

Advanced Higher Maths

Properties of Functions

2001

A function is defined by $f(x) = \frac{x^2 + 6x + 12}{x + 2}$, $x \neq -2$.

(a) Express $f(x)$ in the form $ax + b + \frac{b}{x + 2}$ stating the values of a and b .

(b) Write down an equation for each of the two asymptotes.

(c) Show that $f(x)$ has two stationary points.

Determine the coordinates and the nature of the stationary points.

(d) Sketch the graph of f .

(e) State the range of values of k such that the equation $f(x) = k$ has no solution.

(2, 2, 4, 1, 1 marks)

2002

Express $\frac{x^2}{(x+1)^2}$ in the form $A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, ($x \neq -1$), stating the values of the constants A, B and C .

A curve is defined by $y = \frac{x^2}{(x+1)^2}$, ($x \neq -1$).

(i) Write down equations for its asymptotes.

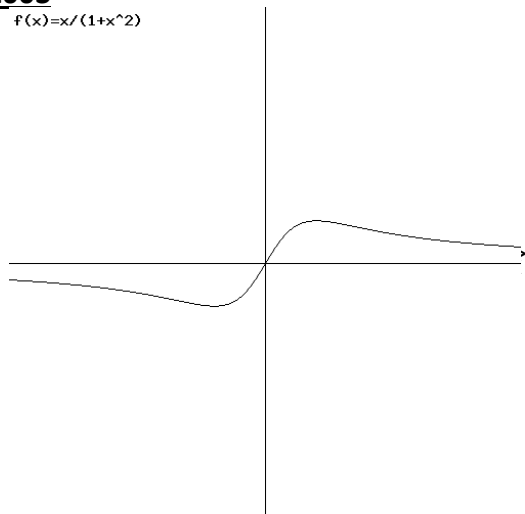
(ii) Find the stationary point and justify its nature.

(iii) Sketch the curve showing clearly the features found in (i) and (ii).

(3, 2, 4, 2 marks)

2003

$$f(x) = \frac{x}{1+x^2}$$



The diagram shows the shape of $y = \frac{x}{1+x^2}$.

Obtain the stationary points of the graph.

Sketch the graph of $y = \left| \frac{x}{1+x^2} \right|$ and identify its

three critical points.

(4, 3 marks)

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2004

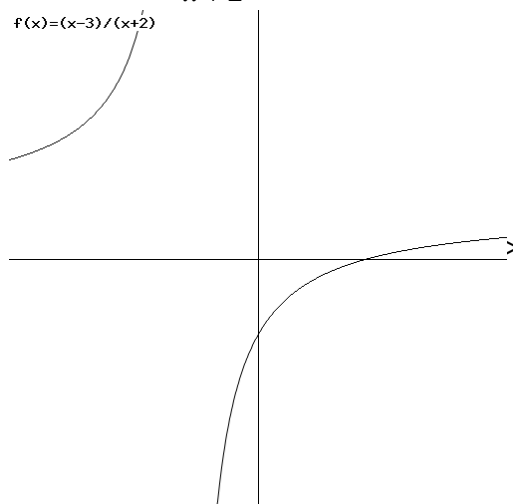
Determine whether the function $f(x) = x^4 \sin 2x$ is odd, even or neither.

Justify your answer.

(3 marks)

2004

The function f is defined by $f(x) = \frac{x-3}{x+2}$, $x \neq -2$, and the diagram shows part of its graph.



- (a) Obtain algebraically the asymptotes of the graph of f .
- (b) Prove that f has no stationary values.
- (c) Does the graph of f have any points of inflexion? Justify your answer.
- (d) Sketch the graph of the inverse function f^{-1} .

