Advanced Higher Maths

Further Differentiation

2001

① Differentiate with respect to x.

(a) $f(x)=(2+x)\tan^{-1}\sqrt{x-1}, \quad x>1$

(b) $g(x)=e^{\cot 2x}, \quad 0<x<\frac{\pi}{2}$ **(4, 2 marks)**

② A curve has equation $xy+y^2=2$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(b) Hence find an equation of the tangent to the curve at the point $(1,1)$.

(3, 2 marks)

2002

① A curve is defined by the parametric equations

$$x=t^2+t-1, \quad y=2t^2-t+2$$

for all t. show that the point $A(-1,5)$ lies on the curve and obtain an equation of the tangent to the curve at the point A.

(6 marks)

② Given $y=(x+1)^2(x+2)^{-4}$ and $x>0$, use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the form $\left(\frac{a}{x+1}+\frac{b}{x+2}\right)y$, stating the values of the constants a and b. **(3 marks)**

2003

① (a) Given $f(x)=x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer.

(b) Given $y=3^x$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x.

(3, 3 marks)

② The equation $y^3+3xy=3x^2-5$ defines a curve passing through the point $A(2,1)$. Obtain an equation for the tangent to the curve at A.

(4 marks)

2004

① (a) Given $f(x)=\cos^2 x\, e^{\tan x}$, $\frac{-\pi}{2}<x<\frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'\left(\frac{\pi}{4}\right)$.

(b) Differentiate $g(x)=\frac{\tan^{-1}2x}{1+4x^2}$. **(4, 3 marks)**

② A curve is defined by the equations $x=5\cos\theta$, $y=5\sin\theta$, $(0\le\theta\le 2\pi)$.

Use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ.

Find the equation of the tangent to the curve at the point where $\theta=\frac{\pi}{4}$.

(2, 3 marks)

2005

Given the equation $2y^2-2xy-4y+x^2=0$ of a curve, obtain the x coordinate of each point at which the curve has a horizontal tangent.

(4 marks)

2006

① Differentiate, simplifying your answers:

(a) $2\tan^{-1}\sqrt{1+x}$, where $x>-1$;

(b) $\frac{1+\ln x}{3x}$, where $x>0$. **(3, 3 marks)**

② Given $xy-x=4$, use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y.

Hence obtain $\frac{d^2y}{dx^2}$ in terms of x and y. **(2, 3 marks)**

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2007

A curve is defined by the parametric equations $x = \cos 2t$, $y = \sin 2t$, $0 < t < \frac{\pi}{2}$.

(a) Use parametric differentiation to find $\frac{dy}{dx}$.

Hence find the equation of the tangent when $t = \frac{\pi}{8}$.

(b) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence show that $\sin 2t \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = k$,

where k is an integer. State the value of k.

(5, 5 marks)

2008

① (a) Differentiate $f(x) = \cos^{-1}(3x)$ where $\frac{-1}{3} < x < \frac{1}{3}$.

(b) Given $x = 2\sec\theta$, $y = 3\sin\theta$, use parametric differentiation to find $\frac{dy}{dx}$

in terms of θ.

(2, 3 marks)

② A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$.

Use implicit differentiation to find $\frac{dy}{dx}$.

Hence find an equation of the tangent to the curve where $x = 1$.

(3, 3 marks)

2009

① (a) Given $f(x) = (x+1)(x-2)^3$, obtain the values of x for which $f'(x) = 0$.

(b) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$.

(3, 4 marks)

② The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point

where $x = 1$.

(5 marks)

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2010

Given $y = t^3 - \frac{5}{2}t^2$ and $x = \sqrt{t}$ for $t > 0$, use parametric differentiation to express $\frac{dy}{dx}$ in terms of t in simplified form.

Show that $\frac{d^2y}{dx^2} = at^2 + bt$, determining the values of the constants a and b.

Obtain an equation for the tangent to the curve which passes through the point of inflexion.

(4, 3, 3 marks)

2011

Obtain $\frac{dy}{dx}$ when y is defined as a function of x by the equation $y + e^y = x^2$.

(3 marks)

2012

① The radius of a cylindrical column of liquid is decreasing at the rate of $0 \cdot 02ms^{-1}$, while the height is increasing at the rate of $0 \cdot 01ms^{-1}$.
Find the rate of change of the volume when the radius is $0 \cdot 6$ metres and the height is 2 metres.
(Recall that the volume of a cylinder is given by $V = \pi r^2 h$.) **(5 marks)**

② A curve is defined parametrically, for all t, by the equations

$$x = 2t + \frac{1}{2}t^2, \qquad y = \frac{1}{3}t^3 - 3t.$$

Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t.
Find the values of t at which the curve has stationary points and determine their nature.
Show that the curve has exactly two points of inflexion.

(5, 3, 2 marks)

2013

A curve has equation $x^2 + 4xy + y^2 + 11 = 0$.

Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(-2, 3)$. **(6 marks)**

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2014

① Differentiate $y = \tan^{-1}(3x^2)$. **(3 marks)**

② Given $x = \ln(1+t^2)$, $y = \ln(1+2t^2)$ use parametric differentiation to find

$\frac{dy}{dx}$ in terms of t. **(3 marks)**

③ Given $e^y = x^3 \cos^2 x$, $x > 0$, show that $\frac{dy}{dx} = \frac{a}{x} + b\tan x$, for some constants a and b

State the values of a and b. **(3 marks)**

2015

① The equation $x^4 + y^4 + 9x - 6y = 14$ defines a curve passing through the point A$(1,2)$.

Obtain the equation of the tangent to the curve at A. **(4 marks)**

② For $y = 3^{x^2}$, obtain $\frac{dy}{dx}$. **(3 marks)**

③ Given $x = \sqrt{t+1}$ and $y = \cot t$, $0 < t < \pi$, obtain $\frac{dy}{dx}$ in terms of t.

(3 marks)