Advanced Higher Maths

Differential Equations

2001

A chemical plant food loses effectiveness at a rate proportional to the amount present in the soil. The amount M grams of plant food effective after t days satisfies the differential equation

$\frac{dM}{dt} = kM$, where k is a constant.

(a) Find the general solution for M in terms of t where the initial amount of food is M_0 grams.

(b) Find the value of k if, after 30 days, only half the initial amount of plant food is effective.

(c) What percentage of the original amount of plant food is effective after 35 days?

(d) The plant food has to be renewed when its effectiveness falls below 25%.
Is the manufacturer of the plant food justified in calling its product "sixty day super food"?

(3, 3, 2, 2 marks)

2003

The volume $V(t)$ of a cell at time t changes according to the law $\frac{dV}{dt} = V(10-V)$ for $0 < V < 10$

Show that $\frac{1}{10}\ln V - \frac{1}{10}\ln(10-V) = t + C$ for some constant C.

Given that $V(0) = 5$, show that $V(t) = \frac{10e^{10t}}{1+e^{10t}}$.

Obtain the limiting value of $V(t)$ as $t \to \infty$. **(4, 3, 2 marks)**

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2007

A garden centre advertises young plants to be used as hedging.
After planting, the growth, G metres (ie the increase in height) after t years is modelled by the differential equation

$$\frac{dG}{dt} = \frac{25k - G}{25}$$

where k is a constant and $G = 0$ when $t = 0$.

(a) Express G in terms of t and k.

(b) Given that a plant grows $0 \cdot 6$ metres by the end of 5 years, find the value of k correct to 3 decimal places.

(c) On the plant labels it states that the expected growth after 10 years is approximately 1 metre. Is this claim justified?

(d) Given that the initial height of the plants was $0 \cdot 3$ m, what is the likely long-term height of the plants? **(4, 2, 2, 2 marks)**

2009

Given that $x^2 e^y \frac{dy}{dx} = 1$ and $y = 0$ when $x = 1$, find y in terms of x. **(4 marks)**

2011

Given that $y > -1$ and $x > -1$, obtain the general solution of the differential equation

$$\frac{dy}{dx} = 3(1 + y)\sqrt{1 + x}$$

expressing your answer in the form $y = f(x)$. **(5 marks)**

2013

In an environment without enough resources to support a population greater than 1000, the population $P(t)$ at time t is governed by Verhurst's law

$$\frac{dP}{dt} = P(1000 - P)$$

Show that $\ln \frac{P}{1000 - P} = 1000t + C$ for some constant C.

Hence show that $P(t) = \frac{1000K}{K + e^{-1000t}}$ for some constant K.

Given that $P(0) = 200$, determine at what time t, $P(t) = 900$. **(4, 3, 3 marks)**

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2015

Vegetation can be irrigated by putting a small hole in the bottom of a cylindrical tank, so that the water leaks out slowly. Torricelli's Law states that the rate of change of volume, V, of the water in the tank is proportional to the square root of the height, h, of the water above the hole.
This is given by the differential equation:

$$\frac{dV}{dt} = -k\sqrt{h},\ k > 0$$

(a) For a cylindrical tank with constant cross-sectional area, A, show that the rate of change of the height of the water in the tank is given by

$$\frac{dh}{dt} = \frac{-k}{A}\sqrt{h}\,.$$

(b) Initially, when the height of the water is 144cm, the rate at which the height is changing is -3 cm/hr.

By solving the differential equation in part (a), show that $h = \left(12 - \frac{1}{80}t\right)^2$.

(c) How many days will it take for the tank to empty?

(d) Given that the tank has radius 20cm, find the rate at which the water was being delivered to the vegetation (in cm^3/hr) at the end of the fourth day.

(2, 4, 2, 3 marks)