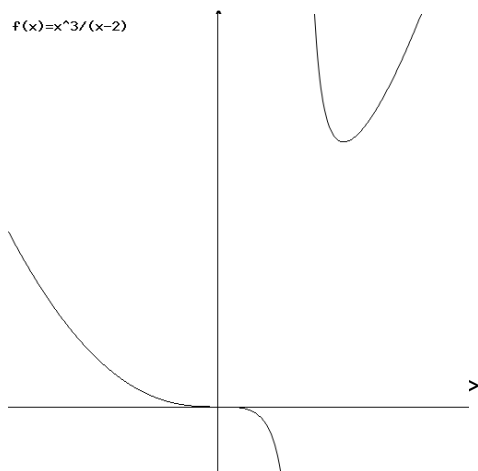
State the asymptotes and domain of f^{-1} .

(3, 2, 2, 3 marks)

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2005

The diagram shows part of the graph of $y = \frac{x^3}{x-2}$, $x \neq 2$.



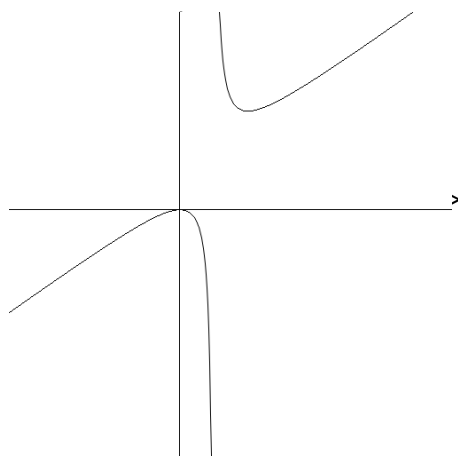
(a) Write down the equation of the vertical asymptote.

(b) Find the coordinates of the stationary points of the graph of $y = \frac{x^3}{x-2}$.

(c) Write down the coordinates of the stationary points of the graph of $y = \left| \frac{x^3}{x-2} \right| + 1$.

(1, 4, 2 marks)

2006



The diagram shows part of the graph of a function f which satisfies the following conditions:

(i) f is an even function;

(ii) two of the asymptotes of the graph $y = f(x)$ are $y = x$ and $x = 1$.

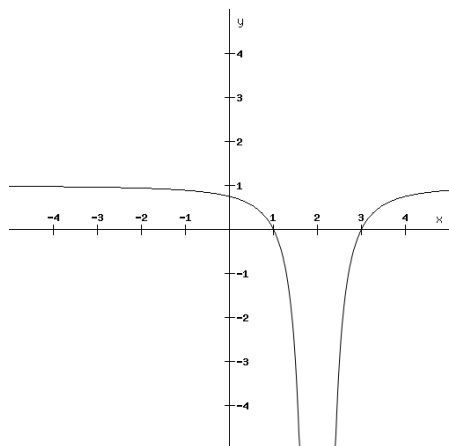
Copy the diagram and complete the graph. Write down equations for the other two asymptotes.

(3 marks)

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2008

Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph of $y = -f(x+1)$ showing its asymptotes. Write down the equations of the asymptotes.



(4 marks)

2009

The function $f(x)$ is defined by $f(x) = \frac{x^2 + 2x}{x^2 - 1}$ ($x \neq \pm 1$).

Obtain equations for the asymptotes of the graph of $f(x)$.

Show that $f(x)$ is a strictly decreasing function.

Find the coordinates of the points where the graph of $f(x)$ crosses

- (i) the x -axis and
- (ii) the horizontal asymptote.

Sketch the graph of $f(x)$, showing clearly all relevant features.

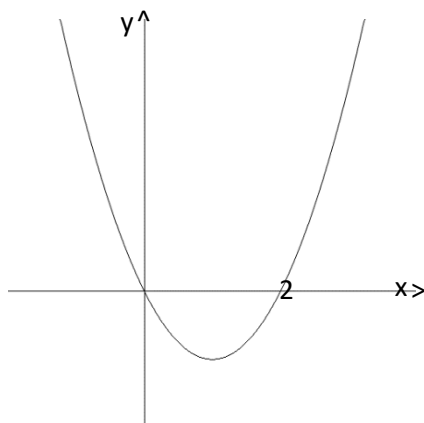
(3, 3, 2, 2 marks)

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2010

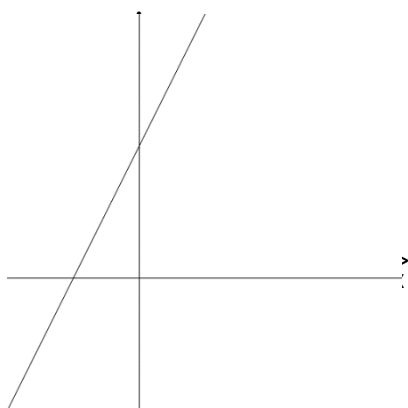
The diagram below shows part of the graph of a function $f(x)$.

