Advanced Higher Maths

Sequences & Series

2002

Define $S_n(x)$ by

$$S_n(x) = 1 + 2x + 3x^2 + \ldots + nx^{n-1}$$

where n is a positive integer.

Express $S_n(1)$ in terms of n.

By considering $(1-x)S_n(x)$, show that

$$S_n(x) = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)}, \quad x \neq 1.$$

Obtain the value of $\lim_{n\to\infty}\left\{\frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots + \frac{n}{3^{n-1}} + \frac{3}{2}\cdot\frac{n}{3^n}\right\}$. **(2, 4, 3 marks)**

2003

Given that $u_k = 11 - 2k, \quad (k \geq 1)$, obtain a formula for $S_n = \sum_{k=1}^{n} u_k$.

Find the values of n for which $S_n = 21$. **(3, 2 marks)**

2004

(a) Obtain the sum of the series $8 + 11 + 14 + \ldots + 56$.

(b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266, Calculate the common ratio.

(c) An arithmetic sequence, A, has first term a and common difference 2, and a geometric sequence, B, has first term a and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of a.

Obtain the smallest value of n such that the sum to n terms for sequence B is more than **twice** the sum to n terms for the sequence A. **(2, 3, 3, 2 marks)**

2005

The sum, $S(n)$, of the first n terms of a sequence, $u_1, u_2, u_3, \ldots$ is given by

$$S(n) = 8n - n^2, \quad n \geq 1.$$

Calculate the values of u_1, u_2, u_3 and state what type of sequence it is.

Obtain a formula for u_n in terms of n, simplifying your answer. **(3, 2 marks)**

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2006

The first three terms of a geometric sequence are

$\frac{x(x+1)}{(x-2)}, \frac{x(x+1)^2}{(x-2)^2}$ and $\frac{x(x+1)^3}{(x-2)^3}$, where $x<2$.

(a) Obtain expressions for the common ratio and the nth term of the sequence.

(b) Find an expression for the sum of the first n terms of the sequence.

(c) Obtain the range of values of x for which the sequence has a sum to infinity and find an expression for the sum to infinity.

(3, 3, 4 marks)

2007

Show that $\sum_{r=1}^{n}(4-6r)=n-3n^2$.

Hence write down a formula for $\sum_{r=1}^{2q}(4-6r)$.

Show that $\sum_{r=q+1}^{2q}(4-6r)=q-9q^2$. **(2, 1, 2 marks)**

2008

The first term of an arithmetic sequence is 2 and the $20th$ term is 97. Obtain the sum of the first 50 terms.

(4 marks)

2009

The first two terms of a geometric sequence are $a_1=p$ and $a_2=p^2$.

Obtain expressions for S_n and S_{2n} in terms of p, where $S_k=\sum_{j=1}^{k}a_j$.

Given that $S_{2n}=65S_n$ show that $p^n=64$.

Given also that $a_3=2p$ and that $p>0$, obtain the exact value of p and hence the value of n.

(1, 1, 2, 1, 1 marks)

2010

The second and third terms of a geometric series are -6 and 3 respectively.
Explain why the series has a sum to infinity and obtain this sum.

(5 marks)

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2011

① Write down an expression for $\sum_{r=1}^{n} r^3 - \left(\sum_{r=1}^{n} r\right)^2$ and an expression for $\sum_{r=1}^{n} r^3 + \left(\sum_{r=1}^{n} r\right)^2$.

(1, 3 marks)

② The first three terms of an arithmetic sequence are $a, \frac{1}{a}, 1$ where $a < 0$.

Obtain the value of a and the common difference.

Obtain the smallest value of n for which the sum of the first n terms is greater than 1000.

(5, 4 marks)

2012

The first and fourth terms of a geometric sequence are 2048 and 256 respectively.
Calculate the value of the common ratio.

Given that the sum of the first n terms is 4088, find the value of n.

(2, 3 marks)

2013

Write down the sums to infinity of the geometric series

$$1 + x + x^2 + x^3 + \ldots\ldots$$

and

$$1 - x + x^2 - x^3 + \ldots\ldots$$

valid for $|x| < 1$.

Assuming that it is permitted to integrate an infinite series term by term, show that, for $|x| < 1$,

$$\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\right)$$

Show how this series can be used to evaluate $\ln 2$.
Hence determine the value of $\ln 2$ correct to 3 decimal places. **(7, 3 marks)**

2015

The sum of the first twenty terms of an arithmetic sequence is 320.

The twenty-first term is 37.

What is the sum of the first ten terms? **(5 marks)**