Advanced Higher Maths

Vectors

2001

Let L_1 and L_2 be the lines

$$L_1: \; x = 8 - 2t, \;\; y = -4 + 2t, \;\; z = 3 + t$$

$$L_2: \; \frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}.$$

(a)(i) Show that L_1 and L_2 intersect and find their point of intersection.

(ii) Verify the acute angle between them is $\cos^{-1}\left(\frac{4}{9}\right)$.

(b) (i) Obtain an equation of the plane Π that is perpendicular to L_2 and passes through the point $(1, -4, 2)$.

(ii) Find the coordinates of the point of intersection of the plane Π and the line L_1.

(4, 2, 3, 2 marks)

2002

(a) Find an equation for the plane π_1 which contains the points $A(1,1,0)$, $B(3,1,-1)$ and $C(2,0,-3)$.

(b) Given that π_2 is the plane whose equation is $x + 2y + z = 3$, calculate the size of the acute angle between the plane π_1 and π_2.

(4, 3 marks)

2003

Find the point of intersection of the line $\frac{x-3}{4} = \frac{y-2}{-1} = \frac{z+1}{2}$

and the plane with equation $2x + y - z = 4$. **(4 marks)**

2004

(a) Find an equation of the plane π_1 containing the points $A(1,0,3)$, $B(0,2,-1)$ and $C(1,1,0)$.

Calculate the size of the acute angle between π_1 and the plane π_2 with equation $x + y - z = 0$.

(b) Find the point of intersection of the plane π_2 and the line $\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}$.

(4, 3, 3 marks)

2005

The equations of two planes are $x - 4y + 2z = 1$ and $x - y - z = -5$. By letting $z = t$ or otherwise, obtain parametric equations for the line of intersection of the planes.

Show that this line lies in the plane with equation $x + 2y - 4z = -11$.

(4, 1 marks)

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2006

Obtain an equation for the plane passing through the point $P(1,1,0)$ which is perpendicular to the line L given by $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z}{-1}$.

Find the coordinates of the point Q where the plane and L intersect.

Hence, or otherwise, obtain the shortest distance from P to L and explain why this is the shortest distance.

(3, 4, 2, 1 marks)

2007

Lines L_1 and L_2 are given by the parametric equations

$$L_1 : x = 2+s,\ \ y=-s,\ \ z=2-s \qquad\qquad L_2 : x=-1-2t,\ \ y=t,\ \ z=2+3t.$$

(a) Show that L_1 and L_2 do not intersect.

(b) The line L_3 passes through the point $P(1,1,3)$ and its direction is perpendicular to the directions of both L_1 and L_2. Obtain parametric equations for L_3.

(c)Find the coordinates of the point Q where L_3 and L_2 intersect and verify that P lies on L_1.

(d) PQ is the shortest distance between the lines L_1 and L_2. Calculate PQ.

(3, 3, 3, 1 marks)

2008

(a) Find an equation of the plane π_1 through the point $A(1,1,1)$, $B(2,-1,1)$ and $C(0,3,3)$.

(b) The plane π_2 has equation $x+3y-z=2$.

Given that the point $(0,a,b)$ lies on both the planes π_1 and π_2, find the values of a and b. Hence find an equation of the line of intersection of the planes π_1 and π_2.

(c) Find the size of the acute angle between the planes π_1 and π_2.

(3, 4, 3 marks)

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2009

(a) Use Gaussian elimination to solve the following system of equations

$$x + y - z = 6$$
$$2x - 3y + 2z = 2$$
$$-5x + 2y - 4z = 1$$

(b) Show that the line of intersection, L, of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations

$$x = \lambda$$
$$y = 4\lambda - 14$$
$$z = 5\lambda - 20.$$

(c) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$.

(5, 2, 4 marks)

2010

Given $\underline{u} = -2\underline{i} + 5\underline{k}$, $\underline{v} = 3\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{w} = -\underline{i} + \underline{j} + 4\underline{k}$.

Calculate $\underline{u}.(\underline{v} \times \underline{w})$. **(4 marks)**

2011

The lines L_1 and L_2 are given by the equations $\frac{x-1}{k} = \frac{y}{-1} = \frac{z+3}{1}$ and $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ respectively.

Find

(a) The value of k for which L_1 and L_2 intersect and the point of intersection.

(b) The acute angle between L_1 and L_2.

(6, 4 marks)

2012

Obtain an equation for the plane passing through the points $P(-2,1,-1)$, $Q(1,2,3)$ and $R(3,0,1)$.

(5 marks)

2013

(a) Find an equation of the plane π_1 through the points $A(0,-1,3)$, $B(1,0,3)$ $C(0,0,5)$.

(b) π_2 is the plane through A with normal in the direction $-\underline{j} + \underline{k}$.

Find an equation of the plane π_2.

(c) Determine the acute angle between the planes π_1 and π_2.

(4, 2, 3 marks)

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2014

Three vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ are given by $\underline{u}, \underline{v}$ and $\underline{w}$ where
$\underline{u} = 5\underline{i} + 13\underline{j},\ \underline{v} = 2\underline{i} + \underline{j} + 3\underline{k},\ \underline{w} = \underline{i} + 4\underline{j} - \underline{k}.$
Calculate $\underline{u}.(\underline{v} \times \underline{w})$.
Interpret your result geometrically.

(3, 1 marks)

2015

A line L_1, passes through the point P$(2,4,1)$ and is parallel to

$$\underline{u}_1 = \underline{i} + 2\underline{j} - \underline{k}$$

and a second line, L_2, passes through Q$(-5,2,5)$ and is parallel to

$$\underline{u}_2 = -4\underline{i} + 4\underline{j} + \underline{k}.$$

(a) Write down the vector equations for L_1 and L_2.

(b) Show that the lines L_1 and L_2 intersect and find their point of intersection.

(c) Determine the equation of the plane containing L_1 and L_2. **(2, 4, 4 marks)**