State whether $f(x)$ is odd, even or neither. Fully justify your answer.



(3 marks)

2011



The diagram shows part of the graph of a function $f(x)$.

Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes.

(4 marks)

2012

A function is defined by $f(x) = |x+2|$ for all x .

(a) Sketch the graph of the function for $-3 \leq x \leq 3$.

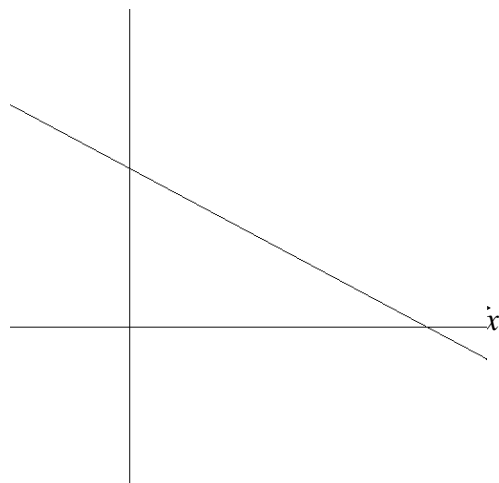
(b) On a separate diagram, sketch the graph of $f'(x)$.

(2, 2 marks)

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2013

Part of the straight line graph of a function $f(x)$ is shown.



- (a) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes.
- (b) State the value of k for which $f(x) + k$ is an odd function.
- (c) Find the value of h for which $|f(x+h)|$ is an even function.

(2, 1, 2 marks)

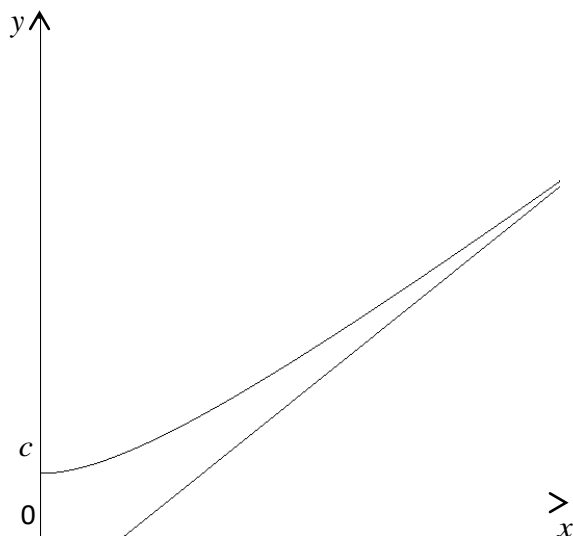
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2014

The function $f(x)$ is defined for all $x \geq 0$.

The graph of $y = f(x)$ intersects the y -axis at $(0, c)$ where $0 < c < 5$.

The graph of the function and its asymptote, $y = x - 5$, are shown below.



(a) Copy the diagram above.

On the same diagram, sketch the graph of $y = f^{-1}(x)$.

Clearly show any points of intersection and any asymptotes.

(b) What is the equation of the asymptote of the graph $y = f(x+2)$?

(c) Why does your diagram show that the equation $x = f(f(x))$ has at least one solution?

(4, 1, 1 marks)

2015

For some function, f , define $g(x) = f(x) + f(-x)$ and

$$h(x) = f(x) - f(-x).$$

Show that $g(x)$ is an even function and that $h(x)$ is an odd function.

Hence show that $f(x)$ can be expressed as the sum of an even and an odd function.

(4 marks)