k, where I is the identity matrix .

Hence obtain (i) the inverse matrix A^{-1} and (ii) the matrix A^2B. **(4 marks)**

2002

① A matrix $A=\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$. Prove by induction that $A^n=\begin{pmatrix}n+1 & n\\ -n & 1-n\end{pmatrix}$, where n is a positive integer. **(6 marks)**

② Write down the 2×2 matrix A representing a reflection in the x-axis and the 2×2 matrix B representing an anti-clockwise rotation of $30°$ about the origin.
Hence show that the image of a point (x, y) under the transformation A followed by the transformation B is $\left(\frac{kx+y}{2}, \frac{x-ky}{2}\right)$, stating the value of k. **(4 marks)**

2003

The matrix A is such that $A^2=4A-3I$ where I is the corresponding identity matrix.

Find integers p and q such that $A^4=pA+qI$. **(4 marks)**

2004

Write down the 2×2 matrix M_1 associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin.

Write down the matrix M_2 associated with reflection in the x-axis.

Evaluate M_2M_1 and describe geometrically the effect of the transformation represented by M_2M_1.

(2, 1, 2 marks)

2005

Given the matrix $A=\begin{pmatrix}0 & 4 & 2\\ 1 & 0 & 1\\ -1 & -2 & -3\end{pmatrix}$, show that $A^2+A=kI$ for some constant k, where I is the 3×3 unit matrix.

Obtain the values of p and q for which $A^{-1}=pA+qI$. **(4, 2 marks)**

Advanced Higher Maths

2006

① Calculate the inverse of the matrix $\begin{pmatrix} 2 & x \\ -1 & 3 \end{pmatrix}$.

For what value of x is this matrix singular? **(4 marks)**

② The square matrices A and B are such that $AB = BA$. Prove by induction that $A^n B = BA^n$ for all integers $n \geq 1$. **(5 marks)**

2007

Matrices A and B are defined by $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}$.

(a) Find the product AB.

(b) Obtain the determinants of A and AB.

Hence or otherwise, obtain an expression for $\det B$. **(2, 2, 1 marks)**

2008

Let the matrix $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$.

(a) Obtain the value(s) of x for which A is singular.

(b) When $x = 2$, show that $A^2 = pA$ for some constant p.

Determine the value of q such that $A^4 = qA$. **(2, 3 marks)**

2009

Given the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.

(a) Find A^{-1} in terms of t when A is non-singular.

(b) Write down the value of t such that A is singular.

(c) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t. **(3, 1, 1 marks)**

Advanced Higher Maths

2010

① Obtain the 2×2 matrix M associated with an enlargement, scale factor 2, followed by a clockwise rotation of $60°$ about the origin.

(4 marks)

② Use Gaussian elimination to show that the set of equations

$$x-y+z=1$$
$$x+y+2z=0$$
$$2x-y+az=2$$

has a unique solution when $a\neq 2\cdot 5$.

Explain what happens when $a=2\cdot 5$.

Obtain the solution when $a=3$.

Given $A=\begin{pmatrix}5 & 2 & -3\\ 1 & 1 & -1\\ -3 & -1 & 2\end{pmatrix}$ and $B=\begin{pmatrix}1\\0\\2\end{pmatrix}$, calculate AB.

Hence or otherwise state the relationship between A and the matrix $C=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 2\\ 2 & -1 & 3\end{pmatrix}$.

(5, 1, 1, 1, 2 marks)

2011

(a) For what value of λ is $\begin{pmatrix}1 & 2 & -1\\ 3 & 0 & 2\\ -1 & \lambda & 6\end{pmatrix}$ singular?

(b) For $A=\begin{pmatrix}2 & 2\alpha+\beta & -1\\ 3\alpha+2\beta & 4 & 3\\ -1 & 3 & 2\end{pmatrix}$ obtain values of α and β such that $A'=\begin{pmatrix}2 & -5 & -1\\ -1 & 4 & 3\\ -1 & 3 & 2\end{pmatrix}$.

(3, 3 marks)

2012

A non-singular $n\times n$ matrix A satisfies the equation $A+A^{-1}=I$ where I is the $n\times n$ identity matrix. Show that $A^3=kI$ and state the value of k.

(4 marks)

Advanced Higher Maths

2013

Matrices A and B are defined by $A=\begin{pmatrix}4 & p\\ -2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}x & -6\\ 1 & 3\end{pmatrix}$

(a) Find A^2.

(b) Find the value of p for which A^2 is singular.

(c) Find the values of p and x if $B=3A'$. **(1, 2, 2 marks)**

2014

Given A is the matrix $\begin{pmatrix}2 & a\\ 0 & 1\end{pmatrix}$,

Prove by induction that $A^n=\begin{pmatrix}2^n & a\left(2^n-1\right)\\ 0 & 1\end{pmatrix}$, $n\geq 1$. **(4 marks)**

2015

① Obtain the value(s) of p for which the matrix $A=\begin{pmatrix}p & 2 & 0\\ 3 & p & 1\\ 0 & -1 & -1\end{pmatrix}$ is singular. **(4 marks)**

② Write down the 2×2 matrix, M_1, associated with a reflection in the y-axis.

Write down a second 2×2 matrix, M_2, associated with an anticlockwise rotation through an angle of $\frac{\pi}{2}$ radians about the origin.

Find the 2×2 matrix, M_3, associated with an anticlockwise rotation through $\frac{\pi}{2}$ radians about the origin followed by a reflection in the y-axis.

What single transformation is associated with M_3? **(4 marks)**