

Advanced Higher Maths

Further Differential Equations

2001

① Find the solution of the following differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = x, \quad x > 0 \quad (4 \text{ marks})$$

② Find the general solution of the following differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1 \quad (5 \text{ marks})$$

2002

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\cos x$$

Hence determine the solution which satisfies $y(0) = 0$ and $y'(0) = 1$.

(6, 4 marks)

2003

Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x$$

given that $y = 2$ and $\frac{dy}{dx} = 1$, when $x = 0$.

(10 marks)

2004

(a) A mathematical biologist believes that the differential equation $x\frac{dy}{dx} - 3y = x^4$ models a process. Find the general solution of the differential equation.

Given that $y = 2$ when $x = 1$, find the particular solution, expressing y in terms of x .

(5, 2 marks)

(b) The biologist subsequently decides that a better model is given by the equation

$$y\frac{dy}{dx} - 3x = x^4.$$

Given that $y = 2$ when $x = 1$, obtain y in terms of x .

(4 marks)

Advanced Higher Maths

2005

Obtain the general solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20\sin x$

Hence find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

(7, 3 marks)

2006

Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 2$.

(6 marks)

2007

Obtain the general solution of the equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$

(6 marks)

2008

Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$, when $x = 0$, find the particular solution.

(7, 3 marks)

2009

(a) Solve the differential equation

$$(x+1)\frac{dy}{dx} - 3y = (x+1)^4$$

given that $y = 16$ when $x = 1$, expressing the answer in the form $y = f(x)$.

(b) Hence find the area enclosed by the graphs of $y = f(x)$, $y = (1-x)^4$ and the x -axis.

(6, 4 marks)

Advanced Higher Maths

2010

Obtain the general solution of the equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$$

Hence obtain the solution for which $y = 3$ when $x = 0$ and $y = e^{-\pi}$ when $x = \frac{\pi}{2}$.

(4, 3 marks)

2011

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12$$

Find the particular solution for which $y = \frac{-3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.

(7, 3 marks)

2012

(a) Express $\frac{1}{(x-1)(x+2)^2}$ in partial fractions.

(b) Obtain the general solution of the differential equation

$$(x-1)\frac{dy}{dx} - y = \frac{x-1}{(x+2)^2}$$

expressing your answer in the form $y = f(x)$.

(4, 7 marks)

2013

Solve the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 4e^{3x}, \text{ given that } y = 1 \text{ and } \frac{dy}{dx} = -1 \text{ when } x = 0.$$

(11 marks)

2014

Find the solution $y = f(x)$ to the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0, \text{ given that } y = 4 \text{ and } \frac{dy}{dx} = 3 \text{ when } x = 0.$$

(6 marks)

Advanced Higher Maths

2015

Solve the second order differential equation

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 10y = 3e^{2x}$$

given that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$.

(10 marks)